Theory errors in the one-jet inclusive cross section

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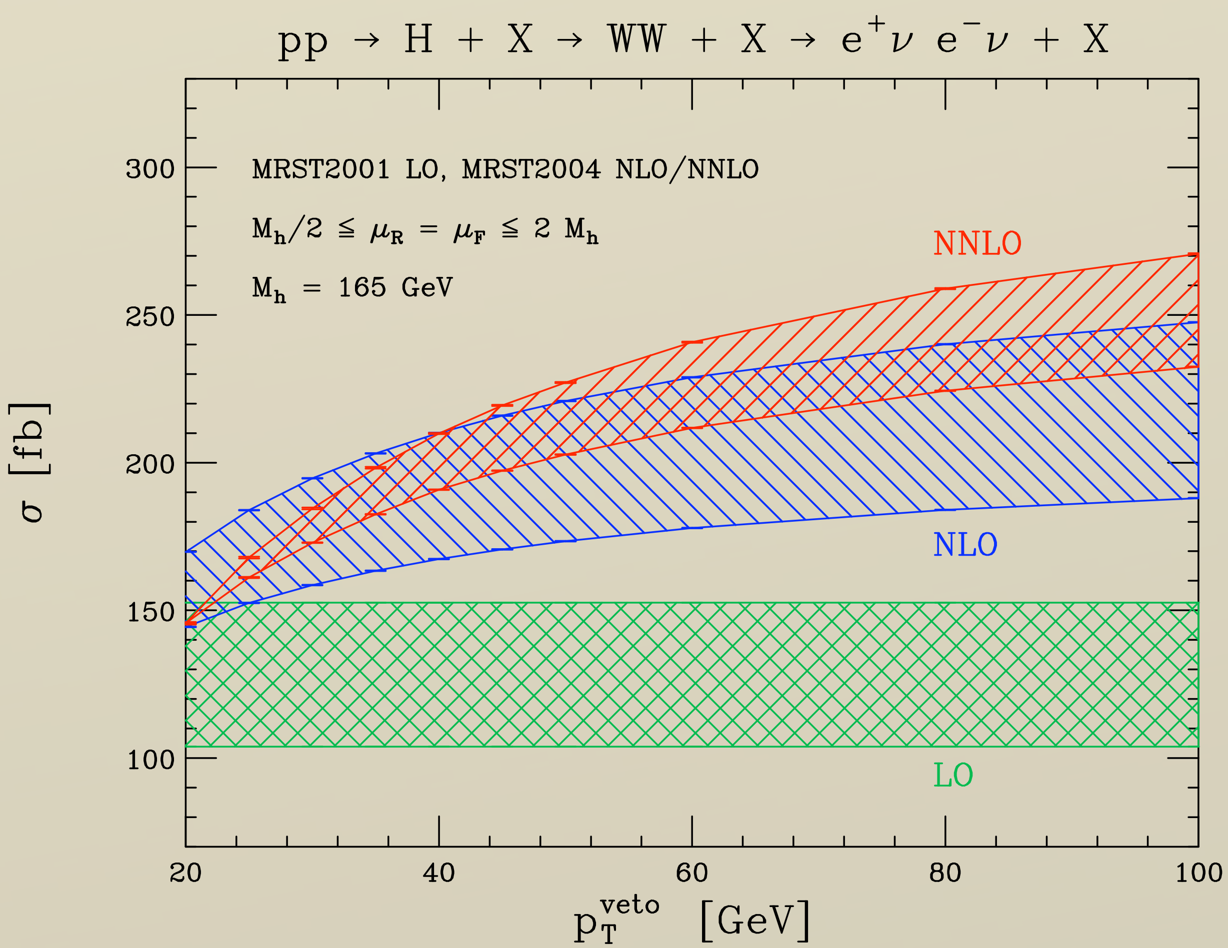
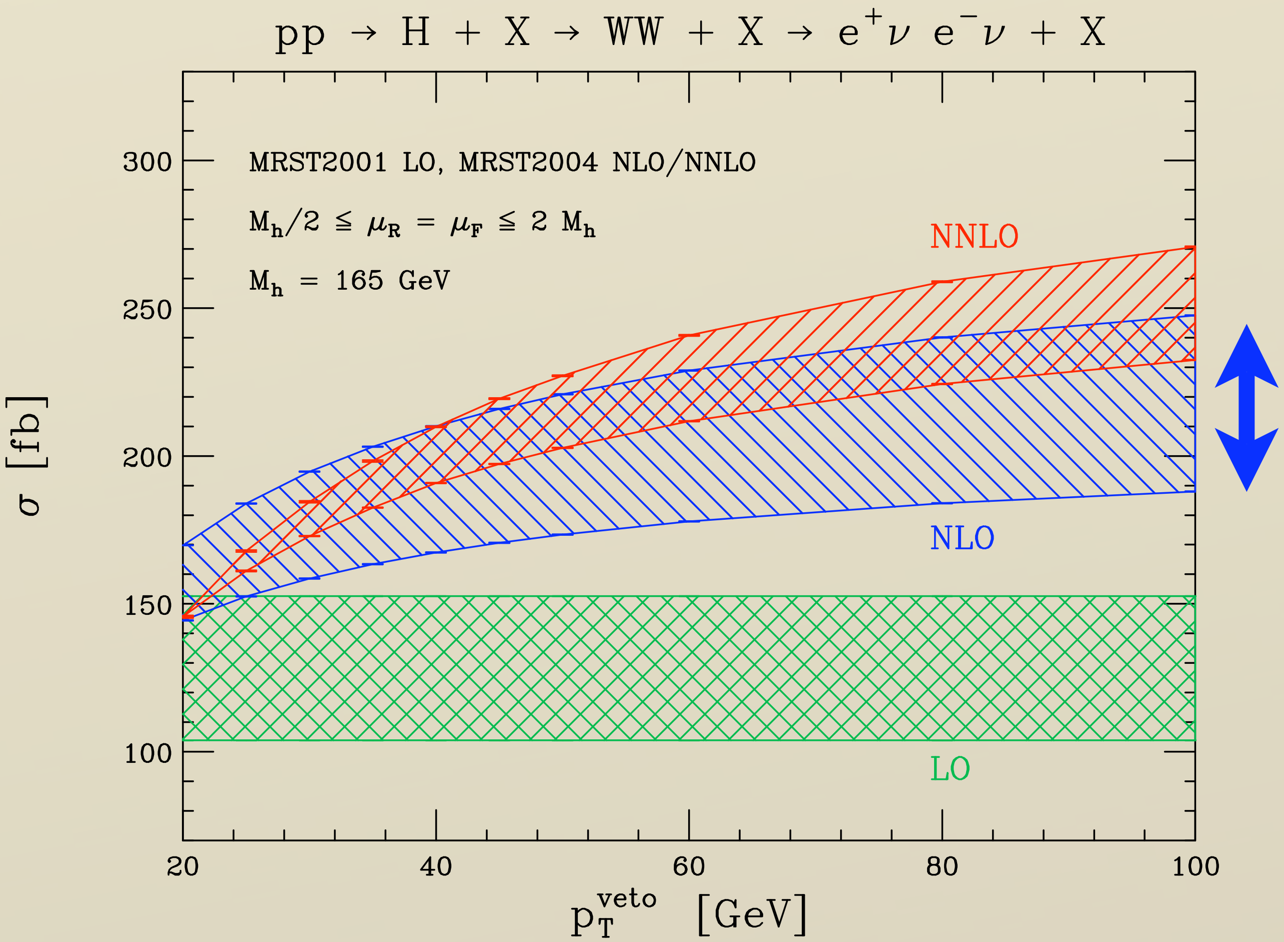
Motivation

