U(1) SUPERSPACE GEOMETRY AND COUPLING OF MATTER TO SUPERGRAVITY

Bruno Zumino Memorial Meeting

CERN

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U(1) SUPERSPACE GEOMETRY

$$E^A = dz^M E_M{}^A \quad , \quad \phi_B{}^A = dz^M \phi_{MB}{}^A \quad , \quad A = dz^M A_M$$

$$T^A = \frac{1}{2} E^C E^D T_{DC}{}^A \quad , \quad R_B{}^A = \frac{1}{2} E^C E^D R_{DCB}{}^A \quad , \quad F = \frac{1}{2} E^C E^D F_{DC}$$

$$T^A = DE^A = dE^A + E^B \phi_B{}^A + w(E^A)\, E^A A$$

$$R_B{}^A = d\phi_B{}^A + \phi_B{}^C \phi_C{}^A$$

$$F = dA$$

Bianchi identities

$$DT^A = E^B R_B{}^A + w(E^A)\, E^A F$$

$$DR_B{}^A = 0$$

$$dF = 0$$

$w(E^A)$ chiral U(1) weights

$$w(E^a) = 0 \quad , \quad w(E^\alpha) = +1 \quad , \quad w(E_{\dot\alpha}) = -1$$

for instance: $\quad w(T_{CB}{}^A) = -w(E^C) - w(E^B) + w(E^A)$

INFINITESIMAL TRANSFORMATIONS

$$\delta E^A = L_\xi E^A + E^B L_B{}^A + w(E^A) E^A \alpha$$
$$\delta \phi_B{}^A = L_\xi \phi_B{}^A - dL_B{}^A - L_B{}^C \phi_C{}^A + \phi_B{}^C L_C{}^A$$
$$\delta A = L_\xi A - d\alpha$$

$$\delta E^A = D\xi^A + \iota_\xi T^A + E^B (L_B{}^A - \iota_\xi \phi_B{}^A) + w(E^A) E^A (\alpha - \iota_\xi A)$$
$$\delta \phi_B{}^A = \iota_\xi R_B{}^A - d(L_B{}^A - \iota_\xi \phi_B{}^A) - (L_B{}^C - \iota_\xi \phi_B{}^C)\phi_C{}^A + \phi_B{}^C (L_C{}^A - \iota_\xi \phi_C{}^A)$$
$$\delta A = \iota_\xi F + d(\alpha - \iota_\xi A)$$

$L_B{}^A = \iota_\xi \phi_B{}^A$, $\alpha = i_\xi A$ result in

$$\delta_{WZ} E^A = D\xi^A + \iota_\xi T^A$$
$$\delta_{WZ} \phi_B{}^A = \iota_\xi R_B{}^A$$
$$\delta_{WZ} A = \iota_\xi F$$

the Wess - Zumino transformations

CONSTRAINTS

CHOICE OF STRUCTURE GROUP : Lorentz - transformations

$$\phi_b{}^a \ , \quad \phi_\beta{}^\alpha = -\tfrac{1}{2}(\sigma^{ba})_\beta{}^\alpha \phi_{ba} \ , \quad \phi^{\dot\beta}{}_{\dot\alpha} = -\tfrac{1}{2}(\bar\sigma^{ba})^{\dot\beta}{}_{\dot\alpha}\phi_{ba}$$

TORSION CONSTRAINTS

dim 0 : $T_{\gamma\beta}{}^a = 0 \ , \quad T^{\dot\gamma\dot\beta a} = 0 \ , \quad T_\gamma{}^{\dot\delta a} = -2i(\sigma^a\epsilon)_\gamma{}^{\dot\beta}$

dim $\frac{1}{2}$:

$$\left.\begin{array}{l} T_{\gamma b}{}^a \ , \ T_{\gamma\beta}{}^\alpha \ , \ T_\gamma{}^{\dot\beta}{}_{\dot\alpha} \\ T^{\dot\gamma}{}_b{}^a \ , \ T^{\dot\gamma}{}_\beta{}^\alpha \ , \ T^{\dot\gamma\dot\beta}{}_{\dot\alpha} \\ T_{\gamma\beta\dot\alpha} \ , \ T^{\dot\gamma\dot\beta\alpha} \end{array}\right\} \text{all vanish}$$

