CENTRAL CHARGE AND CORRELATORS IN STRINGS ON ADS3 WZW

- LIGHTNING REVIEW OF THE ADS3 WZW
- STRINGS ON ADS3 WZW: HOLOGRAPHY BEYOND THE SUPERGRAVITY LIMIT
- THE SPACETIME AFFINE LIE AND VIRASORO ALGEBRAS
- THE IDENTITY OPERATOR
- THE HOLOGRAPHIC DICTIONARY
- A PROBLEM WITH CLUSTER DECOMPOSITION AND WARD IDENTITIES
- THE SOLUTION: “THE CONUNDRUM IS A BUSILLIS”
- CLUSTER DECOMPOSITION RECOVERED
- A FEW FINAL OBSERVATIONS

J.H. KIM M. PORRATI arXiv:1503.07186

TOGETHER WITH SUPERSYMMETRY, BRUNO’S WORKS WITH CALLAN, COLEMAN AND WESS ON NONLINEAR REALIZATIONS OF SYMMETRIES IS PERHAPS THE MOST INFLUENTIAL AND LONG LASTING.

THIS TALK IS ONE MORE MODEST TRIBUTE TO THE IMPORTANCE OF THAT REMARKABLE BODY OF WORK.

LIGHTNING REVIEW OF THE ADS3 WZW

(SEE GIVEON, KUTASOV, SEIBERG)

ADS3 CLASSICAL METRIC

$$ds^2 = k(d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma})$$

CLASSICAL LAGRANGIAN

$$L = k(\partial\phi\bar{\partial}\phi + \bar{\partial}\gamma\partial\bar{\gamma})$$

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AN ANTISYMMETRIC B FIELD HAS BEEN ADDED (WZ TERM)

SYMMETRY OF THEORY IS SL(2,R)XSL(2,R)

CLASSICAL PRIMARY FIELDS BELONG TO REPRESENTATION j=h+1 of SL(2,R) [LEFT-RIGHT SYMMETRIC]

THEY ARE TARGET-SPACE FIELDS OF DIMENSION (h,h)

$$\Phi_h = (|\gamma - x|^2 e^{\phi} + e^{-\phi})^{-2h} \approx \frac{1}{2h-1} e^{2(h-1)\phi} \delta^2(\gamma - x) + \mathcal{O}\left(e^{2(\phi-2)}\right)$$

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HOLOGRAPHICALLY: SOURCES FOR OPERATORS AT BOUNDARY POINT ***x***

IN THE FULL QUANTUM THEORY Φ_h BECOME WORLDSHEET FIELDS LABELED BY h***,x***

THE STRING THEORY VERTICES ARE

$$V(x,\bar{x},h,I) = \int d^2z \Phi_h(x,\bar{x}|z,\bar{z}) O_I \qquad \Delta_I + \Delta_h = 1$$

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WORLDSHEET CURRENT ALGEBRA COMPACTLY WRITTEN AS

$$J(x|z) = 2xJ_3(z) - J^+(z) - x^2 J^-(z)$$

SPACETIME VIRASORO AND AFFINE-LIE ALGEBRAS ARE GENERATED BY STRING VERTICES THAT GIVE RISE TO THE CORRECT WARD IDENTITIES

AFFINE-LIE (SIMPLER TO WRITE)

$$K^a(x) = -\frac{1}{k}\int d^2z k^a(z)\bar{J}(\bar{x}|\bar{z})\Phi_1(x,\bar{x}|z,\bar{z}) = -\frac{1}{\pi}\int d^2z\bar{\partial}[k^a(z)\Lambda(x,\bar{x}|z,\bar{z})]$$

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A BAD OPERATOR WITH log(z) CORRELATOR

WARD IDENTITY IN VEVS OF VERTICES

$$\langle ...K^a(x)K^b(y).....\rangle = \langle .....\frac{1}{(x-y)^2}k_G I + \frac{1}{(x-y)}f_c^{ab}K^c(y)....\rangle$$

IDENTITY OPERATOR:

$$I = \frac{1}{k^2}\int d^2 z J\bar{J}\Phi_1 = -\frac{1}{2\pi i k}\oint dz J\Lambda$$

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INDEPENDENT OF $x, \bar{x}$ BUT NOT SAME CONSTANT ON DIFFERENT VEVS OF PHYSICAL VERTICES

$$\langle I \prod_i \Phi_{h_i}\rangle = \frac{1}{k}[1 - g + \sum_i (h_i - 1)]\langle \prod_i \Phi_{h_i}\rangle$$

