# Black Hole Attractors, Charge Orbits and Moduli Spaces

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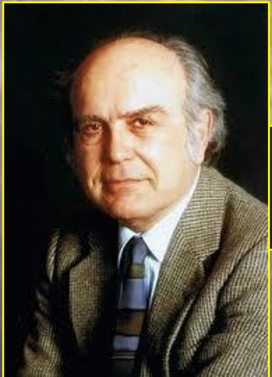


*Bruno Zumino Memorial Meeting,*
*CERN, April 28, 2015*

I met Bruno for the first time in Fall 2008. I had just started my 2° post-doc fellowship at ITP, Stanford, and I was visiting Sergio Ferrara at UC Berkeley.

Bruno was from Roma, and, as soon as he knew that I was from Ostia (the Roman seaplace where I was born and grown up), he started to recall his childhood, when he used to spend his Summer time in Ostia beach…

With this friendly start, a nice collaboration grew up with Bruno, which led in a few months to the first of the papers I had the honour to coauthor with him :

***Duality, Entropy and ADM Mass in Supergravity***,
**B. L. Cerchiai, S. Ferrara, AM, B. Zumino,**
**Phys. Rev. D79, 125010 (2009), arXiv:0902.3973 [hep-th].**

While being post-doc researcher in Stanford (2008-2010), I paid regular visits to Bruno in Berkeley.
We used to meet just after lunch, starting our discussions
with a nice cup of American coffee, but speaking strictly in Italian, better
with Roman accent (that he had almost completely lost, but liked to listen to…).

During my stay in Stanford, our collaboration kept growing, and we wrote a paper devoted to the charge orbits of attractors in 5D (actually, the first complete account for all supergravity theories) :

***Charge Orbits of Extremal Black Holes in 5D Supergravity,***
**B. L. Cerchiai, S. Ferrara, AM, B. Zumino,**
**Phys. Rev. D82, 085010 (2010), arXiv:1006.3101 [hep-th].**

My 3° post-doc was at CERN, where I kept collaborating with Sergio and Bruno, on some topics related to the Ehlers group and Jordan algebras, among others. This collaboration eventually led to the paper :

***Jordan Pairs, E6 and U-Duality in Five Dimensions,***
**S. Ferrara, AM, B. Zumino,**
**J. Phys. A46, 065402 (2013), arXiv:1208.0347 [math-ph].**

Furthermore, Bruno kindly involved me in a project together with two of his PhD students in UCB. This started a nice collaboration on Freudenthal triple systems, giving rise to the paper :

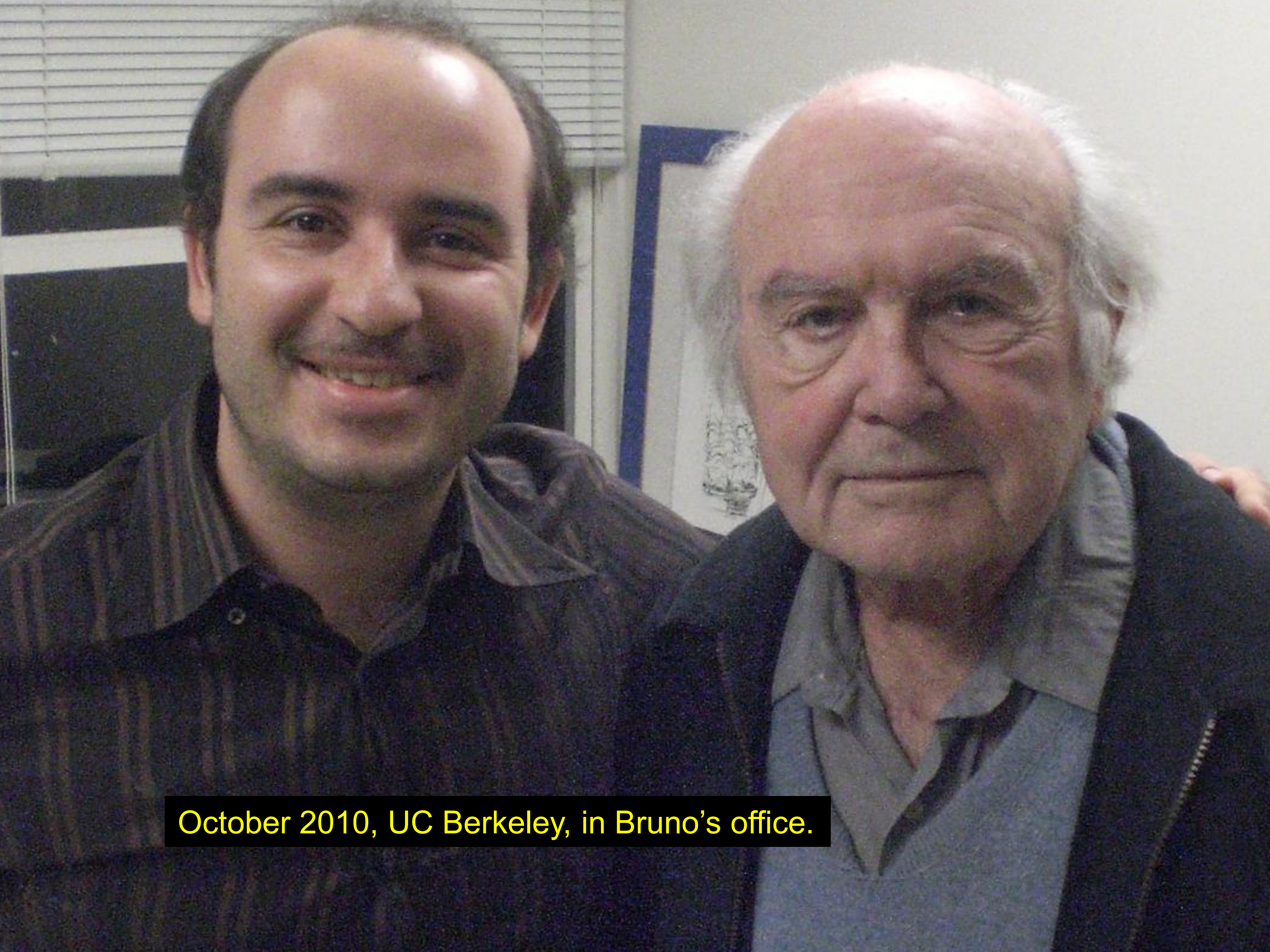
***Freudenthal Gauge Theory,***
**AM, C.-X. Qiu, S.D. Shih, A. Tagliaferro, B. Zumino,**
**JHEP 1303, 132 (2013), arXiv:1208.0013 [hep-th].**

