Beam instabilities (I)

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in CERN Accelerator School, Advanced Level, Warsaw

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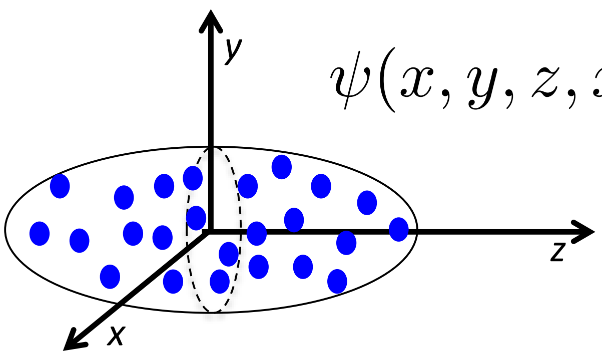
What is a beam instability?

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!



$$\psi(x, y, z, x', y', \delta)$$

$$N = \int_{-\infty}^{\infty} \psi(x, y, z, x', y', \delta) dx dx' dy dy' dz d\delta$$

$$\langle x \rangle = \frac{1}{N} \int_{-\infty}^{\infty} x \psi(x, y, z, x', y', \delta) dx dx' dy dy' dz d\delta$$

$$\sigma_x^2 = \frac{1}{N} \int_{-\infty}^{\infty} (x - \langle x \rangle)^2 \psi(x, y, z, x', y', \delta) dx dx' dy dy' dz d\delta$$

And similar definitions for $\langle y \rangle, \ \sigma_y, \ \langle z \rangle, \ \sigma_z$

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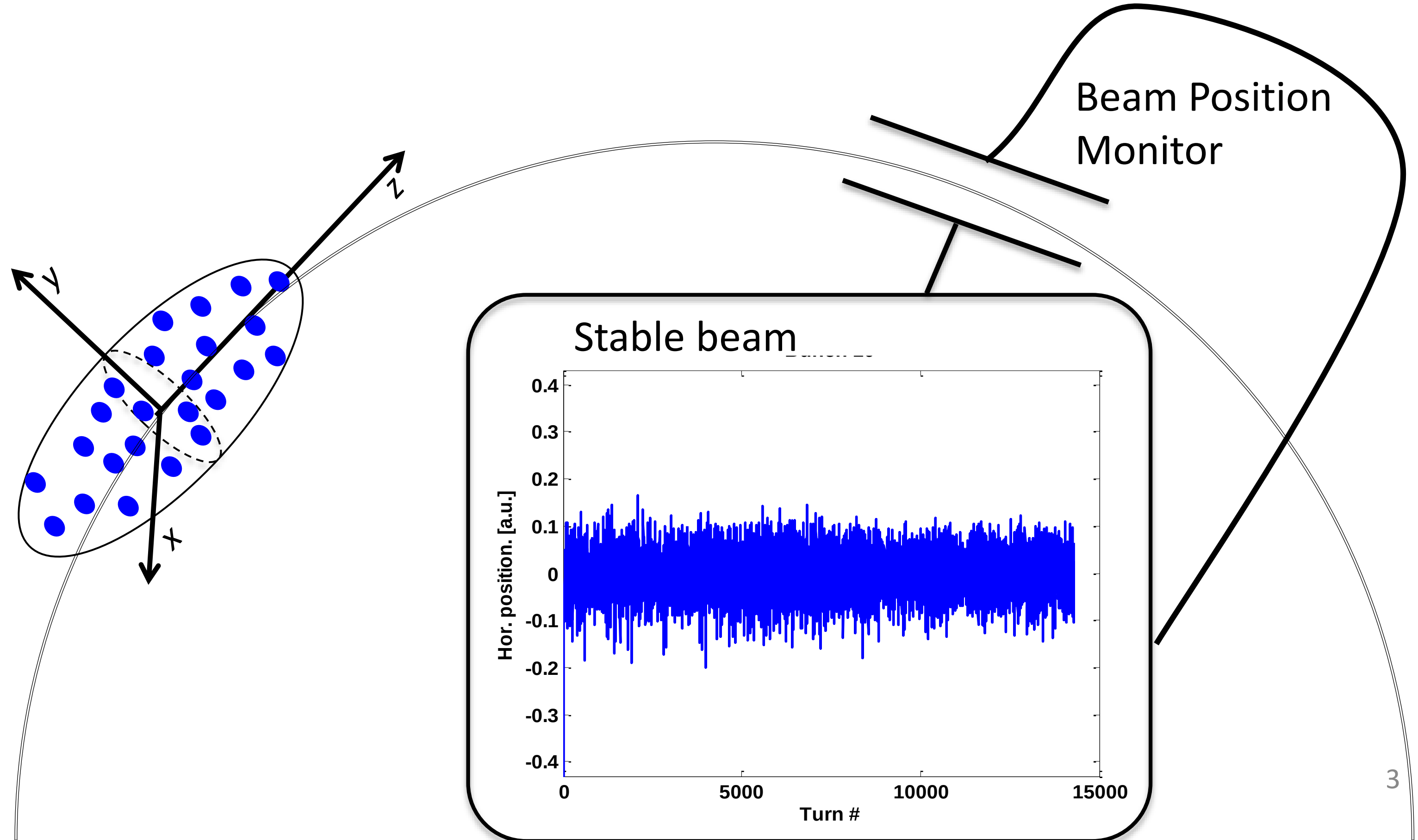

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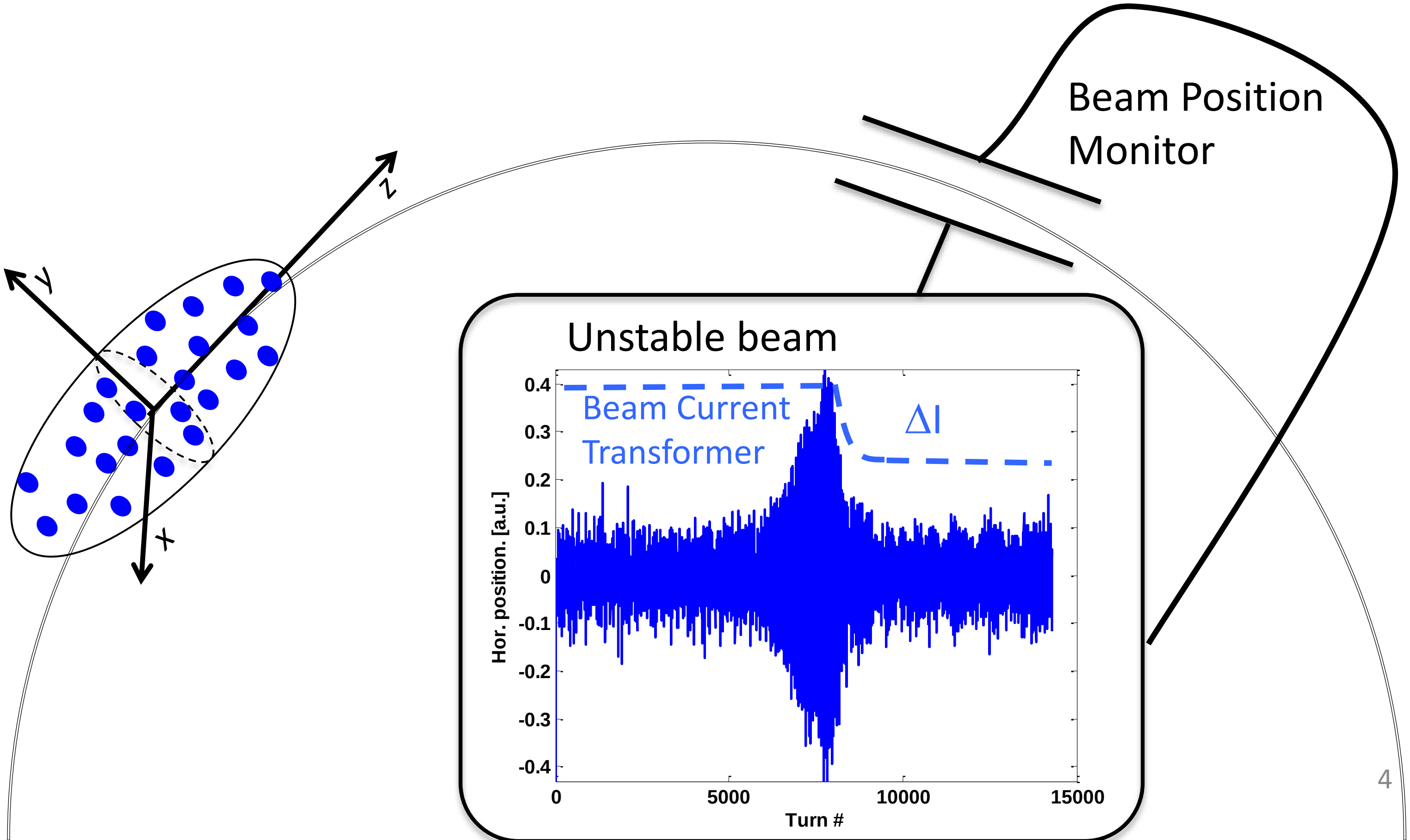

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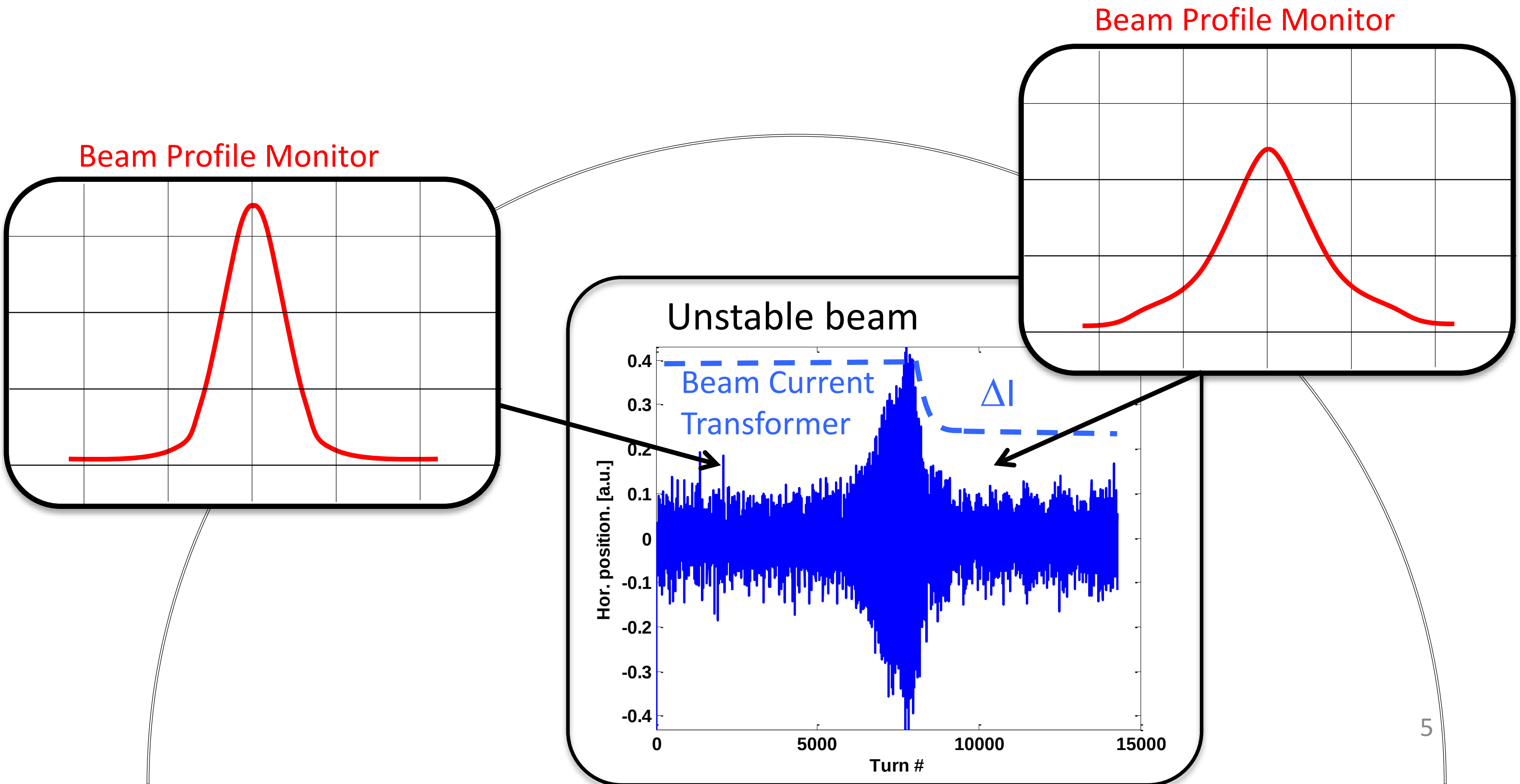

Why study beam instabilities?

- What is a beam instability?
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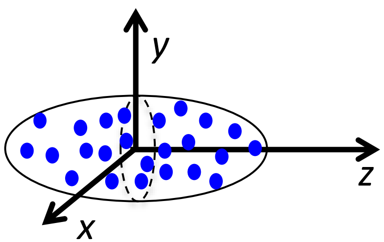
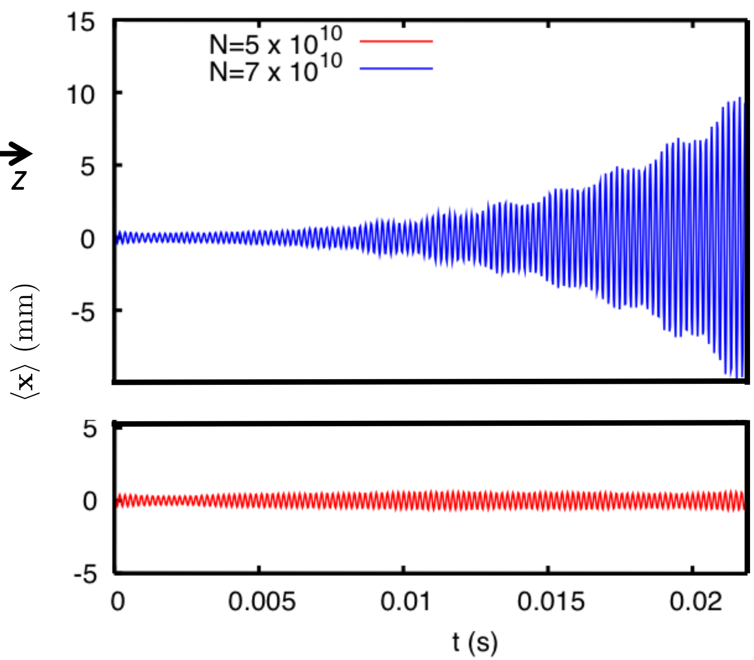
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)





Typical situation

- A beam centroid instability appears when the intensity is raised above a certain **threshold**
- The **threshold** can be optimized by an accurate choice of the machine settings

Why study beam instabilities?

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  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!

- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)

  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

Types of beam instabilities

⇒ Beam instabilities occur in both linear and circular machines

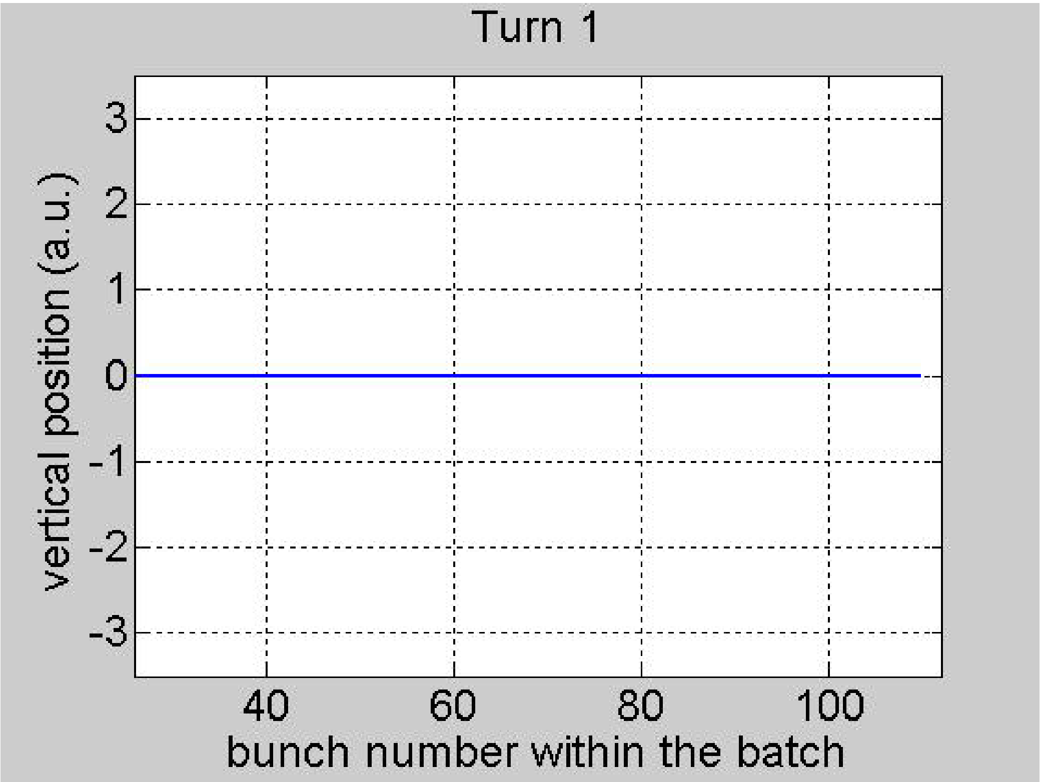
– Longitudinal plane (z,δ)
– Transverse plane (x,y,x',y')

⇒ Beam instabilities can affect the beam on different scales

– Cross-talk between bunches
  → The unstable motion of subsequent bunches is coupled
  → The instability is consequence of another mechanism that builds up along the bunch train
– Single bunch effect
– Coasting beam instabilities

Example of multi-bunch instability

⇒ They can affect the beam on different scales
  - Cross-talk between bunches
    - → **The unstable motion of subsequent bunches is coupled**
    - → The instability is consequence of another mechanism that builds up along the bunch train
  - Single bunch effect



