The Anomalous Bruno Zumino

William A. Bardeen
Scientist Emeritus
Fermilab


William Bardeen
Fermi National Accelerator Laboratory
Bruno Zumino
University of California, Berkeley

Bruno Zumno - Most Cited Works

1. **Supergauge Transformations in Four-Dimensions**
(2175) J. Wess (Karlsruhe U.), B. Zumino (CERN). 1974.
Published in **Nucl.Phys. B70 (1974) 39-50**
DOI: 10.1016/0550-3213(74)90355-1
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 2175 records 1000+

2. **Consequences of anomalous Ward identities**
(2147) J. Wess (CERN & Karlsruhe U.), B. Zumino (CERN). 1971.
Published in **Phys.Lett. B37 (1971) 95**
DOI: 10.1016/0370-2693(71)90582-X
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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Detailed record - Cited by 2147 records 1000+

Origin of Quantum Anomalies

- The Neutral Pion Lifetime
  - H. Fukuda and Y. Miyamoto, Prog.Theor.Phys. IV-3,347(1949)
  - J. Steinberger, Phys.Rev. 76-8,1180(1949)
  - J. Schwinger, Phys.Rev. 82, 664 (1951)

On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay

J. STEINBERGER*
The Institute for Advanced Study, Princeton, New Jersey
(Received June 13, 1949)

The method of subtraction fields in current meson perturbation theory is described, and it is shown that it leads to finite results in all processes. The method is, however, not without ambiguities, and these are stated. It is then applied to the following problems in meson decay: Decay of a neutral meson into two and three γ-rays, into a positron-electron pair, and into another neutral meson and photon; decay of a charged meson into another charged meson and a photon, and into an electron (or μ-meson) and neutrino. The lifetimes are tabulated in Tables I, II and III. The results are quite different from those of previous calculations, in all those cases in which divergent and conditionally convergent integrals occur before subtraction, but identical whenever divergences are absent. The results are discussed in the light of recent experimental evidence.

Foundations in Quantum Field Theory

- Axial-Vector anomalies in quantum electrodynamics, nonrenormalization
  - S.L. Adler, Phys.Rev. 177,2426(1969)
  - S.L. Adler, W. Bardeen, Phys.Rev. 182,1517(1969)

Axial-Vector Vertex in Spinor Electrodynamics

STEPHEN L. ADLER
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 24 September 1968)

Working within the framework of perturbation theory, we show that the axial-vector vertex in spinor electrodynamics has anomalous properties which disagree with those found by the formal manipulation of field equations. Specifically, because of the presence of closed-loop "triangle diagrams," the divergence of axial-vector current is not the usual expression calculated from the field equations, and the axial-vector current does not satisfy the usual Ward identity. One consequence is that, even after the external-line wave-function renormalizations are made, the axial-vector vertex is still divergent in fourth- (and higher-) order perturbation theory. A corollary is that the radiative corrections to $\nu_l l$ elastic scattering in the local current-current theory diverge in fourth (and higher) order. A second consequence is that, in massless electrodynamics, despite the fact that the theory is invariant under γ_5 transformations, the axial-vector current is not conserved. In an Appendix we demonstrate the uniqueness of the triangle diagrams, and discuss a possible connection between our results and the $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$ decays. In particular, we argue that as a result of triangle diagrams, the equations expressing partial conservation of axial-vector current (PCAC) for the neutral members of the axial-vector-current octet must be modified in a well-defined manner, which completely alters the PCAC predictions for the π^0 and the η two-photon decays.

Foundations in Quantum Field Theory

- Chiral symmetry and pion decay in the sigma model
  - J.S. Bell and R. Jackiw, IL Nuovo Cimento, LX-1, 47(1969)

A PCAC Puzzle: $\pi^0 \to \gamma\gamma$ in the σ-Model.

J. S. Bell

CERN - Geneva

R. Jackiw (*)

CERN - Geneva

Jefferson Laboratory of Physics, Harvard University - Cambridge, Mass.

(ricevuto l'11 Settembre 1968)

Summary. — The effective coupling constant for $\pi^0 \to \gamma\gamma$ should vanish for zero pion mass in theories with PCAC and gauge invariance. It does not so vanish in an explicit perturbation calculation in the σ-model. The resolution of the puzzle is effected by a modification of Pauli-Villars-Gupta regularization which respects both PCAC and gauge invariance.