What I most remember of Bruno was his intense commitment to research, always joint to his distinguished approach, relaxed and deeply focussed at the same time.

He contributed with subtle and profound observations to our research interaction, crucially supervising our path with his attitude of considering the general/concep- -tual framework and paying attention to the smallest details, at the same time.

…for a young post-doc researcher like me, it was a real honour and pleasure to meet him, and have the chance to be one of his collaborators (actually the last one with whom he published a paper)…


May 2009, UC Berkeley (with G. Torri, student from Imperial College, London).



October 2010, UC Berkeley, in Bruno’s office.

# Summary of the Talk

- ❖ Basics on the **attractor mechanism**
- ❖ Basics on ungauged N=2, d=4 MESGT
- ❖ Effective BH potential
- ❖ Classes of attractors in extremal BHs
- ❖ Stability of attractors
- ❖ **U-duality orbits** of BH e.m. charges
- ❖ Ineffectiveness of the attractor mechanism : **attractor *moduli spaces***
- ❖ What about N>2, d=4 extended supergravities ?

Final Remarks and Outlook…

# What is the Attractor Mechanism?

→**Bekenstein – Hawking Entropy – Area Formula**
*(macroscopic approach to BH thermodynamics)*

$$S_{BH} = \frac{A_{Horizon}}{4}$$

→***We consider an extremal (T=0), dyonic, asymptotically flat, spherically symmetric, static BH***

$$ds^2_{c=0} = -e^{2U}dt^2 + e^{-2U}\left[dr^2 + (r-r_H)^2 d\Omega^2\right]$$

**A priori the BH entropy will depend on the following variables :**

$$S_{BH} = S_{BH}\left(q_0, q_i, p^0, p^i; z^i_{Horizon}(\{z_\infty\}), \bar{z}^{\bar{i}}_{Horizon}(\{z_\infty\})\right)$$

**BH electric charges**

**BH magnetic charges**

**Values of the scalar fields at the event horizon of the BH:**

$$z^i_{Horizon} = lim_{r\to r_H^+} z^i(r)$$

**Notice :**

$z^i_\infty$ are unconstrained ; they can take any possible complex value

they will in general depend on the **initial data** of their classical evolution dynamics, *i.e.* on the **asymptotical values**

$$z^i_\infty = lim_{r\to\infty} z^i(r)$$

*Can the scalars be stabilized at the BH event horizon ?*

*Can they be made independent on the unconstrained asymptotical values* $z^i_\infty$ *?*

*S. Ferrara, R. Kallosh, A. Strominger, '95*
*S. Ferrara, R. Kallosh, '96*

*S. Ferrara, G. Gibbons, R. Kallosh, '97*

## Attractor Mechanism :

**In approaching the event horizon, the scalars completely *lose memory* of the initial data, and take values dependent only on the electric/magnetic charges of the BH:**

$$lim_{r\to r_H^+} z^i\left(r;\{z_\infty\}\right) = z^i_{Horizon}\left(\{q\},\{p\}\right)$$

*Regardless of the initial conditions, the horizon values depend only on the charges!*

$$\begin{aligned} S_{BH} &= S_{BH}\left(q_0,q_i,p^0,p^i;z^i_{Horizon}\left(\{q\},\{p\}\right),\overline{z}^{\overline{i}}_{Horizon}\left(\{q\},\{p\}\right)\right) \\ &= S_{BH}\left(\{q\},\{p\}\right) \end{aligned}$$

Conserved charges, from $(U(1))^{n_V+1}$ gauge-inv.