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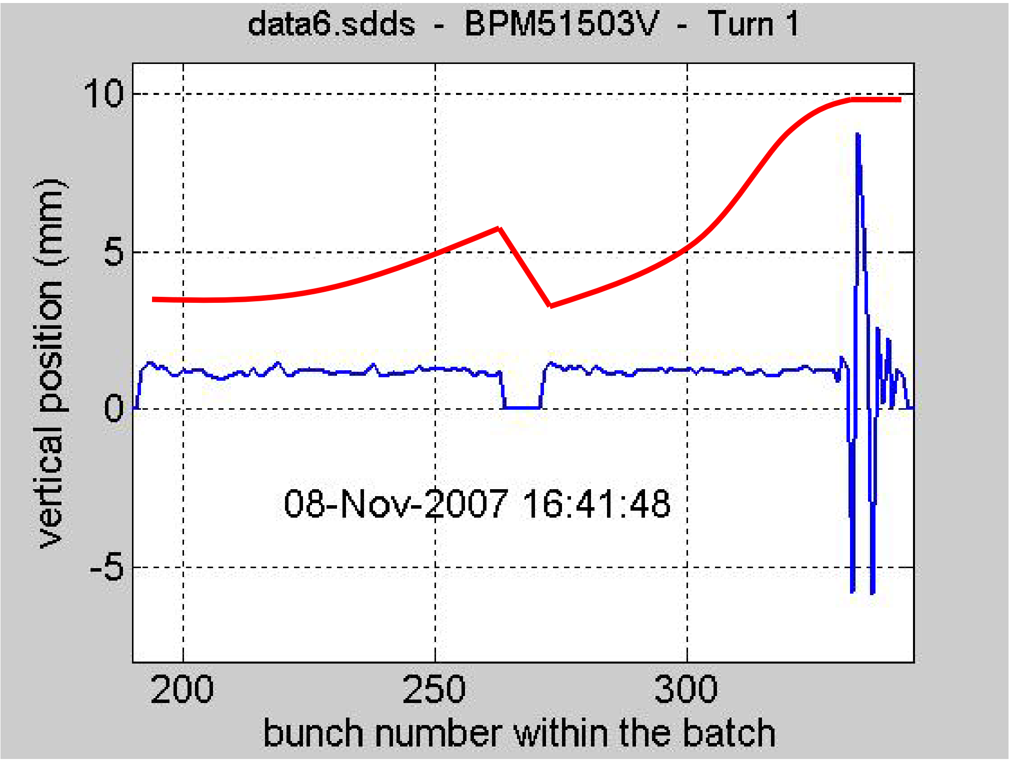


Example of single bunch instability

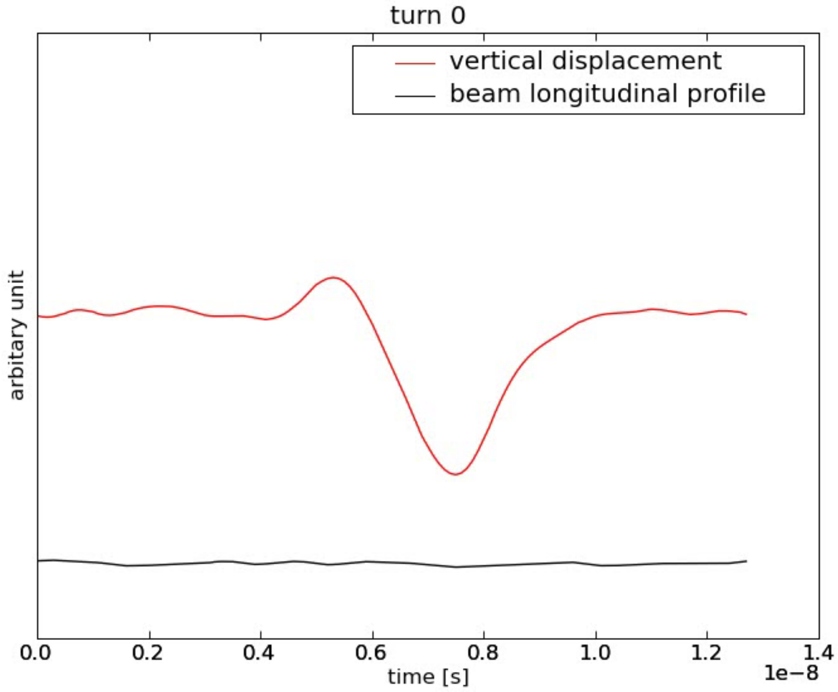
⇒ They can affect the beam on different scales

– Cross-talk between subsequent bunches

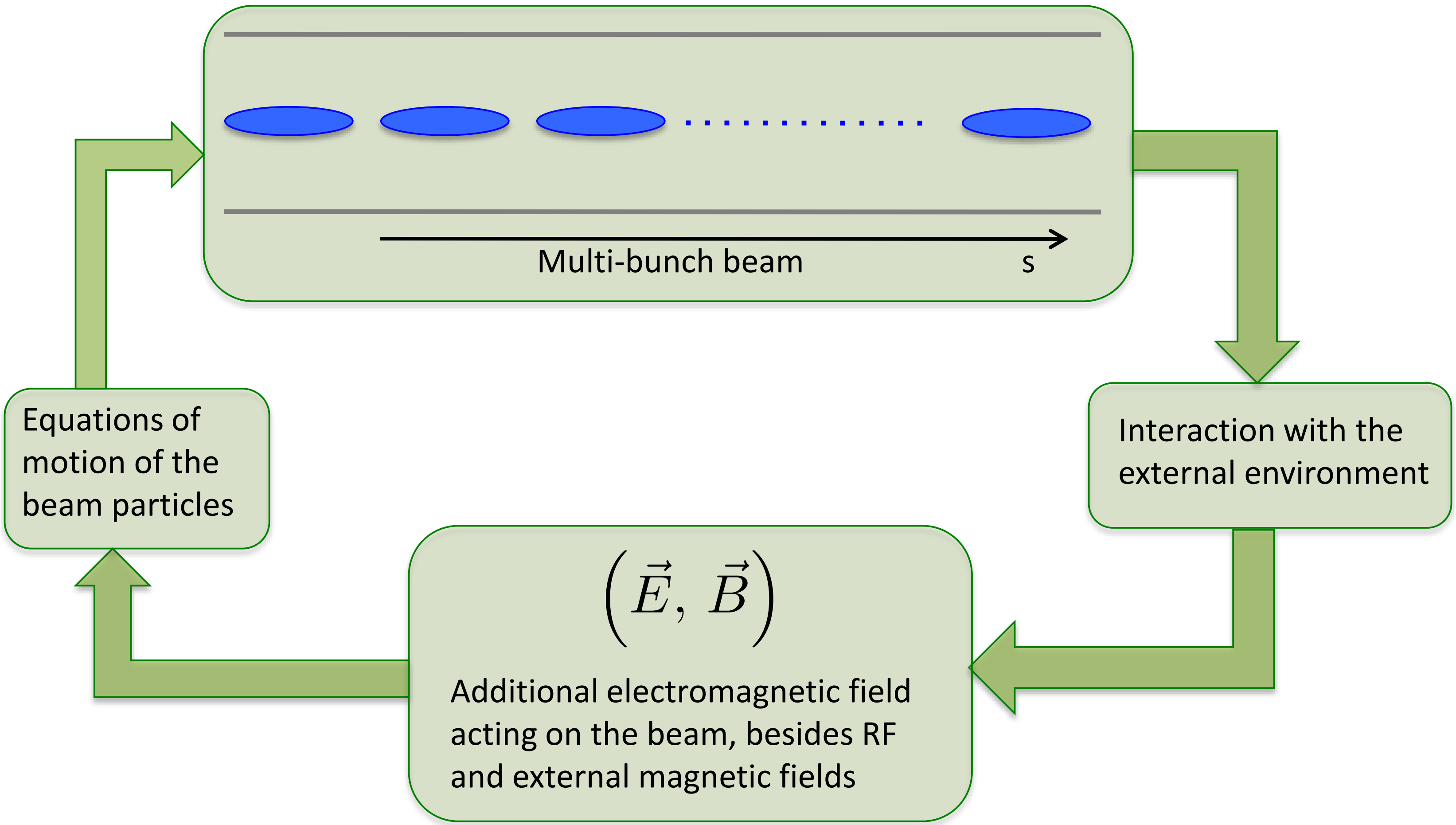
→ The instability builds up along a bunch train

→ The instability is consequence of another mechanism that builds up along the bunch train

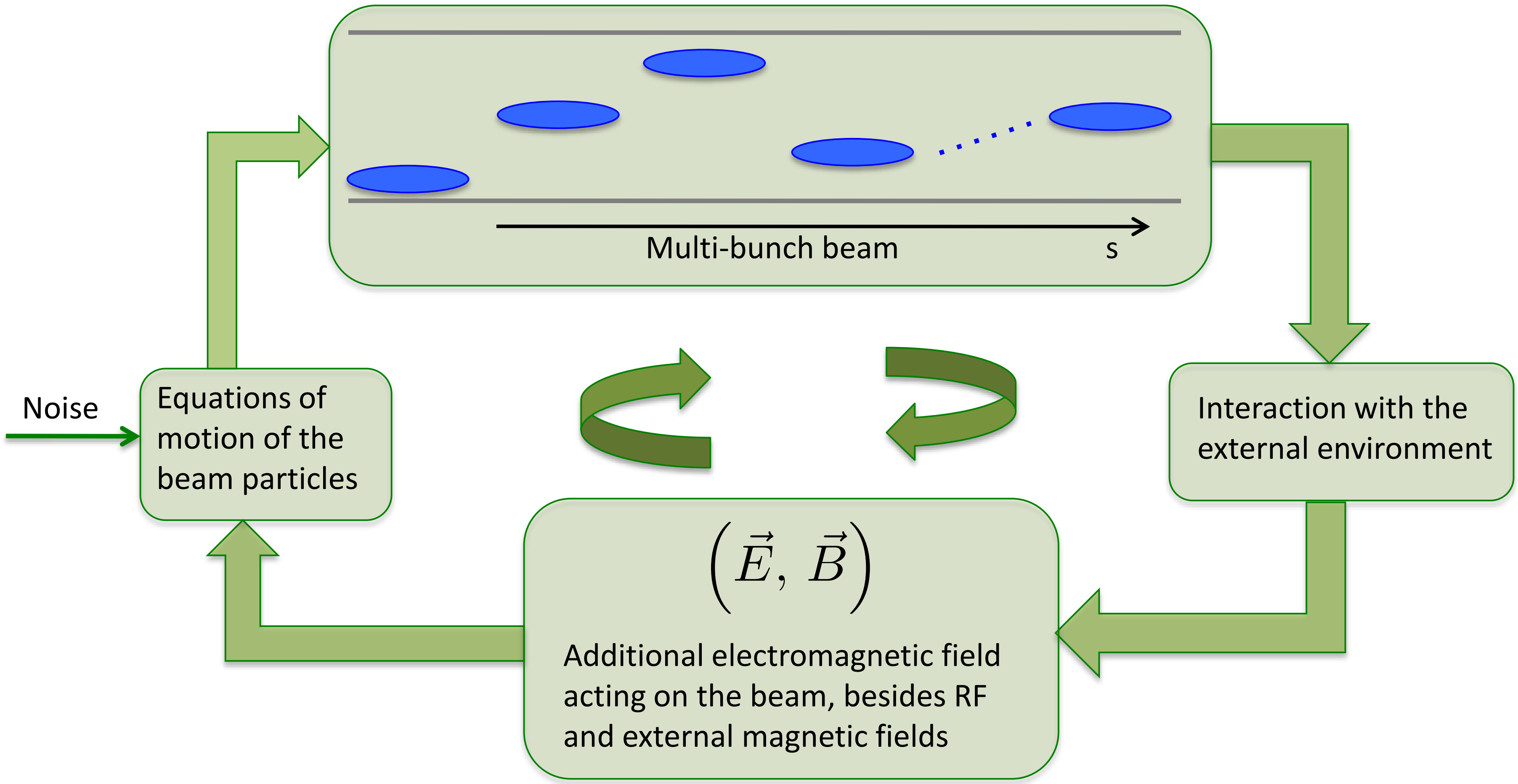
– **Single bunch effect**



Instability loop



Instability loop



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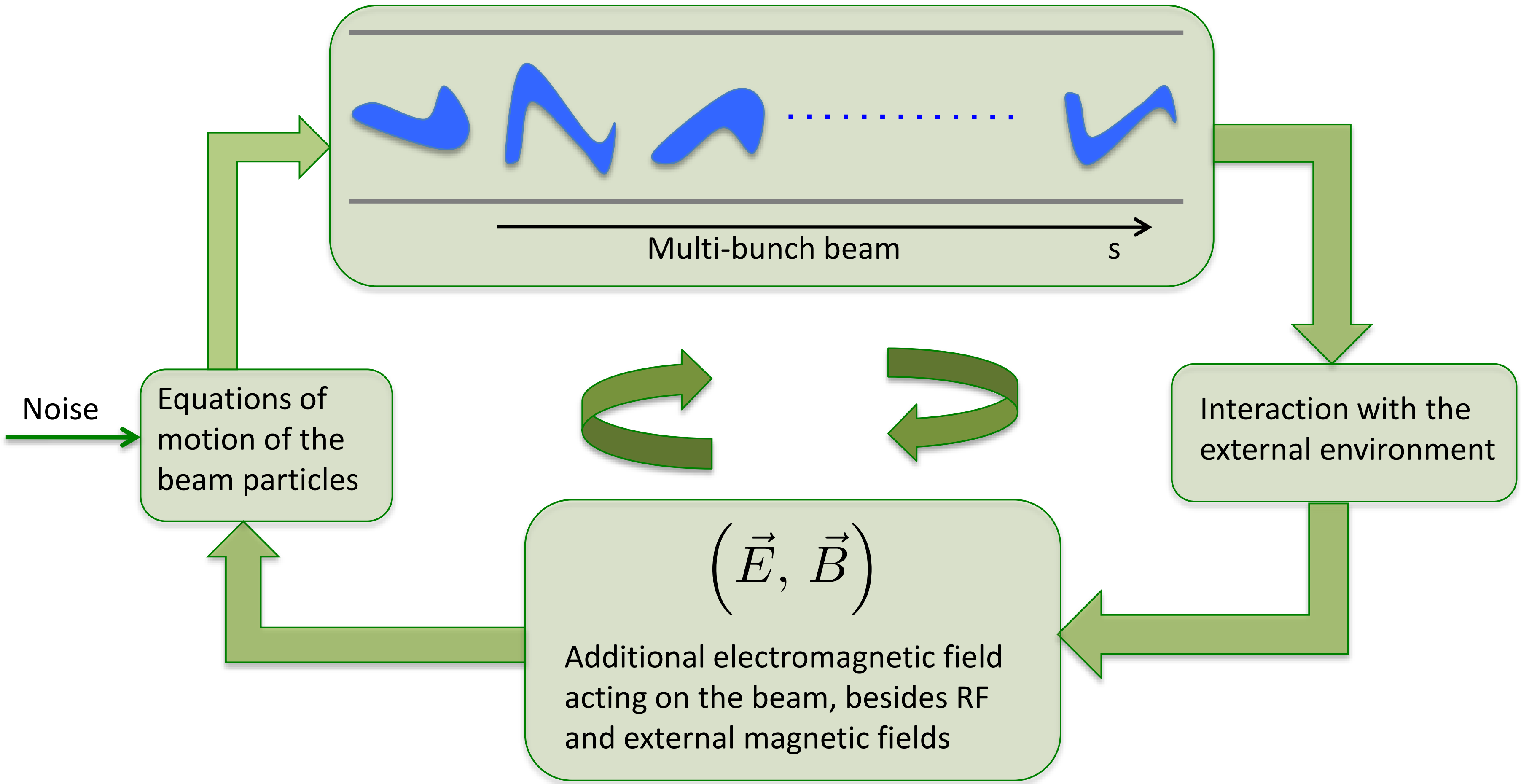


Some examples of beam-environment interaction

Interaction of the beam with the external environment

Beam self-fields

- Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Fields from another beam

- Set of Maxwell's equations with
  - The other beam as the source term
  - Boundary conditions given by the chamber in which the encounter between the beams takes place

$\left(\vec{E}, \vec{B}\right)$

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

The electron/ion cloud

- Electron/ion production and accumulation
- Poisson's equation with
  - The electron cloud as the source term
  - Boundary conditions given by the chamber in which the electron cloud builds up

Electro-magnetic beam-environment interaction

Interaction of the beam with the external environment

Beam self-fields

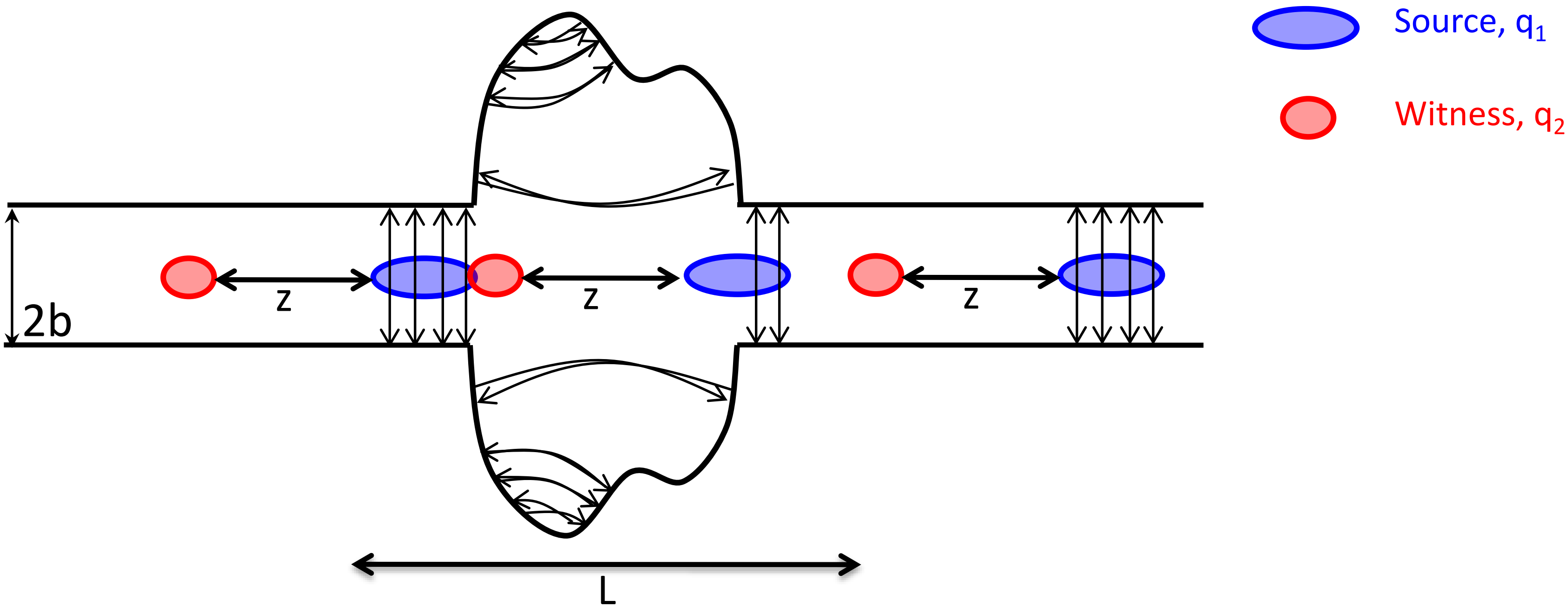
– Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Need of a mathematical framework to describe the effects of the self-induced fields on the beam