Fundamental Structure of Chiral Anomalies

- General form of nonabelian anomalies.
  - W. Bardeen, Phys.Rev. 184,1848(1969)

Anomalous Ward Identities in Spinor Field Theories

WILLIAM A. BARDEEN*†
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 24 February 1969)

We consider the model of a spinor field with arbitrary internal degrees of freedom having arbitrary nonderivative coupling to external scalar, pseudoscalar, vector, and axial-vector fields. By carefully defining the S matrix in the interaction picture, the vector and axial-vector currents associated with the external vector and axial-vector fields are found to satisfy anomalous Ward identities. If we require that the vector currents satisfy the usual Ward identities, the divergence of the axial-vector current contains well-defined anomalous terms. These terms are explicitly calculated.

Fundamental Structure of Chiral Anomalies

Consistency conditions, Wess-Zumino terms
- J. Wess and B. Zumino, Phys.Lett. 37B-1, 95(1971)

CONSEQUENCES OF ANOMALOUS WARD IDENTITIES

J. WESS
CERN, Geneva, Switzerland
and
University of Karlsruhe, Karlsruhe, Germany

and

B. ZUMINO
CERN, Geneva, Switzerland

Received 7 September 1971

The anomalies of Ward identities are shown to satisfy consistency or integrability relations, which restrict their possible form. For the case of SU(3) × SU(3) we verify that the anomalies given by Bardeen satisfy the consistency relations. A solution of the anomalous Ward identities is also given which describes concisely all anomalous contributions to low energy theorems. The contributions to strong five pseudoscalar interactions, to K_{l4}, to one- and two-photon interactions with three pseudoscalars are explicitly exhibited.

Wess-Zumino Consistency Conditions

In this note we observe that the anomalies must satisfy consistency or integrability relations which follow from the structure of the gauge group and which are non trivial in the case of non-abelian groups. We also give explicitly a local functional satisfying the anomalous Ward identities. This functional can be interpreted as an effective action which describes all modifications to the low energy theorems due to the presence of the anomalies. We treat in detail the case of

$$X_i(x)\,G_j(x') = f_{ijk}\,\delta(x - x')G_k(x)\ ,$$
$$Y_i(x)\,G_j(x') - Y_j(x')\,G_i(x) = 0\ .$$

The first of these integrability conditions states simply that G_i belongs to an octet and is obviously satisfied by Bardeen's expression. The second is more subtle; it can be verified, by explicit calculation, that it is also satisfied. Conversely, one can show that this consistency relation can be used to determine all other terms in G_i if one knows its first term, that proportional to $\epsilon_{\mu\nu\sigma\tau}$ $\mathrm{tr}\lambda_i V_{\mu\nu} V_{\sigma\tau}$, which must occur, as we know from Adler's abelian case. In this precise sense Bar-

Wess-Zumino Nonlinear Chiral Lagrangian

Assuming $W[0,\ V_\mu, A_\mu] = 0$, we find the solution

$$W[\xi,\ V_\mu, A_\mu] = \frac{1 - \exp(\xi\cdot U)}{\xi\cdot U}\,\xi\cdot G[V_\mu, A_\mu] =$$

$$= \int_0^1 d\ell\, \exp(-\ell\,\xi\cdot U)\,\xi\cdot G\ .$$

We have also verified explicitly, using the infinitesimal transformation properties of the fields ξ, V_μ and A_μ, that this local functional satisfies the anomalous Ward identities. In performing this check the consistency conditions for G_i have to be used.

The above explicit solution for $W[\xi, V_\mu, A_\mu]$ can be used as an effective Lagrangian giving directly all many particle vertices which describe the low energy consequences of the anomalous terms in the Ward identities. We work out these

Wess-Zumino Term

observe that our effective action contains vertices describing purely strong interactions among pseudoscalar mesons. The simplest of these is the five pseudoscalar vertex, given by

$$\frac{1}{6\pi^2 F_\pi^5}\epsilon_{\mu\nu\sigma\tau}\,\mathrm{tr}(\Pi\partial_\mu\Pi\partial_\nu\Pi\partial_\sigma\Pi\partial_\tau\Pi), \quad \Pi = \tfrac{1}{2}\Pi_i\lambda_i\ .$$

Vertices with more than five pseudoscalars can be worked out just as easily. Notice that this vertex cannot be obtained as part of a chiral invariant.

Instantons and Anomalies

- A.M. Polyakov, Phys.Lett. 59B, 82 (1975)
- A.A. Belavin, et al, Phys.Lett. 59B, 85 (1975)
- G. ‘t Hooft, Phys.Rev. D14, 3432 (1976)

Computation of the quantum effects due to a four-dimensional pseudoparticle*

G. 't Hooft†

Physics Laboratories, Harvard University, Cambridge, Massachusetts 02138

(Received 28 June 1976)

A detailed quantitative calculation is carried out of the tunneling process described by the Belavin-Polyakov-Schwarz-Tyupkin field configuration. A certain chiral symmetry is violated as a consequence of the Adler-Bell-Jackiw anomaly. The collective motions of the pseudoparticle and all contributions from single loops of scalar, spinor, and vector fields are taken into account. The result is an effective interaction Lagrangian for the spinors.