# N=2, d=4 Supergravity coupled to $n_V$ Abelian vector multiplets: Maxwell-Einstein Supergravity Theory (MESGT)

**Field content :**

→ **gravity multiplet**

$$\left(V_\mu^a, \psi^A, \psi_A, A^0\right)$$

*graviphoton*

*Vielbein*

*SU(2) doublet of gravitinos*

→ **$n_V$ Abelian vector multiplets**

$$\left(A^I, \lambda^{iA}, \overline{\lambda}_A^{\overline{i}}, z^i\right)$$

*complex scalar fields*

*U(1) gauge boson*

*doublet of gauginos*

*overall* $\left(U(1)\right)^{n_V+1}$ *gauge symmetry*

*No hypermultiplets will be considered : they decouple from the system*

**Only scalars from vector multiplets are relevant for the Attractor Mechanism**

Thus, which is the criterion to determine the purely charge-dependent configs. $z^i_{Horizon}$ of the scalars? <u>How can the Attractor Mechanism be implemented?</u>

*"Critical implementation"* (Ferrara, Gibbons, Kallosh, '97:

$z^i_{Horizon}(\{q\},\{p\})$ actually are non-degenerate critical points of an *"effective black hole potential"*

$$V_{BH}(z,\bar{z};\{q\},\{p\}) \equiv |Z|^2(z,\bar{z};\{q\},\{p\}) + G^{i\bar{j}}(z,\bar{z}) D_i Z(z,\bar{z};\{q\},\{p\}) \overline{D_{\bar{j}} Z}(z,\bar{z};\{q\},\{p\})$$

*Z* is the *N=2 central charge* : $\left\{Q^A_\alpha, Q^B_\beta\right\} = \epsilon_{\alpha\beta} Z^{[AB]} = \epsilon_{\alpha\beta}\epsilon^{AB} Z$

regular contravariant metric of the <u>Special Kaehler</u> scalar manifold :

$$G_{i\bar{j}} = \partial_i \overline{\partial}_{\bar{j}} K(z,\bar{z})$$

covariant derivative of *Z*:

$$D_i Z = \partial_i Z + \frac{1}{2}(\partial_i K) Z$$

horizon scalars' configs.
→characterized as <u>critical pts.</u> of $V_{BH}$

$$\{z_{Horizon}(\{q\},\{p\})\} : \begin{cases} (\partial_i V_{BH})(z_{Horizon}, \bar{z}_{Horizon}; \{q\},\{p\}) = 0; \\ V_{BH}(z_{Horizon}, \bar{z}_{Horizon}; \{q\},\{p\}) \neq 0. \end{cases}$$

***Classical BH entropy***

$$S_{BH}(\{q\},\{p\}) = \pi V_{BH}(\{z_{Horizon}(\{q\},\{p\})\},\{\bar{z}_{Horizon}(\{q\},\{p\})\};\{q\},\{p\}) = \pi \left. V_{BH}\right|_{\partial V_{BH}=0}$$

## *Classification of BH attractors in N=2, d=4 MESGT:*

1. ***½-BPS attractors*:** they preserve the maximum number of SUSYs (4 out of 8), and they do saturate the Bogomol'ny – Prasad – Sommerfeld (BPS) bound:

$$V_{BH,\frac{1}{2}-BPS} = |Z|^2_{\frac{1}{2}-BPS} = M^2_{BH}$$

Defining conditions:

$$Z_{\frac{1}{2}-BPS} \neq 0, \quad (D_i Z)_{\frac{1}{2}-BPS} = 0 \ \ \forall i = 1, ..., n_V$$

**Known since the mid 90's, starting from the cited seminal papers by Ferrara, Gibbons, Kallosh, Strominger**

**2. *non-BPS attractors with non-vanishing central charge*:** they do not preserve any SUSY of the asymptotical Minkowski bkgd.,and do **NOT saturate** the BPS bound:

$$M^2_{BH} = V_{BH,non-BPS,Z\neq 0} = \left(|Z|^2 + G^{i\bar{j}} D_i Z \overline{D}_{\bar{j}} \overline{Z}\right)_{non-BPS,Z\neq 0} > |Z|^2_{non-BPS,Z\neq 0}$$

Defining conditions:

$$Z_{non-BPS,Z\neq 0} \neq 0, \quad (D_i Z)_{non-BPS,Z\neq 0} \neq 0 \text{ for some values of } i$$

Discovered 10 yrs ago (Goldstein *et al*., '05, Tripathy and Trivedi '*06*, and many others…). They correspond to BH backgrounds breaking all SUSYs, but in the framework of a SUSY theory.

**3. *non-BPS Attractors with vanishing central charge*:** they do not preserve any SUSY of the asymptotical Minkowskian background,and do not saturate the BPS bound:

$$M_{BH}^2 = V_{BH,non-BPS,Z=0} = \left(G^{i\bar{j}} D_i Z \overline{D}_{\bar{j}} \overline{Z}\right)_{non-BPS,Z=0} > |Z|^2_{non-BPS,Z=0} = 0$$

It can be traced back to the ***regularity*** of the SKG of the scalar manifold

Defining conditions:

$$Z_{non-BPS,Z=0} = 0, \quad (D_i Z)_{non-BPS,Z=0} \neq 0 \text{ for some values of } i$$

Until June 2006, the unique explicit example of such a kind of extremal BH attractors was given by Giryavets in '06 .