Time domain: Wake fields

Frequency domain: Beam coupling impedances

Wake fields (general)

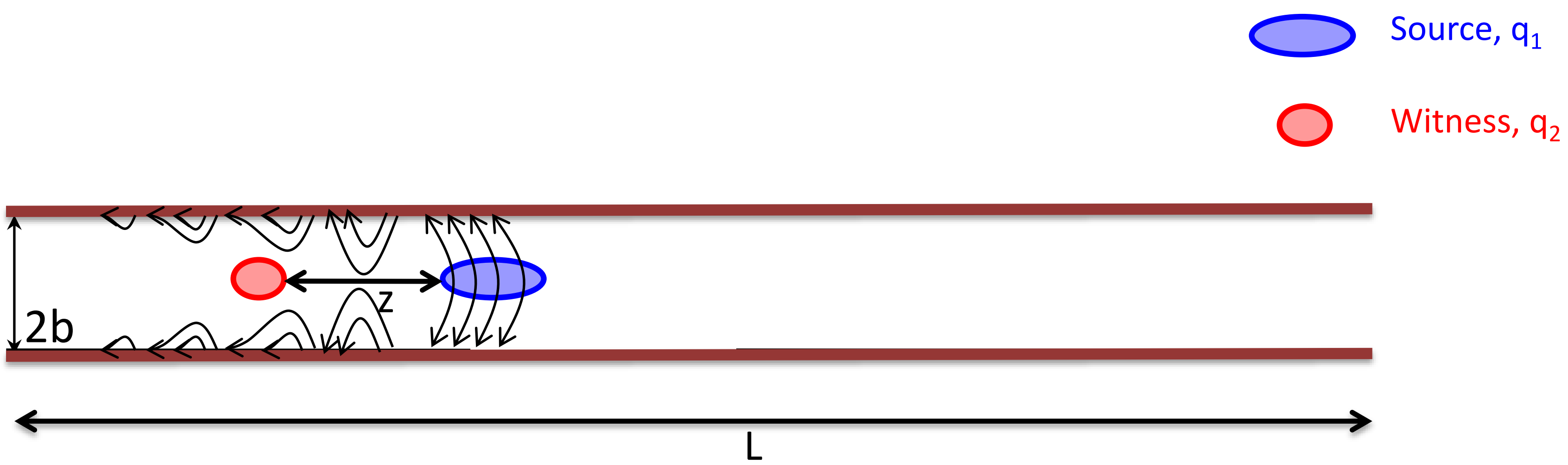


– While source and witness ($q_i\ \delta(s-ct)$), distant by $z<0$, move in a perfectly conducting chamber, the witness does not feel any force ($\gamma >> 1$)

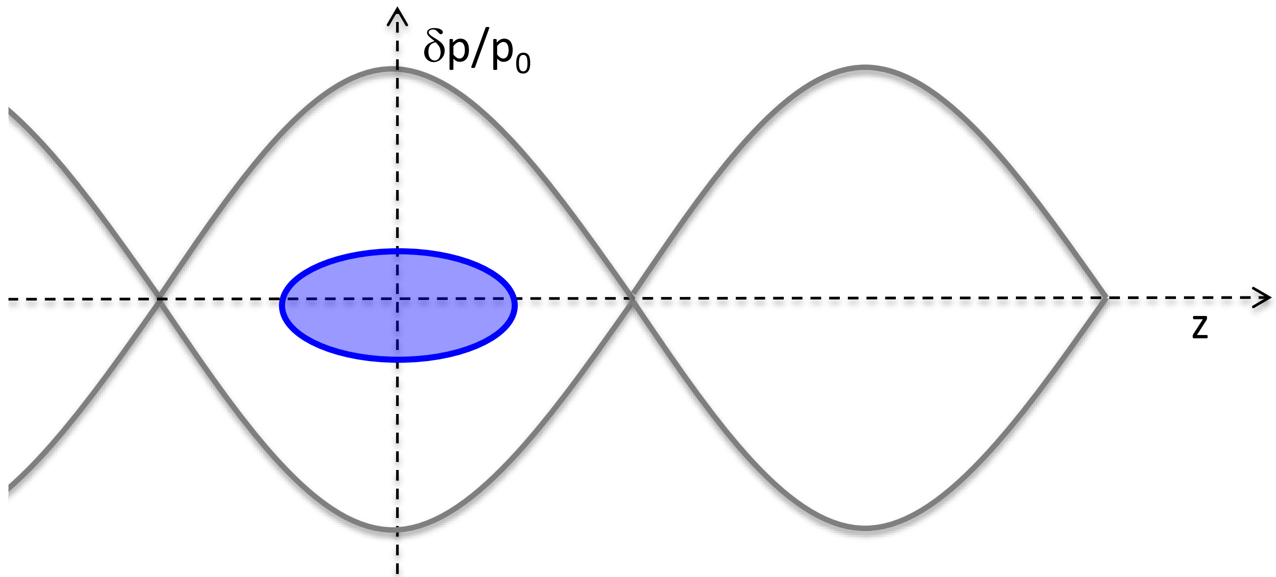
– When the source encounters a discontinuity (e.g., transition, device), it produces an electromagnetic field, which trails behind (wake field)

  - The source loses energy
  - The witness feels a net force all along an effective length of the structure, L

Wake fields (general)

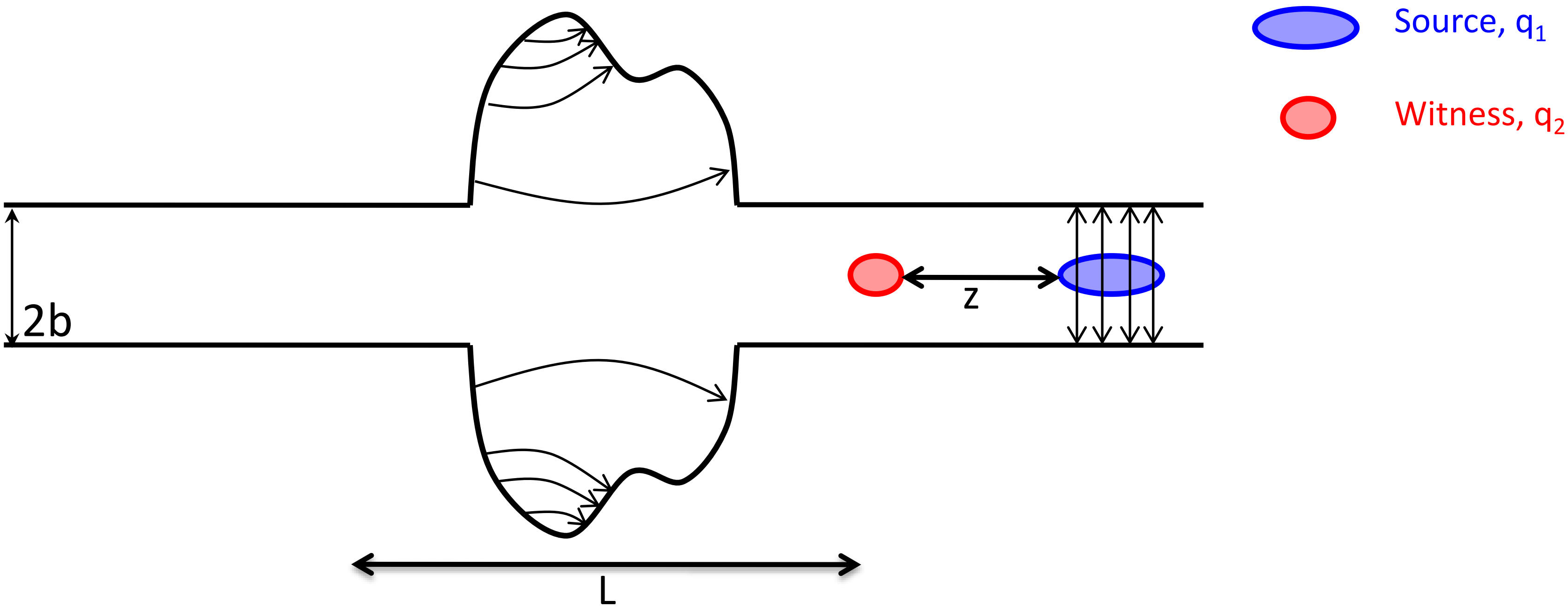


- Not only geometric discontinuities cause electromagnetic fields trailing behind sources traveling at light speed.
- For example, a pipe with finite conductivity causes a delay in the induced currents, which also produces delayed electromagnetic fields
  - No ringing, only slow decay
  - The witness feels a net force all along an effective length of the structure, L
- In general, also electromagnetic boundary conditions other than PEC can be the origin of wake fields.



1. The longitudinal plane

Longitudinal wake function: definition



$$\int_0^L F_{\|}(s,z)ds = -q_1 q_2 W_{\|}(z)$$

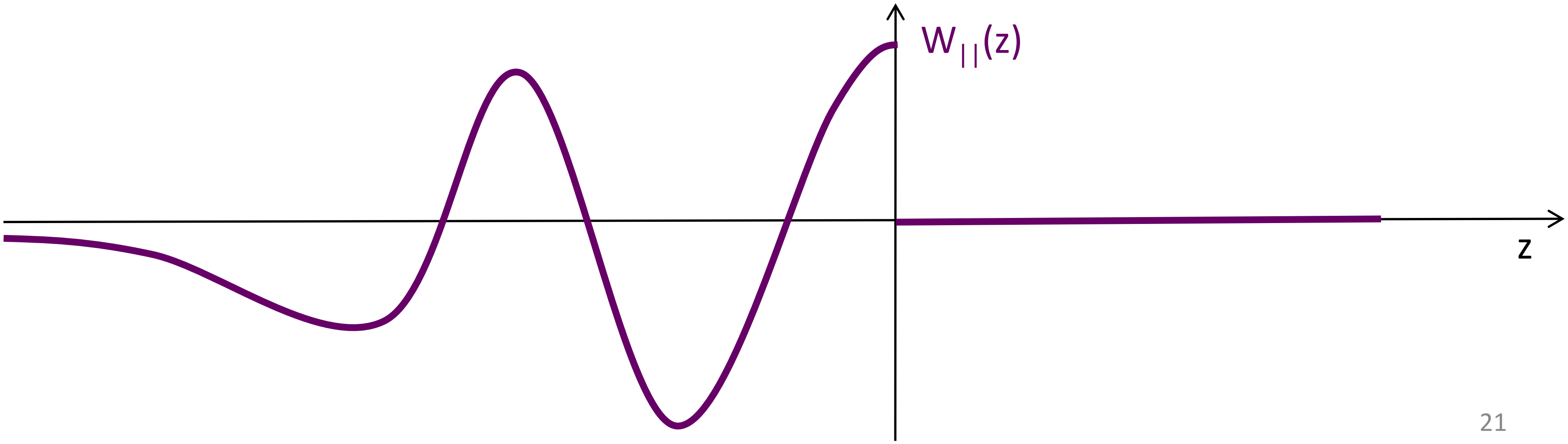
$$F_{\|}(s,z) = q_2 E_z(s,z)$$

$$\Delta E_2 \quad > \quad \frac{\Delta E_2}{E_0} = \left(\frac{\gamma^2 - 1}{\gamma^2}\right)\frac{\Delta p_2}{p_0}$$

Longitudinal wake function: properties

$$W_{||}(z) = -\frac{\Delta E_2}{q_1 q_2} \qquad \begin{array}{c} z \to 0 \\ q_2 \to q_1 \end{array} \qquad W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in 0, ***$W_{//}(0)$***, is related to the energy lost by the source particle in the creation of the wake
- ***$W_{//}(0)>0$*** since ***$\Delta E_1<0$***
- ***$W_{//}(z)$*** is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation

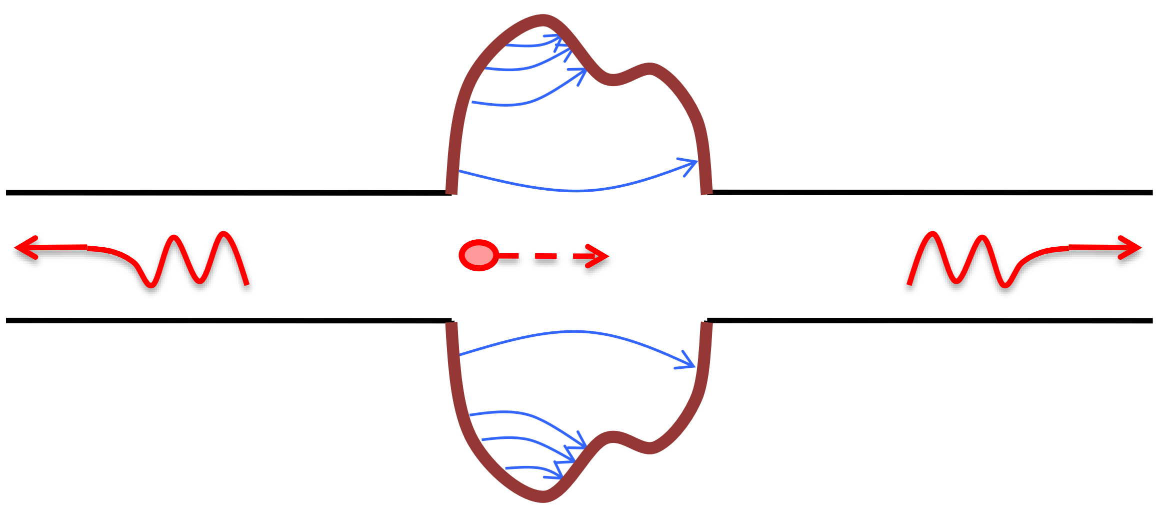


The energy balance

$$W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into
  - Electromagnetic energy of the modes that remain trapped in the object
    - ⇒ Partly dissipated on lossy walls or into purposely designed inserts or HOM absorbers
    - ⇒ Partly transferred to following particles (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of modes that propagate down the beam chamber (above cut-off), which will be eventually lost on surrounding lossy materials



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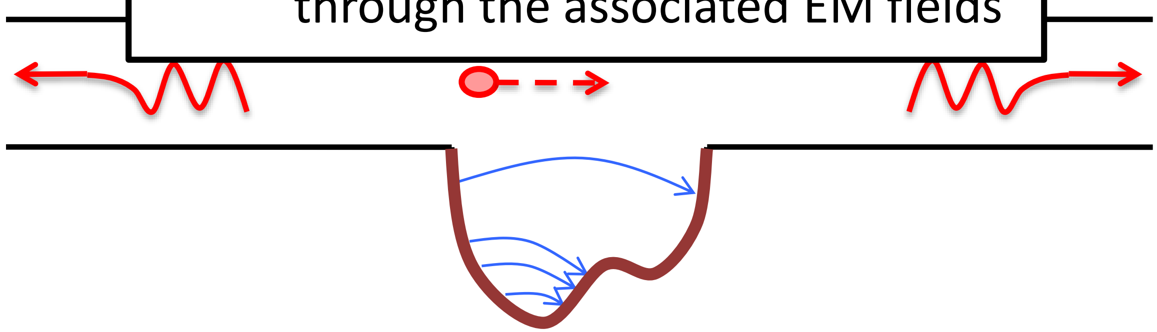
The energy loss is very important because

⇒ It causes **beam induced heating** of the beam environment (damage, outgassing)

⇒ It feeds into both **longitudinal and transverse instabilities** through the associated EM fields



Longitudinal impedance

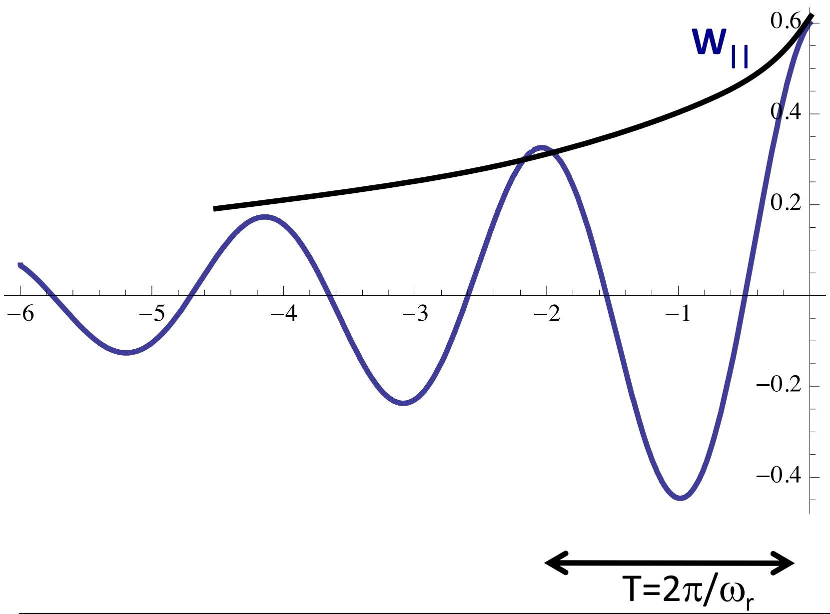
- The wake function of an accelerator component is basically its Green function in time domain (i.e., its response to a pulse excitation)
  - ⇒ Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

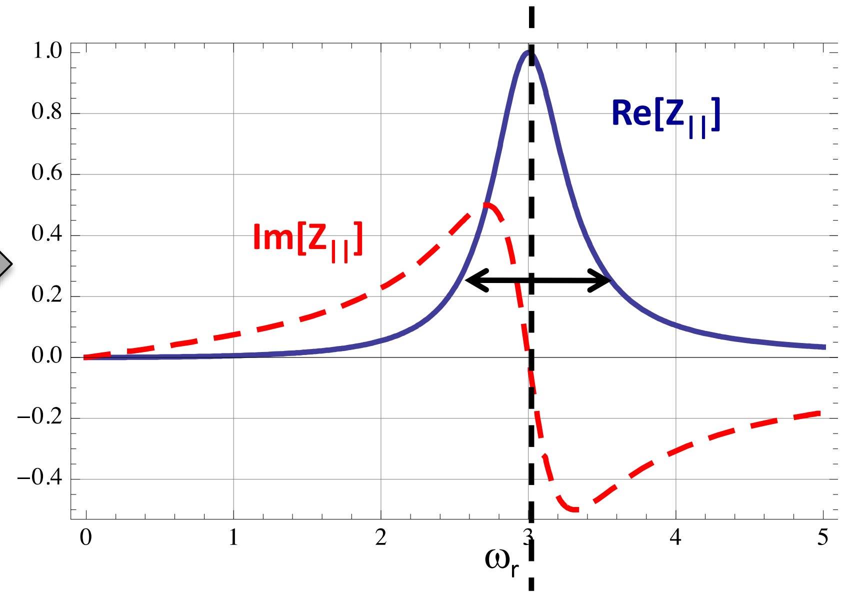
- We can also describe it as a transfer function in frequency domain
- This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\underbrace{\boxed{Z_{\|}(\omega)}}_{[\Omega]} = \int_{-\infty}^{\infty} \underbrace{\boxed{W_{\|}(z)}}_{[\Omega/\mathrm{s}]} \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