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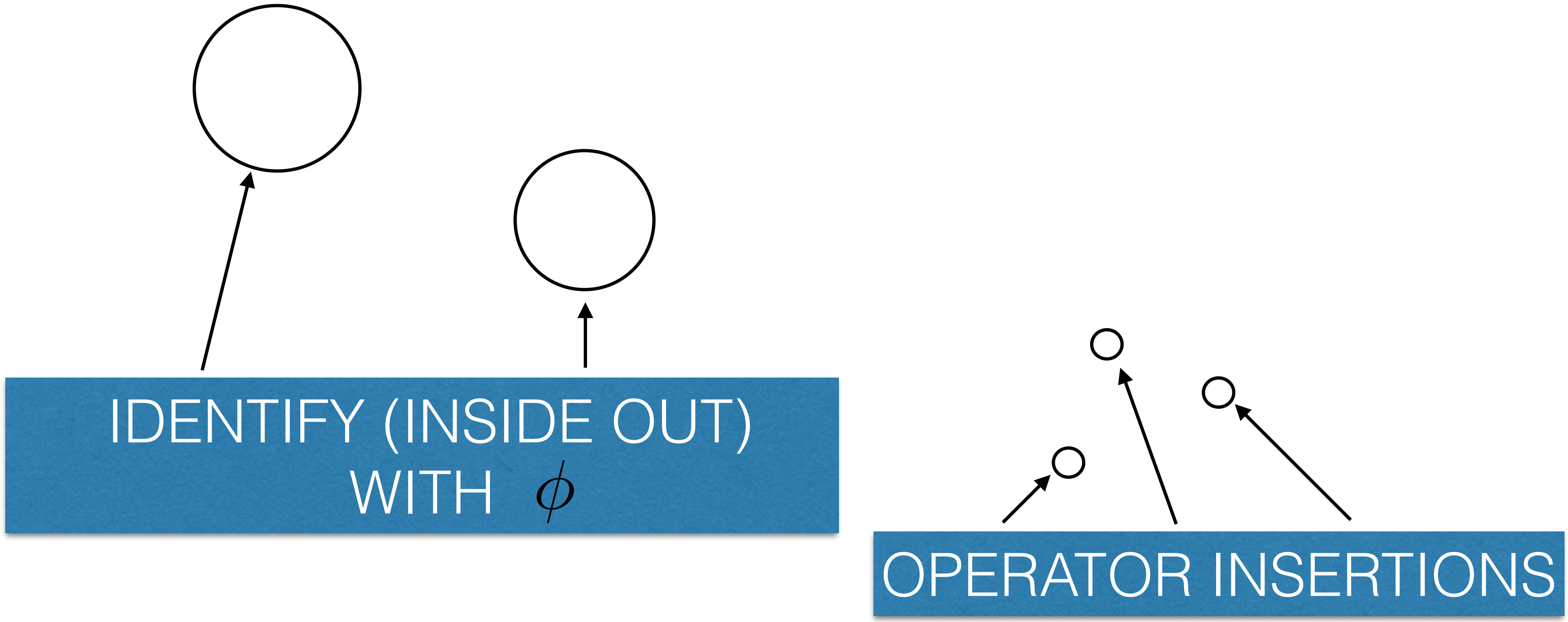
g=0 GIVEON-KUTASOV, ALL g, KIM-PORRATI

$$I = \frac{1}{k^2}\int_\Sigma d^2z J\bar{J}\Phi_1 = -\frac{1}{2\pi i k}\oint_{\cup_i C_i} dz J\Lambda$$



ANOMALOUS TRANSFORMATION UNDER $z \to z' = \phi(z)$

$$J\Lambda(x|z) \to (J\Lambda)'(x|z') = \partial_z\phi(z)\left[J\Lambda(x|z) + \frac{1}{2}\frac{\partial_\phi^2 z}{\partial_\phi z}\right]$$

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SCHOTTKY PARAMETRIZATION OF SURFACE: $\phi(z) = \dfrac{az+b}{cz+d}$

$$-\frac{1}{2\pi i k} \oint_{C \cup \phi(C)} dz J\Lambda = -\frac{1}{2\pi i k} \oint_C dz \partial_z \phi(z) \frac{1}{2} \frac{\partial_\phi^2 z}{\partial_\phi z} = \frac{1}{2\pi i} \oint_{\phi(C)} dz \frac{c}{cz+a}$$

COUNTERCLOCKWISE INTEGRAL: -1

CONTRIBUTION OF OPERATOR INSERTION FROM OPE:

$$-\frac{1}{2\pi i} \oint_C dw J\Lambda(x|w) \Phi_h(x, \bar{x}|z, \bar{z}) = (h-1) \Phi(x, \bar{x}|z, \bar{z})$$