Since then, the **non-BPS, Z=0 attractors** have been studied in a number of frameworks:

**→ Homogeneous symmetric Special Kaehler geometries [see next slides];**

**→ Special Kaehler scalar manifolds arising from compactifications of d=10 superstrings on Fermat CY3' s;**

**→ Peculiar homogeneous symmetric models, the so-called st$^2$ ($n_V$ =2) and stu ($n_V$ =3) models.**

**→ …and various other models….**

## *What about the stability of the extremal BH attractors?*

→ It is crucial for the **physical meaning** of the BH electric/magnetic charge configuration(s) supporting the BH attractor

→ Determined by the signature of the $2n_V$ x $2n_V$ covariant Hessian matrix of the *"effective BH potential"* $V_{BH}$, evaluated at the considered critical point (*attractor*)

$$H^{V_{BH}}_{\widehat{i}\widehat{j}}\Big|_{\partial V_{BH}=0} \equiv D_{\widehat{i}}D_{\widehat{j}}V_{BH}\left(z,\overline{z};\{q\},\{p\}\right)\Big|_{\partial V_{BH}=0}$$

$$H^{V_{BH}}_{\widehat{i}\widehat{j}}\Big|_{\partial V_{BH}=0} \underset{pos.\ def.}{>} 0$$

**All *eigenvalues* are strictly positive → the attractor is *stable***

(local *minimum* of $V_{BH}$)

$$H^{V_{BH}}_{\widehat{i}\widehat{j}}\Big|_{\partial V_{BH}=0} \underset{neg.\ def.}{\leqslant} 0$$

**All *eigenvalues* are strictly negative → the attractor is *unstable***

(local *maximum* of $V_{BH}$)

$$H^{V_{BH}}_{\widehat{i}\widehat{j}}\Big|_{\partial V_{BH}=0} \underset{not\ def.}{\gtrless} 0$$

**Eigenvalues have any sign : some >0, some <0**
**(eventually some = 0)**
**→ the attractor is a flex point of $V_{BH}$ , not properly stable**

***→The higher-order covariant ders. of $V_{BH}$ (at the considered crit. pts.) have to be studied to check stability***

## ❖ **Scalar manifolds** of symmetric N=2, d=4 MESGTs

| | $\frac{G_V}{H_V}$ | $r$ | $dim_{\mathbb{C}} \equiv n_V$ |
|---|---|---|---|
| *quadratic sequence* $n \in \mathbb{N}$ | $\frac{SU(1,n)}{U(1)\otimes SU(n)}$ | 1 | $n$ |
| $\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$ | $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,n)}{SO(2)\otimes SO(n)}$ | $2\ (n=1)$ <br> $3\ (n \geqslant 2)$ | $n+1$ |
| $J_3^{\mathbb{O}}$ | $\frac{E_{7(-25)}}{E_{6(-78)}\otimes U(1)}$ | 3 | 27 |
| $J_3^{\mathbb{H}}$ | $\frac{SO^*(12)}{U(6)}$ | 3 | 15 |
| $J_3^{\mathbb{C}}$ | $\frac{SU(3,3)}{S(U(3)\otimes U(3))} = \frac{SU(3,3)}{SU(3)\otimes SU(3)\otimes U(1)}$ | 3 | 9 |
| $J_3^{\mathbb{R}}$ | $\frac{Sp(6,\mathbb{R})}{U(3)}$ | 3 | 6 |

**SKG :**

$$R_{i\bar{j}k\bar{l}} = -g_{i\bar{j}}g_{k\bar{l}} - g_{i\bar{l}}g_{k\bar{j}} + C_{ikm}\overline{C}_{\overline{jlp}}g^{m\bar{p}}$$

$J_3^{\mathbb{A}}, \quad \mathbb{A} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ is the ***Jordan algebra*** of degree 3 of Hermitian 3x3 matrices over the 4 *division algebras* of real (**R**), complex (**C**), quaternions (**H**), octonions (**O**)

$\Gamma_{m,n}$ is the Jordan algebra of degree 2 with a quadratic form with Lorentzian signature (m,n), *i.e.* the Clifford algebra of SO(m,n)

Jordan algebras were completely classified by Jordan, Von Neumann and Wigner in an attempt to generalize *Quantum Mechanics* beyond **C**

***They are related to the Magic Square***

Freudenthal, Rozen, Tits

Gunaydin, Sierra, Townsend

| $J$ | $\mathrm{Aut}(J)$ | $\mathrm{Str}_0(J)$ | $\mathrm{Conf}(J)$ | $\mathrm{QConf}(J)$ |
|---|---|---|---|---|
| $\mathbb{R}$ | $1$ | $1$ | $Sl(2,\mathbb{R})$ | $G_{2(2)}$ |
| $\mathbb{R} \oplus \Gamma_{n-1,1}$ | $SO(n-1)$ | $SO(n-1,1)$ | $Sl(2) \times SO(n,2)$ | $SO(n+2,4)$ |
| $J_3^{\mathbb{R}}$ | $SO(3)$ | $Sl(3,\mathbb{R})$ | $Sp(6)$ | $F_{4(4)}$ |
| $J_3^{\mathbb{C}}$ | $SU(3)$ | $Sl(3,\mathbb{C})$ | $SU(3,3)$ | $E_{6(+2)}$ |
| $J_3^{\mathbb{H}}$ | $USp(6)$ | $SU^*(6)$ | $SO^*(12)$ | $E_{7(-5)}$ |
| $J_3^{\mathbb{O}}$ | $F_4$ | $E_{6(-26)}$ | $E_{7(-25)}$ | $E_{8(-24)}$ |

## Algebraic definition of symmetricity :

$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{k},$

Cartan decomposition of a Lie algebra *g*

*h* = compact Lie subalgebra

*k* = complementary algebra of h in g (*tangent space at the identity coset*)

$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}$ from the definition of subalgebra

$[\mathfrak{h}, \mathfrak{k}] \subset \mathfrak{k}$ by the adjoint, *h* acts on *k* as a repr. R whose real dim. is dim(G/H) (it holds in any coset G/H)

$[\mathfrak{k}, \mathfrak{k}] \subset \mathfrak{h}$ symmetricity condition; in gen. it simply holds $[\mathfrak{k}, \mathfrak{k}] \subset \mathfrak{g}$

## Differential definition of symmetricity :

$D_\lambda R_{\mu\nu\rho\sigma} = 0$ the Riemann tensor is covariantly constant

**rank r** = maximal real dimension of a flat *(i.e.* vanishing Riemann tensor), totally geodesic submanifold

All symmetric scalar manifolds in mupergravity are actually
**(I)RGS** = (**Irreducible) Riemannian Globally Symmetric Spaces**:

- strictly positive definite metric;
- *Einstein spaces*, with (constant) *negative* scalar curvature.

