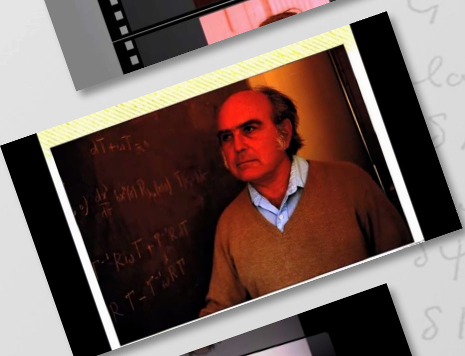
Duality, Gauge Fields and Supersymmetry Breaking



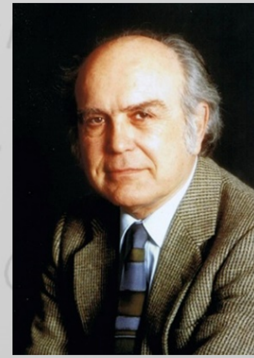
Sergio FERRARA

Bruno Zumino Memorial Meeting
CERN, April 28 2015

The last time I met Bruno was in 2013, when the Berkeley Center for Theoretical Physics organized a conference celebrating his 90th birthday.

A collection of very distinguished speakers gave talks on their current research, with focus on Bruno's influence on their past and present scientific activity.

I had the privilege of sitting between him and Mary K at the opening Conference banquet, when Bruno complained about the quality of the wine offered at the dinner. I did not find the wine particularly bad, and indeed the Conference Organizers told me that it was supposed to be a rather good wine. But it was not so for Bruno.



Berkeley Center For Theoretical Physics Presents
Celebrating Bruno Zumino's 90th Birthday
BrunoFest 2013
May 2 - 4 2013

Invited Speakers

•Edward Witten (IAS)
•Steven Weinberg (University of Texas at Austin)
•Peter van Nieuwenhuizen (Stony Brook)
•Nathan Seiberg (IAS)
•John H. Schwarz (Caltech)
•Albert S. Schwarz (UC Davis)
•Gigi Rolandi (CERN)
•Luciano Maiani (U. Rome La Sapienza)
•Renata Kallosh (Stanford)
•David Gross (KITP/UCSB) to be confirmed
•Michael B. Green (Cambridge University)
•Fabiola Gianotti (CERN)
•Dan Freedman (MIT)
•Sergio Ferrara (CERN)
•John Ellis (CERN)
•Savas Dimopoulos (Stanford)
•Stanley Deser (Brandeis and Caltech)
•Nima Arkani-Hamed (IAS)

Bruno, being tired, left the banquet early, before several closing speeches on his behalf took place.

Being, just after Julius Wess, the most prolific collaborator and a close friend of him (the archives indicate 14 joint publications, four of which are ranked "famous"), I had the honor of being invited to write three obituaries for Bruno that appeared in *"Il Nuovo Saggiatore",* a magazine of the *Italian Physical Society*, in the *CERN Courier* and in *Physics Today*.

In these articles I tried to describe in a concise manner his scientific life, his profound influence in the field of Theoretical Physics and my interaction with him. Fortunately, the latter occurred at two very diverse stages of my career, as a young researcher (CERN post-doc) in the 70s, and as a Senior Scientist across the first decade of the third Millennium.

I am glad that some of his younger collaborators of these last years, P. Aschieri and A. Marrani, are speakers at this Meeting.

Since this is a Memorial Conference dedicated to Bruno, a very sad occasion for me to be invited to speak, I have decided to devote a portion of this talk to personal recollections.

In retrospect I met him for the first time in 1967, when I was a Physics student at the University of Rome “La Sapienza” (at the time the only one), and he came driving a Porsche all the way from Geneva to Rome to give a seminar on "Current Algebra".

I still remember that the seminar room at the Physics Institute "Guglielmo Marconi“ was crowded, as expected for a famous figure as he was already at the time.

Later, in 1971, when I was at Frascati National Labs, working with Gatto, Grillo and Parisi on the application of Conformal Invariance to short distance phenomena (Bjorken scaling), he invited me to give a seminar at CERN. At that time he was the Theory Division leader.

