Heavy quark energy loss in finite length SYM plasma

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based on F. Dominguez, C. Marquet, A. Mueller, B. Wu and B.-W. Xiao, arXiv:0803.3234

Motivations

- it is unclear if the perturbative QCD approach can describe the suppression of high-p_T particles in Au+Au collisions at RHIC, in particular for heavy-quark energy loss:

  high-p_T electrons from c and b decays indicate similar suppression for light and heavy quarks, while the dead-cone effect in pQCD implies a weaker suppression for heavier quarks

  $\Rightarrow$ this motivates to think about a strongly-coupled plasma

- for the *N*=4 SYM theory, the AdS/CFT correspondence allows to investigate the strong coupling regime

  limited tools to address the QCD dynamics at strong coupling

  $\Rightarrow$ the results for SYM may provide insight on strongly-coupled gauge theories, some aspects may be universal

in this work, we consider the trailing string picture of heavy-quark energy loss by Herzog et al., and address the question of finite-extend matter

Contents

- A few ingredients of AdS/CFT correspondence
  classical gravity in the AdS_5 black-hole metric
- The partonic picture of the strongly-coupled plasma
  DIS off the SYM plasma, the saturation scale
  Y. Hatta, E. Iancu and A. Mueller, arXiv:0710.5297
- The trailing string picture of heavy-quark energy-loss
  Herzog et al, JHEP 0607 (2006) 013
- Interpretation in partonic picture
  the energy loss is determined by the saturation scale
- The case of finite-extend plasma

The AdS/CFT correspondence

- The $\mathcal{N}$=4 SYM theory: 1 gauge field, 4 fermions, 6 scalars, all adjoint

in the large Nc limit, the ‘t Hooft coupling λ controls the theory

$$N_c \to \infty \text{ and } g_{YM} \to 0 \text{ with } \lambda \equiv g_{YM}^2 N_c \text{ finite}$$

strong coupling means ‘t Hooft limit in gauge theory: $\lambda \gg 1$

- The equivalent string theory in AdS_5 x S_5 : weak coupling and small curvature

$$g_{YM} \ll 1 \Leftrightarrow g_s \ll 1 \qquad \lambda \gg 1 \Leftrightarrow R \gg l_s$$

classical gravity is a good approximation

- The AdS_5 black-hole metric

$$ds^2 = R^2 \left[\frac{du^2}{h(u)} - h(u)dt^2 + u^2(dx^i)^2\right] = G_{\mu\nu} dX^\mu dX^\nu$$

curvature radius of AdS_5

$$h(u) = u^2\left(1 - \frac{u_h^4}{u^4}\right)$$

$u = r/R^2$ fifth dimension

horizon $u_h = \pi T$

T = Hawking temperature of the black hole = temperature of the SYM plasma

the SYM theory lives on the boundary at r = infinity

DIS off the SYM plasma

- The retarded current-current correlator

$$\Pi_{\mu\nu}(q) = i \int d^4y e^{-iq.y} \Theta(y^0) \langle [J_\mu(y), J_\nu(0)] \rangle_T$$

its imaginary part gives the plasma structure functions

R-current, equivalent of EM current for SYM theory

the current-plasma interaction is described by the propagation of a vector field which obeys Maxwell equations in AdS_5

- A partonic picture:

assume high energy high virtuality: $q^\mu = (\omega, 0_\perp, -Q^2/(2\omega)) \qquad \omega \gg Q \gg T$

coherence time of the current $t_c \sim \omega/Q^2$

probes plasma fluctuations with energy fraction $x = Q^2/(2\omega T) \simeq 1/(t_c T)$

for $x > T/Q$, the vector field is prevented to penetrate AdS space by a potential barrier

$\Rightarrow$ structure functions exponentially small, no large-*x* partons

decreasing *x* at fixed Q^2, the barrier disappears for $x = T/Q$

$\Rightarrow$ structure functions saturated, all the partons at small *x*

when a heavy quark goes through the SYM plasma, the relevant fluctuations in its wavefunction have a lifetime Q_s/T^2

A heavy quark in the plasma

- a heavy quark lives on a brane at $u = u_m = 2\pi M/\sqrt{\lambda} \to \infty$ with a string attached to it, hanging down to the horizon
- the string dynamics is given by the Nambu-Goto action:

$$S = -\frac{\sqrt{\lambda}}{2\pi R^2}\int d\tau d\sigma \sqrt{-det\ g_{ab}}$$ area of the string worldsheet

$$g_{ab} = G_{\mu\nu}(\partial_a X^\mu)(\partial_b X^\nu)$$ induced metric on the worldsheet

$$\pi^a_\mu = \frac{\delta \mathcal{L}}{\delta \partial_a X^\mu}$$ canonical momenta conjugate to X^μ

