Neutrino Theory: Some Recent Developments

Ernest Ma

Physics and Astronomy Department
University of California
Riverside, CA 92521, USA

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Brief History of Neutrino Tribimaximal Mixing

1978: Cabibbo and Wolfenstein conjectured independently that

$$U_{l\nu}^{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This should dispel the myth that everybody expected small mixing angles in the lepton sector as in the quark sector.

2002: Harrison/Perkins/Scott proposed the tribimaximal mixing matrix, i.e.

$$U_{l\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0)$$

2004: I discovered the simple connection:

$$U_{l\nu}^{HPS} = (U_{l\nu}^{CW})^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This means that if

$$\mathcal{M}_l = U_{l\nu}^{CW} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} (U_R^l)^\dagger$$

and $\mathcal{M}_\nu$ has $2-3$ reflection symmetry, with zero $1-2$ and $1-3$ mixing, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix},$$

$U_{l\nu}^{HPS}$ will be obtained, but **how?**.

Tribimaximal mixing means:

$\theta_{13} = 0$, $\sin^2 2\theta_{23} = 1$, $\tan^2\theta_{12} = 1/2$.

In 2002 (when HPS proposed it), world data were not precise enough to test this idea.

In 2004 (when I derived it), SNO data implied $\tan^2\theta_{12} = 0.40 \pm 0.05$, which was not so encouraging.

Then in 2005, revised SNO data obtained
$\tan^2\theta_{12} = 0.45 \pm 0.05$,
and tribimaximal became a household word, unleashing a glut of papers.

Tetrahedral Symmetry A_4

For 3 families, we should look for a group with a $\underline{3}$ representation, the simplest of which is A_4, the group of the even permutation of 4 objects.

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	–1

$$\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$$

Multiplication rule:

$$\underline{3} \times \underline{3} = \underline{1}(11 + 22 + 33) + \underline{1}'(11 + \omega^2 22 + \omega 33)$$

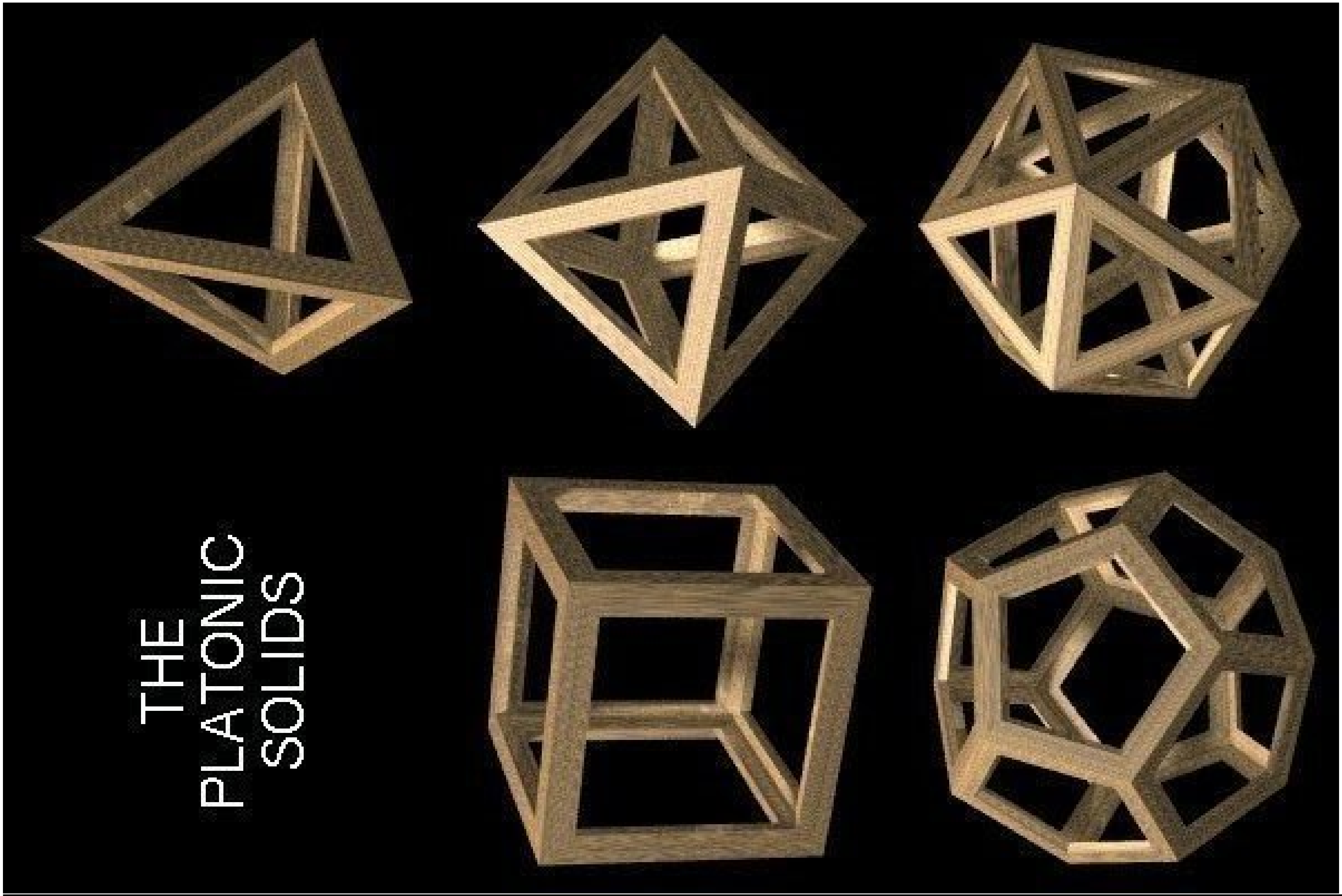
$$+\underline{1}''(11 + \omega 22 + \omega^2 33) + \underline{3}(23, 31, 12) + \underline{3}(32, 13, 21).$$

Note that $\underline{3} \times \underline{3} \times \underline{3} = \underline{1}$ is possible in A_4,
i.e. $a_1 b_2 c_3$ + permutations,
and $\underline{2} \times \underline{2} \times \underline{2} = \underline{1}$ is possible in S_3,
i.e. $a_1 b_1 c_1 + a_2 b_2 c_2$.