- Nonperturbative quantum chromodynamics
- Zero modes and anomaly structure

Wess-Zumino Nonlinear Chiral Lagrangian
- E. Witten, Nucl.Phys. 223,422, (1983)

GLOBAL ASPECTS OF CURRENT ALGEBRA

Edward WITTEN*

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 4 March 1983

A new mathematical framework for the Wess-Zumino chiral effective action is described. It is shown that this action obeys an a priori quantization law, analogous to Dirac's quantization of magnetic change. It incorporates in current algebra both perturbative and non-perturbative anomalies.

- Topology in five dimensional space-time
- Topological index in quantum chromodynamics: n=N_colors

Wess-Zumino Nonlinear Chiral Lagrangian
- J. Maldacena, Adv.Theor.Math.Phys. 2, 231 (1998)

The Large N Limit of Superconformal field theories and supergravity [1]

Juan Maldacena[2]

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low energy limit where the field theory on the brane decouples from the bulk. We observe that, in this limit, we can still trust the near horizon geometry for large N. The enhanced supersymmetries of the near horizon geometry correspond to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincare group). The 't Hooft limit of 3+1 $\mathcal{N} = 4$ super-Yang-Mills at the conformal point is shown to contain strings: they are IIB strings. We conjecture that compactifications of M/string theory on various Anti-deSitter spacetimes is dual to various conformal field theories. This leads to a new proposal for a definition of M-theory which could be extended to include five non-compact dimensions.

- AdS Dual representation of conformal field theories
- Full five dimensional representation of Wess-Zumino action.

Physical Implications of Anomaly Structure

- Neutral pion decay – anomalous currents
- Evidence for quarks with color and the origins of QCD
- U(1) Problem in QCD
  - the eta' mass
  - accidental symmetries
- Nontrivial anomaly cancelation in the Standard Model
- Model building constraints – anomaly cancelation conditions
- Baryon asymmetries
  - anomalous baryon number current
  - leptogensis
- Axions and the strong CP problem

Mathematical Implications of Anomaly Structure

- Topology, Geometry

Index theorems, spectral properties of Dirac operator relate anomaly to topological structure of gauge fields.

- M.F. Atiyah and I.M. Singer, Ann. of Math. 87, 485 (1968)
- M.F. Atiyah and G.B. Segal, Ann. of Math. 87, 531(1968)
- M.F. Atiyah and I.M. Singer, Ann. Math. 97, 119, 139 (1972)

Anomaly = toplogical index.

$$\partial^{\mu} J_{5\mu} = \frac{N_f}{8\pi^2} \cdot tr\{\ G^{\mu\nu}(gA) \cdot {}^*G_{\mu\nu}(gA)\ \}$$

Differential geometry used to analyze structure of anomalies in arbitrary dimensions of space-time. Descent equations.

- B. Zumino, Les Houches (1983); R. Stora, Cargése (1983);
- L. Baulieu, Nucl. Phys. B241, 557 (1984)

Mathematical Implications of Anomaly Structure

- Index theorems, gravitational anomalies
  - L. Alvarez-Gaumé and E. Witten, Nucl.Phys. B234, 269(1983)

GRAVITATIONAL ANOMALIES

Luis ALVAREZ-GAUMÉ[1]

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Edward WITTEN[2]

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA

Received 7 October 1983

It is shown that in certain parity-violating theories in $4k+2$ dimensions, general covariance is spoiled by anomalies at the one-loop level. This occurs when Weyl fermions of spin-$\frac{1}{2}$ or -$\frac{3}{2}$ or self-dual antisymmetric tensor fields are coupled to gravity. (For Dirac fermions there is no trouble.) The conditions for anomaly cancellation between fields of different spin is investigated. In six dimensions this occurs in certain theories with a fairly elaborate field content. In ten dimensions there is a unique theory with anomaly cancellation between fields of different spin. It is the chiral $n=2$ supergravity theory, which is the low-energy limit of one of the superstring theories. Beyond ten dimensions there is no way to cancel anomalies between fields of different spin.