BIANCHI IDENTITIES

$$E^B E^C E^D \left(\mathcal{D}_D T_{CB}{}^A + T_{DC}{}^F T_{FB}{}^A - R_{DCB}{}^A \right) = 0$$

CONSEQUENCES OF CONSTRAINTS

dim 1 : $T_{\gamma b \dot\alpha} = -i(\sigma_b)_{\gamma\dot\alpha} R^{+}$, $T^{\dot\gamma}{}_{b}{}^{\alpha} = -i(\bar\sigma_b)^{\dot\gamma\alpha} R$

$T_{\gamma b}{}^{\alpha} = \frac{i}{2}(\sigma_c\bar\sigma_b)_{\gamma}{}^{\alpha} G^{c}$, $T^{\dot\gamma}{}_{b\dot\alpha} = -\frac{i}{2}(\bar\sigma_c\sigma_b)^{\dot\gamma}{}_{\dot\alpha} G^{c}$

$T_{cb}{}^{a} = 0$

curvatures $R_{\underline{\delta}\underline{\gamma} b}{}^{a}$, $F_{\underline{\delta}\underline{\gamma}}$ are given in terms of R, R^{+} and G_a

dim $\frac{3}{2}$: tersions $T_{cb}{}^{\alpha}$, $T_{cb\dot\alpha}$

curvatures $R_{\underline{\delta} cb}{}^{a}$, $F_{\underline{\delta} c}$

zoom on R, R^{+}

$\mathcal{D}^{\dot\alpha} R = 0$, $\mathcal{D}_{\alpha} R = -\frac{1}{3} X_{\alpha} - \frac{2}{3} T_{cb}{}^{\varphi} (\sigma^{cb}\epsilon)_{\varphi\alpha}$

$\mathcal{D}_{\alpha} R^{+} = 0$, $\mathcal{D}^{\dot\alpha} R^{+} = -\frac{1}{3} \bar X^{\dot\alpha} - \frac{2}{3} T_{cb\dot\varphi} (\bar\sigma^{cb}\epsilon)^{\dot\varphi\dot\alpha}$

$\mathcal{D}^{2} R + \bar{\mathcal{D}}^{2} R^{+} = -\frac{2}{3} R_{ba}{}^{ba} + 4 G^{a} G_{a} + 32 R^{+} R - \frac{2}{3} \mathcal{D}^{\alpha} X_{\alpha}$

GENERIC CONSTRUCTION OF INVARIANT ACTION

r : superfield of weight $w(r) = +2$, $\mathcal{D}^{\dot\alpha} r = 0$

$\bar r$: superfield of weight $w(\bar r) = -2$, $\mathcal{D}_\alpha \bar r = 0$

$$r = r| \,,\quad r_\alpha = \frac{1}{\sqrt2}\mathcal{D}_\alpha r| \,,\quad f = -\frac14 \mathcal{D}^\alpha \mathcal{D}_\alpha r|$$

$$\bar r = \bar r| \,,\quad \bar r^{\dot\alpha} = \frac{1}{\sqrt2}\mathcal{D}^{\dot\alpha}\bar r| \,,\quad \bar f = -\frac14 \mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha}\bar r|$$

component field density:

$$\begin{aligned}\mathcal{L}(r,\bar r) = \; & e f + \frac{ie}{\sqrt2}(\bar\psi_m \bar\sigma^m r) - e(\bar M + \bar\psi_m \bar\sigma^{mn}\bar\psi_n) r \\ & + e\bar f + \frac{ie}{\sqrt2}(\psi_m \sigma^m \bar r) - e(M + \psi_m \sigma^{mn}\psi_n)\bar r\end{aligned}$$

the identification $r = -3R$, $\bar r = -3R^+$

gives rise to

$$\begin{aligned}\mathcal{L}(R,R^+) = & -\frac{e}{2}\mathcal{R} + \frac{e}{2}\varepsilon^{mnpq}(\bar\psi_m\bar\sigma_n\mathcal{D}_p\psi_q - \psi_m\sigma_n\mathcal{D}_p\bar\psi_q) \\ & -\frac{e}{3}M\bar M + \frac{e}{3}b^a b_a + eD\end{aligned}$$

with $\quad D = -\frac12 \mathcal{D}^\alpha X_\alpha| + \frac{i}{2}(\psi_m\sigma^m)_{\dot\alpha}\bar X^{\dot\alpha}| + \frac{i}{2}(\bar\psi_m\bar\sigma^m)^\alpha X_\alpha|$