In N=2, d=4 MESGT, there are no massless Hessian modes at ½-BPS crit pts of $V_{BH}$

Define

$$H_{\widetilde{ij}}^{V_{BH}} \equiv \begin{pmatrix} D_i D_j V_{BH} & D_i \overline{D}_{\bar{j}} V_{BH} \\ \overline{D}_{\bar{i}} D_j V_{BH} & \overline{D}_{\bar{i}} \overline{D}_{\bar{j}} V_{BH} \end{pmatrix}$$

$2n_V$ x $2n_V$ Hessian matrix of $V_{BH}$

At ½-BPS crit pts :

Ferrara, Gibbons, Kallosh

$$H_{\widetilde{ij},\frac{1}{2}-BPS}^{V_{BH}} \equiv 2|Z|^2 \begin{pmatrix} 0 & g_{i\bar{j}} \\ g_{j\bar{i}} & 0 \end{pmatrix}$$

$2n_V$ eigenvs. >0

→ ***Stability OK***

At non-BPS Z<>0 crit pts : general result for cubic special Kaeheler geometries

Tripathy, Trivedi

$$\mathcal{F}(z) = d_{ijk} z^i z^j z^k, \;\; i,j,k = 1,\dots,n_V$$

SPLIT : $2n_V$→ $n_V$ +1 eigenvs >0 + $n_V$ -1 eigenvs =0 (massless Hessian modes)

Vanishing eigenvalues (*i.e. massless Hessian modes*) are *ubiquitous* at non-BPS crit pts of $V_{BH}$, *whose actual* ***stability*** *must be checked*

The fundamental identities of **Special Kaehler Geometry** read as follows :

$$\begin{aligned} \overline{D}_{\bar{i}} Z &= 0; \\ D_i D_j Z &= i C_{ijk} g^{k\bar{k}} \overline{D}_{\bar{k}} \overline{Z}; \\ \overline{D}_{\bar{j}} D_i Z &= g_{i\bar{j}} Z, \end{aligned}$$

$$\begin{aligned} \overline{D}_{\bar{l}} C_{ijk} &= 0; \\ D_{[l} C_{i]jk} &= 0. \end{aligned}$$

Duality invariant quantities are those that remain unchanged under symultaneous action of the U-duality group G on the BH charges as well as on the scalar fields. The complete basis of local duality invariants *in any* SKG has been found in the first paper with Bruno :

Cerchiai, Ferrara, AM, Zumino

$$i_1 = Z\overline{Z}$$

$$i_2 = g^{i\bar{j}} Z_i \overline{Z}_{\bar{j}} \qquad \left( Z_i = D_i Z \ , \ \overline{Z}_{\bar{i}} = \overline{D}_{\bar{i}} \overline{Z} \right),$$

$$i_3 = \frac{1}{6} \left[ Z N_3(\overline{Z}) + \overline{Z} \overline{N}_3(Z_i) \right], \qquad i_4 = \frac{i}{6} \left[ Z N_3(\overline{Z}) - \overline{Z} \overline{N}_3(Z) \right],$$

$$i_5 = g^{i\bar{i}} C_{ijk} C_{\bar{i}\bar{j}\bar{k}} \overline{Z}^j \overline{Z}^k Z^{\bar{j}} Z^{\bar{k}},$$

where the cubic norms are given by

$$N_3(\overline{Z}) = C_{ijk} \overline{Z}^i \, \overline{Z}^j \, \overline{Z}^k, \qquad \overline{N}_3(Z) = C_{\bar{i}\bar{j}\bar{k}} Z^{\bar{i}} \, Z^{\bar{j}} \, Z^{\bar{k}}.$$

A nice result, also obtained in the first paper with Bruno, concerns the determination of a combination of such local duality invariants which actually is independent on scalar fields, and thus can be identified with the unique quartic invariant polynomial $I_4$ of the representation R of the U-duality group G in which the BH charges sit :

$$I_4 = (i_1 - i_2)^2 + 4i_4 - i_5 \, ,$$

Cerchiai, Ferrara, AM, Zumino

Or, equivalently :

$$I_4 = \left(Z\overline{Z} - Z_i\overline{Z}^i\right)^2 + \frac{2}{3}i\left[ZN_3\left(\overline{Z}\right) - \overline{ZN_3}\left(Z\right)\right] + g^{i\overline{j}}C_{ikl}C_{\overline{jmn}}\overline{Z}^k\overline{Z}^l Z^{\overline{m}}Z^{\overline{n}}$$