Longitudinal impedance: resonator

$$Z_{||}(\omega) = \int_{-\infty}^{\infty} W_{||}(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

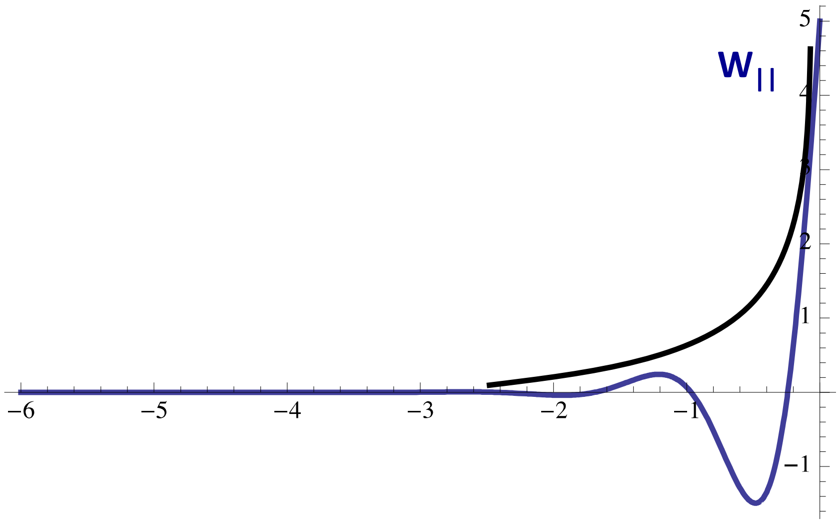


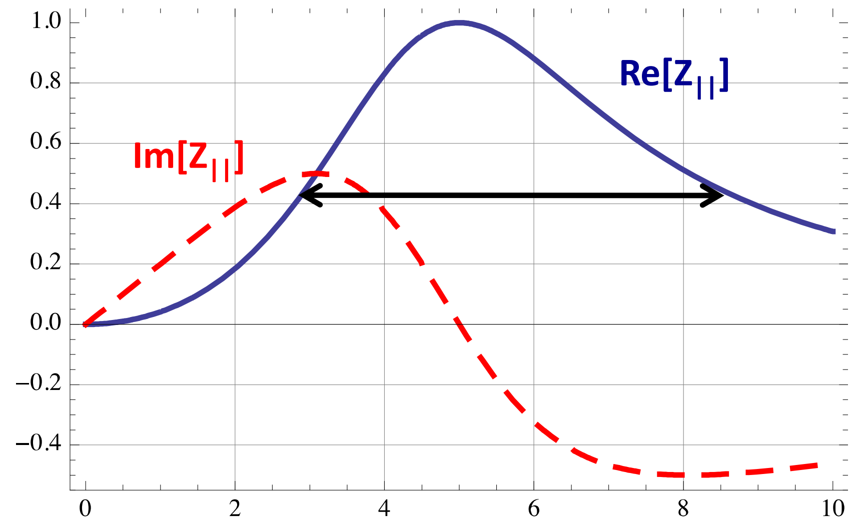


– The frequency ω_r is related to the oscillation of E_z, and therefore to the frequency of the mode excited in the object
– The decay time depends on how quickly the stored energy is dissipated (quantified by a quality factor Q)

Longitudinal impedance: resonator

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} W_{\parallel}(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$





- In general, the impedance will be composed of several of these resonator peaks
- Other contributors to the global impedance can also have different shapes, e.g. the resistive wall

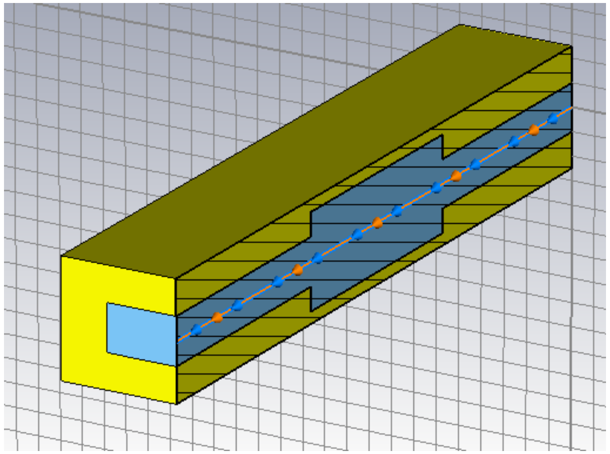
Longitudinal wake & impedance Equations of the resonator

$$Z_{||}^{\mathrm{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\dfrac{\omega}{\omega_r} - \dfrac{\omega_r}{\omega}\right)}$$

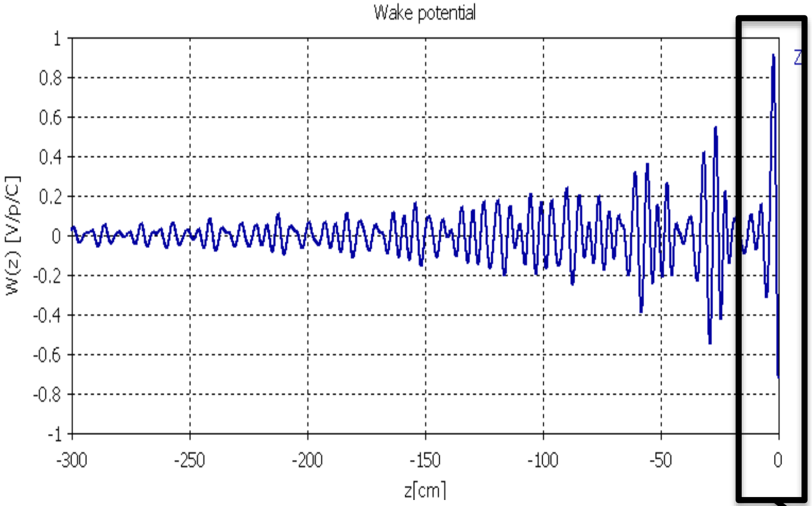
$$W_{||}^{\mathrm{Res}}(z) = \begin{cases} 2\alpha_z R_{s||} \exp\left(\dfrac{\alpha_z z}{c}\right)\left[\cos\left(\dfrac{\bar{\omega} z}{c}\right) + \dfrac{\alpha_z}{\bar{\omega}} \sin\left(\dfrac{\bar{\omega} z}{c}\right)\right] & \text{if} \quad z < 0 \\ \alpha_z R_{s||} \quad \text{if} \quad z = 0 \\ 0 \quad \text{if} \quad z \overset{>}{<} 0 \end{cases}$$

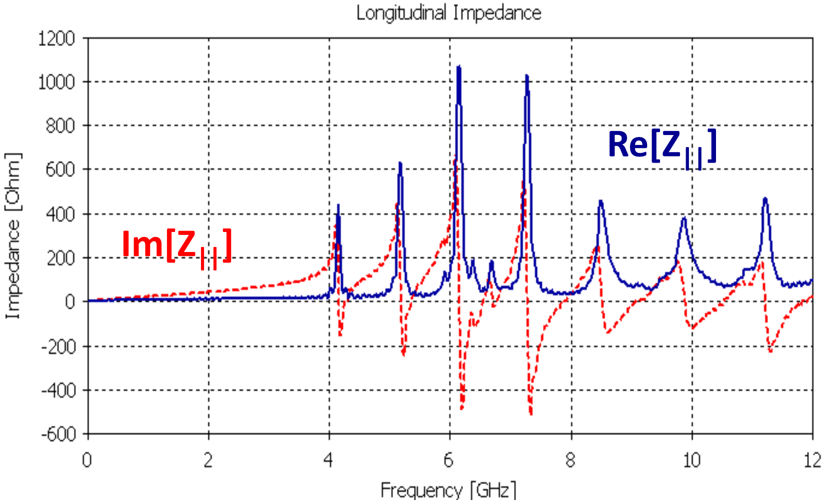
$$\alpha_z = \frac{\omega_r}{2Q} \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha_z^2}$$

Longitudinal impedance: cavity



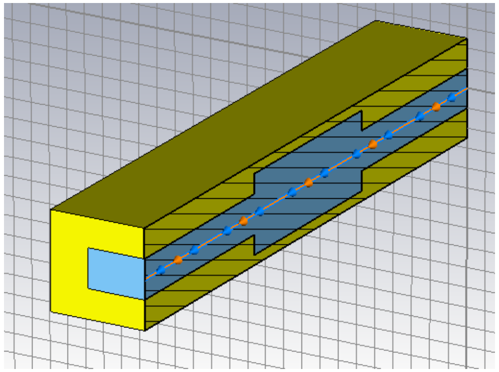
- A more complex example: a simple pill-box cavity with walls having finite conductivity
- Several modes can be excited
  - Below the pipe cut-off frequency the width of the peaks is only determined by the finite conductivity of the walls
  - Above, losses also come from propagation in the chamber



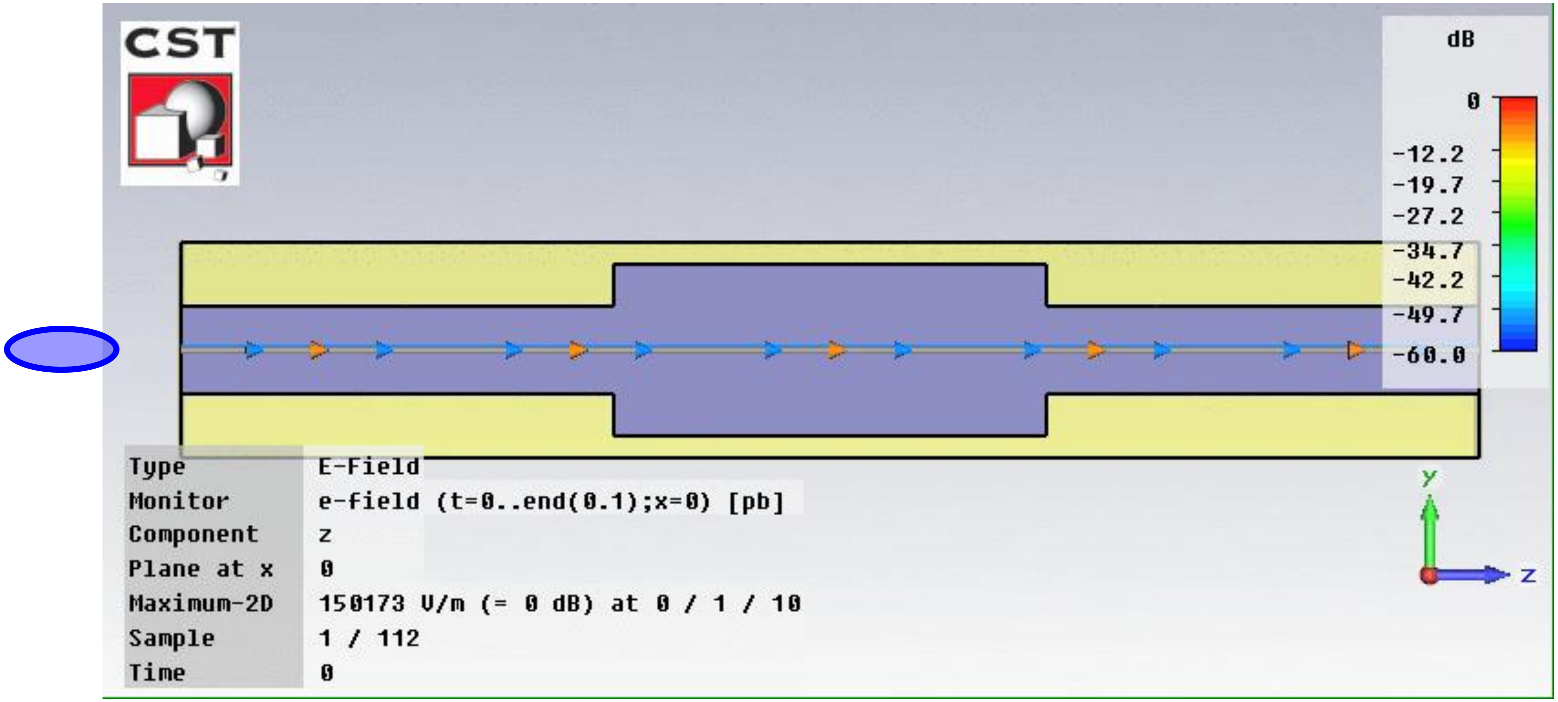


Note $W_{||}(0)<0$, **CST** uses opposite convention!!

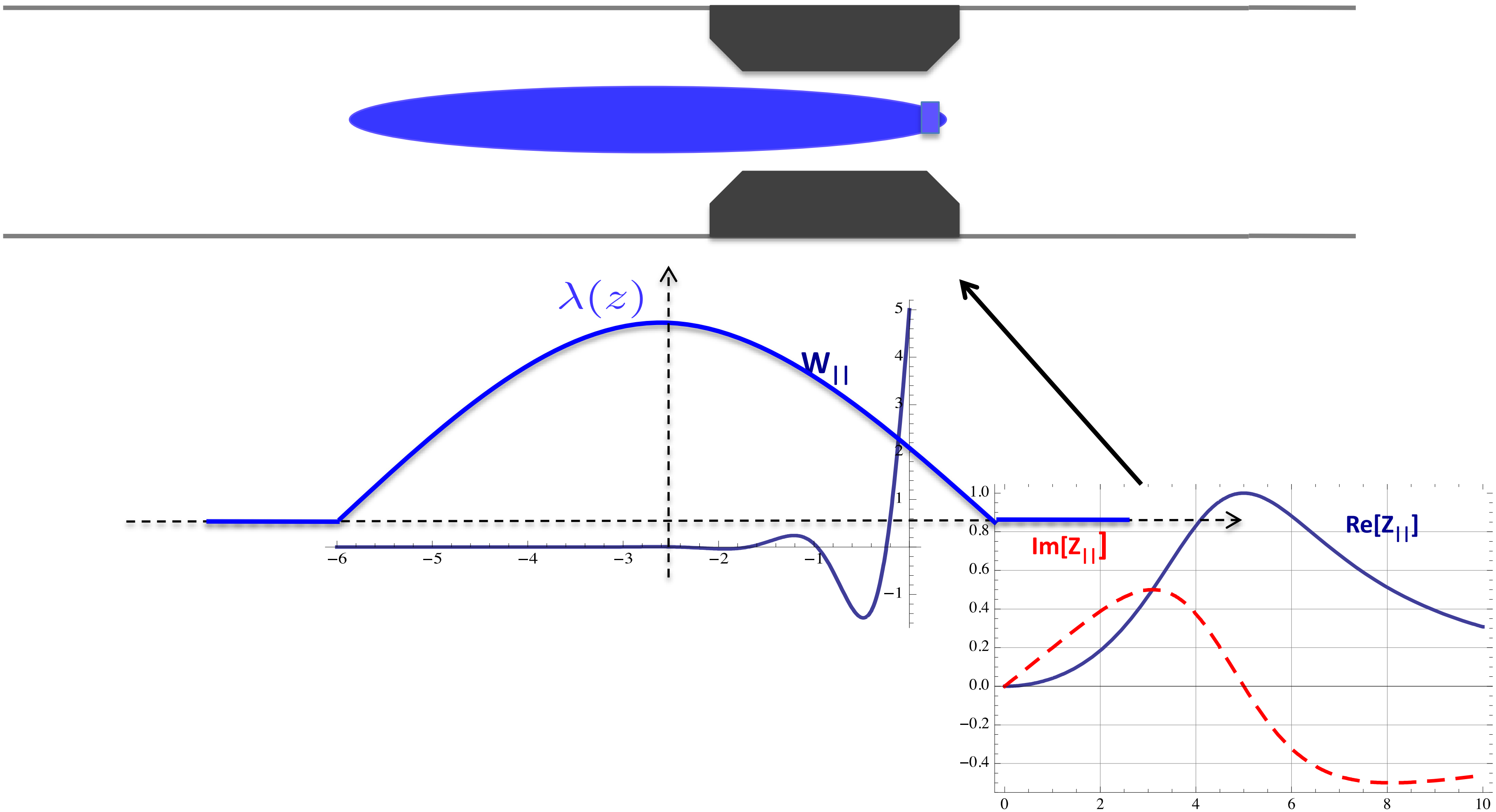
Longitudinal impedance: cavity



– Evolution of the electromagnetic fields (E_z) in the cavity while and after the beam has passed