- Perturbative calculations are usually presented with an error estimate.
- For example, Anastasiou, Dissertori, and Stockli, JHEP 0709, 018 (2007):



- Suppose that we have only NLO.



- We hope our NLO error band gives a range where NNLO will fall.

- For cross sections used for parton distributions, we should include the estimated theory error in the fitting procedure.

- The one jet inclusive cross section is used in parton fitting.
- This cross section has relatively large theory errors since it is known only to NLO.
- We therefore provide an estimate of the theory error in a form suitable for fitting.
- *Caveat*: a theory error is a guess. Opinions can differ.

$$\frac{d\sigma}{dE_T} = \frac{1}{2(y_{\rm max} - y_{\rm min})} \left[\int_{-y_{\rm max}}^{-y_{\rm min}} dy + \int_{y_{\rm min}}^{y_{\rm max}} dy \right] \frac{d\sigma}{dE_T\, dy}$$

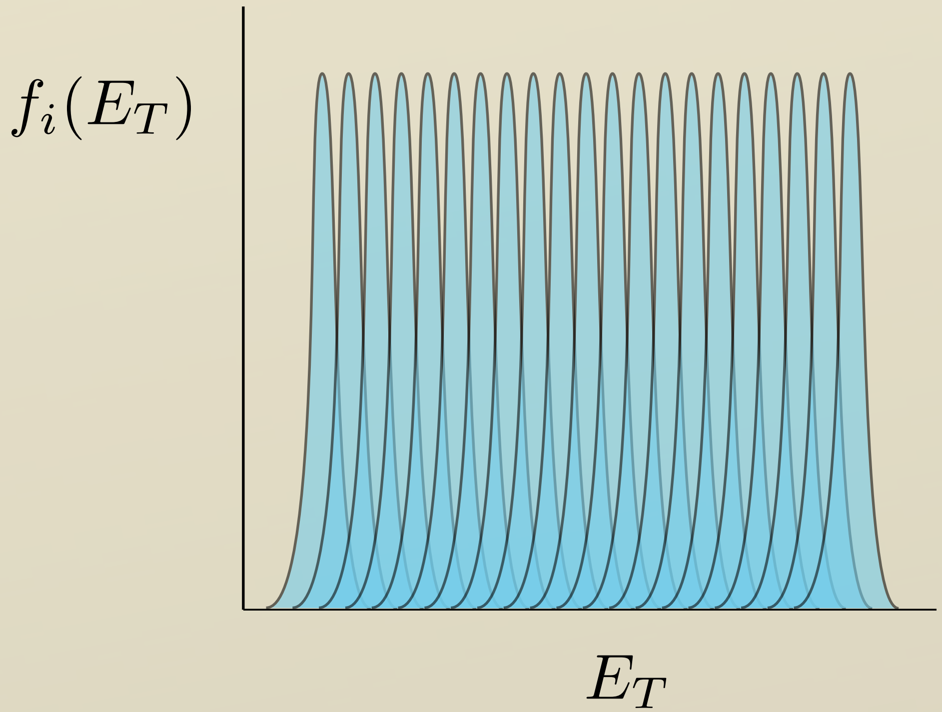
- In this talk, I present our results for $\sqrt{s} = 1960$ GeV, $y_{\rm min} = 0$, $y_{\rm max} = 1.0$, with a cone algorithm using $R = 0.7$ and $R_{\rm sep} = 1.3$.