THE D - TERM

$$D(x) = -\frac{1}{2}\mathcal{D}^\alpha X_\alpha| + \frac{i}{2}(\psi_m \sigma^m)_{\dot\alpha}\bar{X}^{\dot\alpha}| + \frac{i}{2}(\bar\psi_m \bar\sigma^m)^\alpha X_\alpha|$$

The superfields $X_\alpha, \bar{X}^{\dot\alpha}$ appear in the $U(1)$ field strength coefficients (see Bianchi-identities!)

$$F_{\beta a} = +\frac{i}{2}\sigma_{a\beta\dot\beta}\bar{X}^{\dot\beta} \quad , \quad F^{\dot\beta}{}_a = -\frac{i}{2}\bar\sigma_a{}^{\dot\beta\beta}X_\beta \quad ,$$

they satisfy the equations

$$\mathcal{D}_\alpha \bar{X}^{\dot\alpha} = 0 \quad , \quad \bar{\mathcal{D}}^{\dot\alpha} X_\alpha = 0$$

and

$$\mathcal{D}^\alpha X_\alpha = \bar{\mathcal{D}}_{\dot\alpha}\bar{X}^{\dot\alpha} .$$

The F_{BA} constraints

$$F_{\beta\alpha} = \mathcal{D}_\beta A_\alpha + \mathcal{D}_\alpha A_\beta = 0$$

$$F^{\dot\beta\dot\alpha} = \bar{\mathcal{D}}^{\dot\beta} A^{\dot\alpha} + \bar{\mathcal{D}}^{\dot\alpha} A^{\dot\beta} = 0$$

are solved by

$$A_\alpha = -T^{-1}\mathcal{D}_\alpha T$$

$$A^{\dot\alpha} = -U^{-1}\bar{\mathcal{D}}^{\dot\alpha} U$$

where $U^{-1} = \bar{T}$

The prepotentials T and U, defined in

$$A_\alpha = -T^{-1}D_\alpha T \quad , \quad A^{\dot\alpha} = -U^{-1}D^{\dot\alpha}U$$

should reproduce the gauge transformations

$$A_\alpha \to A_\alpha - g^{-1}D_\alpha g \quad , \quad A^{\dot\alpha} \to A^{\dot\alpha} - g^{-1}D^{\dot\alpha} g$$

of the potentials,

$$T \longrightarrow \bar{P}Tg \quad , \quad D_\alpha \bar{P} = 0$$

$$U \longrightarrow QUg \quad , \quad D^{\dot\alpha}Q = 0$$

where $\bar{P}$ and Q, the pregauge transformations, are constrained by chirality conditions

for instance, the superfield $W = TU^{-1}$ transforms as

$$W \longrightarrow \bar{P}Tgg^{-1}U^{-1}Q^{-1} = \bar{P}WQ \, ,$$

it is inert under g - transformations

consider now Ψ, a superfield of weight $w(\Psi)$ which transforms as

$$\Psi \to g^{w(\Psi)} \Psi ,$$

the quantity $T^a U^b$, a, b constants transforms as

$$T^a U^b \longrightarrow \bar{P}^a Q^b (T^a U^b) g^{a+b}$$

definition: $\quad \Psi(a,b) = (T^a U^b)^{-w(\Psi)} \Psi$

$$\Psi(a,b) \longrightarrow (\bar{P}^a Q^b)^{-w(\Psi)} g^{-(a+b-1)w(\Psi)} \Psi(a,b)$$