Two years later, in 1973, I came to CERN as a Postdoctoral fellow, and that was the time Bruno gave birth to Supersymmetry with Julius Wess.

In 1974 we wrote together three papers, having to do respectively with the construction of Supersymmetric Yang-Mills theories (and, in particular, with their asymptotic freedom properties), with non renormalization theorems (with Jean Iliopoulos), and with chiral superspace (with Wess).

FERMI - BOSE SYMMETRY

eal fields ; ψ Majorana spinor
anticommuting

$-\frac{i}{2}\bar{\psi}\gamma^a\partial_a\psi + \frac{1}{2}F^2 + \frac{1}{2}G^2$

$\partial_a = \frac{\partial}{\partial x^a}$

Nuclear Physics B79 (1974) 413–421. North-Holland Publishing Company

SUPERGAUGE INVARIANT YANG-MILLS THEORIES

S. FERRARA and B. ZUMINO
CERN, Geneva

Received 27 May 1973

Abstract: We construct Lagrangian theories which are simultaneously invariant under supergauge transformations and under Yang-Mills transformations. The simplest of these turns out to be just the usual theory describing the interaction of a Yang-Mills field with a Majorana spinor belonging to the regular representation of the internal symmetry group. This theory is asymptotically free. Other examples, involving in addition to spinors also scalar and pseudoscalar fields are described. They also are asymptotically free, provided the number of scalar supermultiplets is not too high and supergauge invariance, as expected, is preserved by renormalization.

$\gamma^a\partial_a\psi = 0$

$F = 0$

$G = 0$

"Scalar" = "chiral" sup

Nuclear Physics B77 (1974) 413–419. North-Holland Publishing Company

SUPERGAUGE INVARIANCE AND THE GELL-MANN-LOW EIGENVALUE

S. FERRARA
CERN, Geneva

J. ILIOPOULOS*
Laboratoire de Physique theorique, Orsay

B. ZUMINO
CERN, Geneva

Received 28 March 1974

Abstract: The connections among supergauge, scale and conformal invariance are elucidated on the example of a renormalizable field theory model. For the massless model, at the Gell-Mann-Low eigenvalue, a non-perturbative argument leads to the contradictory result that it can only describe a free field theory. A study of the Callan-Symanzik equations for the massive model clarifies the situation from the point of view of perturbation theory. It is argued that the eigenvalue equation has no non-trivial solution and that the effective coupling constant increases without bound with energy.

Volume 51B, number 3 PHYSICS LETTERS 5 August 1974

SUPERGAUGE MULTIPLETS AND SUPERFIELDS

S. FERRARA and B. ZUMINO
CERN, Geneva, Switzerland

J. WESS
Karlsruhe University, Germany

Received 3 May 1974

Superfields are defined as functions of the space-time variable x and of anticommuting two-component spinors θ and $\bar{\theta}$. They have definite transformation properties under supergauge transformations. Their expansion in θ and $\bar{\theta}$ generates a finite number of ordinary fields forming a multiplet. A number of operations are defined which allow the construction of new superfields from given ones. The corresponding set of multiplets is complete, in the sense that the product of any two can be decomposed as a sum of multiplets belonging to the same set.

anticommuting

In the subsequent years we wrote together two more papers. One, in 1975, on the "Supercurrent Multiplet“, and the other, in 1977, with Joel Scherk on the emergence of electric magnetic duality in (N=extended) Supergravity Theories.

Mary K and Bruno then wrote together, in 1981, a fundamental paper on electric-magnetic dualities in more general theories, including non linear theories for spin one gauge fields, then generalizing the famous Born-Infeld example.

Regretfully, that was also the time that Bruno moved from CERN to Berkeley, really a great loss for the Theory Division.

The subject of electric-magnetic duality permeated Bruno’s research in subsequent years, and was also the subject of my scientific interactions with him when I visited Berkeley, in the Fall of 2008, as a Miller Visiting Professor.