Format for theory errors

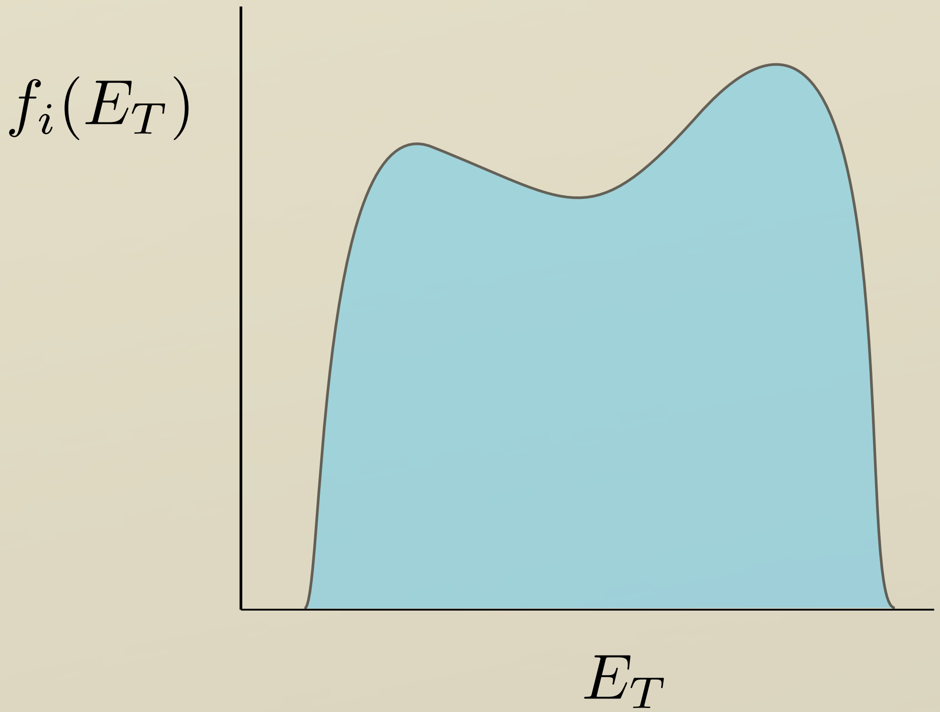
$$\frac{d\sigma}{dE_T} = \left[\frac{d\sigma}{dE_T}\right]_{\rm NLO} \left\{ 1 + \sum_i \lambda_i f_i(E_T) \right\}$$

- $f_i(E_T)$ are functions to be specified.
- λ_i are Gaussian random variables with standard deviation 1.
- The size of the functions $f_i(E_T)$ gives the size of the errors.
- This gives the complete error matrix as for experimental systematic errors.

$$\frac{d\sigma}{dE_T} = \left[\frac{d\sigma}{dE_T}\right]_{\mathrm{NLO}} \left\{ 1 + \sum_i \lambda_i f_i(E_T) \right\}$$



uncorrelated errors



correlated errors

Scale dependence

- Use dependence on renormalization and factorization scales.
- If we had an NNLO calculation, the dependence on the scales would be cancelled to that order.
- Thus the dependence on the scales gives an estimate of the error induced by truncating the perturbative expansion at one loop order.

- We use the standard central choice

$$\mu_{\text{uv}} = \mu_{\text{co}} = E_T/2$$

- Define

$$x_1 = \log_2(2\mu_{\text{uv}}/E_T)$$
$$x_2 = \log_2(2\mu_{\text{co}}/E_T)$$

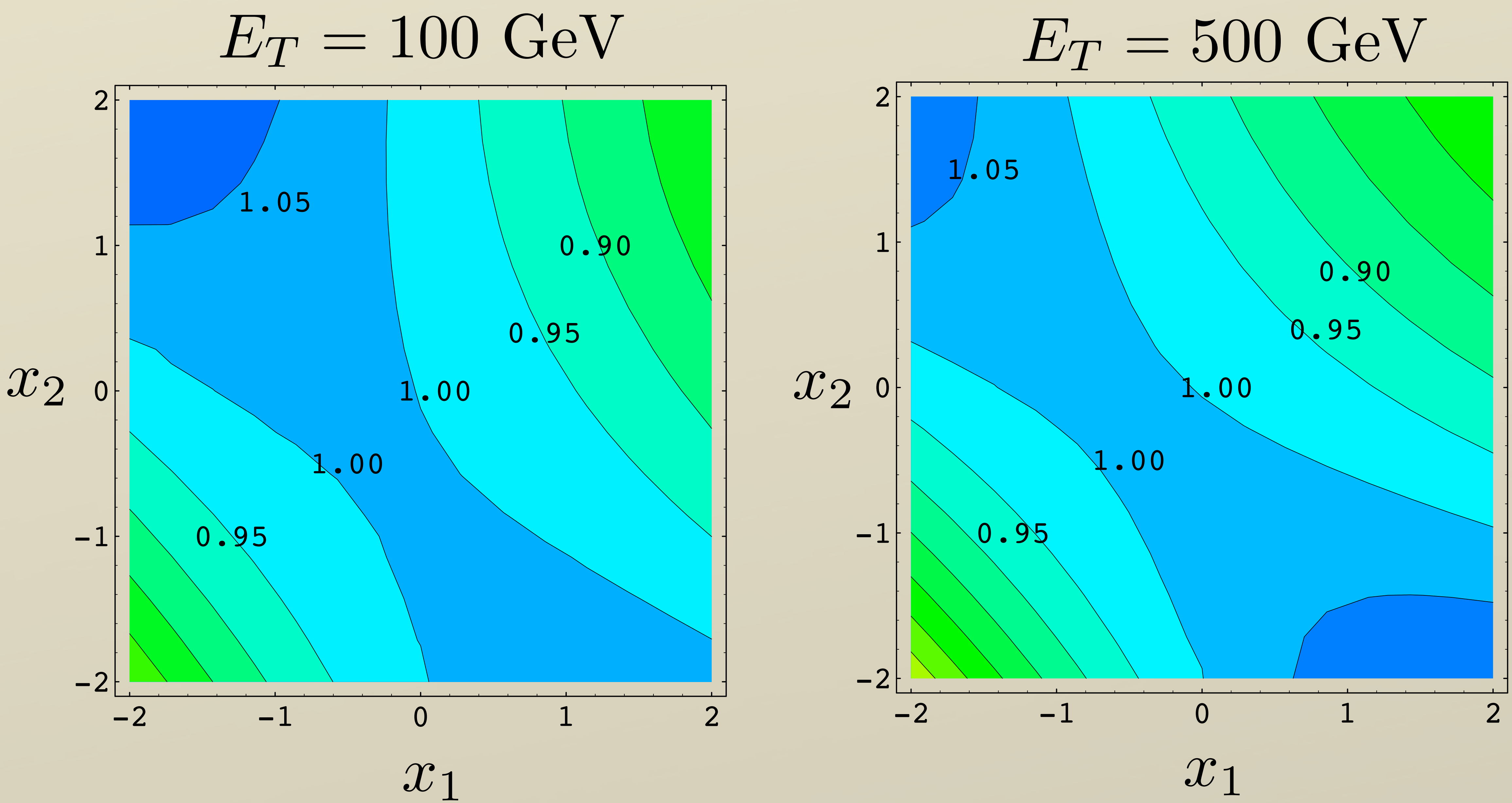
- Fit to

$$\left[\frac{d\sigma(x_1, x_2)}{dE_T}\right]_{\text{NLO}} \approx \left[\frac{d\sigma(0,0)}{dE_T}\right]_{\text{NLO}} P(\vec{x})$$

$$P(\vec{x}) = 1 + \sum_J x_J A_J + \sum_{J,K} x_J M_{JK} x_K$$

$$\left[\frac{d\sigma(x_1,x_2)}{dE_T}\right]_{\rm NLO} \approx \left[\frac{d\sigma(0,0)}{dE_T}\right]_{\rm NLO} P(\vec{x})$$