THE HOLOGRAPHIC DICTIONARY (de Boer, H. Ooguri, H. Robins, J. Tannenhauser)

$$W = \sum_{g=0}^{\infty} g_S^{2g-2} \langle \exp \left[\int d^2x J(x,\bar{x},h,I) V(x,\bar{x},h,I) + \int d^2x \lambda(x,\bar{x}) I \right] \rangle_g$$

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SO THE IDENTITY OPERATOR HAS NONZERO CONNECTED CORRELATORS E.G.

$$\langle I\Phi_h\Phi_h\rangle_{g=0} = \frac{1}{k}(2h-1)\langle\Phi_h\Phi_h\rangle_{g=0}$$

ADD SOURCES FOR VIRASORO AND AFFINE-LIE ALGEBRAS

SOURCES

$$W = \sum_{g=0}^{\infty} g_S^{2g-2} \langle \exp \left[\int d^2x \, (J(x,\bar{x},h,I)V(x,\bar{x},h,I) + K_{ax}(x) A^a_{\bar{x}}(x,\bar{x}) + T_{xx}(x) g_{\bar{x}\bar{x}}(x,\bar{x}) + \lambda(x,\bar{x}) I) \right] \rangle_g$$

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VERTICES

WARD IDENTITIES RECOVERED BY TRANSFORMING SOURCES AS

$$\delta_\epsilon J = TJ, \qquad \delta_\epsilon A_{\bar{x}} = D^a_{\bar{x}} \epsilon_a, \qquad \delta_\epsilon \lambda = -\pi k_G \epsilon_a \partial_x A^q_{\bar{x}}$$

ADD SOURCES FOR VIRASORO AND AFFINE-LIE ALGEBRAS

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λ IS A GREEN-SCHWARZ FIELD

WARD IDENTITY

$$\delta_\epsilon J * \frac{\delta W}{\delta J} + \delta_\epsilon A * \frac{\delta W}{\delta A} = - \int d^2 x \delta_\epsilon \lambda \frac{dW}{d\lambda_0}$$

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PROBLEM: THE CORRECT WARD IDENTITY IS

$$\delta_\epsilon J * \frac{\delta W}{\delta J} + \delta_\epsilon A * \frac{\delta W}{\delta A} = \text{constant } \times \int d^2x \delta_\epsilon \lambda$$

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PROBLEM: CLUSTER FACTORIZATION IS LOST

$$\langle \Phi_{h_1}(x_1)\Phi_{h_2}(x_2)\Phi_{h_3}(x_3)\Phi_{h_4}(x_4)\rangle \rightarrow \langle \Phi_{h_1}(x_1)\Phi_{h_2}(x_2)I\rangle \frac{1}{\langle II\rangle} \langle I\Phi_{h_3}(x_3)\Phi_{h_4}(x_4)\rangle + ...$$

I INDEPENDENT OF x

“THE CONUNDRUM IS A BUSILLIS”

AN ENTRY IN THE HOLOGRAPHIC DICTIONARY MUST BE CORRECTED

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$$\langle I \rangle \sim \text{number of F-strings creating the } AdS_3 \text{ background} \sim c$$

IT MUST BE KEPT FIXED, WHILE THE “FREE ENERGY” W KEEPS THE CHEMICAL POTENTIAL λ FIXED

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SOLUTION: DEFINE GENERATOR OF CONNECTED CORRELATORS BY LEGENDRE-TRANSFORMING IN λ

$$\Gamma[\langle I \rangle, J] = W[\lambda_0, J] - \lambda_0 \langle I \rangle, \text{ computed at } \frac{dW}{d\lambda_0} = \langle I \rangle$$

PROPERTIES OF THE “EFFECTIVE ACTION”

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CORRECT VALUE OF THE CENTRAL CHARGE

$$\Gamma_{1234} = W_{1234} - W_{12\lambda} \frac{1}{W_{\lambda\lambda}} W_{\lambda 34} + ...$$