Mathematical Implications of Anomaly Structure

- Index theorems, gravitational anomalies
  - O. Alvarez, I.M. Singer and B. Zumino, Comm.Math.Phys. 96,409 (1984)

Gravitational Anomalies and the Family's Index Theorem*

Orlando Alvarez[1] * *, I. M. Singer[2], and Bruno Zumino[1]

1 Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

2 Department of Mathematics and Department of Physics, University of California, Berkeley, CA 94720, USA

Abstract. We discuss the use of the family's index theorem in the study of gravitational anomalies. The geometrical framework required to apply the family's index theorem is presented and the relation to gravitational anomalies is discussed. We show how physics necessitates the introduction of the notion of *local cohomology* which is distinct from the ordinary topological cohomology. The recent results of Alvarez-Gaumé and Witten are derived by using the family's index theorem.

Mathematical Implications of Anomaly Structure

- Structure of gauge and gravitational anomalies
  - W. Bardeen and B. Zumino, Nucl.Phys. B244, 421 (1984)

CONSISTENT AND COVARIANT ANOMALIES IN GAUGE AND GRAVITATIONAL THEORIES*

William A. BARDEEN

Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL 60510, USA

Bruno ZUMINO

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, CA 94720, USA

Received 25 April 1984

The gauge structure of anomalies and the related currents is analyzed in detail. We construct the covariant forms for both the currents and the anomalies for general gauge theories in even-dimensional space-time. The results are then extended to determine the structure of gravitational anomalies. These can always be interpreted as anomalies for local Lorentz transformations.

Mathematical Implications of Anomaly Structure

- Algebraic Structure.
  - J. Mañes, R. Stora and B. Zumino, Comm.Math.Phys. 102, 157 (1985)

Algebraic Study of Chiral Anomalies*

Juan Mañes[1], Raymond Stora[2], and Bruno Zumino[1,3]

1 Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA
2 LAPP, Annecy-le-Vieux, France
3 Department of Physics, University of California, Berkeley, CA 94720, USA

Abstract. The algebraic structure of chiral anomalies is made globally valid on non-trivial bundles by the introduction of a fixed background connection. Some of the techniques used in the study of the anomaly are improved or generalized, including a systematic way of generating towers of "descent equations".

Birth of Consistent String Theories

- Anomalies and String Theory
  - M. Green and J. Schwarz, Phys.Lett. 149B, 117 (1984)
  - M. Green and J. Schwarz, Phys.Lett. 151B, 21 (1985)

ANOMALY CANCELLATIONS IN SUPERSYMMETRIC $D = 10$ GAUGE THEORY AND SUPERSTRING THEORY ☆

Michael B. GREEN
Queen Mary College, University of London, London E1 4NS, UK
and California Institute of Technology, Pasadena, CA 91125, USA

and

John H. SCHWARZ
California Institute of Technology, Pasadena, CA 91125, USA

Received 10 September 1984

Supersymmetric ten-dimensional Yang–Mills theory coupled to $N = 1$, $D = 10$ supergravity has gauge and gravitational anomalies that can be partially cancelled by the addition of suitable local interactions. The remaining pieces of all the anomalies cancel if the gauge group is SO(32) or $E_8 \times E_8$. These cancellations are automatically incorporated in the type I superstring theory based on SO(32). A superstring theory for $E_8 \times E_8$ has not yet been constructed.

Birth of Consistent String Theories

- Summer at the Aspen Center for Physics.
  - M. Green and J. Schwarz, ” Aspen and String Theory”
    - ACP Science History Project

The two of us explored the anomaly problem for type I superstring theory off and on for almost two years until the crucial breakthroughs were made in August 1984 at the Aspen Center for Physics. That summer JHS was the organizer of an ACP workshop entitled “Physics in Higher Dimensions.” This workshop attracted many participants, even though string theory was not yet fashionable, because by that time there was considerable interest in supergravity theories in higher dimensions and Kaluza–Klein compactification. Our research benefitted from the presence of many leading experts including Bruno Zumino, Bill Bardeen, Dan Friedan, and Steve Shenker.

Conclusions

- Anomalies – a curious inconsistency?
- Anomalous currents, nonrenormalization theorem.
- Wess-Zumino Consistency conditions.
- Nonlinear realizations and Wess-Zumino terms.
- The many roles of anomalies in the Standard Model.
- Instantons and topological structure.
- Index theorems and geometry of gauge fields theories

* String theory and the fundamental nature of space and time.

!Thank you!

Bruno

Addendum

- The Neutral Pion lifetime

Prediction from the proton loop (Steinberger):

Γ= 7.6 ev, τ= 0.86 10^-16 sec

- same result for three color quark theory

Experiment

G. Belletitini, et al, NuovoCim XL, A-4, 1139(1965)
"Primakoff Effect and the Neutral Pion Lifetime
τ= 0.73(0.105) 10^-16 sec

PDG τ= 0.84(0.5) 10^-16 sec