Theoretical Concepts in Particle Physics (5)

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Yesterday...

- Symmetries
- $\mathcal{L}$ is:
  - The most general one that is invariant under some symmetries
  - We work up to some order (usually 4)
- How do we “built” invariants?

Invariant of complex numbers

- $U(1)$ is rotation in 1d complex space
- Each complex number comes with a q that tells us how much it rotates
- When we rotate the space by an angle θ, the number rotate as

$$X \to e^{iq\theta} X$$

- Consider $q_X = 1,\ q_Y = 2,\ q_Z = 3$ and write 3rd and 4th order invariants

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$$XX^*YY^* \qquad X^2Y^* \qquad XYZ^* \qquad X^3Z^* \qquad Y^2X^*Z^*$$

$SU(2)$

- $U(2)$ is rotation in 2d complex space. We have $U(2) = SU(2) \times U(1)$
- $SU(2)$ is localy the same as rotation in 3d real space
- Rotations in this space are non-Abelian (non-commutative)
- It depends on the representation: scalar, spinor, vector
- Spin in QM is described by SU(2) rotations, so we use the same language to describe it
- For the SM all we care is that $1/2 \times 1/2 \ni 0$ so we know how to generate singlets
- How can we generate invarints from spin $1/2$ and spin $3/2$?

$SU(3)$

- $U(3)$ is rotation in 3d complex space. We have $U(3) = SU(3) \times U(1)$
- The representations we care about are singlets, triplets and octets
- Unlike $SU(2)$, in $SU(3)$ we have complex representations, 3 and $\bar{3}$
- The three quarks form a triplet (the three colors)
- To form a singlet we need to know that

$$3 \times \bar{3} \ni 1 \qquad 3 \times 3 \times 3 \ni 1$$

- This is why we have baryons and mesons

A game

A game calls "building invariants"

- Symmetry is $SU(3) \times SU(2) \times U(1)$
  - $U(1)$: Add the numbers ($\bar{X}$ has charge $-q$)
  - $SU(2)$: $2 \times 2 \ni 1$ and recall that 1 is a singlet
  - $SU(3)$: we need $3 \times \bar{3} \ni 1$ and $3 \times 3 \times 3 \ni 1$
- Fields are

$$Q(3,2)_1 \qquad U(3,1)_4 \qquad D(3,1)_{-2} \qquad H(1,2)_3$$

- What 3rd and 4th order invariants can we built?

$$(HH^*)^2 \qquad H^3 \qquad UDD \qquad QUD \qquad HQU^*$$

- HW: Find more invariants

Lorentz invariants

Lorentz invariants

- The representations we care about are
  - Singlet: Spin zero (scalars, denote by ϕ)
  - LH and RH fields: Spin half (fermions, $\psi_L, \ \psi_R$)
  - Vector: Spin one (gauge boson, denote by A_μ)
- Fermions are more complicated

$$\mathcal{L} \sim \bar{\psi}\partial_\mu\gamma^\mu\psi$$

- Since $\mathcal{L}$ has dimension 4, ψ is dimension $3/2$
- For fermions when we expand up to 4th order we can have at most two fermion fields
- Under Lorentz, the basic fields are left-handed and right-handed. A mass term must involve both $m\bar{\psi}_L\psi_R$

Local symmetires

Local symmetry

Basic idea: rotations depend on x and t

$$\phi(x_\mu) \to e^{iq\theta}\phi(x_\mu) \quad \xrightarrow{local} \quad \phi(x_\mu) \to e^{iq\theta(x_\mu)}\phi(x_\mu)$$

- It is kind of logical and we think that all imposed symmetries in Nature are local
- The kinetic term $|\partial_\mu \phi|^2$ in not invariant
- We want a kinetic term (why?)
- We can save the kinetic term if we add a field that is
  - Massless
  - Spin 1
  - Adjoint representation: $q = 0$ for $U(1)$, triplet for $SU(2)$, and octet for $SU(3)$

Gauge symmetry

- Fermions are called matter fields. What they are and their representation is an input
- Gauge fields are known as force fields

Local symmetries $\Rightarrow$ force fields

Gauge symmetry

- The coupling of the new field is via the kinetic term. Recall classical electromagnetism

$$H = \frac{p^2}{2m} \Rightarrow H = \frac{(p - qA_i)^2}{2m}$$

- In QFT, for a local $U(1)$ symmetry and a field with charge q

$$\partial_\mu \to D_\mu \qquad D_\mu = \partial_\mu + iqA_\mu$$

- We get interaction from the kinetic term

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni qA\phi^2 + q^2 A^2 \phi^2$$