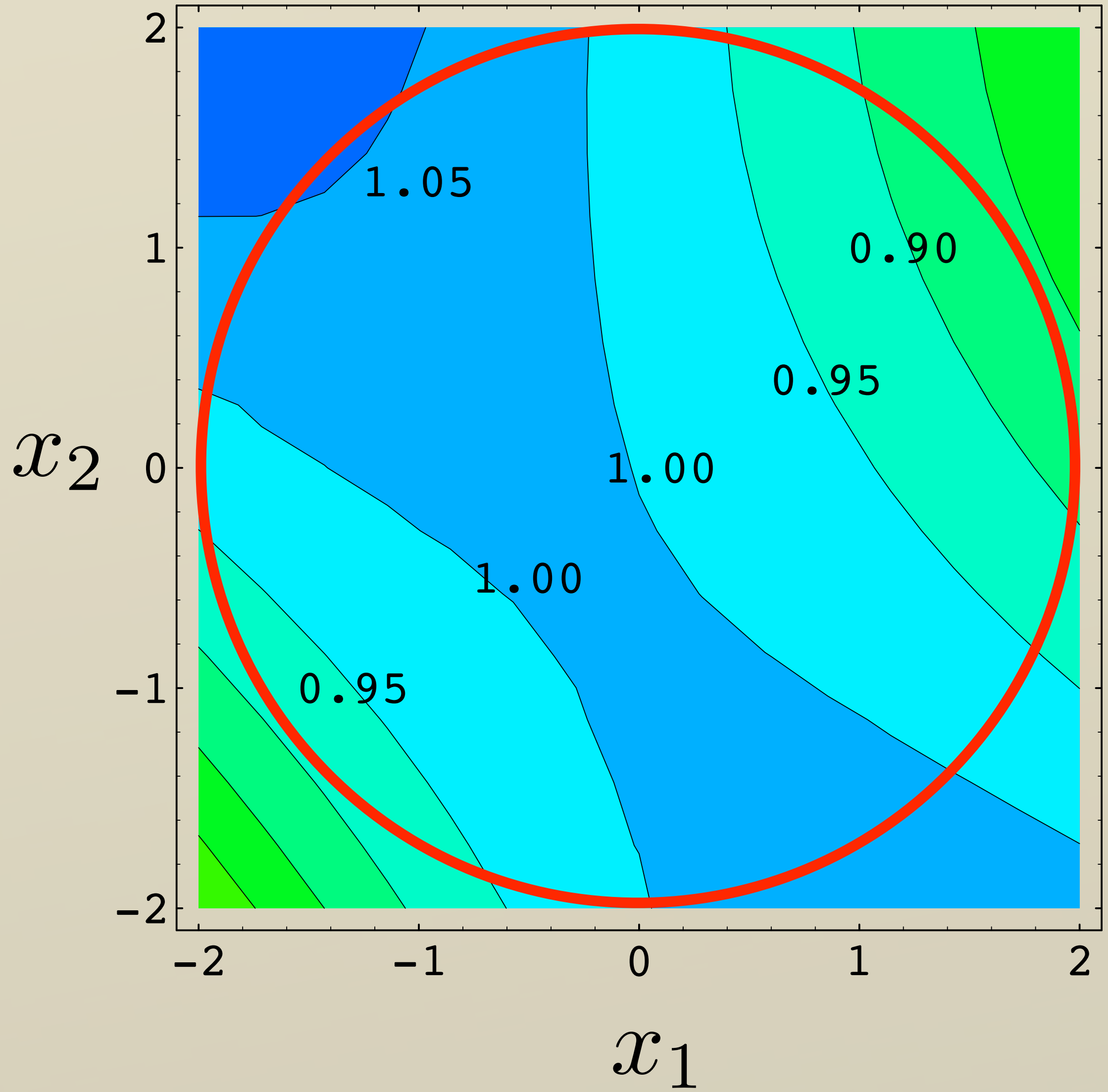
- Graphs of $P(\vec{x})$ (approximate: this is for $d\sigma/(dE_T\,dy)$ at $y=0$).