- parameterization: $X^\mu = (t, x, y, z, u) = (\tau, x(\tau,\sigma), 0, 0, \sigma)$

$$S = -\frac{\sqrt{\lambda}}{2\pi}\int dt du \sqrt{1 - \frac{u^2\dot{x}^2}{h(u)} + u^2 h(u) x'^2}$$

equation of motion: $$\frac{\partial}{\partial u}\left(\frac{u^2 h(u) x'}{\sqrt{-g}}\right) - \frac{u^2}{h(u)}\frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0$$

rate at which energy flows down the string: $$\frac{dE}{dt} = -\pi^1_t = -\frac{\sqrt{\lambda}R^2}{\sqrt{-g}} u^2 h(u)\dot{x}x'$$

The trailing string solution

- assume the quark is being pulled at a constant velocity *v*:

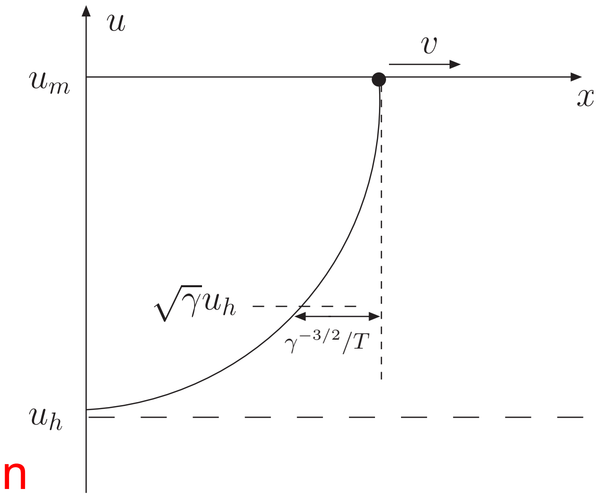
$$x(t,u) = x_0 + vt + F(u)$$

solution (known as the trailing string) : $F(u) = \frac{1}{2u_h}\left[\frac{\pi}{2} - tan^{-1}\left(\frac{u}{u_h}\right) - cot^{-1}\left(\frac{u}{u_h}\right)\right]$

corresponding rate of energy flow down the string:

$$-\frac{dE}{dt} = \frac{\sqrt{\lambda}}{2\pi}(\pi T)^2\frac{v^2}{\sqrt{1-v^2}}$$

Herzog et al (2006)
Gubser et al (2006)
Liu et al (2006)



- key observation:
  the part of string above $u = \sqrt{\gamma}\ u_h = Q_s$
  is genuinely part of heavy quark
  the part of string below $\sqrt{\gamma}\ u_h$ is emitted radiation

the picture is valid for $u_m > \sqrt{\gamma}\ u_h$ meaning $M > \sqrt{\lambda\gamma}\ T$

it naturally explains why $-\frac{dE}{dt} \propto \sqrt{\lambda}\ Q_s^2$

Energy loss in the partonic picture

- this picture is obtained from several results

  - the part of the string below Qs is not causally connected with the part of the string above: Qs corresponds to a horizon in the rest frame of the string

  - when computing the stress-tensor on the boundary: $T^{\mu\nu} = T^{\mu\nu}_{HQ} + T^{\mu\nu}_{plasma} + \Delta T^{\mu\nu}$
    the trailing string is a source of metric perturbations in the bulk which give $\Delta T^{\mu\nu}$

    Gubser et al (2006), Chesler and Yaffe (2007)

    one gets $\Delta T^{00} \ll T^{00}_{HQ}$ for $Q_s|\vec{x}| \ll 1$

  the energy density is unchanged around the heavy quark up to distances ~ 1/Qs

- simple derivation of the energy loss:

the radiated partons in the wavefunction have $k_T < Q_s$ and $\omega < \gamma Q_s$
giving the maximum (dominant) values $k_T = Q_s$ and $\omega = Q_s \gamma$
and therefore a coherence time $t_c = \gamma / Q_s$

then $-\dfrac{dE}{dt} \propto \sqrt{\lambda}\,\dfrac{\omega}{t_c} \propto \sqrt{\lambda}\; Q_s^2$

this does not give the overall coefficient
but it gets the right *v* and *T* dependences