Despite its various terms do depend on scalars, $I_4$ as a whole ***does NOT*** **:**

$$\boxed{\partial_\varphi I_4 = 0 = \partial_{\overline{\varphi}} I_4}$$

For instance, an equivalent expression (manifestly scalar-independent) reads (in the so-called 4D/5D special coordinates' symplectic frame) :

$$I_4 = -\left(p^0 q_0 + p^i q_i\right)^2 + 4q_0 I_3(p) - 4p^0 I_3(q) + 4\frac{\partial I_3(p)}{\partial p^i}\frac{\partial I_3(q)}{\partial q_i}$$

$$I_3(p) := \frac{1}{3!}d_{ijk}p^i p^j p^k, \quad I_3(q) := \frac{1}{3!}d^{ijk}q_i q_j q_k$$

Ferrara, Gunaydin

**symmetric** scalar manifolds **G/H** (including symm. SKGs of N=2, D=4 sugra) :

The various classes of crit pts of $V_{BH}$ are supported by charges belonging to **non-degenerate charge orbits** of the relevant representation R of G, determining its *embedding* into the *symplectic group* $Sp(2n_V+2,\mathbf{R})$

The G-representation space R of the BH em charges gets **stratified**, under the action of G, in G-orbits (non-symmetric cosets **G/$\mathcal{H}$** ). Ferrara, Gunaydin

$\mathcal{H}$ is the **stabilizer** (isotropy) group of the orbit = symmetry of the charge configs., it relates equivalent BH charge configs

each G-orbit supports a class of crit. pts. of $V_{BH}$ , corresponding to specific SUSY-preserving properties of the near-horizon geometry

[ We will be considering the so-called “large” G-orbits, corresponding to extremal BHs with classical non-vanishing entropy ]

When $\mathcal{H}$ is **non-compact**, there is a residual compact symmetry linearly acting on scalars, such that the scalars belonging to the ***“moduli space” $\mathcal{H}$/mcs($\mathcal{H}$)*** (symmetric submanifold of **G/H**) are **not** stabilized in terms of BH charges at the event horizon of the extremal BH

Ferrara, AM

The Attractor Mechanism is ***inactive*** on these unstabilized scalar fields, which are ***flat directions*** of $V_{BH}$ at its critical points.

**symmetric** scalar manifolds **G/H** (cont' d) :

The **absence** of flat directions at **N=2 ½-BPS attractors** can thus be explained by the fact that the stabilizer of the ½-BPS orbit is **compact** : **$\mathcal{H}$=H/U(1)**, where H is the stabilizer of the scalar manifold **G/H**

The massless Hessian modes, ubiquitous at non-BPS crit pts of $V_{BH}$, are actually **all flat directions** of $V_{BH}$ itself at the considered class of crit. pts.

In other words, *at each class of its crit pts*, $V_{BH}$, and thus the classical **Bekenstein-Hawking BH entropy**, <u>does **not** depend on a certain subset of the scalars</u>

Such a set of scalars is thus **not stabilized** at the BH event horizon. Nevertheless…

***BH entropy is independent on all unstabilized scalars***

Thus, the **flat directions** of $V_{BH}$ at its critical points span various "***moduli spaces***", related to the solutions of the *classical Attractor Eqs*.

## ❖ "Large" **charge orbits** of symmetric N=2, d=4 MESGTs

**Ferrara,Gunaydin**
**Bellucci,Ferrara,Gunaydin,AM**

| | $\frac{1}{2}$-BPS orbits $\mathcal{O}_{\frac{1}{2}-BPS}=\frac{G}{H_0}$ | non-BPS, $Z\neq 0$ orbits $\mathcal{O}_{non-BPS,Z\neq 0}=\frac{G}{\hat{H}}$ | non-BPS, $Z=0$ orbits $\mathcal{O}_{non-BPS,Z=0}=\frac{G}{\tilde{H}}$ |
|---|---|---|---|
| *Quadratic Sequence* $(n=n_V\in\mathbb{N})$ | $\frac{SU(1,n)}{SU(n)}$ | $-$ | $\frac{SU(1,n)}{SU(1,n-1)}$ |
| $\mathbb{R}\oplus\Gamma_n$ $(n=n_V-1\in\mathbb{N})$ | $\frac{SU(1,1)\otimes SO(2,n)}{SO(2)\otimes SO(n)}$ | $\frac{SU(1,1)\otimes SO(2,n)}{SO(1,1)\otimes SO(1,n-1)}$ | $\frac{SU(1,1)\otimes SO(2,n)}{SO(2)\otimes SO(2,n-2)}$ |
| $J_3^{\mathbb{O}}$ | $\frac{E_{7(-25)}}{E_6}$ | $\frac{E_{7(-25)}}{E_{6(-26)}}$ | $\frac{E_{7(-25)}}{E_{6(-14)}}$ |
| $J_3^{\mathbb{H}}$ | $\frac{SO^*(12)}{SU(6)}$ | $\frac{SO^*(12)}{SU^*(6)}$ | $\frac{SO^*(12)}{SU(4,2)}$ |
| $J_3^{\mathbb{C}}$ | $\frac{SU(3,3)}{SU(3)\otimes SU(3)}$ | $\frac{SU(3,3)}{SL(3,\mathbb{C})}$ | $\frac{SU(3,3)}{SU(2,1)\otimes SU(1,2)}$ |
| $J_3^{\mathbb{R}}$ | $\frac{Sp(6,\mathbb{R})}{SU(3)}$ | $\frac{Sp(6,\mathbb{R})}{SL(3,\mathbb{R})}$ | $\frac{Sp(6,\mathbb{R})}{SU(2,1)}$ |