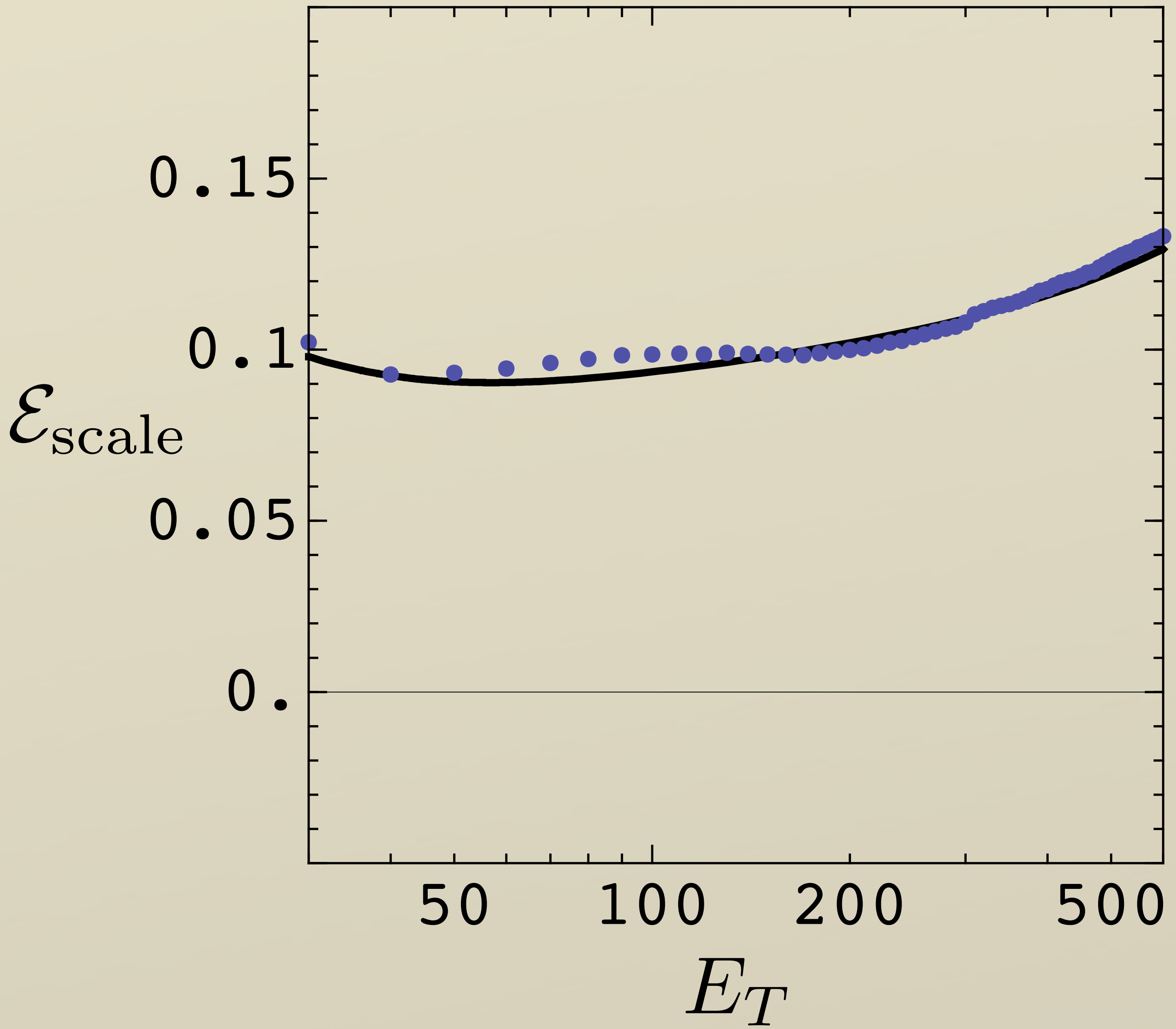
- Define estimated error

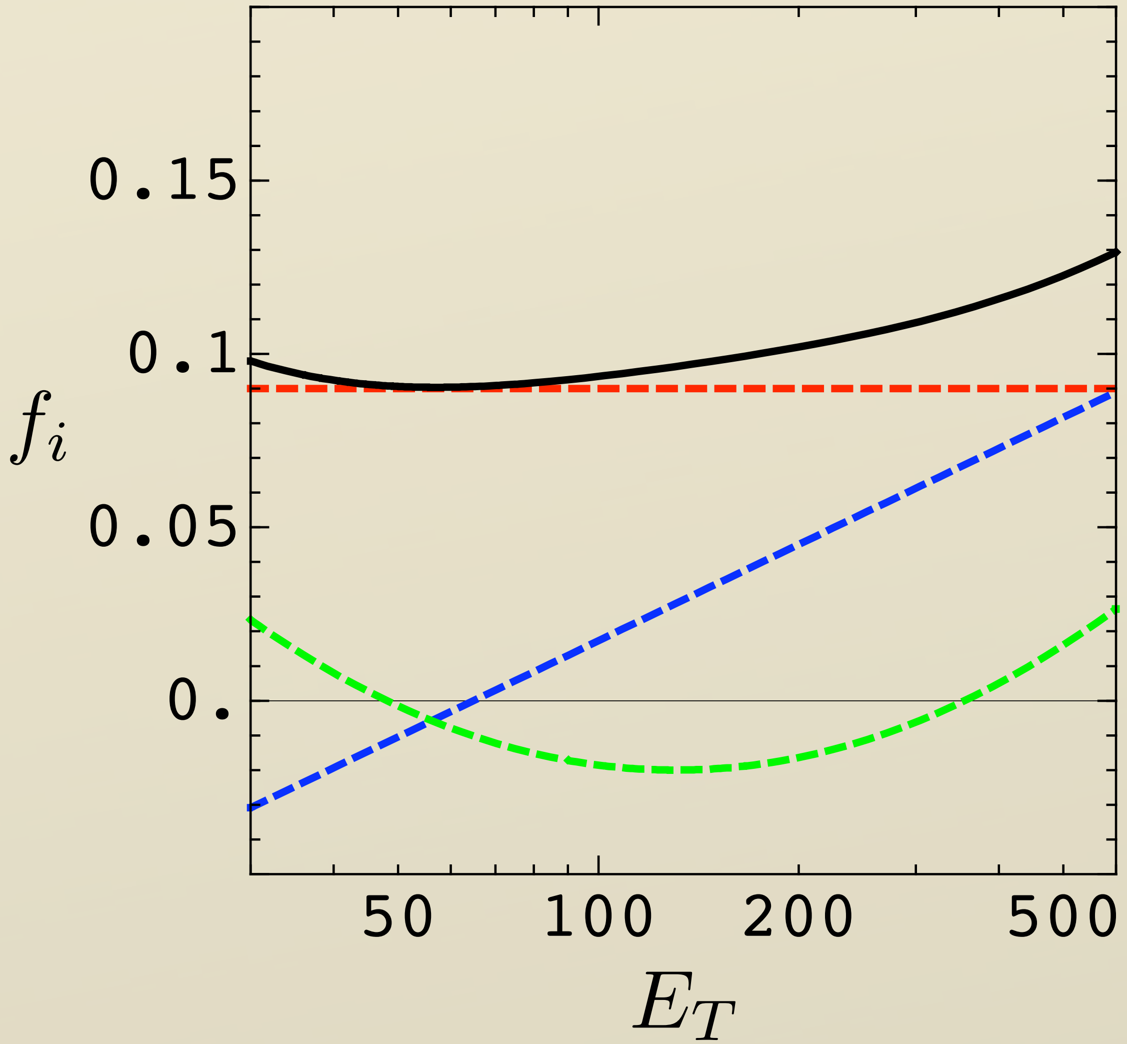
$$\mathcal{E}^2_{\rm scale} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\ P(|\vec{x}|\cos\theta, |\vec{x}|\sin\theta)^2$$

$E_T = 100\ \mathrm{GeV}$



- We find about a 10% error, slowly increasing with E_T.