Nuclear Physics B87 (1975) 207 – 220

TRANSFORMATION PROPERTIES OF THE SUPERCURRENT

S. FERRARA and B. ZUMINO
CERN, Geneva

Received 18 November 1974

It is shown that the spinor current, if correctly defined, belongs to a supermultiplet, together with the energy-momentum tensor and the axial-vector current. The transformation properties of this supermultiplet under both restricted and general supersymmetry transformations are given. The generators of special supersymmetry transformations can be obtained as first moments of the time component of the spinor current. The transformation laws of the supermultiplet containing the spinor current provide the local version of the supersymmetry algebra.

-BOSE SYMMETRY

lds ; ψ Majorana spinor anticommuting

$\gamma^a \partial_a \psi + \frac{1}{2} F^2 + \frac{1}{2} G^2$

$\partial_a = \frac{\partial}{\partial x^a}$

$F = 0$

$G = 0$

"Scalar" = "chiral" sup

$\delta A = i \bar{\alpha} \psi$

$\delta B = i \bar{\alpha} \gamma_5 \psi$

$\delta \psi = [\gamma^a \partial_a (A + \gamma_5 B)$

$\delta F = i \bar{\alpha} \gamma^a \partial_a \psi$

$\delta G = i \bar{\alpha} \gamma_5 \gamma^a \partial_a \psi$

α infinitesimal consta
spinorial param

Nuclear Physics B121 (1977) 393 -402

ALGEBRAIC PROPERTIES OF EXTENDED SUPERGRAVITY THEORIES

S. FERRARA
Frascati National Laboratories, Frascati, Italy

J. SCHERK
*Laboratoire de Physique Théorique * de l'Ecole Normale Supérieure*

B. ZUMINO
CERN, Geneva

Received 13 December 1976

Chirality transformations and generalized duality transformations are used to show that the extended O(2) and O(3) supergravity theories in fact have a U(2) and a U(3) invariance respectively. The coupling of these supergravity theories to matter multiplets having global extended supersymmetry is considered. In one case where the matter multiplet has a non-vanishing central charge, one finds a minimal coupling of the vector field of the supergravity multiplet.

SUPERSYMMETRY = FERMI-BOSE SYMMETRY
SUSY
A, B, F, G real fields ; ψ Majorana spinor
anticommuting

Nuclear Physics B193 (1981) 221–244

DUALITY ROTATIONS FOR INTERACTING FIELDS*

Mary K. GAILLARD

LAPP, Annecy-le-Vieux, France

Bruno ZUMINO

CERN, Geneva, Switzerland

Received 26 May 1981

We study the properties of interacting field theories which are invariant under duality rotations which transform a vector field strength into its dual. We consider non-abelian duality groups and find that the largest group for n interacting field strengths is the non-compact $\mathrm{Sp}(2n,\mathrm{R})$, which has $\mathrm{U}(n)$ as its maximal compact subgroup. We show that invariance of the equations of motion requires that the lagrangian change in a particular way under duality. We use this property to demonstrate the existence of conserved currents, the invariance of the energy-momentum tensor and the S-matrix, and also in the general construction of the lagrangian. Finally we comment on the existence of zero-mass spin-one bound states in $N = 8$ supergravity, which possesses a non-compact E_7 dual invariance.

$\delta G = i\,\bar{\alpha}\,\gamma_5\,\gamma^a \partial_a \psi$

α infinitesimal constant anticommuting
spinorial parameter.

In those days electric-magnetic duality, and its realizations in Supergravity and Superstrings as non compact symmetries acting also on scalar fields, had an influence on the physics of Black Holes and on their classifications, especially in the extremal case. And also on the AdS/CFT correspondence.

In 2008 we published a review on electric-magnetic duality with Paolo Aschieri. This work followed an invitation by Luisa Cifarelli, President of the Italian Physical Society, when we were awarded the 2005 "Enrico Fermi Prize" together with Gabriele Veneziano.

One of my last papers with Bruno (and his younger collaborators A. Marrani and B.L. Cerchiai) was dated 2009 and treated universal properties of extremal (both BPS and nBPS) black holes in N=2 theories, such as AdM mass, Entropy and Attractors.