- The interaction is proportional to q

Accidental symmetries

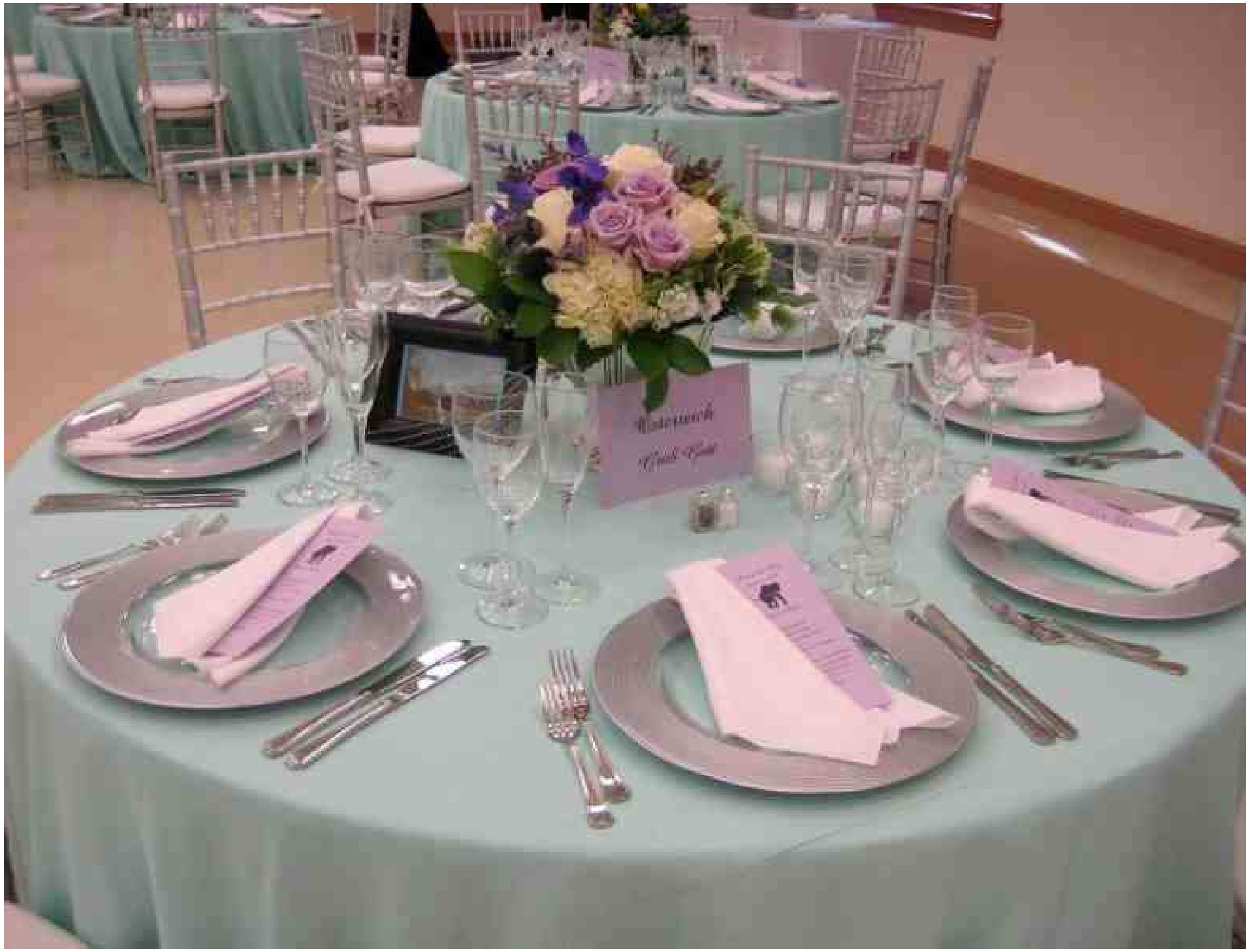
- We only impose local symmetries
- Yet, because we truncate the expansion, we can get symmetries as output
- They are global, and are called accidental
- Example: $U(1)$ with $X(q=1)$ and $Y(q=-4)$

$$V(XX^*, YY^*) \Rightarrow U(1)_X \times U(1)_Y$$

  - X^4Y breaks this symmetry
- In the SM baryon and lepton numbers are accidental symmetries

SSB

Breaking a symmetry



SSB

- By choosing a ground state we break the symmetry
- We choose to expend around a point that does not respect the symmetry
- PT only works when we expand around a minimum

What is the different between a broken symmetry and no symmetry?

SSB implies relations between parameters

SSB

Symmetry is $x \to -x$ and we keep up to x^4

$$f(x) = a^2x^4 - 2b^2x^2 \qquad x_{\min} = \pm b/a$$

We choose to expand around $+b/a$ and use $u \to x - b/a$

$$f(x) = 4b^2u^2 + 4bau^3 + a^2u^4$$

- No $u \to -u$ symmetry
- The $x \to -x$ symmetry is hidden
- A general function has 3 parameters $c_2u^2 + c_3u^3 + c_4u^4$
- SSB implies a relation between them

$$c_3^2 = 4c_2c_4$$

SSB in QFT

- When we expand the field around a minimum that is not invariant under a symmetry

$$\phi \to v + H$$

- It breaks the symmetries that ϕ is not a singlet under
- Masses to other fields via Yukawa interactions

$$\phi X^2 \to (v + H) X^2 = v X^2 + \ldots$$

- Gauge fields of the broken symmetries also get mass

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni A^2 \phi^2 \to v^2 A^2$$

The SM

The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1}$$

  - One scalar

$$\phi(1,2)_{+1/2}$$

Then Nature is described by

- Output: the most general $\mathcal{L}$ up to dim 4
- This model has a $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry
- Initial set of measuremnts to find the parameters
  - SSB: $SU(2)_L \times U(1)_Y \;\to\; U(1)_{EM}$
  - Fermion masses, gauge couplings and mixing angles

The SM pass (almost) all of it tests

Nature

Infinities and all that

Why we "cut" at ϕ^4?

- Any theory have limits: We call them UV and IR limits
- Higher dim. operators are more sensitive to the UV
- $d = 3$ we can claculate. $d = 4$ we need to be carefull
- Just like the energy of a point like charged particle

$$U \propto \int |E|^2 d^V \propto \int \frac{\alpha^2}{r^4} r^2 dr \sim \frac{1}{r}\Big|_0^\infty \to \infty$$

- We can still calculate it if we care about change of energy of a test particle
- What really takes care of this infinity?
- Higher dim operators must point into a deeper theory