CLUSTER DECOMPOSITION VIOLATING TERMS (INTERNAL λ LINES) CANCEL

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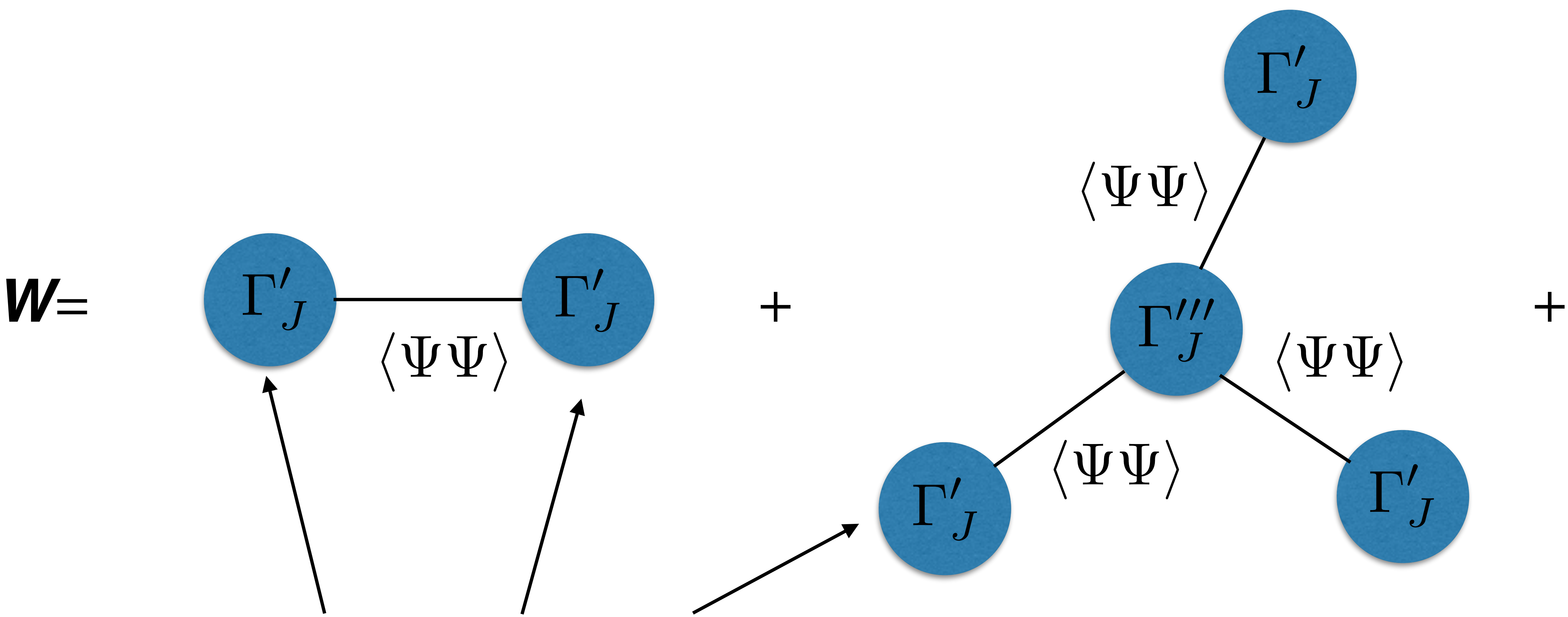
THIS PROPERTY HOLDS IN GENERAL BECAUSE THE EFFECTIVE ACTION IS 1-PI IN λ

RELATION BETWEEN EFFECTIVE ACTION Γ AND FREE ENERGY ***W***



TO RECOVER THE CORRECT EXPANSION, TADPOLES MUST VANISH AT ***J***=0

A FEW FINAL COMMENTS

THE CORRECT HOLOGRAPHIC PRESCRIPTION IS

$$Z = Ce^{\Gamma[\langle I \rangle, J]}$$

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IN THE LEGENDRE-TRANSFORMED THEORY, THE VEV IS FIXED BY THE CONDITION OF NO TADPOLE AT ZERO SOURCE ***J***:

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OUR PRESCRIPTION CLOSELY RESEMBLES DEFINING LIOUVILLE THEORY AT FIXED AREA (IN WAKIMOTO REPRESENTATION ***I*** IS SIMILAR TO THE AREA OPERATOR)

WHEN COMPUTING ENTROPIES OF BPS BLACK HOLE MICROSTATES BEYOND THE LEADING APPROXIMATION, SIMILAR ISSUES ARISE. IN THAT CASE THEY CENTER ON THE CORRECT DEFINITION OF THE SETS OF CHARGES AND CHEMICAL POTENTIALS TO KEEP FIXED

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WE WORKED IN THE ***k > 1*** CASE. FOR ***k<1*** THE IDENTITY IS NOT A PHYSICAL OPERATOR AND DOES NOT HAVE 3-POINT FUNCTIONS WITH PHYSICAL VERTICES. SO, IT IS NOT CLEAR IF ONE MUST LEGENDRE-TRANSFORM EVEN IN THE LATTER CASE.