Single bunch effects



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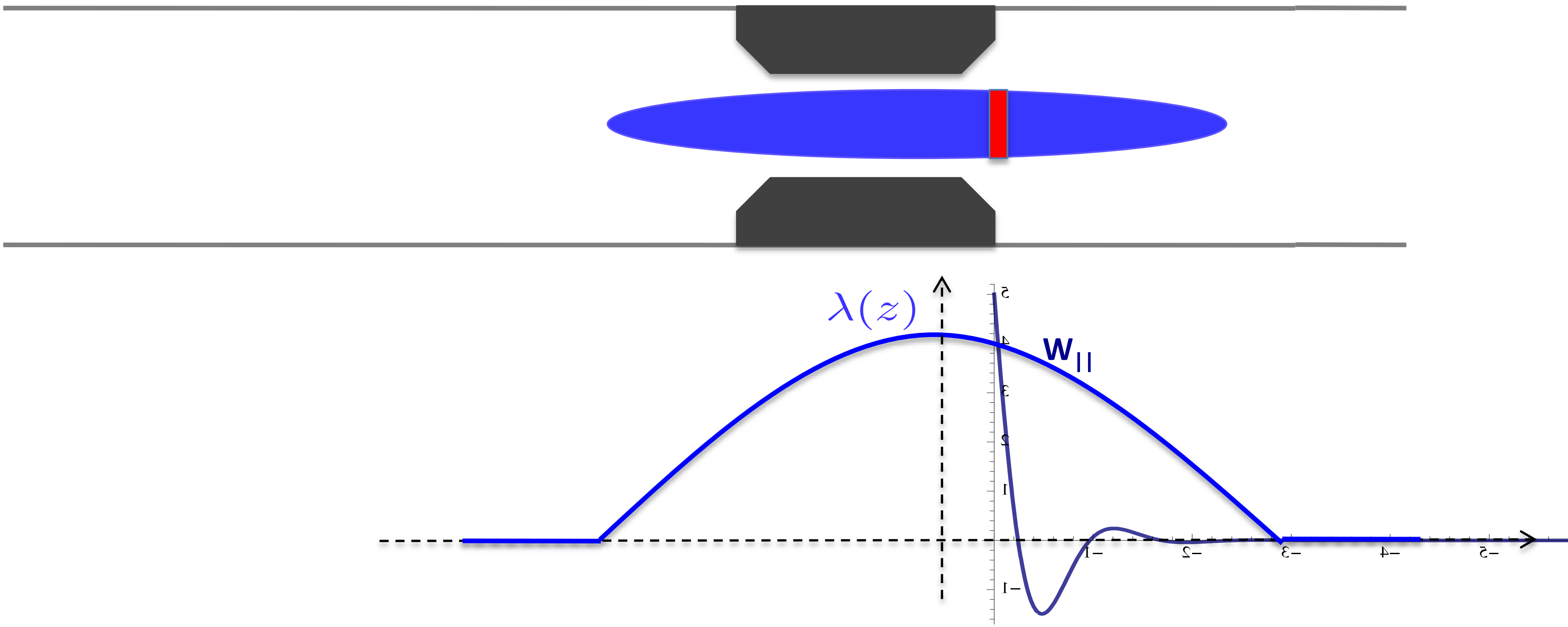
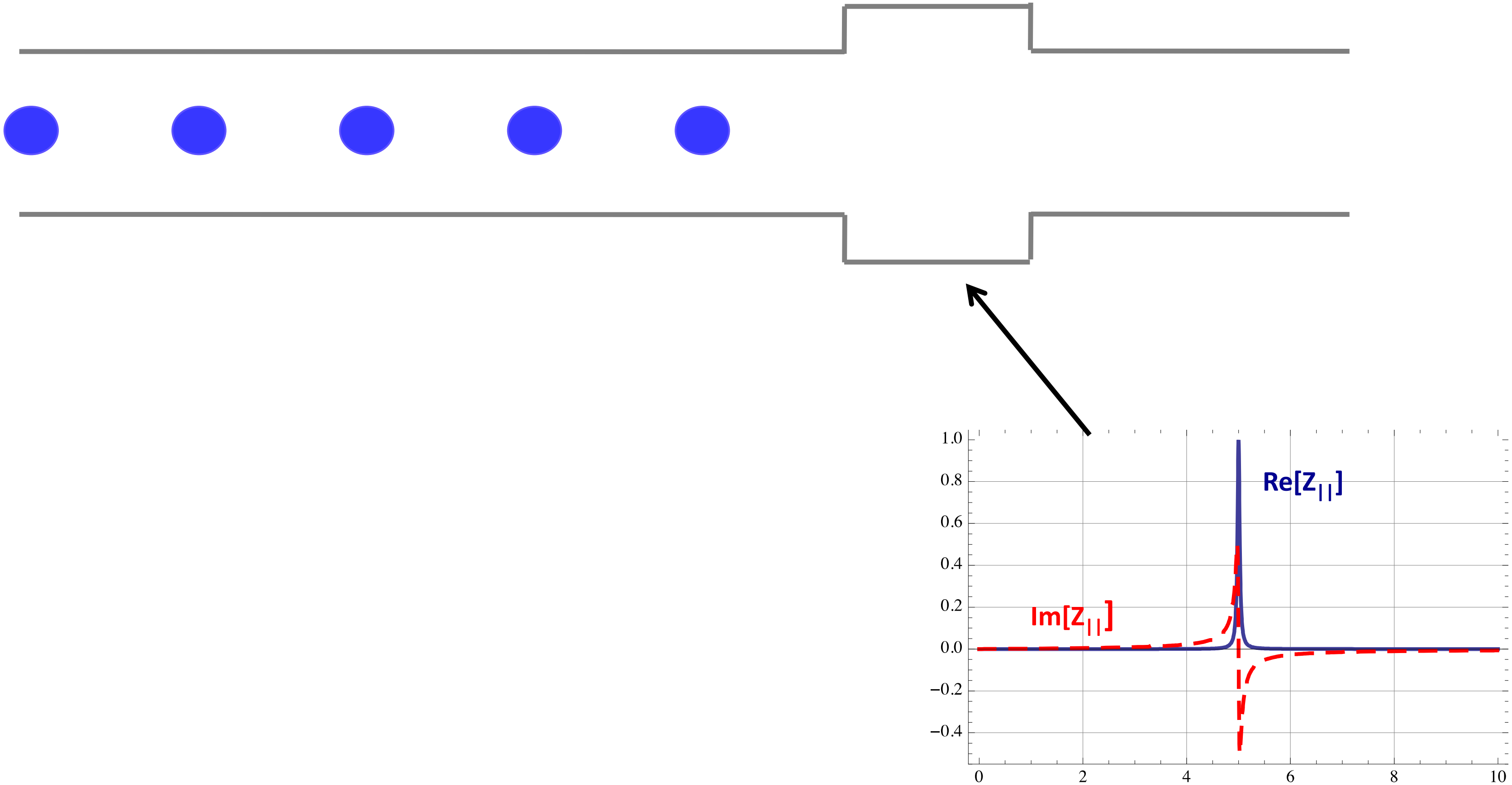
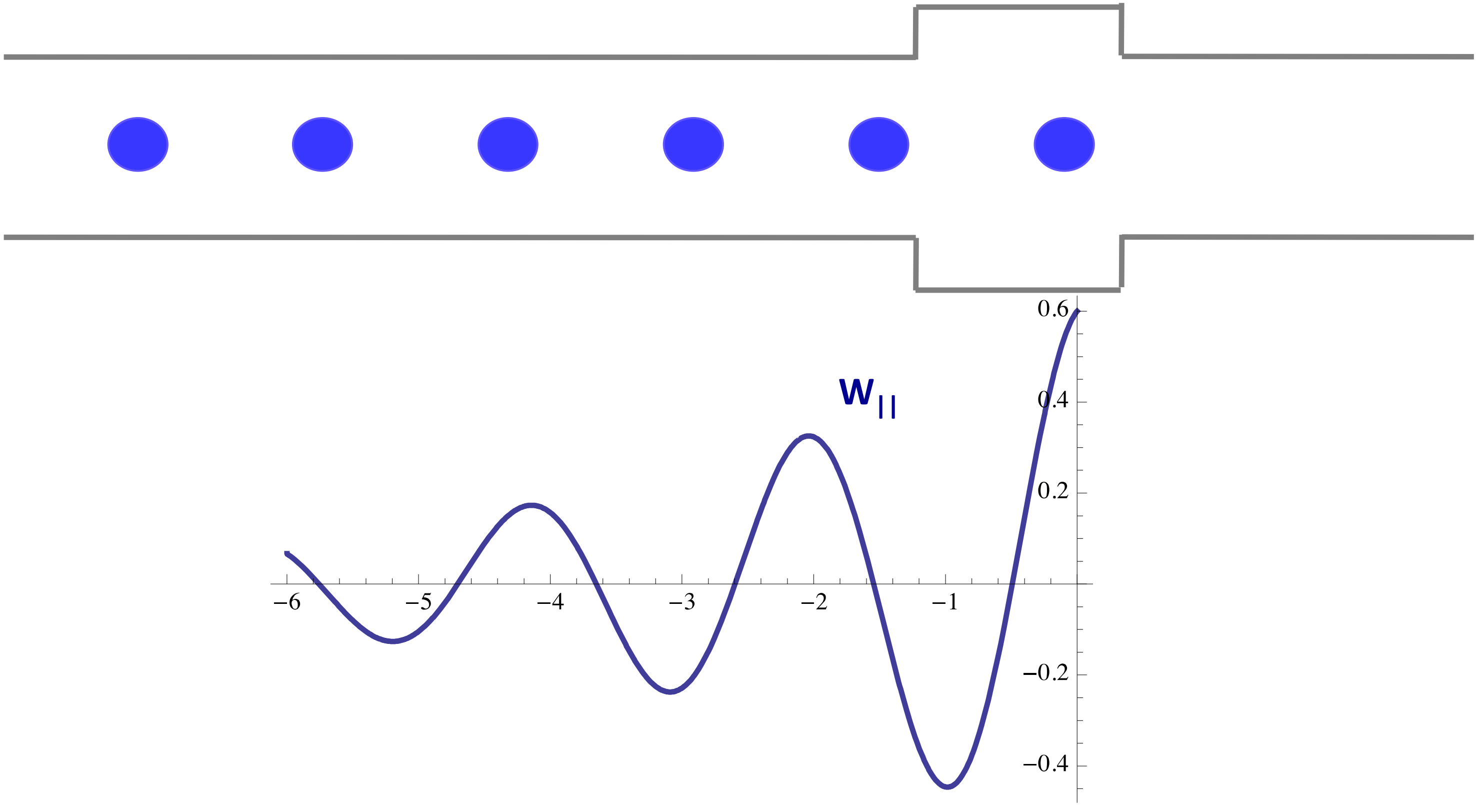


$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{||}(z - z') dz'$$

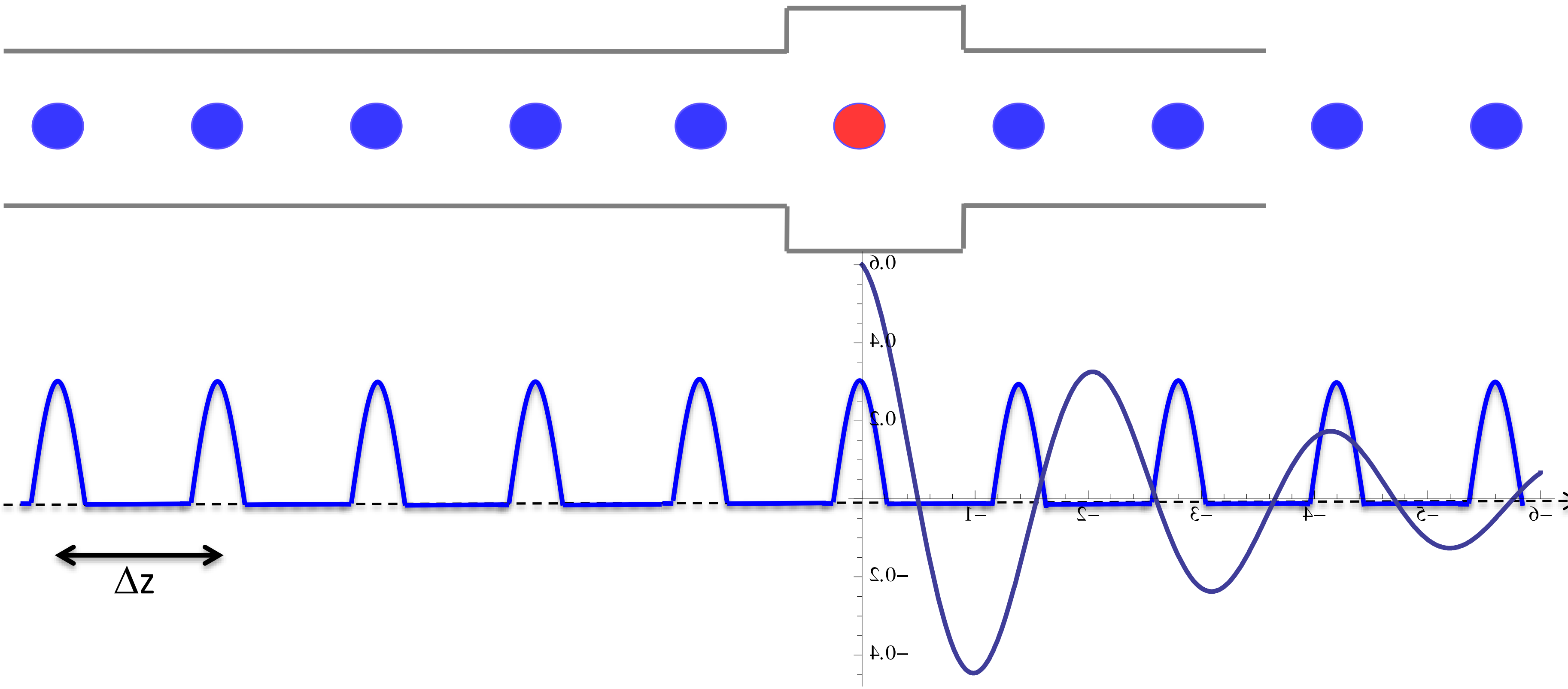
Multi bunch effects



Multi bunch effects



Multi bunch effects

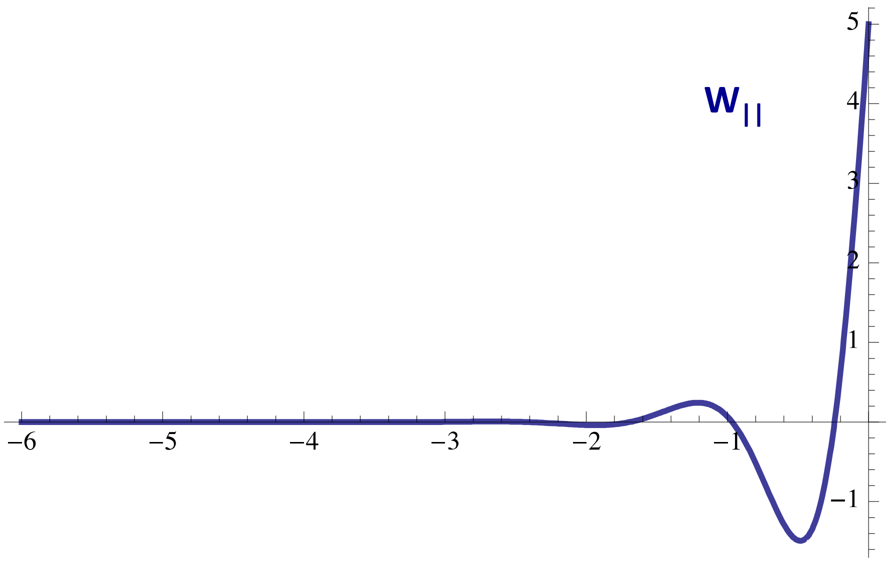


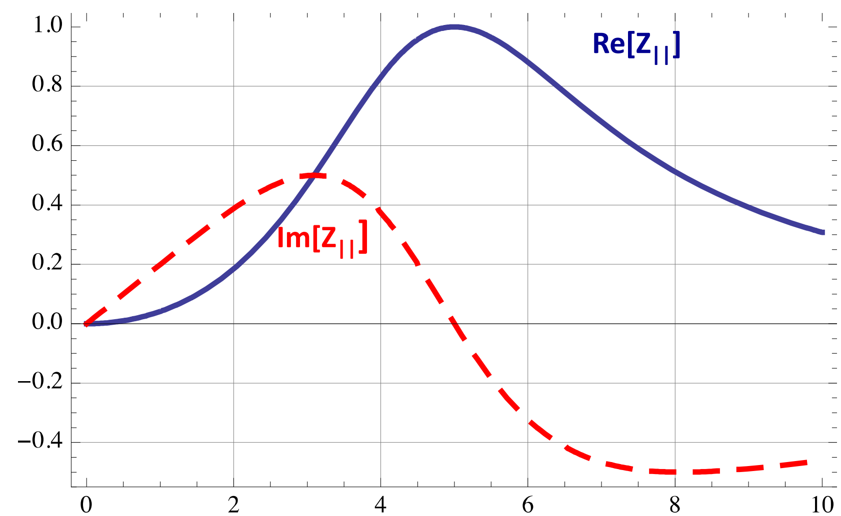
$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z') W_{||}(z_j - z') dz' \approx N_j e^2 \sum_{i=0}^{M} N_i W[(j-i)\Delta z]$$

$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z') W_{||}(z_j - z') dz' \approx N_j e^2 \sum_{k=0}^{\infty} \sum_{i=0}^{M} N_i W[(j-i)\Delta z + kC]$$

Single bunch vs. Multi bunch

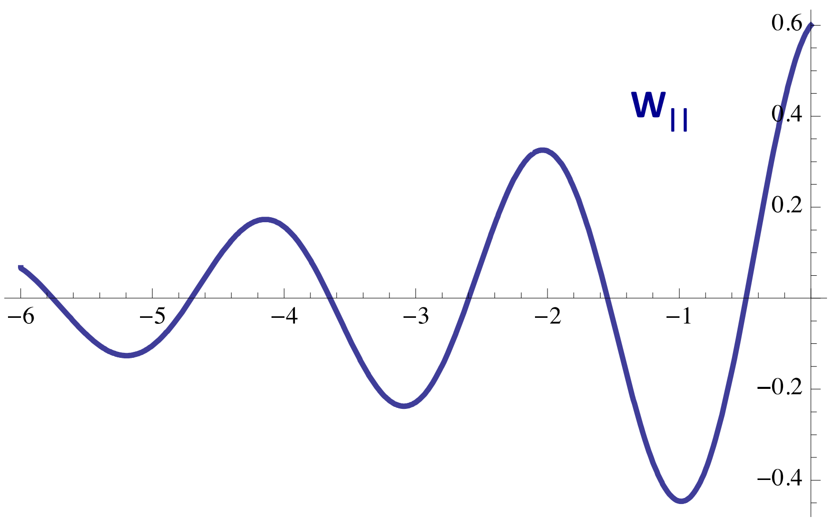
- A short-lived wake, decaying over the length of one bunch, can only cause intra-bunch (head-tail) coupling
- It can be therefore responsible for single bunch instabilities

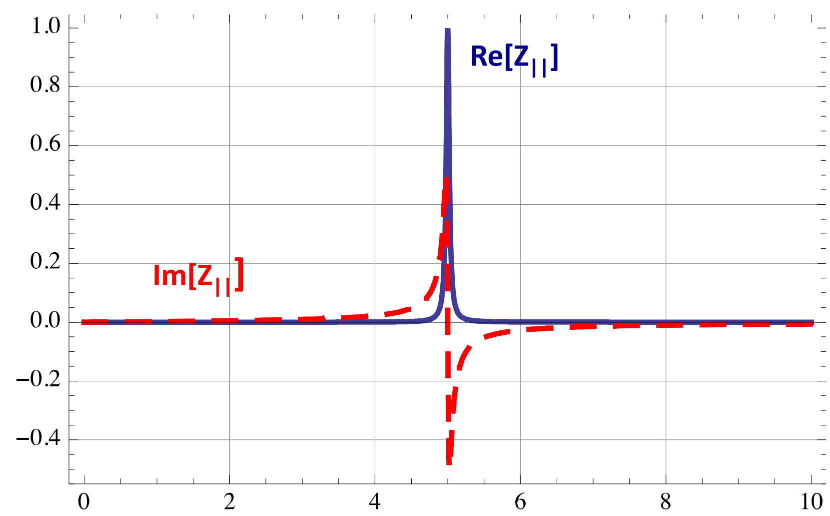




Single bunch vs. Multi bunch

- A long-lived wake field, decaying over the length of a bunch train or even more than a turn, causes bunch-to-bunch or multi-turn coupling
- It can be therefore responsible for multi-bunch or multi-turn instabilities