- Divide this into parts.
- The unknown contributions can have a shape.
- Higher order polynomials have smaller coefficients.

$$f_1(E_T) = 0.09$$

$$f_2(E_T) = 0.04\left\{\log(15E_T/\sqrt{s}) + 0.7\right\}$$

$$f_3(E_T) = 0.02\left\{[\log(15E_T/\sqrt{s})]^2 - 1.0\right\}$$

- Net error,

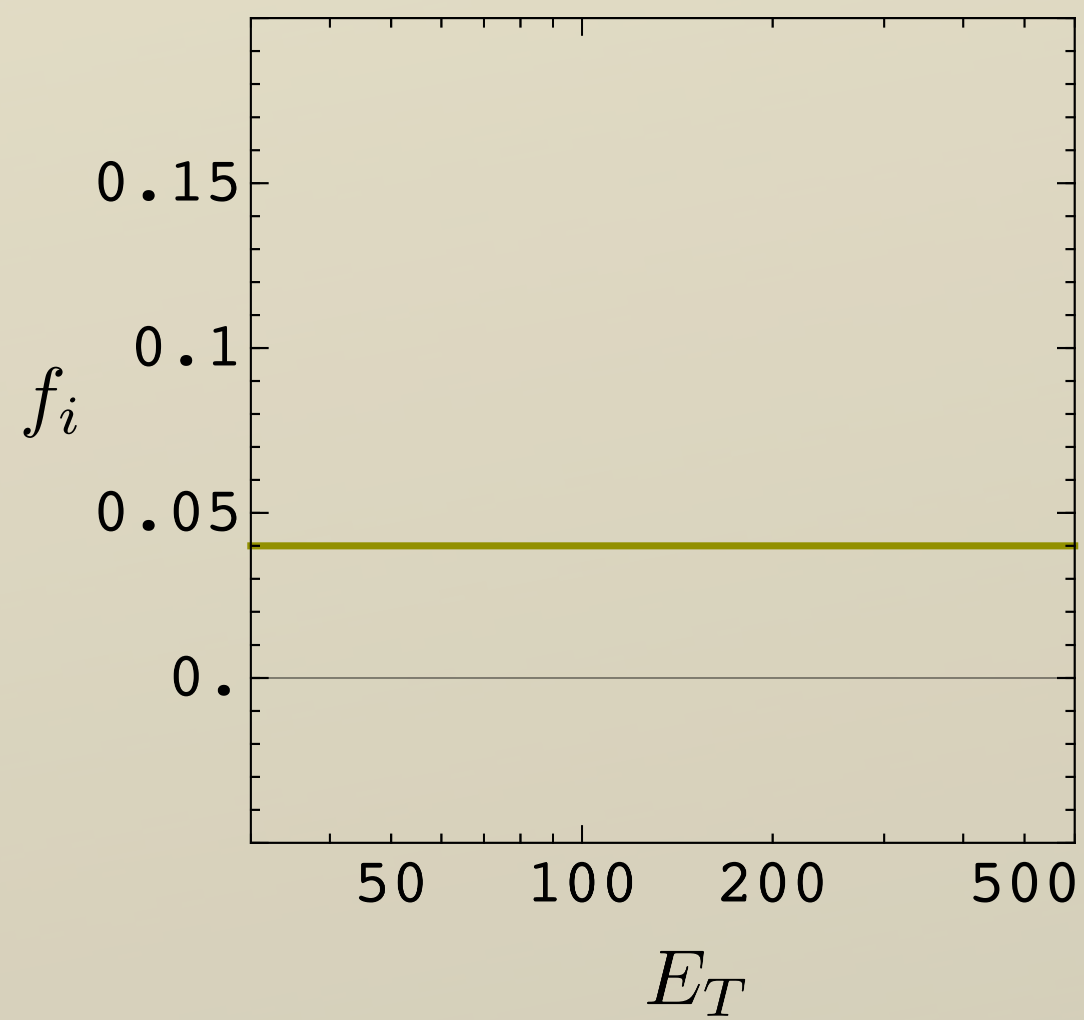
$$\mathcal{E}_{\rm scale} = \sqrt{f_1(E_T)^2 + f_2(E_T)^2 + f_3(E_T)^2}$$

Summation of threshold logs

- Kidonakis, Owens, and Sterman have shown how to sum “threshold logs” in the jet cross section.
- The threshold logs are important when the variation of the parton distributions with x is large.
- Since they represent terms beyond NLO, we can use the summed logs as an error estimate.

- The summed logs are available as part of "FastNLO" (Kluge, Rabbertz, Wobisch).
- For $\mu_{\mathrm{uv}} = \mu_{\mathrm{co}} = E_T/2$, the threshold logs contribution is about 4%, not strongly dependent on E_T.
- So we take

$$f_4(E_T) = 0.04$$



Power suppressed corrections

- Some E_T can be lost from the jet when the partons hadronize.
- Some E_T can be gained by the jet from the underlying event.
- Dasgupta, Magnea and Salam have estimated these effects.
- Our estimate based on this work is