**in N=2 :** $\left\{Q_\alpha^A,Q_\beta^B\right\}=\epsilon_{\alpha\beta}Z^{[AB]}=\epsilon_{\alpha\beta}\epsilon^{AB}Z$

## ❖ Non-BPS Z<>0 **moduli spaces** of symmetric N=2, d=4 MESGTs

| | $\frac{\hat{H}}{\hat{h}}$ | r | $dim_{\mathbb{R}}$ |
|---|---|---|---|
| $\mathbb{R} \oplus \Gamma_n$ $(n = n_V - 1 \in \mathbb{N})$ | $SO(1,1) \otimes \frac{SO(1,n-1)}{SO(n-1)}$ | $1 (n=1)$ $2 (n \geqslant 2)$ | $n$ |
| $J_3^{\mathbb{O}}$ | $\frac{E_{6(-26)}}{F_{4(-52)}}$ | 2 | 6 |
| $J_3^{\mathbb{H}}$ | $\frac{SU^*(6)}{USp(6)}$ | 2 | 14 |
| $J_3^{\mathbb{C}}$ | $\frac{SL(3,C)}{SU(3)}$ | 2 | 8 |
| $J_3^{\mathbb{R}}$ | $\frac{SL(3,\mathbb{R})}{SO(3)}$ | 2 | 5 |

$\hat{h}$ =mcs $\hat{H}$

**Ferrara,AM**

They are nothing but the *real special* scalar manifolds of symmetric N=2, **d=5** MESGTs

## ❖ Non-BPS Z=0 moduli spaces of symmetric N=2, d=4 MESGTs

| | $\frac{\tilde{H}}{\tilde{h}} = \frac{\tilde{H}}{\tilde{h}' \otimes U(1)}$ | r | $dim_{\mathbb{C}}$ |
|---|---|---|---|
| $Quadratic\ \ Sequence$<br>$(n = n_V \in \mathbb{N})$ | $\frac{SU(1,n-1)}{U(1) \otimes SU(n-1)}$ | 1 | $n-1$ |
| $\mathbb{R} \oplus \Gamma_n$<br>$(n = n_V - 1 \in \mathbb{N})$ | $\frac{SO(2,n-2)}{SO(2) \otimes SO(n-2)}, n \geqslant 3$ | $1(n=3)$<br>$2(n \geqslant 4)$ | $n-2$ |
| $J_3^{\mathbb{O}}$ | $\frac{E_{6(-14)}}{SO(10) \otimes U(1)}$ | 2 | 16 |
| $J_3^{\mathbb{H}}$ | $\frac{SU(4,2)}{SU(4) \otimes SU(2) \otimes U(1)}$ | 2 | 8 |
| $J_3^{\mathbb{C}}$ | $\frac{SU(2,1)}{SU(2) \otimes U(1)} \otimes \frac{SU(1,2)}{SU(2) \otimes U(1)}$ | 2 | 4 |
| $J_3^{\mathbb{R}}$ | $\frac{SU(2,1)}{SU(2) \otimes U(1)}$ | 1 | 2 |

Generally, they are non-special symmetric Kaehler manifolds

**Ferrara, AM**

To recap :

**<u>all</u>** non-degenerate crit pts of $V_{BH}$ in *symmetric* N=2,d=4 MESGTs are **<u>stable</u>** (and thus determine extremal BH attractors in strict sense):

✓<u>with **no** flat directions at all</u> in ½-BPS class (indeed, the stabilizer of the corresponding supporting charge orbits is compact);

✓<u>with some flat directions</u>, spanning the related **<u>moduli space</u>** of <u>unstabilized</u> scalar degrees of freedom, in non-BPS (with Z<>0 and Z=0) classes.

# What about Extended Supergravities ?

In **<u>N>2-extended</u>**, d=4 supergravities, <u>also</u> non-degenerate 1/N-BPS extremal BH attractors exhibit a related ***moduli space***.
The same reasoning as above can be made, because **all** N>2-extended, d=4 supergravities have symmetric scalar manifolds.

There are three classes of *non-degenerate* crit. pts. of $V_{BH}$ :

- ❑ 1/N-BPS;
- ❑ non-BPS with non-vanishing <u>central charge matrix</u> $Z_{AB}$ (A,B=1,…,N);
- ❑ non-BPS with $Z_{AB}$=0.

Once again, **all** classes of crit pts of $V_{BH}$ are **<u>stable</u>**, up to some ubiquitous <u>flat directions</u>, spanning the related symmetric **<u>moduli spaces</u>**.

## ❖ **Scalar manifolds** of N>2-extended, d=4 supergravities

| $\mathcal{N}$ | $G_{\mathcal{N},4}/H_{\mathcal{N},4}$ |
|---|---|
| 3 | $\mathbf{III}_{3,n} : \frac{SU(3,n)}{SU(3)\otimes SU(n)\otimes U(1)}, \quad n \in \mathbb{N}$ |
| 4 | $\mathbf{III}_{1,1} \otimes IV_{6,n} : \frac{SU(1,1)}{U(1)} \otimes \frac{SO(6,n)}{SO(6)\otimes SO(n)}, \quad n \in \mathbb{N}\cup\{0\} \quad (\mathbb{R}\oplus\mathbf{\Gamma}_{n-1,5})$ |
| 5 | $\mathbf{III}_{1,5} : \frac{SU(1,5)}{SU(5)\otimes U(1)} \quad (M_{1,2}(\mathbb{O}))$ |
| 6 | $\mathbf{V}_6 : \frac{SO^*(12)}{SU(6)\otimes U(1)} \quad (J_3^{\mathbb{H}})$ |
| 8 | $\mathbf{5} : \frac{E_{7(7)}}{SU(8)} \quad \left(J_3^{\mathbb{O}_s}\right)$ |