In the second part of this talk I will consider some modern applications of Bruno's work on duality, in connection with the Supersymmetric Born-Infeld theory, its relation to partial spontaneous supersymmetry breaking and generalizations to other gauge fields and to other space-time dimensions.

In particular I will describe multi-field non-linear theories with non trivial electric-magnetic dualities.

Electric-magnetic duality is one of the most fascinating symmetries of non-linear theories.

Free Maxwell theory is the prototype of a duality invariant theory. The relativistic form of its equations in vacuum is

$$
\begin{aligned}
\partial_\mu F^{\mu\nu} &= 0 \qquad & \Box A_\mu - \partial_\mu \partial \cdot A &= 0 \\
\partial_\mu \widetilde{F}^{\mu\nu} &= 0 \qquad & F_{\mu\nu} = \partial_\mu A_\nu &- \partial_\nu A_\mu
\end{aligned}
$$

Letting $F_{\mu\nu} = \left(\vec{E}, \vec{B}\right)$ one recovers the conventional form

$$
\begin{aligned}
\partial_t \vec{E} &= \nabla \times \vec{B} \qquad & \nabla \cdot \vec{B} &= 0 \\
\partial_t \vec{B} &= - \nabla \times \vec{E} \qquad & \nabla \cdot \vec{E} &= 0
\end{aligned}
$$

$$
(F_{0i} = E_i \ , \quad F_{ij} = \epsilon_{ijk} B_k)
$$

The **energy-momentum tensor** is

$$T_{\mu\nu}: \quad T_{ij} = E_i\, E_j \; + \; B_i\, B_j \;, \quad T_{0i} \;=\; \epsilon_{ijk} E_j B_k \;, \quad T_{00} \;= \vec{E}^2 \;+\; \vec{B}^2$$

and **the equations of motion**, together with $T_{\mu\nu}$ (and in particular the Hamiltonian) **are invariant under U(1) rotations**

$$\begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}' = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\,\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

$$\delta_\alpha T_{\mu\nu} \;=\; 0$$

Note that the Lagrangian $\mathcal{L} \;=\; \frac{1}{2}\left(\vec{E}^2 \;-\; \vec{B}^2\right)$ is not invariant.

Moreover ***e.m.* duality is not an internal symmetry, since it rotates a tensor into a pseudo-tensor** (a sort of bosonic chiral transformation)

Non-linear theories of electromagnetism (with possible addition of other matter fields and electric and/or magnetic sources) are obtained introducing an **“electric displacement”** $\vec{D}$ and a **“magnetic field”** $\vec{H}$ that only for the free Maxwell theory coincide with the **electric field** $\vec{E}$ and the **magnetic induction** $\vec{B}$

$$\vec{E} = \vec{D}\,, \qquad \vec{B} = \vec{H}$$

In a medium (with possible sources) the non—linear *e.m.* equations become

$$\partial_t \vec{B} + \nabla \times \vec{E} = 0 \left(4\pi \vec{J}_m\right) \qquad \nabla \cdot \vec{B} = 0\,(4\pi \rho_m)$$

$$-\,\partial_t \vec{D} + \nabla \times \vec{H} = 4\pi \vec{J}_e \qquad \nabla \cdot \vec{D} = 4\pi \rho_e$$

where $\left(\vec{E}, \vec{B}, \vec{D}, \vec{H}\right)$ are linked by the **constitutive relations**

$$\vec{D} = \vec{D}\left(\vec{E}\,, \vec{B}\right) \quad \vec{H} = \vec{H}\left(\vec{E}\,, \vec{B}\right)$$

with $\vec{D} = \vec{E} + \ldots \,, \quad \vec{H} = \vec{B} + \ldots$

we observe that **the non-linear equations are invariant under rotations**

$$\left(\vec{B}, -\vec{D}\right) \to \left(\vec{B}, -\vec{D}\right)_\alpha \qquad \left(\vec{E}, \vec{H}\right) \to \left(\vec{E}, \vec{H}\right)_\alpha$$

In a relativistic notation one can write

$$F_{\mu\nu} = \left(\vec{E}, \vec{B}\right) \qquad G_{\mu\nu} = \left(\vec{H}, -\vec{D}\right)$$

and the non-linear equations with sources become

$$\partial_\mu \widetilde{G}^{\mu\nu} = J_e^\nu \,, \qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0 \, (J_m^\nu)$$

and are invariant under rotations $(F_{\mu\nu} \; G_{\mu\nu}) \to (F_{\mu\nu} \,, G_{\mu\nu})_\alpha$
provided sources rotate accordingly.

