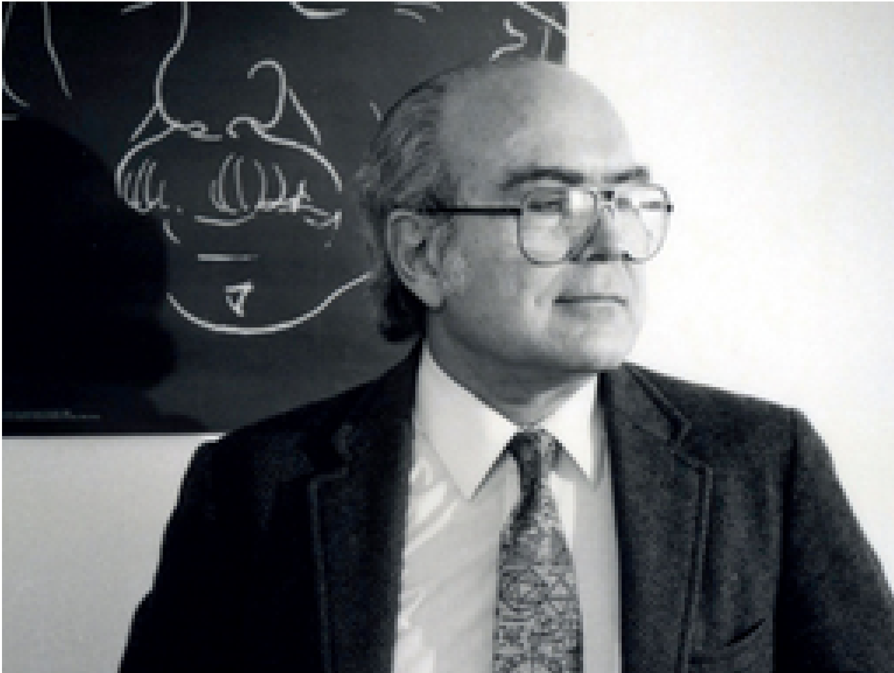
BRUNO ZUMINO

Memorial Meeting

CERN, April 27-28 2015

John Iliopoulos

ENS Paris



BRUNO ZUMINO

1923-2014

He graduated from the University of Rome in 1945

Sixty years of intense life in Theoretical Physics

Probably more than 180 publications (The exact number is not known!)

The earliest paper I found

Relativistic Heisenberg picture

by **Bruno Zumino.**
Centro di Studio per la Fisica Nucleare del Consiglio Nazionale delle Ricerche, Roma.

Internationaler Kongress über Kernphysik und Quantenelektrodynamik,
Basel, 5. bis 9. September 1949
Helvetica physica acta, 23, Suppl. 3 (1950) 243-247

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- Other problems in Theoretical Physics:
  "Some Questions in Relativistic Hydromagnetics" (1957)
  "Formal Solution of the Equations of Statistical Equilibrium" (1959)

CPT

Zumino's contributions have been largely ignored:

- "On the equivalence of invariance under time reversal and under particle-antiparticle conjugation in relativistic field theories"
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- Some Consequences of TCP-Invariance
  Gerhart Lüders (MIT), Bruno Zumino (Stevens Tech.)
  Apr 1957. 2 pp.
  Phys.Rev. 106 (1957) 385-386

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  - Current Algebra and Algebra of Fields
  - Phenomenological Lagrangians
  - Chiral dynamics
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- **1990-2014**
  - SUSY and SUGRA
  - String theory
  - Quantum groups and deformations
  - Non commutative geometry

GAUGE THEORIES
AND

NON-COMMUTATIVE GEOMETRY

Motivation

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- 4) Large N gauge theories and matrix models.

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- 3) Seiberg-Witten map.
- 4) Large N gauge theories and matrix models.
- 5) The construction of gauge theories using the techniques of non-commutative geometry.

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  $\partial^\mu x_\nu = \delta^\mu_\nu \qquad [x_\mu, f(x)] = i\theta_{\mu\nu}\partial^\nu f(x)$
- Define a * product
  $f * g = e^{\frac{i}{2}\frac{\partial}{x_\mu}\theta_{\mu\nu}\frac{\partial}{y_\nu}} f(x) g(y)|_{x=y}$

All computations can be viewed as expansions in θ

expansions in the external field

More efficient ways?

Large N field theories

- $\phi^i(x)\ i = 1, ..., N\ ;\ N \to \infty$

  $\phi^i(x) \to \phi(\sigma, x)\ 0 \leq \sigma \leq 2\pi$

  $\sum_{i=1}^{\infty} \phi^i(x)\phi^i(x) \to \int_0^{2\pi} d\sigma(\phi(\sigma, x))^2$

  but

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- For a Yang-Mills theory, the resulting expression is local

Gauge theories on surfaces

E.G. Floratos and J.I.