$t_c = \gamma / Q_s$ and $Q_s = t_c\; T^2$ give $Q_s = \sqrt{\gamma}\; T$

The case of finite-extend matter

we would like to know the medium length L dependence of the energy loss

exact calculation difficult to set up, need another scale in the metric

using the partonic picture, we can get the L dependence

possible setup: a fully dressed quark going through the medium
(like in the trailing string picture)

one easily gets $-\frac{dE}{dt} \propto \sqrt{\lambda}\,\frac{\omega}{L} \propto \sqrt{\lambda}\,\frac{\gamma Q_s}{L}$ but this is not relevant

in reality, the heavy quark is bare when produced

and then builds its wavefunction while interacting with the medium

how to set this up in AdS ? our proposal:

describe the creation with a brief acceleration to the desired speed

then stopping the acceleration triggers the building of the wavefunction

key issue: the time it takes to build the fluctuations which dominate the energy loss

if the ones that dominate in the infinite matter case have time to build
before the heavy quark escapes the plasma, then the result is as before;
if not, the hardest fluctuations which could be build dominate

The accelerating string

the equation of motion at zero temperature: $\frac{\partial}{\partial u}\left(\frac{u^4 x'}{\sqrt{-g}}\right) - \frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0$ Xiao (2008)

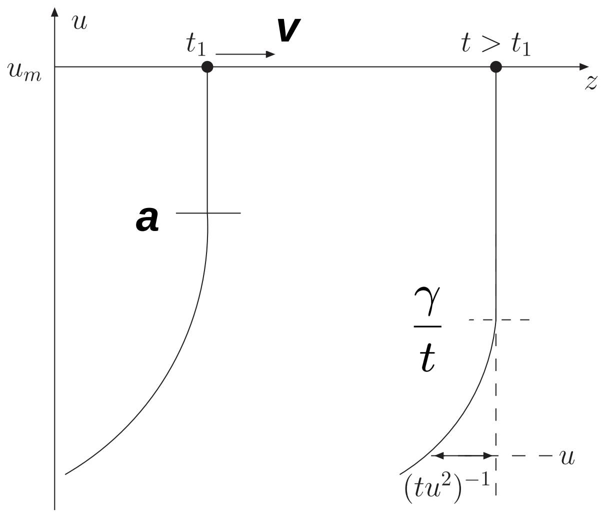
a can be interpreted as the acceleration of the quark $a = \text{const.} = \frac{\partial}{\partial t}(\gamma \dot{x})\Big|_{u\to\infty}$

solution $x^2 = t^2 - \frac{1}{u^2} + \frac{1}{a^2}$

the acceleration acts like an effective temperature (Unruh effect): the part of string below *u* =*a* is not causally connected with the part of the string above

at finite T, this separation is not affected, provided T << a

when stopping the acceleration, this separation goes down as γ/t :
the heavy quark is building its wavefunction



when $t = \gamma/Q_s$ ($= t_c$, the time it takes to build the fluctuations which dominate the energy loss in the infinite matter case), the separation crosses Qs, hence:

If $t_c < L$, the result is as before, *v* is small enough for the system to quickly adjust to a trailing string

If $t_c > L$, then softer fluctuations dominate:

for $u \ll \gamma/t$, only soft components $\omega < tu^2$ contribute to the heavy quark

$$\Rightarrow \quad -\frac{dE}{dt} \propto \sqrt{\lambda}\, Q_s^2 \text{ with } Q_s = LT^2$$

Summary

results for energy loss	QCD at weak coupling	SYM at strong coupling
heavy-quark energy loss	$-\frac{dE}{dt} \propto \alpha_s N_c\ Q_s^2$	$-\frac{dE}{dt} \propto \sqrt{\lambda}\ Q_s^2$
infinite matter or $t_c < L$	$Q_s^2 = \hat{q}\ t_c$ $t_c = \gamma^{2/3}/T$ $Q_s^2 = T^2\gamma^{2/3}$	$Q_s^2 = T^4\ t_c^2$ $t_c = \gamma^{1/2}/T$ $Q_s^2 = T^2\gamma$
finite matter with $L < t_c$	$Q_s^2 = \hat{q}\ L$	$Q_s^2 = T^4\ L^2$

coherence time $t_c \sim \gamma/Q_s$

results for p_T broadening $\quad \frac{dp_T^2}{dt} \propto \sqrt{\lambda}\frac{dQ_s^2}{dt_c}\ ,\sqrt{\lambda}\frac{dQ_s^2}{dL}$

one easily gets $dp_T^2/dt \propto \sqrt{\lambda\gamma}\ T^3$ Gubser (2007), Solana and Teaney (2007)

in pQCD, $\alpha_s N_c\ dQ_s^2/dt$ is radiative p_T broadening << collisionnal p_T broadening $\quad dQ_s^2/dt = \hat{q}$

in SYM, $\lambda \gg 1$ so radiative p_T broadening is dominant