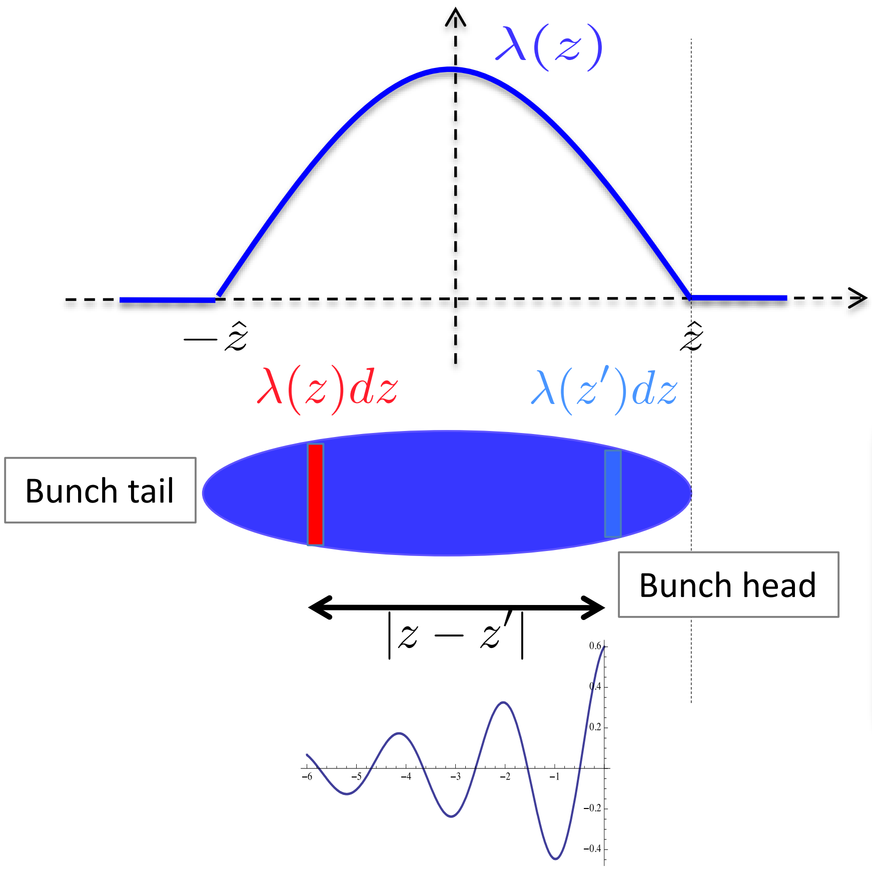
- Detailed calculations:
  - Energy loss
  - Robinson instability

- Qualitative descriptions:
  - Coupled bunch instabilities
  - Single bunch modes

[1] "Physics of Collective Beam Instabilities in High Energy Accelerators", A. W. Chao

Energy loss of a bunch (single pass)

- The energy kick ΔE(z) on each particle e in the witness slice λ(z)dz is the integral of the contributions from the wakes left behind by all the preceding eλ(z')dz slices (sources)
- The total energy loss ΔE of the bunch can then be obtained by integrating ΔE(z) λ(z) over the full bunch extension



$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{||}(z - z') dz'$$

$$\Delta E = \int_{-\hat{z}}^{\hat{z}} \lambda(z) \Delta E(z) dz$$

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \tilde{\lambda}(\omega) \tilde{\lambda}^*(\omega) Z_{||}(\omega) d\omega = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{||}(\omega)\right] d\omega$$

Energy loss of a bunch (multi-pass)

- The total energy loss ΔE of the bunch can still be obtained by integrating ΔE(z) over the full bunch extension
- ΔE(z) this time also includes contributions from all previous turns, spaced by multiples of the ring circumference C

$$\Delta E = -\frac{e^2}{2\pi}\int_{-\infty}^{\infty}\lambda(z)dz\int_{-\infty}^{\infty}dz'\lambda(z')\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z')dz'$$

$$\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z') = \frac{c}{C}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

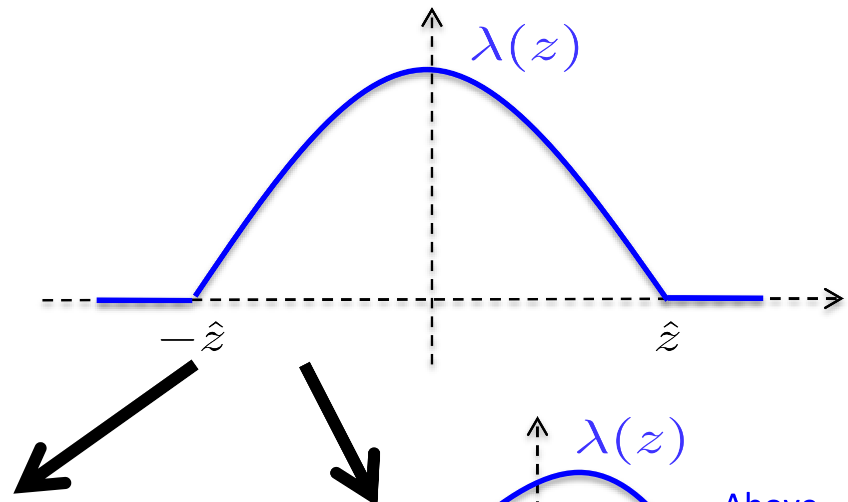
$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\underbrace{\int_{-\infty}^{\infty}\lambda(z)\exp\left(\frac{-ip\omega_0 z}{c}\right)dz}_{\tilde{\lambda}(p\omega_0)}\underbrace{\int_{-\infty}^{\infty}\lambda(z')\exp\left(\frac{ip\omega_0 z'}{c}\right)dz'}_{\tilde{\lambda}^*(p\omega_0)}$$

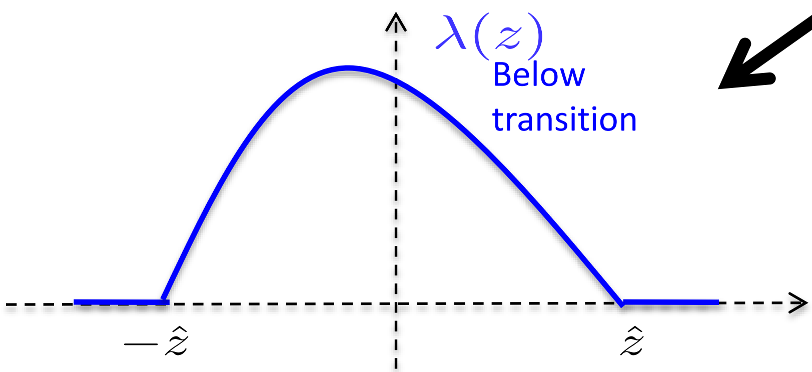
$$\boxed{\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\mathrm{Re}\left[Z_{||}(p\omega_0)\right]}$$

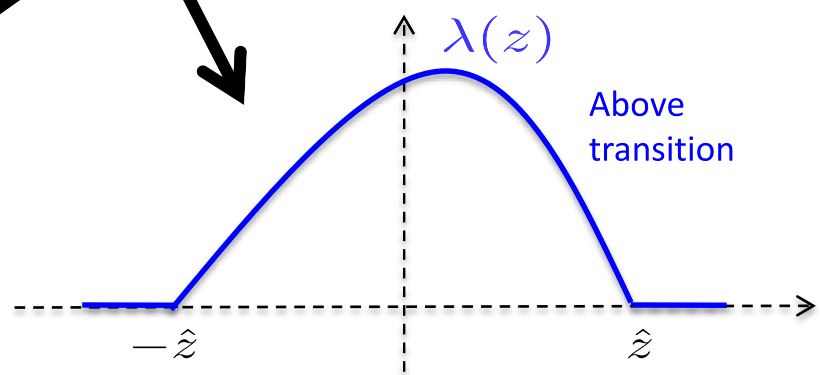
Energy loss per turn: stable phase shift

- The RF system has to compensate for the energy loss by imparting a net acceleration to the bunch
- Therefore, the bunch readjusts to a new equilibrium distribution in the bucket and moves to a synchronous angle $\Delta\Phi_s$

$$\sin \Delta\Phi_s = \frac{\Delta E_{\mathrm{turn}}}{NeV_m}$$







Example
Gaussian bunch and power loss with a broad-band resonator impedance

$$\lambda(z) = \frac{N}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \tilde{\lambda}(\omega) = N \exp\left(-\frac{\omega^2\sigma_z^2}{2c^2}\right)$$

$$\int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{||}(\omega)\right] d\omega$$

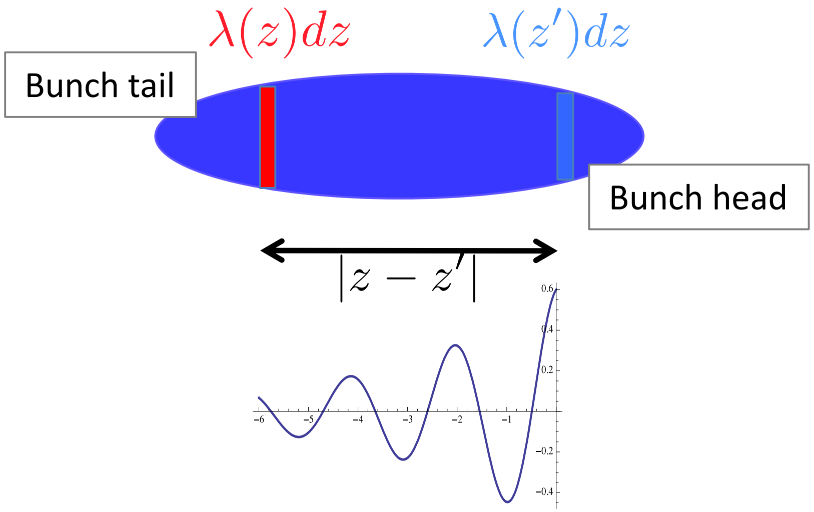
can be calculated with $Z_{||}(\omega) = Z_{||}^{Res}(\omega)$ from previous slide in the two limiting cases

$\sigma_z \gg \frac{c}{\omega_r}$ Need to expand $\mathrm{Re}[Z_{||}(\omega)]$ for small ω

$\sigma_z \ll \frac{c}{\omega_r}$ Can assume $|\lambda(\omega)|$ constant over $\mathrm{Re}[Z_{||}(\omega)]$

Single particle equations of the longitudinal motion in presence of wake fields

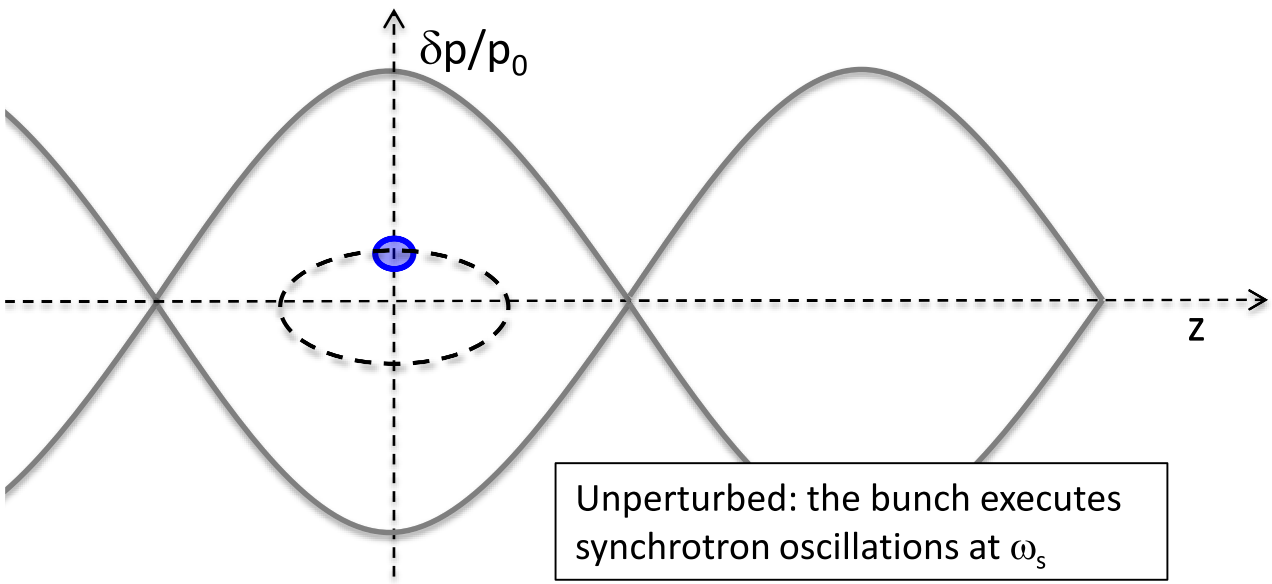
- The single particle in the witness slice λ(z)dz will feel the force from the RF, from the bunch own space charge, and that associated to the wake
- The wake contribution can extend to several turns



$$\underbrace{\frac{d^2z}{dt^2}+\frac{\eta e V_{\mathrm{rf}}(z)}{m_0\gamma C}}_{\text{External RF}}=\underbrace{\frac{\eta e^2}{m_0\gamma C}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\lambda(z'+kC)W_{||}(z-z'-kC)dz'}_{\text{Wake fields}}$$

The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ The bunch additionally feels the effect of a multi-turn wake



Unperturbed: the bunch executes synchrotron oscillations at ω_s

The Robinson instability

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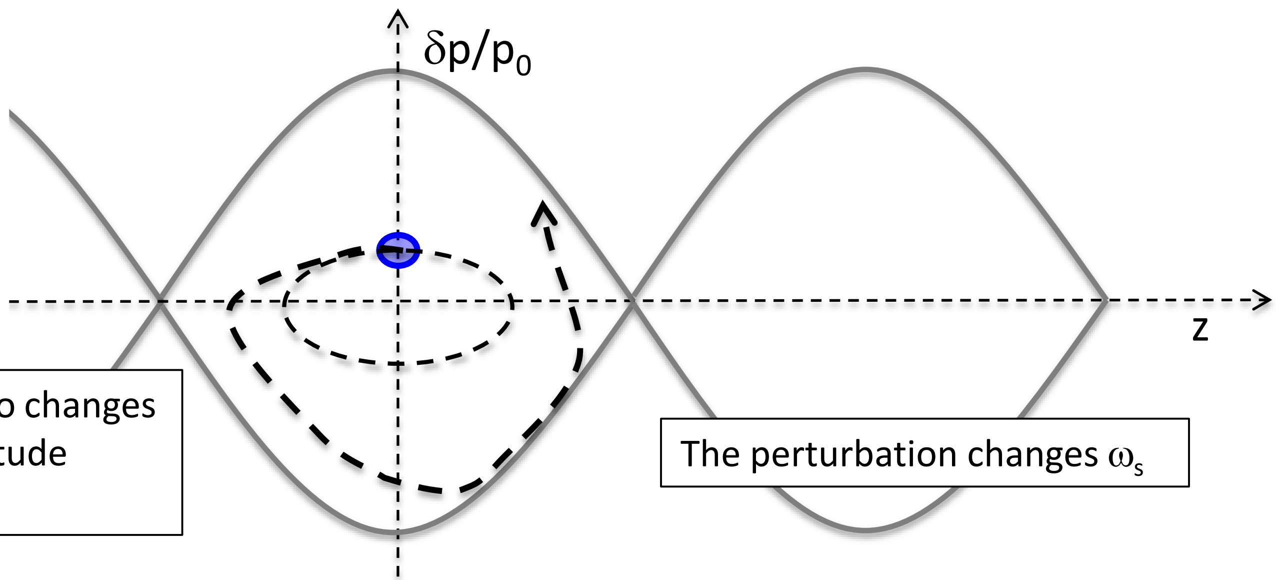
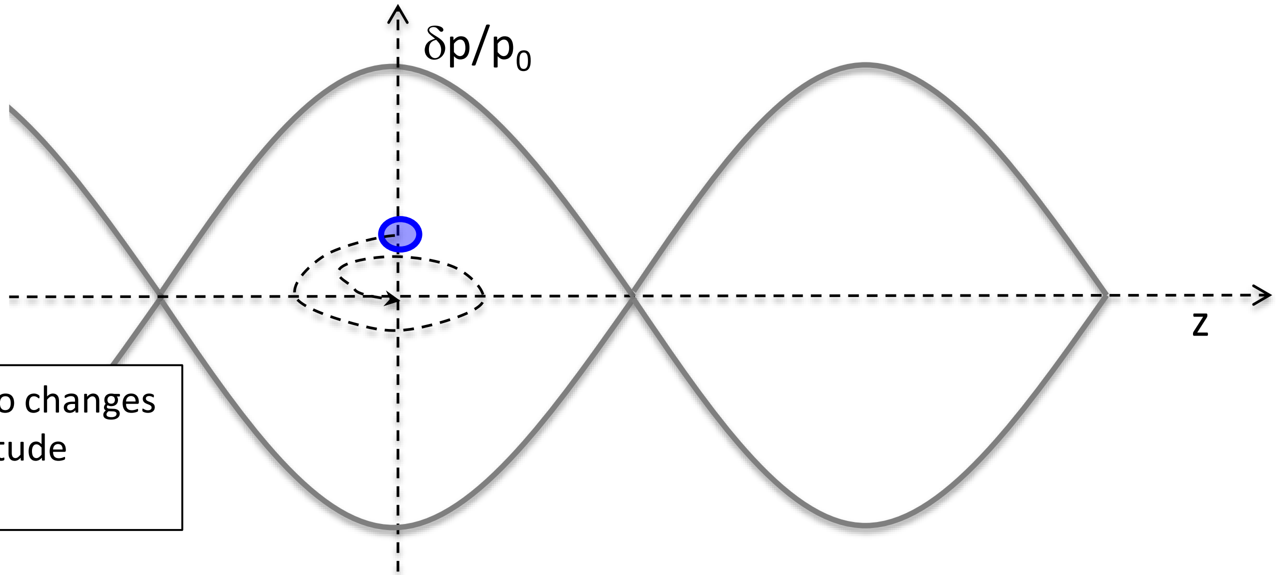