- Given an $SU(N)$ Yang-Mills theory in a $d-$dimensional space

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Gauge theories on surfaces

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- Given an $SU(N)$ Yang-Mills theory in a $d-$dimensional space

$$A_\mu(x) \ = \ A_\mu^a(x) \ t_a$$

- there exists a reformulation in $d+2$ dimensions

$$A_\mu(x) \to \mathcal{A}_\mu(x, z_1, z_2) \qquad F_{\mu\nu}(x) \to \mathcal{F}_{\mu\nu}(x, z_1, z_2)$$

with $$[z_1, z_2] = \frac{2i}{N}$$

$$[A_\mu(x), A_\nu(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \mathcal{A}_\nu(x, z_1, z_2)\}_{Moyal}$$

$$[A_\mu(x), \Omega(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \dot{}(x, z_1, z_2)\}_{Moyal}$$

$$\int d^4x \;\; Tr\,(F_{\mu\nu}(x) F^{\mu\nu}(x)) \quad \to \quad \int d^4x dz_1 dz_2 \; \mathcal{F}_{\mu\nu}(x, z_1, z_2) * \mathcal{F}^{\mu\nu}(x, z_1, z_2)$$

I. Large N

-A simple algebraic result:

At large N

The $SU(N)$ algebra $\rightarrow$ The algebra of the area preserving diffeomorphisms of a closed surface. (sphere or torus).

-The structure constants of $[SDiff(S^2)]$ are the limits for large N of those of $SU(N)$.

-Alternatively: For the sphere

$$x_1 = cos\phi \; sin\theta, \quad x_2 = sin\phi \; sin\theta, \quad x_3 = cos\theta$$

$$Y_{l,m}(\theta,\phi) = \sum_{\substack{i_k=1,2,3\\ k=1,\dots,l}} \alpha^{(m)}_{i_1\dots i_l} \; x_{i_1}\dots x_{i_l}$$

where $\alpha^{(m)}_{i_1\dots i_l}$ is a symmetric and traceless tensor.
For fixed l there are $2l+1$ linearly independent tensors $\alpha^{(m)}_{i_1\dots i_l}$, $m=-l,\dots,l$.

Choose, inside $SU(N)$, an $SU(2)$ subgroup.

$$[S_i, S_j] = i\epsilon_{ijk} S_k$$

A basis for $SU(N)$:

$$S^{(N)}_{l,m} = \sum_{\substack{i_k=1,2,3 \\ k=1,\dots,l}} \alpha^{(m)}_{i_1\dots i_l}\ S_{i_1}\dots S_{i_l}$$

$$[S^{(N)}_{l,m},\ S^{(N)}_{l',m'}] = i f^{(N)l'',m''}_{l,m;\ l',m'}\ S^{(N)}_{l'',m''}$$

The three $SU(2)$ generators S_i, rescaled by a factor proportional to $1/N$, will have well-defined limits as N goes to infinity.

$$S_i \to T_i = \frac{2}{N} S_i$$
$$[T_i, T_j] = \frac{2i}{N} \epsilon_{ijk} T_k$$
$$T^2 = T_1^2 + T_2^2 + T_3^2 = 1 - \frac{1}{N^2}$$

In other words: under the norm $\|x\|^2 = Trx^2$, the limits as N goes to infinity of the generators T_i are three objects x_i which commute and are constrained by

$$x_1^2 + x_2^2 + x_3^2 = 1$$

$$\frac{N}{2i}\,[f,g] \to \quad \epsilon_{ijk}\; x_i\; \frac{\partial f}{\partial x_j}\frac{\partial g}{\partial x_k}$$

$$\frac{N}{2i}\,[T^{(N)}_{l,m}, T^{(N)}_{l',m'}] \to \;\{Y_{l,m}, Y_{l',m'}\}$$

$$N[A_\mu, A_\nu] \to \quad \{A_\mu(x,\theta,\phi), A_\nu(x,\theta,\phi)\}$$

II. To all orders

We can parametrise the T_i's in terms of two operators, z_1 and z_2.

$$T_+ = T_1 + iT_2 = e^{\frac{iz_1}{2}} \, (1 - z_2^2)^{\frac{1}{2}} \, e^{\frac{iz_1}{2}}$$
$$T_- = T_1 - iT_2 = e^{-\frac{iz_1}{2}} \, (1 - z_2^2)^{\frac{1}{2}} \, e^{-\frac{iz_1}{2}}$$
$$T_3 = z_2$$

If we assume that z_1 and z_2 satisfy:

$[z_1, z_2] = \frac{2i}{N}$

The T_i's satisfy the $SU(2)$ algebra.

If we assume that the T_i's satisfy the $SU(2)$ algebra, the z_i's satisfy the Heisenberg algebra

The techniques of non-com. geometry

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- Question: Is there a space on which Internal symmetry transformations act as Diffeomorphisms?
- Answer: Yes, but it is a space with non-commutative geometry. A space defined by an algebra of matrix-valued functions

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- *New predictions for the B.E.H. mass?*

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- Answer: NO! There is no fixed point in the renormalisation group equations.
- Related question: Is there a B.R.S. symmetry for this model?

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- Whether it will turn out to be convenient for us to use is still questionable.
- It will depend on our ability to simplify the mathematics sufficiently, or to master them deeply, in order to get new insights
- We need somebody with knowledge and imagination