covariant derivative of Ψ,

$$\mathcal{D}\Psi = d\Psi + w(\Psi) A \Psi$$

definition: $\quad A(a,b) = A + (T^a U^b)^{-1} d(T^a U^b)$

covariant derivative in (a,b) - basis

$$\mathcal{D}\Psi(a,b) = d\Psi(a,b) + w(\Psi) A(a,b) \Psi(a,b)$$

satisfies

$$\mathcal{D}\Psi(a,b) = (T^a U^b)^{-w(\Psi)} \mathcal{D}\Psi$$

properties of $A(a,b) = A + (T^a U^b)^{-1} d (T^a U^b)$

using $A_\alpha = -T^{-1} D_\alpha T$, $A^{\dot\alpha} = -U^{-1} D^{\dot\alpha} U$ as well as $W = T U^{-1}$

one obtains for $A_\alpha(a,b)$ and $A^{\dot\alpha}(a,b)$ the expressions

$$A_\alpha(a,b) = -\frac{1}{2} W^{-1} D_\alpha W + (T^{a-\frac{1}{2}} U^{b-\frac{1}{2}})^{-1} D_\alpha (T^{a-\frac{1}{2}} U^{b-\frac{1}{2}})$$

$$A^{\dot\alpha}(a,b) = +\frac{1}{2} W^{-1} D^{\dot\alpha} W + (T^{a-\frac{1}{2}} U^{b-\frac{1}{2}})^{-1} D^{\dot\alpha} (T^{a-\frac{1}{2}} U^{b-\frac{1}{2}})$$

while $A_{\alpha\dot\alpha} = \frac{i}{2} (D_\alpha A_{\dot\alpha} + D_{\dot\alpha} A_\alpha)$, in the (a,b)-basis

the transformation law of $T^{a-\frac{1}{2}} U^{b-\frac{1}{2}}$ is given as

$$T^{a-\frac{1}{2}} U^{b-\frac{1}{2}} \longrightarrow \bar{P}^{a-\frac{1}{2}} Q^{b-\frac{1}{2}} T^{a-\frac{1}{2}} U^{b-\frac{1}{2}} g^{a+b-1}$$

the g-transformations appear with a power $(a+b-1)$, similar as in $\psi(a,b) \longrightarrow \bar{P}^a Q^b g^{-(a+b-1)w(\psi)} \psi(a,b)$

as a consequence, for $a+b=1$, the g-transformations drop out completely from the geometric structure !

they are replaced by the $\bar{P}$ and Q transformations in addition to $a+b=1$ we choose now $a=\frac{1}{2}$, $b=\frac{1}{2}$

and parametrize $W=e^{-\frac{K}{2}}$, $\bar{P}=e^{-\frac{\bar{F}}{2}}$, $Q=e^{\frac{F}{2}}$

in this case we find $K \longrightarrow K+F+\bar{F}$,

a superfield $\Psi(\frac{1}{2},\frac{1}{2})$ of weight $w(\Psi)$ transforms as

$$\Psi(\tfrac{1}{2},\tfrac{1}{2}) \longrightarrow e^{-\frac{i}{2}w(\Psi)\,\mathrm{Im}\,F}\,\Psi(\tfrac{1}{2},\tfrac{1}{2})$$

the gauge potential $A(\frac{1}{2},\frac{1}{2})$ takes the form

$$A(\tfrac{1}{2},\tfrac{1}{2})=\frac{1}{4}E^{\alpha}D_{\alpha}K-\frac{1}{4}E_{\dot\alpha}D^{\dot\alpha}K+\frac{i}{16}E^{a}\bar{\sigma}_{a}^{\dot\alpha\alpha}[D_{\alpha},D_{\dot\alpha}]K$$

with transformation law

$$A(\tfrac{1}{2},\tfrac{1}{2}) \longrightarrow A(\tfrac{1}{2},\tfrac{1}{2})+\frac{i}{2}\,d\,\mathrm{Im}\,F$$

in what follows any geometric quantity is taken to be in the $a=\frac{1}{2}$, $b=\frac{1}{2}$ basis and we therefore omit the $(\frac{1}{2},\frac{1}{2})$ notation

furthermore we shall consider

$$K=K(\phi,\bar{\phi}), \quad F=F(\phi), \quad \bar{F}=\bar{F}(\bar{\phi})$$

to be the Kähler potential and the Kähler transformations the superspace geometry where the $U(1)$ prepotential K is replaced by the Kähler potential $K(\phi, \bar{\phi})$ is called $U_K(1)$ superspace or Kähler superspace