$$\delta E_T = 0.5 \pm 0.7 \text{ GeV}$$

- To see the effect on the cross section, define

$$\frac{d\sigma(E_T)}{dE_T} = g(E_T)$$

$$\frac{1}{g(E_T)} \frac{dg(E_T)}{dE_T} = -\frac{n}{E_T} \qquad n \approx 7$$

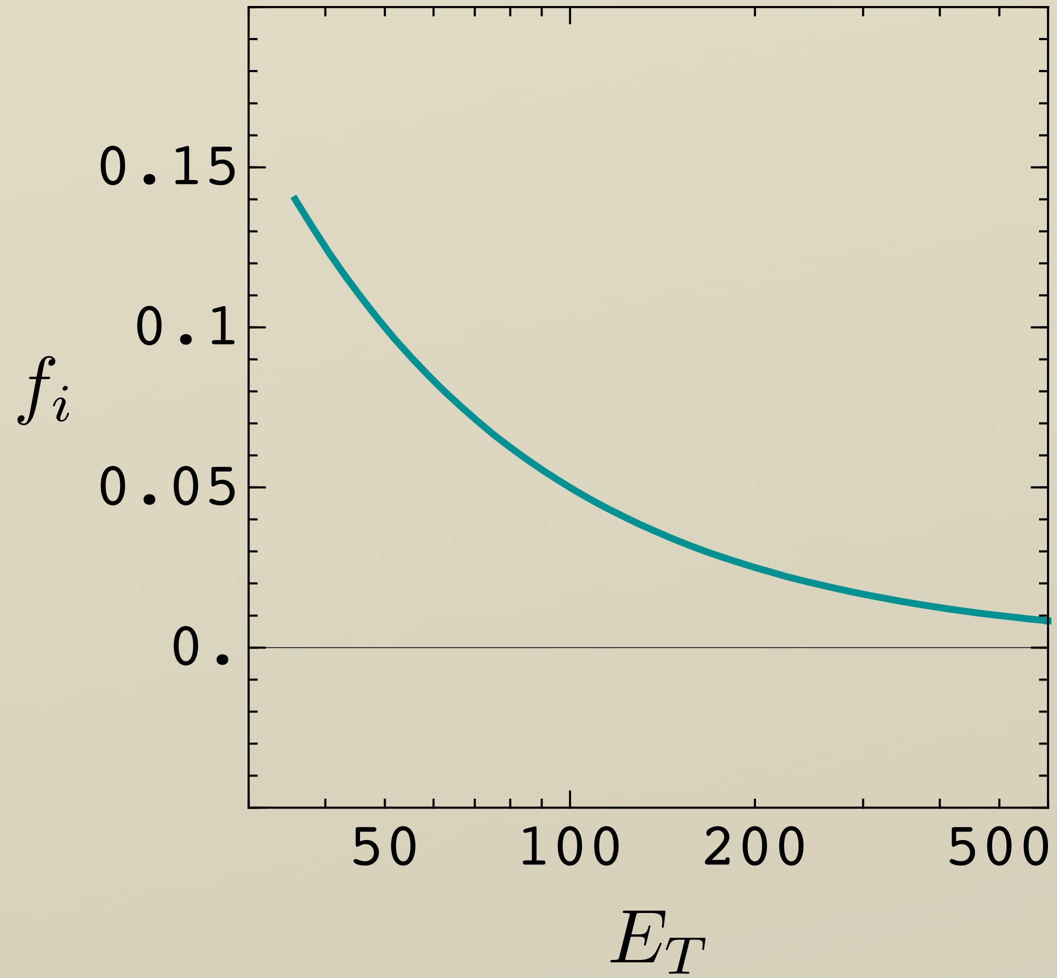
$$\begin{aligned} g(E_T) &= g_{\text{pert}}(E_T - \delta E_T) \\ &\approx g_{\text{pert}}(E_T) \left\{ 1 + n \frac{\delta E_T}{E_T} \right\} \end{aligned}$$

The estimated error

$$\frac{d\sigma}{dE_T} \approx \frac{d\sigma_{\rm pert}}{dE_T}\left\{1 + n\frac{\delta E_T}{E_T}\right\}$$

$$\delta E_T = 0.5 \pm 0.7\ \text{GeV} \qquad n \approx 7$$

$$f_5(E_T) = n\,\frac{0.7\ \text{GeV}}{E_T} = \frac{5\ \text{GeV}}{E_T}$$



Assembled errors

$$\frac{d\sigma}{dE_T} = \left[\frac{d\sigma}{dE_T}\right]_{\rm NLO} \left\{1 + \sum_i \lambda_i f_i(E_T)\right\}$$