Here $G_{\mu\nu} = G_{\mu\nu}(F_{\mu\nu})$, and in the vacuum $F_{\mu\nu} = -\widetilde{G}_{\mu\nu}$

The **source-free stress tensor** now becomes *(Gaillard and Zumino, 1981)*

$$T^{\mu}{}_{\lambda} \;=\; -\,\partial_\lambda \varphi^i\,\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^i} \;+\; \delta^{\mu}_{\lambda}\,\mathcal{L} \;+\; \widetilde{G}^{\mu\nu}\,F_{\nu\lambda}$$

(when matter fields φ^i are included)

$T^{\mu}{}_{\lambda}$ **is invariant under duality rotations also acting on matter fields, and the first term is absent in a pure non-linear theory for** $F_{\mu\nu}$

Generalizing to **n Maxwell fields** is straightforward, and the original U(1) rotations become at most **U(n) rotations**:

$$\begin{aligned}\delta F &= a\cdot F \;+\; b\cdot G\\ \delta G &= c\cdot F \;+\; d\cdot G\end{aligned}$$

with matrices a, b, c, d such that

$$d \;=\; a \;=\; -\,a^T\,, \quad b \;=\; b^T\,, \quad c \;=\; -\,b$$

The above invariance under electric-magnetic duality is far from trivial, since **the (F,G) rotation must be consistent with the constitutive relation** $G_{\mu\nu} = G_{\mu\nu}(F_{\mu\nu})$, and it is highly non trivial to find functionals that satisfy such compatibility constraints.

In addition, the request of having a Lagrangian formulation for such theories demands that

$$\widetilde{G}_{\mu\nu} = 2\,\frac{\partial \mathcal{L}}{\partial F^{\mu\nu}}$$

or, in non-covariant form

$$\vec{D} = 2\,\frac{\partial \mathcal{L}\left(\vec{E},\vec{B}\right)}{\partial \vec{E}} \qquad \vec{H} = -2\,\frac{\partial \mathcal{L}\left(\vec{B},\vec{B}\right)}{\partial \vec{E}}$$

with the integrability condition

$$\frac{\partial \widetilde{G}^{\mu\nu}}{\partial F^{\rho\sigma}} = \frac{\partial \widetilde{G}_{\rho\sigma}}{\partial F_{\mu\nu}}$$

It can be shown that **invariance under U(n) rotations requires that the constitutive relations satisfy the conditions**

(Gaillard and Zumino)

$$G^\Lambda \, \widetilde{G}^\Sigma \; + \; F^\Lambda \, \widetilde{F}^\Sigma \; = \; 0$$
$$G^\Lambda \, \widetilde{F}^\Sigma \; - \; G^\Sigma \, \widetilde{F}^\Lambda \; = \; 0$$

In particular, **for n=1 there is just one condition**

$$G \, \widetilde{G} \; + \; F \, \widetilde{F} \; = \; 0$$

which is trivially satisfied in Maxwell theory, where $F \; = \; - \, \widetilde{G}$

A non-trivial solution of the constraints is provided by the **Born-Infeld Theory *("Foundations of the new field theory", Proc. R. Soc. London A144 (1934) 425)*** (later reconsidered by Schrödinger, Dirac and many others), which takes the form

$$\mathcal{L}_{BI} \; = \; \mu^2 \left[1 \; - \; \sqrt{-det \left(\eta_{\mu\nu} \; + \; \frac{1}{\mu} \, F_{\mu\nu} \right)} \; \right]$$
$$= \; \mu^2 \left[\; 1 \; - \; \sqrt{1 \; + \; \frac{1}{2\,\mu^2} \, F^2 \; - \; \frac{1}{16\,\mu^4} \, \left(F \cdot \widetilde{F} \right)^2} \; \right] \; = \; \mathcal{L}_{BI} \left(\vec{B}^2 \; - \; \vec{E}^2, \vec{E} \cdot \vec{B} \right)$$