in this case the gauge potential A takes the form

$$A = \frac{1}{4}\left(K_k\, d\phi^k - K_{\bar{k}}\, d\bar{\phi}^{\bar{k}}\right) + \frac{i}{8} E^a \left(12\, G_a + \bar{\sigma}_a^{\dot{\alpha}\alpha} g_{k\bar{k}}\, \mathcal{D}_\alpha \phi^k\, \mathcal{D}_{\dot{\alpha}} \bar{\phi}^{\bar{k}}\right)$$

a fonction of N superfields $\phi^k, \phi^{\bar{k}}$ with Kähler metric $g_{k\bar{k}}$

the superfields
$$X_\alpha = -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8R\right) \mathcal{D}_\alpha K(\phi, \bar{\phi})$$
$$\bar{X}^{\dot{\alpha}} = -\frac{1}{8}\left(\mathcal{D}^2 - 8R^\dagger\right) \bar{\mathcal{D}}^{\dot{\alpha}} K(\phi, \bar{\phi})$$

become now

$$X_\alpha = -\frac{i}{2} g_{k\bar{k}}\, \sigma^a_{\alpha\dot{\alpha}}\, \mathcal{D}_a \phi^k\, \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\phi}^{\bar{k}} + \frac{1}{2} g_{k\bar{k}}\, \mathcal{D}_\alpha \phi^k\, \bar{F}^{\bar{k}}$$
$$\bar{X}^{\dot{\alpha}} = -\frac{i}{2} g_{k\bar{k}}\, \bar{\sigma}^{a\dot{\alpha}\alpha}\, \mathcal{D}_a \bar{\phi}^{\bar{k}}\, \mathcal{D}_\alpha \phi^k + \frac{1}{2} g_{k\bar{k}}\, \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\phi}^{\bar{k}}\, F^k$$

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and the expression for the Kähler D-term reads

$$-\frac{1}{2} \mathcal{D}^\alpha X_\alpha = -g_{k\bar{k}}\, \eta^{ab}\, \mathcal{D}_a \phi^k\, \mathcal{D}_b \bar{\phi}^{\bar{k}} - \frac{i}{4} g_{k\bar{k}}\, \sigma^a_{\alpha\dot{\alpha}} \left(\mathcal{D}^\alpha \phi^k\, \mathcal{D}_a \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\phi}^{\bar{k}} + \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\phi}^{\bar{k}}\, \mathcal{D}_a \mathcal{D}^\alpha \phi^k\right)$$
$$+ g_{k\bar{k}}\, F^k \bar{F}^{\bar{k}} + \frac{1}{16} R_{k\bar{k}l\bar{l}}\, \mathcal{D}^\alpha \phi^k\, \mathcal{D}_\alpha \phi^l\, \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\phi}^{\bar{k}}\, \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\phi}^{\bar{l}}$$

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$\bar{r}$: superfield of weight $w(\bar{r}) = -2$, $\mathcal{D}_\alpha \bar{r} = 0$

$$r = r| \,, \quad r_\alpha = \frac{1}{\sqrt{2}} \mathcal{D}_\alpha r| \,, \quad f = -\frac{1}{4} \mathcal{D}^\alpha \mathcal{D}_\alpha r \,|$$

$$\bar{r} = \bar{r}| \,, \quad \bar{r}^{\dot{\alpha}} = \frac{1}{\sqrt{2}} \mathcal{D}^{\dot{\alpha}} \bar{r}| \,, \quad \bar{f} = -\frac{1}{4} \mathcal{D}_{\dot{\alpha}} \mathcal{D}^{\dot{\alpha}} \bar{r} \,|$$

component field density:

$$\mathcal{L}(r, \bar{r}) = ef + \frac{ie}{\sqrt{2}} (\bar{\psi}_m \bar{\sigma}^m r) - e(\bar{M} + \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n) r$$
$$+ e\bar{f} + \frac{ie}{\sqrt{2}} (\psi_m \sigma^m \bar{r}) - e(M + \psi_m \sigma^{mn} \psi_n) \bar{r}$$