The perturbation also changes the oscillation amplitude
Unstable motion

The perturbation changes ω_s

The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ The bunch additionally feels the effect of a multi-turn wake



The perturbation also changes the oscillation amplitude
Damped motion

The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  - ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - ⇒ The bunch additionally feels the effect of a multi-turn wake

$$\frac{d^2 z}{dt^2} + \omega_s^2 z = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} W_{||}\left[z(t) - z(t-kT_0) - kC\right]$$

We assume that the wake can be linearized on the scale of a synchrotron oscillation

$$W_{||}\left[z(t) - z(t-kT_0) - kC\right] \approx W_{||}(kC) + W_{||}'(kC) \cdot \left[z(t) - z(t-kT_0)\right]$$

The Robinson instability

$$W_{||}\left[z(t) - z(t - kT_0) - kC\right] \approx \cancel{W_{||}(kC)} + W'_{||}(kC) \cdot \left[z(t) - z(t - kT_0)\right]$$

⇒ The term $\Sigma\, W_{||}(kC)$ only contributes to a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z_0 and not around 0. This term represents the stable phase shift that compensates for the energy loss

⇒ The dynamic term proportional to $z(t)-z(t-kT_0) \approx kT_0 dz/dt$ will introduce a “friction” term in the equation of the oscillator, which can lead to instability!

$$z(t) \propto \exp\left(-i\Omega t\right)$$

$$\Omega^2 - \omega_s^2 = -\frac{Ne^2\eta}{Cm_0\gamma} \boxed{\sum_{k=-\infty}^{\infty} \left[1 - \exp\left(-ik\Omega T_0\right)\right] \cdot W'_{||}(kC)}$$

$$\boxed{i \cdot \frac{1}{C} \sum_{p=-\infty}^{\infty} \left[p\omega_0 Z_{||}(p\omega_0) - (p\omega_0 + \Omega) Z_{||}(p\omega_0 + \Omega)\right]}$$

The Robinson instability

⇒ We assume a small deviation from the synchrotron tune
⇒ Re(Ω - ω_s) → Synchrotron tune shift
⇒ Im(Ω - ω_s) → Growth/damping rate, only depends on the dynamic term, if it is positive there is an instability!

$$\Omega^2 - \omega_s^2 \approx 2\omega_s \cdot \boxed{(\Omega - \omega_s)}$$

Complex frequency shift

$$\Delta\omega_s = \mathrm{Re}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \times$$

$$\times \sum_{p=-\infty}^{\infty} \left[p\omega_0 \mathrm{Im} Z_{||}(p\omega_0) - (p\omega_0 + \omega_s) \mathrm{Im} Z_{||}(p\omega_0 + \omega_s)\right]$$

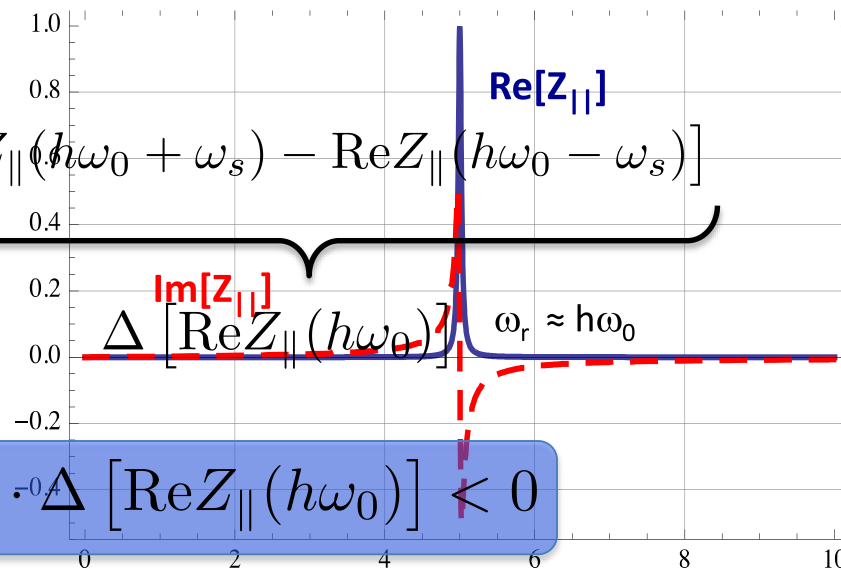
$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{||}(p\omega_0 + \omega_s)$$

The Robinson instability

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{\parallel}(p\omega_0 + \omega_s)$$

- ⇒ We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 >> \omega_s$ (e.g. RF cavity fundamental mode or HOM)
- ⇒ Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
- ⇒ Stability requires that η and $\Delta[\mathrm{Re}\, Z_{||}(h\omega_0)]$ have different signs

$$\tau^{-1} = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta h\omega_0}{2\gamma T_0^2 \omega_s} \underbrace{\left[\mathrm{Re} Z_{\parallel}(h\omega_0 + \omega_s) - \mathrm{Re} Z_{\parallel}(h\omega_0 - \omega_s)\right]}_{\Delta \left[\mathrm{Re} Z_{\parallel}(h\omega_0)\right]}$$



Stability criterion → $\eta \cdot \Delta \left[\mathrm{Re} Z_{\parallel}(h\omega_0)\right] < 0$

The Robinson instability

Stability criterion → $\eta \cdot \Delta\left[\mathrm{Re}Z_{||}(h\omega_0)\right] < 0$

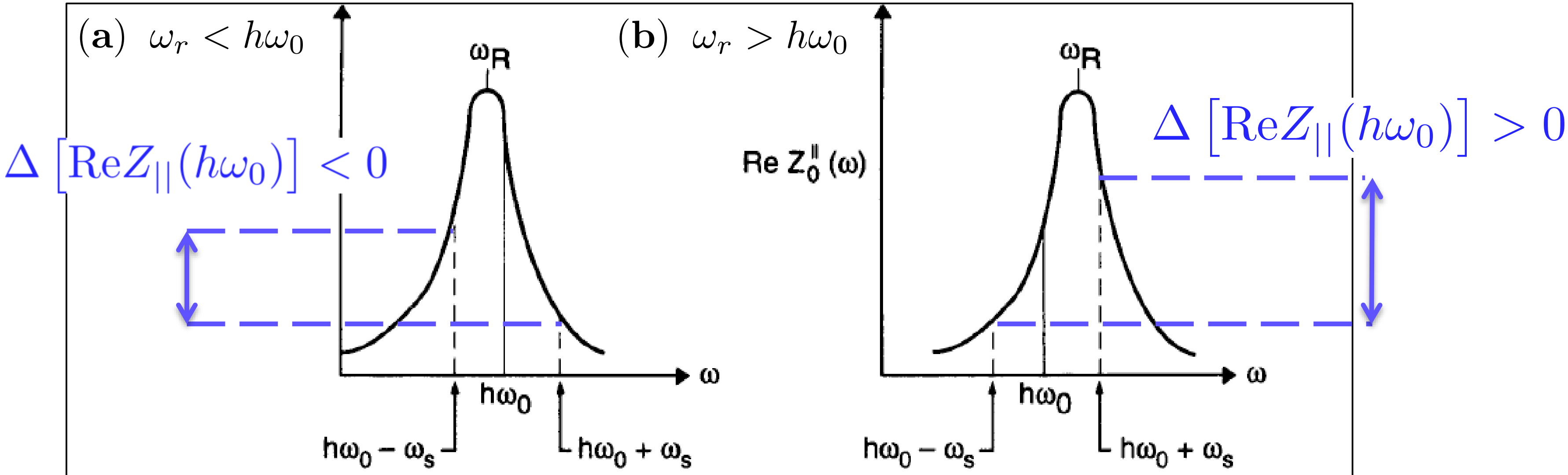


Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r < h\omega_0$	$\omega_r > h\omega_0$
Above transition (η>0)	stable	unstable
Below transition (η<0)	unstable	stable

The Robinson instability

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{\parallel}(p\omega_0 + \omega_s)$$

⇒ Other types of impedances can also cause instabilities through the Robinson mechanism

⇒ However, a smooth broad-band impedance with no narrow structures on the ω_0 scale cannot give rise to an instability

- ✓ Physically, this is clear, because the absence of structure on ω_0 scale in the spectrum implies that the wake has fully decayed in one turn time and the driving term in the equation of motion also vanishes

$$\sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{\parallel}(p\omega_0 + \omega_s) \rightarrow \frac{1}{\omega_0} \int_{-\infty}^{\infty} \omega \mathrm{Re} Z_{\parallel}(\omega) d\omega \rightarrow 0$$

Other longitudinal instabilities

- The Robinson instability occurs for a single bunch under the action of a multi-turn wake field
  - It contains a term of coherent synchrotron tune shift
  - It results into an unstable rigid bunch dipole oscillation
  - It does not involve higher order moments of the bunch longitudinal phase space distribution

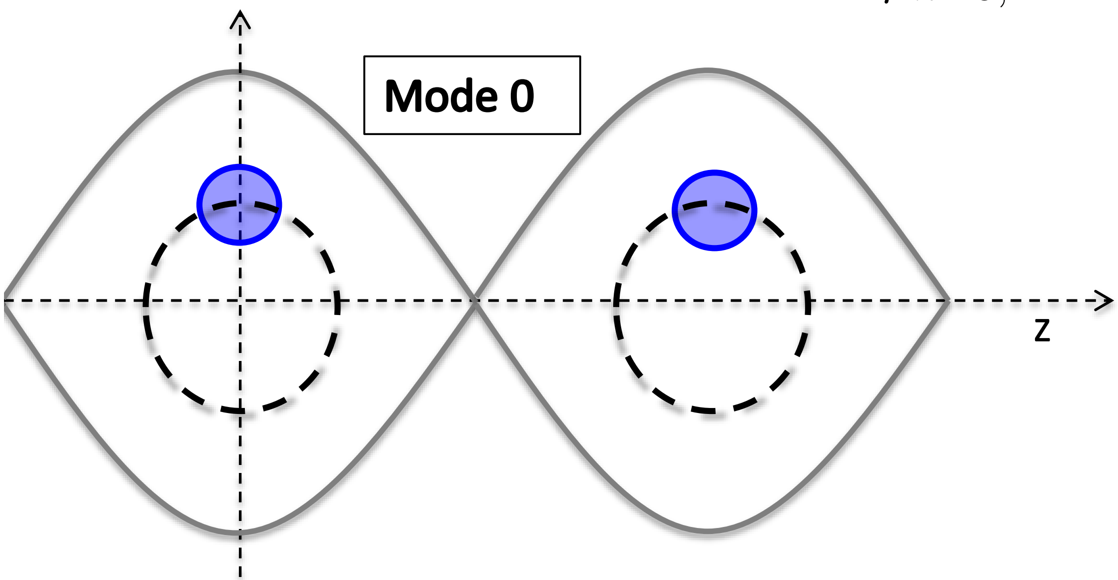
- Other important collective effects can affect a bunch in a beam
  - Potential well distortion (resulting in synchronous phase shift, bunch lengthening or shortening, synchrotron tune shift/spread)
  - Coupled bunch instabilities
  - High intensity single bunch instabilities (e.g. microwave instability)
  - Coasting beam instabilities (e.g. negative mass instability)

- To be able to study these effects we would need to resort to a more detailed description of the bunch(es)
  - Vlasov equation (kinetic model)
  - Macroparticle simulations

Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane

$$\Delta\varphi_{n=0,1} = \frac{2\pi n}{N_b}$$



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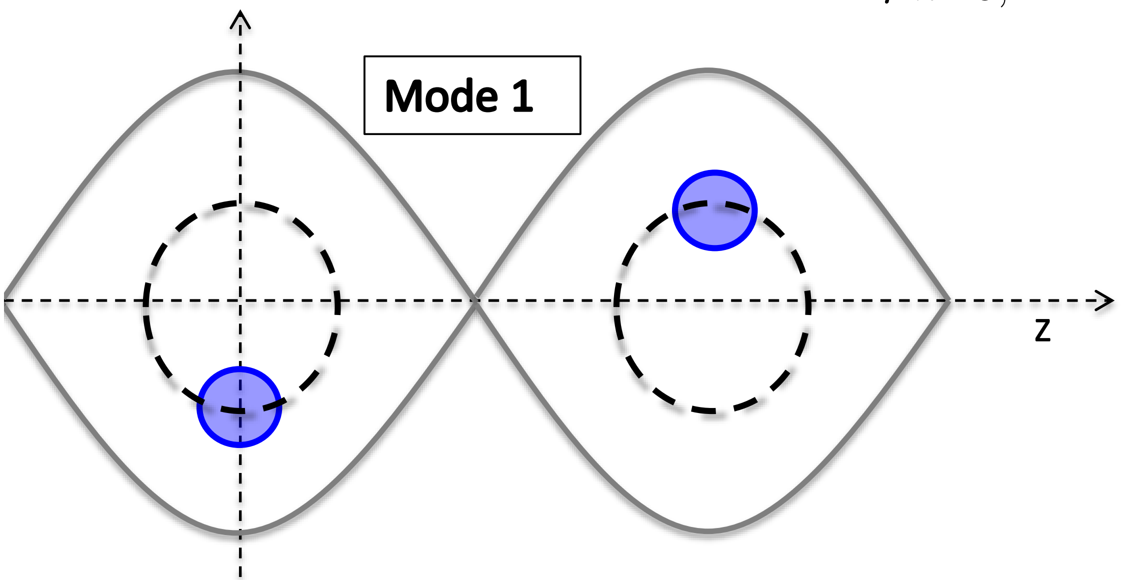
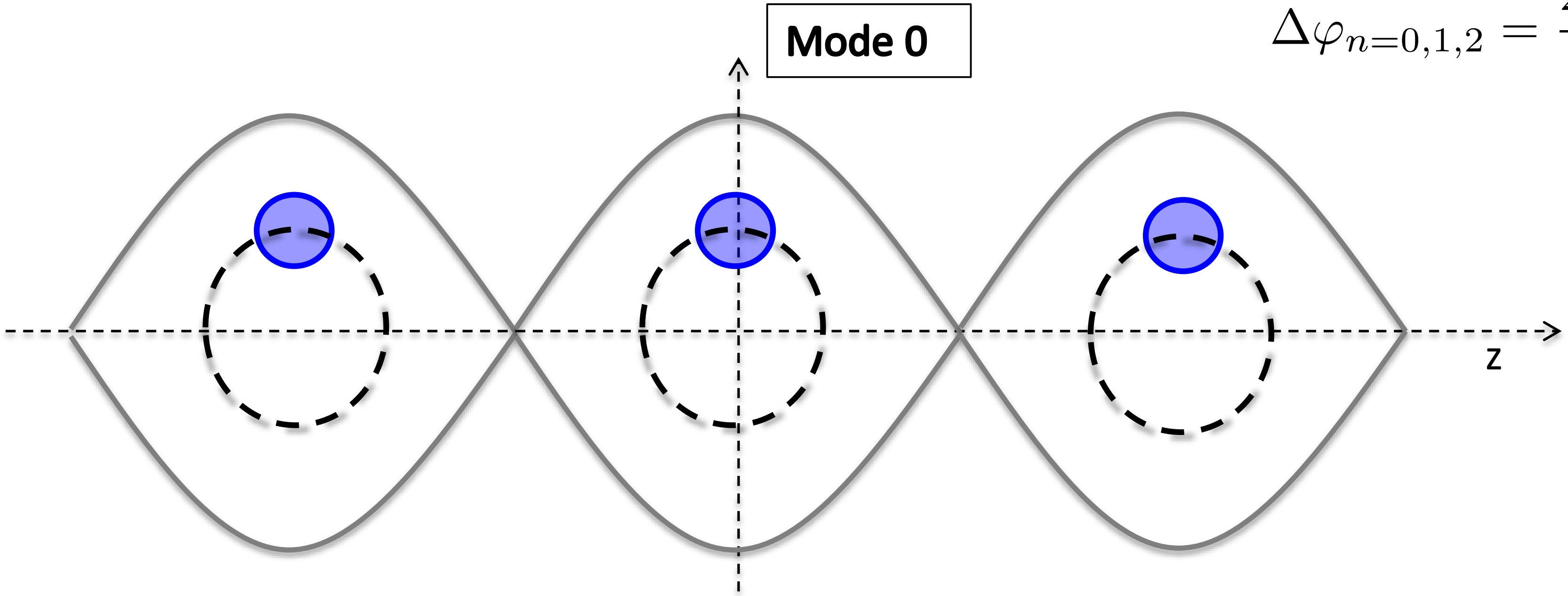


Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes

$$\Delta\varphi_{n=0,1,2} = \frac{2\pi n}{N_b}$$



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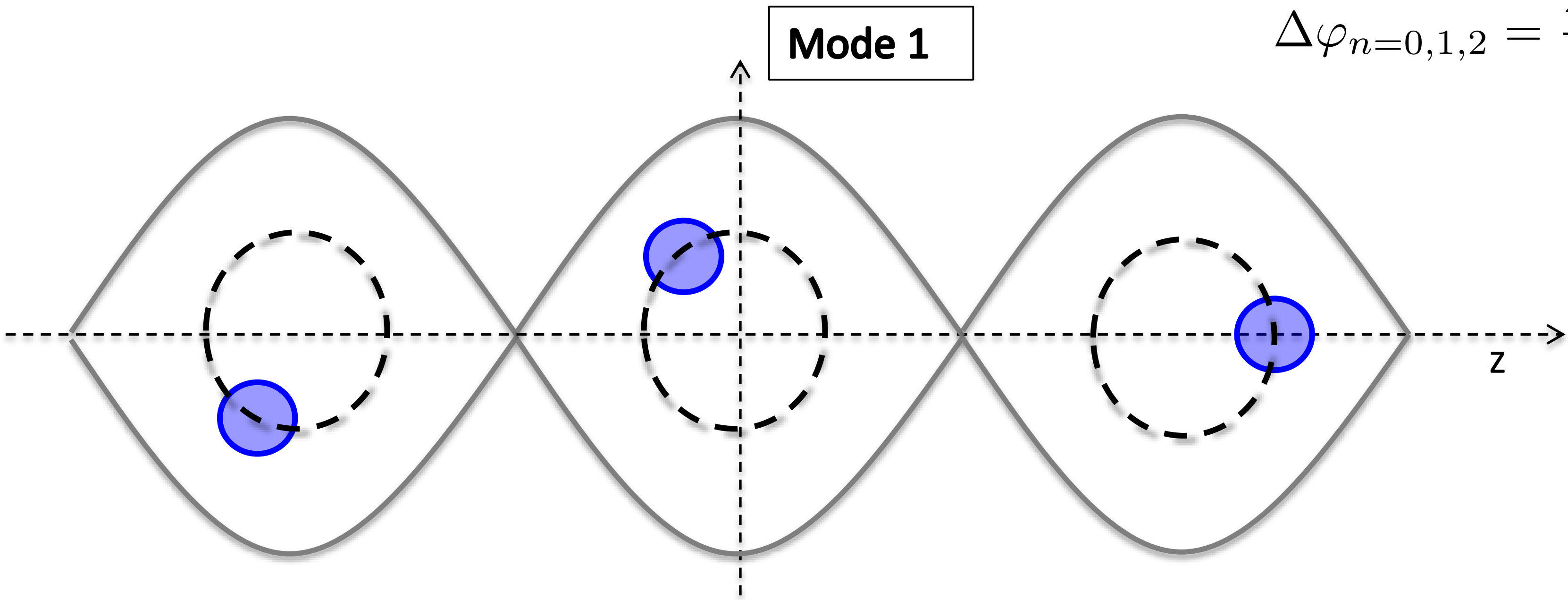
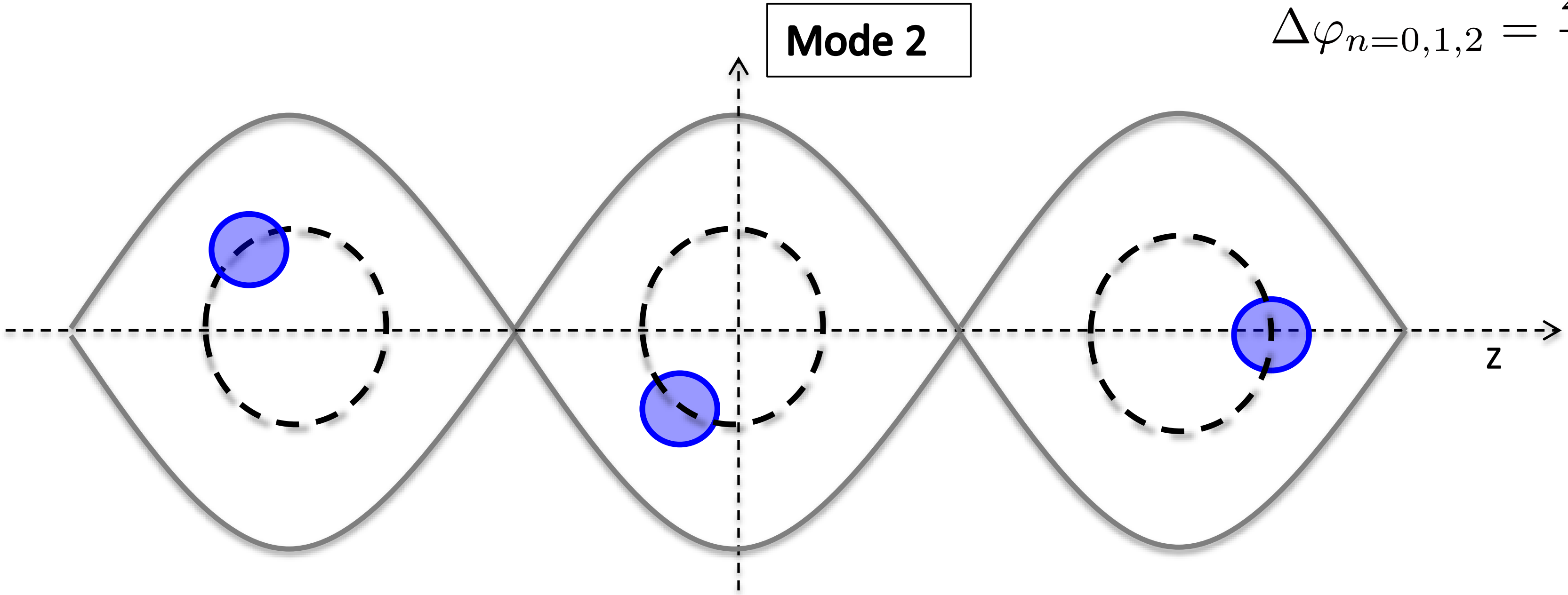


Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes

⇒ **This modes can become unstable under the effect of long range wake fields**

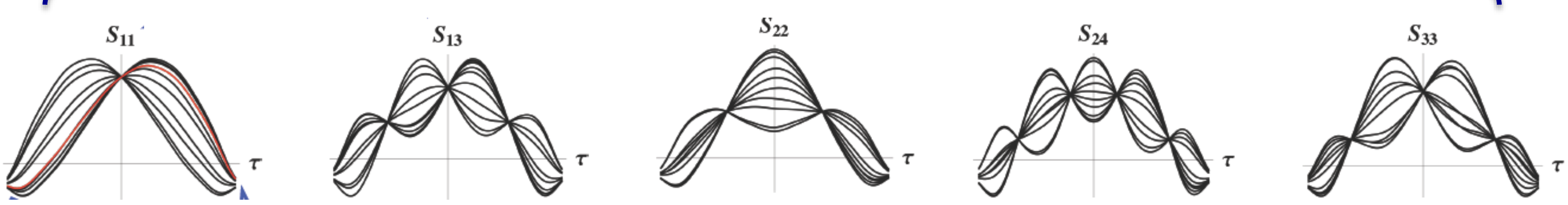
$$\Delta\varphi_{n=0,1,2} = \frac{2\pi n}{N_b}$$



Single bunch modes

- In a similar fashion, a single bunch exhibits a double infinity of natural modes of oscillation, with rather complicated phase space portraits.
- Whatever perturbation on the bunch phase space distribution can be expanded as a series of these modes

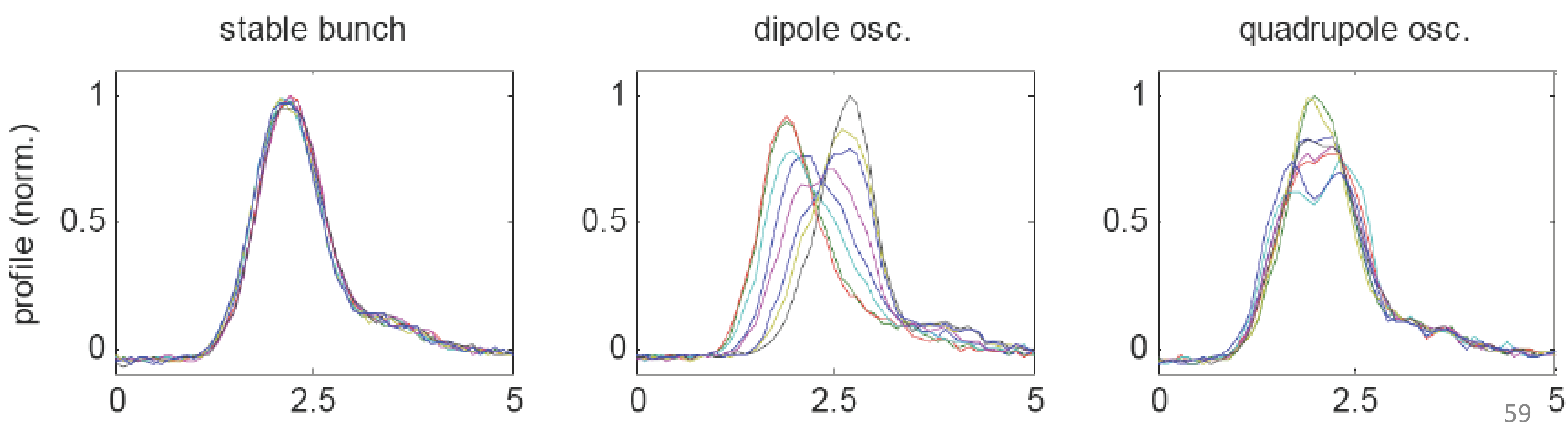
Oscillation modes as observed at a wall current monitor



Single bunch modes

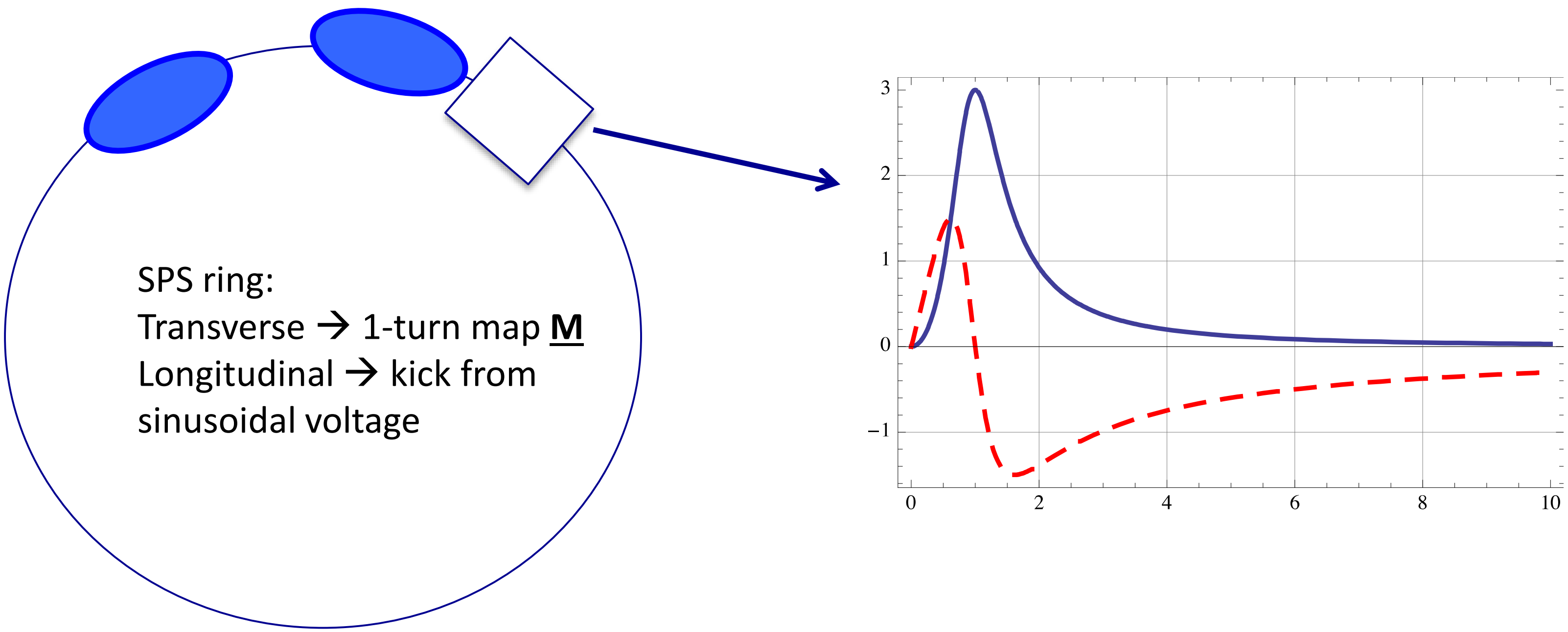
- In a similar fashion, a single bunch exhibits a double infinity of natural modes of oscillation, with rather complicated phase space portraits.
- Whatever perturbation on the bunch phase space distribution can be expanded as a series of these modes
- One of these modes or a combination of them can become unstable under the effect of a short range wake field
- In particular, the frequencies of these modes shift with intensity, and two of the modes can merge above a certain threshold, causing a microwave instability!

Observations in the CERN SPS in 2007



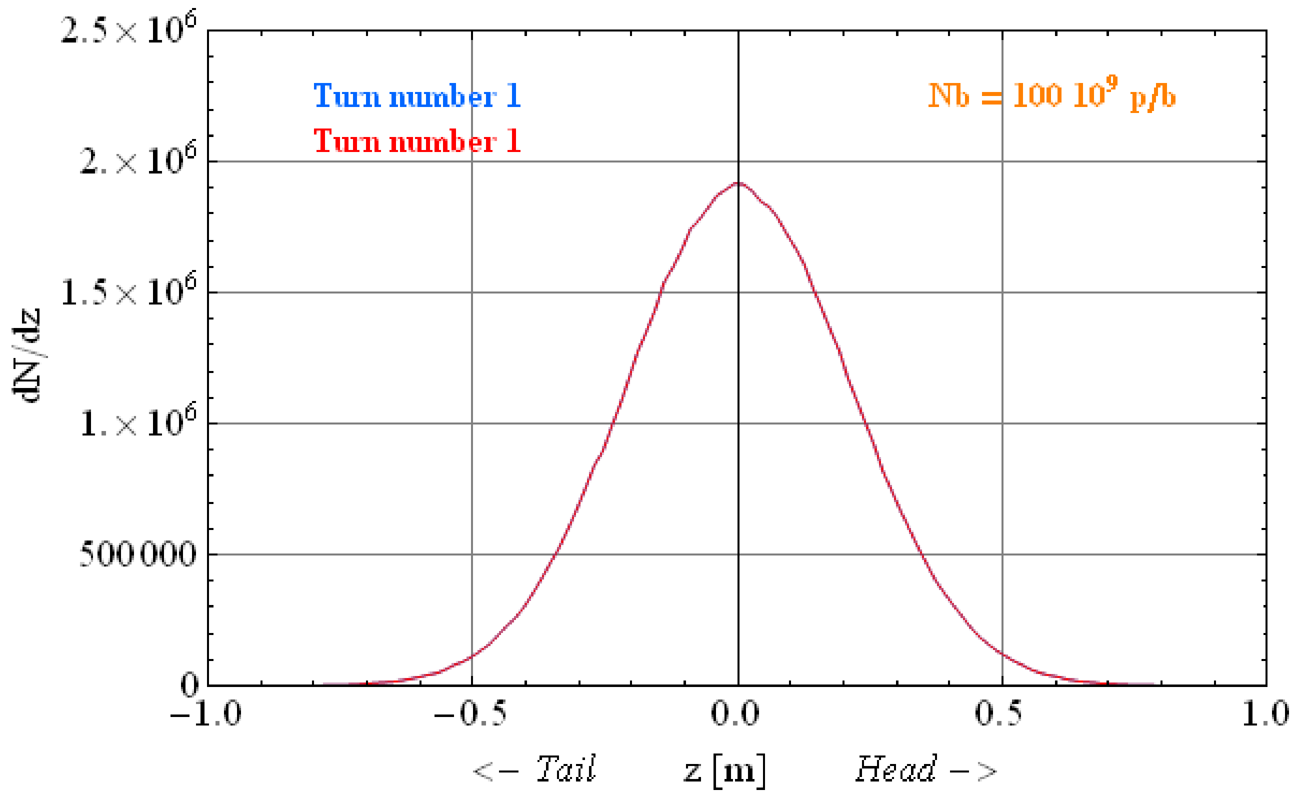
Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring



Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring
- Two different intensity values have been simulated
  - **Low intensity → below instability threshold, but potential well distortion is visible in terms of stable phase shift and bunch lengthening**



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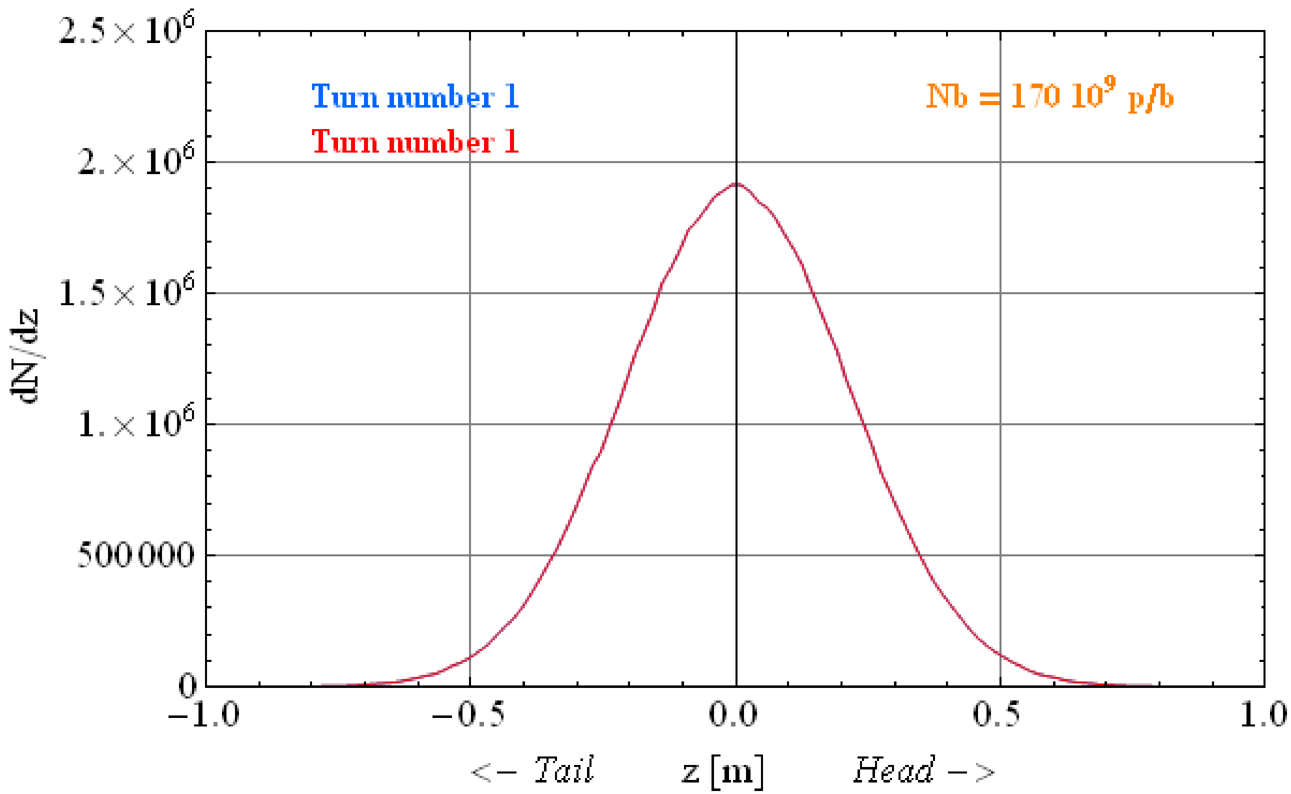
- Two different intensity values have been simulated
  - Low intensity → below instability threshold, but potential well distortion is visible in terms of stable phase shift and bunch lengthening
  - **High intensity → above microwave instability threshold**



Conclusions (part I)

- Beam instability
  - Manifests itself like an exponential coherent motion resulting in beam loss or emittance blow up
  - Can be caused by self induced EM fields
  - Can be described in the framework of wake fields/beam coupling impedances
- Longitudinal effects
  - Energy loss
  - Dipole instability (Robinson), excitation of coupled bunch & single bunch modes
- Tomorrow → transverse wake fields/beam coupling impedances and instabilities