Note the analogy with the relativistic γ factor for the velocity:

$$\mathcal{L}_{BI}\left(\vec{B}=0\right) \;=\; \mu^2\left[\,1\;-\;\sqrt{1\;-\;\frac{1}{\mu^2}\;\vec{E}^2}\,\right]$$

which sets an upper bound for $\vec{E}:\quad \left|\vec{E}\right| \;<\; \mu$

This Lagrangian has in fact unique properties against instabilities created by the medium. Not many generalizations are known in the multi-field case.

The link to modern theories comes from **higher-order curvature terms** produced by loop corrections or by α' corrections in **Superstring Theory**.

In addition, **in Supergravity electric-magnetic duality is a common feature in the Einstein approximation**, and it is natural to enforce it when the theory is deformed by higher (Einstein or Maxwell) curvature terms *(SF, Scherk, Zumino, Cremmer, Julia, Kallosh, Stelle, Bossard, Howe, Nicolai, …)*

The Born-Infeld action emerges in brane dynamics as the bosonic sector of the Goldstino action for N=2 supersymmetry spontaneously broken to N=1.

In this setting the **Maxwell field is the partner of the spin – ½ goldstino** in an N=1 vector multiplet.

(Cecotti, SF, Deser, Puzalowski, Hughes, Polchinski, Liu, Bagger, Galperin, Rocek, Tseytlin, Kuzenko, Theisen, …)

A multi-field extension is possible, in the N=2 setting, starting from an N=2 vector multiplet where suitable Fayet-Iliopoulos terms are introduced to integrate out N=1 chiral multiplets

(Antoniadis, Partouche, Taylor)

For a single N=2 vector multiplet the Born-Infeld action emerges as a solution of the **quadratic constraint**

$$G_{+}^{2} + \mathcal{F}\left(m - \overline{\mathcal{F}}\right) = 0$$

where G_{+} is the self-dual curvature of the Maxwell field

$$G_{+,\,\mu\nu} = F_{\mu\nu} + i\,\widetilde{F}_{\mu\nu}$$

and $\mathcal{F}$ is an auxiliary field such that

$$\mathcal{L}_{BI} = m\ Re\ \mathcal{F}$$

In the multi-field case **the generalization rests on the constraints**

(SF, Porrati, Sagnotti)

$$d_{ABC}\left[G_+^B\,G_+^C \;+\; \mathcal{F}^B\left(m^c \;-\; \overline{\mathcal{F}}^c\right)\right] \;=\; 0$$

where the d_{ABC} are the coefficients of the **cubic term of the holomorphic prepotential of rigid special geometry**, whose classification rests on the singularity structure of cubic varieties.

(FPS + Stora, Yeranyan)

Further generalizations of electric-magnetic dualities can be obtained coupling gauge vectors to scalar fields, as it occurs in Supergravity, where they can be partners of the graviton (in N $\geq$ 4 extended Supergravity) or additional matter (for N $\leq$ 4)

The general situation was again studied by Gaillard and Zumino, **who showed that, in the presence of scalars, the most general electric-magnetic duality rotation for n vector fields belongs to a subgroup of Sp(2n,R).**

In the infinitesimal these transformations act as

$$\begin{aligned}\delta F &= a\cdot F + b\cdot G\\ \delta G &= c\cdot F - a^T\cdot G\\ b &= b^T\,,\quad c = c^T\end{aligned}$$

Under such transformations (which contain U(n) in the particular case $a = -a^T \;,\quad b = -c$), the Lagrangian transforms as

$$\delta\mathcal{L} \;=\; \frac{1}{4}\Big(F\,c\,\widetilde{F} \;+\; G\,b\,\widetilde{G}\Big)$$

Therefore, **when b≠0 (*i.e.* when the electric field transforms into the magnetic one) the duality *is not* an invariance of the Lagrangian, although *it is* an invariance of the stress tensor**. The invariance can only hold for b=0 (c $\neq$ 0 implies a total derivative term), so for matrices of the form

$$\begin{pmatrix} a & 0\\ c & -a^T\end{pmatrix}$$

For n=1, one recovers a notorious example, the SL(2,R) duality of the dilaton-axion system coupled to a Maxwell field.