## ❖ "Large" **Charge orbits** of N>2-extended, d=4 supergravities

| | $\frac{1}{\mathcal{N}}$-BPS orbits $\frac{G}{\mathcal{H}}$ | non-BPS, $Z_{AB} \neq 0$ orbits $\frac{G}{\mathcal{H}}$ | non-BPS, $Z_{AB} = 0$ orbits $\frac{G}{\mathcal{H}}$ |
|---|---|---|---|
| $\mathcal{N} = 3$ | $\frac{SU(3,n)}{SU(2,n)}$ | $-$ | $\frac{SU(3,n)}{SU(3,n-1)}$ |
| $\mathcal{N} = 4$ | $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(6,n)}{SO(4,n)}$ | $\frac{SU(1,1)}{SO(1,1)} \otimes \frac{SO(6,n)}{SO(5,n-1)}$ | $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(6,n)}{SO(6,n-2)}$ |
| $\mathcal{N} = 5$ | $\frac{SU(1,5)}{SU(3) \otimes SU(2,1)}$ | $-$ | $-$ |
| $\mathcal{N} = 6$ | $\frac{SO^*(12)}{SU(4,2)}$ | $\frac{SO^*(12)}{SU^*(6)}$ | $\frac{SO^*(12)}{SU(6)}$ |
| $\mathcal{N} = 8$ | $\frac{E_{7(7)}}{E_{6(2)}}$ | $\frac{E_{7(7)}}{E_{6(6)}}$ | $-$ |

n=# *matter* (*vector*) *multiplets* (*matter coupling* possible only for N=3,4)

N=6 *pure* sugra is "dual" to N=2 *matter coupled* magic sugra on quaternions H

# ❖ **Moduli spaces** of attractors in N>2-extended, d=4 supergravities

| | $\frac{1}{\mathcal{N}}$-BPS moduli space $\frac{\mathcal{H}}{\mathfrak{h}}$ | non-BPS, $Z_{AB} \neq 0$ moduli space $\frac{\widehat{\mathcal{H}}}{\widehat{\mathfrak{h}}}$ | non-BPS, $Z_{AB} = 0$ moduli space $\frac{\widetilde{\mathcal{H}}}{\widetilde{\mathfrak{h}}}$ |
|---|---|---|---|
| $\mathcal{N}=3$ | $\frac{SU(2,n)}{SU(2)\otimes SU(n)\otimes U(1)}$ | $-$ | $\frac{SU(3,n-1)}{SU(3)\otimes SU(n-1)\otimes U(1)}$ |
| $\mathcal{N}=4$ | $\frac{SO(4,n)}{SO(4)\otimes SO(n)}$ | $SO(1,1)\otimes\frac{SO(5,n-1)}{SO(5)\otimes SO(n-1)}$ | $\frac{SO(6,n-2)}{SO(6)\otimes SO(n-2)}$ |
| $\mathcal{N}=5$ | $\frac{SU(2,1)}{SU(2)\otimes U(1)}$ | $-$ | $-$ |
| $\mathcal{N}=6$ | $\frac{SU(4,2)}{SU(4)\otimes SU(2)\otimes U(1)}$ | $\frac{SU^*(6)}{USp(6)}$ | $-$ |
| $\mathcal{N}=8$ | $\frac{E_{6(2)}}{SU(6)\otimes SU(2)}$ | $\frac{E_{6(6)}}{USp(8)}$ | $-$ |

$\mathfrak{h}$, $\widehat{\mathfrak{h}}$ and $\widetilde{\mathfrak{h}}$ are maximal compact subgroups of

$\mathcal{H}$, $\widehat{\mathcal{H}}$ and $\widetilde{\mathcal{H}}$, respectively, and $n$ is the number of matter multiplets

Many Developments on **Attractors and Supergravity** not touched in the talk, *e.g.* :

✓ Explicit solutions to the classical and quantum corrected attractor equations…

✓ Stringy realization of supergravities, and solution of attractor eqs. in non-homogeneous SKG [*Basins of attraction*, *Area codes*,…] …

✓ Connection between BH entropy and quantum entanglement, and the role of attractor mechanism (Black Hole – Qubit Correspondence)…

✓ First order formalism for non-BPS extremal and non-extremal BHs…

✓ Multi-centered BH solutions, and *"horizontal symmetry"*…

✓ Gauged supergravities and asymptotically non-flat (AdS) BHs…

*…and many others…*

# Recent Results and Open Questions…

- *Higher Derivative Contributions* to BH entropy, and relation with *attractor mechanism*...
  ***…is actually attractor mechanism still there?...***

- *The gauging of N=2 MESGT and the role of hypermultiplets in the attractor mechanism…*

- *BHs, attractors and entanglement in Quantum Information Theory* : ***mere mathematical analogy or something deeper?...***

- Multi-centered BHs and intersecting black p-branes' configurations:
  → *explicit solutions for non-BPS attractor flows…*
  → *existence of marginal stability walls for non-BPS flows…*
  → *SUSY-preserving features in marginal decays…*

***…and many others...***
***….Bruno's legacy and intellectual heritage is still rich and alive…***

# *Grazie Bruno!*

…and thank you !