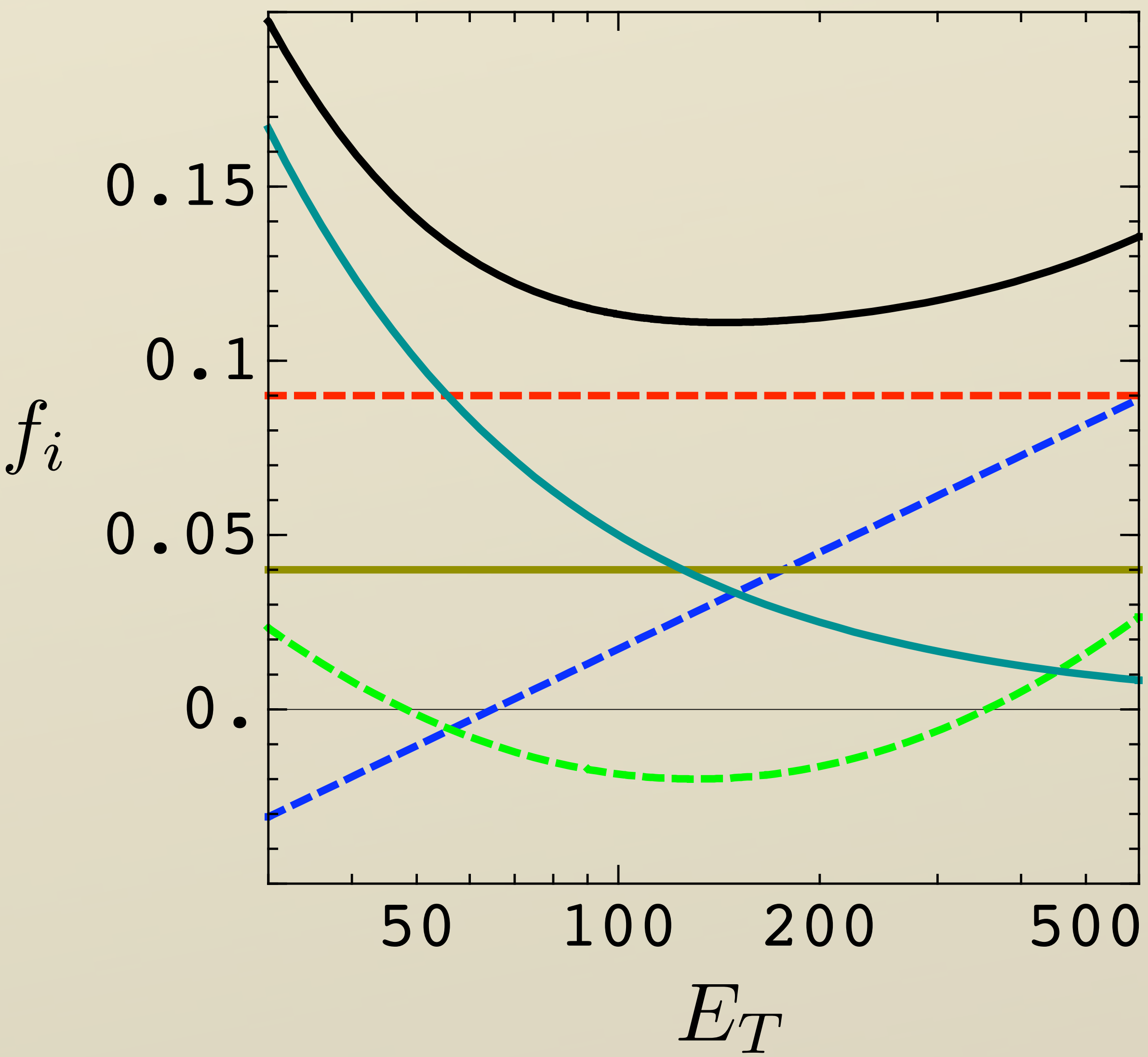
$$f_1(E_T) = 0.09$$

$$f_2(E_T) = 0.04\,\{\,\log(15E_T/\sqrt{s}) + 0.7\}$$

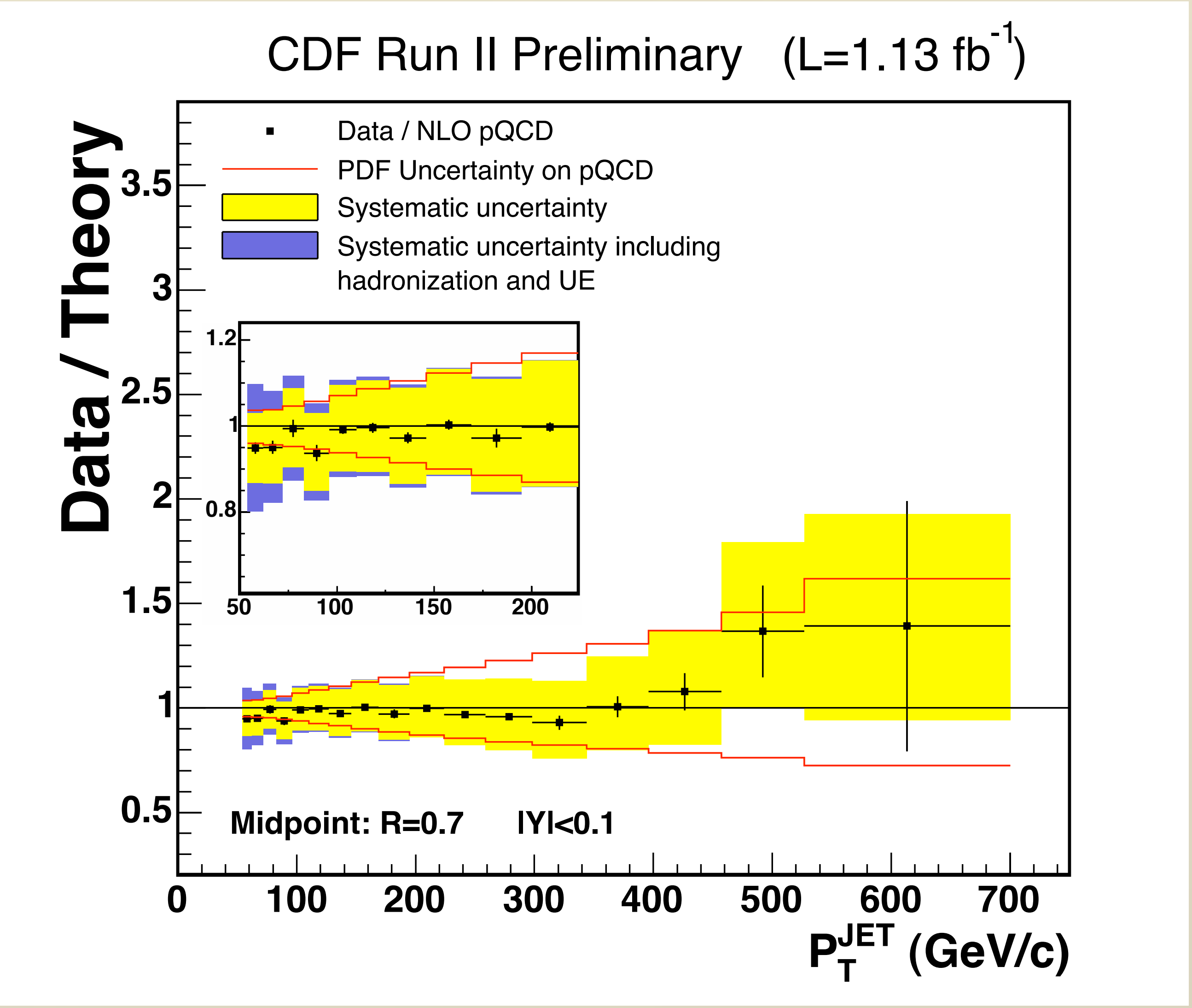
$$f_3(E_T) = 0.02\,\{[\log(15E_T/\sqrt{s})]^2 - 1.0\}$$

$$f_4(E_T) = 0.04$$

$$f_5(E_T) = \frac{5\ \mathrm{GeV}}{E_T}$$



Theory errors



Experimental errors (CDF)