Notice that the duality symmetry must not necessarily be the full Sp(2n, R), but at least a subgroup possessing a 2n-dimensional symplectic representation R, *i.e.* such that $R \times R \supset 1_a$ (as is the case, for instance, for the **56** of $E_{7,7}$ in N=8 Supergravity).

I would like to conclude this talk discussing **other examples of non-linear (square-root) Born-Infeld type Lagrangians in D dimensions** that were suggested from alternative 4D goldstino actions studied by *Bagger, Galperin, Rocek, Tseytlin, Kuzenko, Theisen*

The first class of examples are **D-dimensional generalizations containing pairs of form field strengths** of degrees (p+1,D-p-1), gauge fields that couple to (p-1,D-p-3) branes). These Lagrangians generalize the D=4, p=2 case corresponding to a non-linear Lagrangian for a tensor multiplet regarded as an N=2 → N=1 goldstino multiplet in rigid Supersymmetry.

(Kuzenko, Theisen; SF, Sagnotti, Yeranyan)

These action read

$$\mathcal{L} \;=\; \mu^2 \left[1 \;-\; \sqrt{1 \;+\; \frac{1}{\mu^2}\; X \;-\; \frac{1}{\mu^4}\; Y^{\,2}} \right]$$

$$X \;=\; \star\left[H_{p+1} \wedge \star H_{p+1} \;+\; V_{D-p-1} \wedge \star V_{D-p-1} \right]$$

$$Y \;=\; \star\left[H_{p+1} \wedge V_{D-p-1} \right]$$

and have the property of being **doubly self-dual** under

$$V'_{D-p-1} \;=\; \widetilde{H}_{p+1} \;, \quad H'_{p+1} \;=\; \widetilde{V}_{D-p-1}$$

Moreover, after a single duality the action ends up with two forms of the same degree, in a manifestly U(1) invariant combination

$$\mathcal{L} \;=\; \mu^2 \left[1 \;-\; \sqrt{1 + \frac{W_{D-p-1} \cdot \bar{W}_{D-p-1}}{\mu^2} \;+\; \frac{\left(W_{D-p-1} \cdot \bar{W}_{D-p-1} \right)^2 - W^2_{D-p-1}\, \bar{W}^2_{D-p-1}}{4\,\mu^4}} \right]$$

which has only a $\mathrm{U(1)}_{\mathrm{em}}$ duality.

A complexification of the n=1 Born-Infeld action with an SU(2) duality is

$$\mathcal{L} \;=\; \mu^2 \left[1 \;-\; \sqrt{1 + \frac{F_{p+1}\cdot \overline{F}_{p+1}}{\mu^2} \;-\; \frac{\left(\star\left[F_{p+1}\wedge F_{p+1}\right]\right)\,\left(\star\left[\overline{F}_{p+1}\wedge \overline{F}_{p+1}\right]\right)}{\mu^4}}\;\right]$$

Actions with the full U(n) duality group were proposed by *Aschieri, Brace, Morariu, Zumino* but they are not available in closed form even for n=2.

These non-linear actions can be made massive introducing Green-Schwarz terms *(SF, Sagnotti)* , *i.e.* couplings to another gauge field of the form

$$m H_{p+1} \wedge A_{D-p-1}$$

A Stueckelberg mechanism is then generated, and **the non-linear mass terms take the same form as the original non-linear curvature action**, whose (D-p-1)-form gauge field has been eaten to give mass to the original p-form.

The **simplest example is the four-dimensional Born-Infeld action used to make an antisymmetric field** $b_{\mu\nu}$ **massive**. The mass term comes exactly from the Born-Infeld term with $F_{\mu\nu}$ replaced by $m\, b_{\mu\nu}$

This type of mechanism could be relevant when the system is coupled to N=2 Supergravity, in which case one of the two gravitini would belong to a massive spin-3/2 multiplet.