the identification $r = -3R$, $\bar{r} = -3R^+$

gives rise to

$$\mathcal{L}(R, R^+) = -\frac{e}{2} \mathcal{R} + \frac{e}{2} \varepsilon^{mnpq} (\bar{\psi}_m \bar{\sigma}_n \mathcal{D}_p \psi_q - \psi_m \sigma_n \mathcal{D}_p \bar{\psi}_q)$$
$$- \frac{e}{3} M \bar{M} + \frac{e}{3} b^a b_a + eD$$

with $\quad D = -\frac{1}{2} \mathcal{D}^\alpha X_\alpha | + \frac{i}{2} (\psi_m \sigma^m)_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} | + \frac{i}{2} (\bar{\psi}_m \bar{\sigma}^m)^\alpha X_\alpha |$

THE SUPERGRAVITY-MATTER LAGRANGIEN

$$\mathcal{L} = -\frac{e}{2}\mathcal{R} + \frac{e}{2}\varepsilon^{nmpq}\left(\bar{\Psi}_m\bar{\sigma}_n\mathcal{D}_p\Psi_q - \Psi_m\sigma_n\mathcal{D}_p\bar{\Psi}_q\right)$$

$$-\frac{e}{3}M\bar{M} + \frac{e}{3}b^a b_a + e\,\mathcal{D}_{\text{Matter}}$$

$$\begin{aligned}
\mathcal{D}_{\text{Matter}} = & -g_{k\bar{k}}\,g^{mn}\mathcal{D}_m A^k \mathcal{D}_n \bar{A}^{\bar{k}} - \frac{i}{2}g_{k\bar{k}}\left(\chi^k\sigma^m\mathcal{D}_m\bar{\chi}^{\bar{k}} - \mathcal{D}_m\chi^k\sigma^m\bar{\chi}^{\bar{k}}\right) \\
& + g_{k\bar{k}}F^k\bar{F}^{\bar{k}} + \frac{1}{4}R_{k\bar{k}\ell\bar{\ell}}\left(\chi^k\chi^\ell\right)\left(\bar{\chi}^{\bar{k}}\bar{\chi}^{\bar{\ell}}\right) \\
& -\frac{1}{\sqrt{2}}\left(\bar{\Psi}_m\bar{\sigma}^n\sigma^m\bar{\chi}^{\bar{k}}\right)g_{k\bar{k}}\mathcal{D}_n A^k - \frac{1}{\sqrt{2}}\left(\Psi_n\sigma^n\bar{\sigma}^m\chi^k\right)g_{k\bar{k}}\mathcal{D}_n\bar{A}^{\bar{k}} \\
& -\frac{1}{2}g_{k\bar{k}}\,\varepsilon^{pqmn}\left(\chi^k\sigma_p\bar{\chi}^{\bar{k}}\right)\left(\Psi_q\sigma_m\bar{\Psi}_n\right) - \frac{1}{2}g_{k\bar{k}}\,g^{mn}\left(\Psi_m\chi^k\right)\left(\bar{\Psi}_n\bar{\chi}^{\bar{k}}\right)
\end{aligned}$$

SUPERFIELD ACTIONS

$$\mathcal{A}_{\text{supergravity + matter}} = -3 \int E$$

$$\mathcal{A}_{\text{superpotential}} = \frac{1}{2} \int \frac{E}{R} e^{K/2} W(\Phi) + \frac{1}{2} \int \frac{E}{R^\dagger} e^{K/2} \bar{W}(\bar{\Phi})$$

$$\mathcal{A}_{\text{super Yang-Mills}} =$$

$$= \frac{1}{8} \int \frac{E}{R} f_{(r)(s)}(\Phi) W^{(r)\alpha} W^{(s)}_{\alpha} + \frac{1}{8} \int \frac{E}{R^\dagger} \bar{f}_{(r)(s)}(\bar{\Phi}) W^{(r)}_{\dot{\alpha}} W^{(s)\dot{\alpha}}$$

ALL THE ESSENTIAL AND BEAUTIFUL IDEAS APPEARING IN THE WORK(*) PRESENTED IN THIS TALK ARE DUE TO BRUNO ZUMINO AND JULIUS WESS.

We have just written down their formulas

THANK YOU BRUNO! TANK YOU JULIUS!

(*) with Pierre Binétry, Georges Girardi and Martin Müller