Perfect Three-Dimensional Geometric Solids:

solid	faces	vertices	Plato	group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
cube	6	8	earth	S_4
icosahedron	20	12	water	A_5
dodecahedron	12	20	quintessence	A_5

Amusingly, there are also 5 string theories in 10 dimensions: Type I is dual to Heterotic SO(32), Type IIA is dual to Heterotic $E_8 \times E_8$, and Type IIB is self-dual.


THE
PLATONIC
SOLIDS

Two steps to tribimaximal mixing using A$_4$:
(I) Choose (A) Ma/Rajasekaran(2001): $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $(\phi_i^0, \phi_i^-) \sim \underline{3}$ with $v_1 = v_2 = v_3$, or (B) Ma(2006): $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{3}$, and $(\phi_i^0, \phi_i^-) \sim \underline{1}, \underline{3}$, with $v_1 = v_2 = v_3$. The diagonalization of $\mathcal{M}_l$ yields $U_{l\nu}^{CW}$ automatically, with arbitrary $m_{e,\mu,\tau}$. Here A$_4 \to Z_3$.
(II) In the neutrino sector, use $(\xi_i^{++}, \xi_i^+, \xi_i^0) \sim \underline{1}, \underline{3}$, with $u_2 = u_3 = 0$. Here A$_4 \to Z_2$. Ma(2004); Altarelli/Feruglio(2005):

$$\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}.$$

This is the simplest realization of tribimaximal mixing, with neutrino mass eigenvalues $a + d$, a, $-a + d$, allowing only normal hierarchy.

The keys to tribimaximal mixing are (1) the choice of symmetry, i.e. A_4 or S_4, (2) the choice of lepton and Higgs representations, (3) residual symmetries Z_3 and Z_2 in two different sectors.

This is an example of how group theory alone could determine mixing angles, but leaves all masses free.

The Cabibbo angle $= \pi/14$ could come from $D_7 \to Z_2$: [Lam(2007); Blum/Hagedorn/Lindner(2008).]

Seesaw Variations and the Nonunitarity of the Neutrino Mixing Matrix

With 1 doublet neutrino ν and 1 singlet neutrino N, their 2×2 mass matrix is the well-known

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

resulting in the famous seesaw formula $m_\nu \simeq -{m_D}^2/m_N$. Hence $\nu - N$ mixing $\simeq m_D/m_N \simeq \sqrt{m_\nu/m_N} < 10^{-6}$, for $m_\nu < 1$ eV and $m_N > 1$ TeV.

Consider now 1 ν and 2 singlets: $N_{1,2}$. Their 3×3 mass matrix is then

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix},$$

resulting in $m_\nu \simeq {m_D}^2 m_2 / {m_N}^2$. Since the limit $m_1 = 0$ and $m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for ν and N_2, $L = -1$ for N_1), their smallness is natural $\Rightarrow$ the inverse seesaw [Mohapatra/Valle(1986)]. Here $\nu - N_1$ mixing is still small, but $\nu - N_2$ mixing $\simeq m_D/m_N$ may be big $\Rightarrow$ observable nonunitarity of $U_{l\nu}$.

Consider next $\nu_{1,2}$ and $N_{1,2}$, where

$$\mathcal{M}_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1 & b_2 \end{pmatrix},$$

and $\mathcal{M}_N = \mathrm{diag}(M_1', M_2')$. In that case, the arbitrary imposed condition $b_1^2/M_1' + b_2^2/M_2' = 0$ renders all two light neutrinos massless, and yet $\nu - N$ mixing may be large, because $a_i b_j$ need not be small. Small deviations from this texture will allow small neutrino masses and retain the large $\nu - N$ mixing. This is an active topic of study: see e.g. Pilaftsis(2005);Kersten/Smirnov(2007).

He/Ma(2009): To understand the mechanism and symmetry of the texture hypothesis, change the neutrino basis to

$$\mathcal{M}_D = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad \mathcal{M}_N = \begin{pmatrix} M_1 & M_3 \\ M_3 & M_2 \end{pmatrix}.$$

Then the condition is $m_1 = M_1 = 0$, so that ν_1 and $\nu_2' = (M_3\nu_2 - m_2 N_1)/\sqrt{M_3^2 + m_2^2}$ are massless, showing how large mixing actually occurs, i.e. through the inverse seesaw mechanism. However, lepton number conservation would not only forbid M_1 but also M_2, which is in fact arbitrary here. Where is the symmetry which does this?

Let $\nu_{1,2}$, $N_{1,2}$ have $L = 1, 1, 3, -1$.

Add the usual Higgs doublet (ϕ_1^+, ϕ_1^0) with $L = 0$ and the Higgs singlet χ_2 with $L = 2$. Then m_2 comes from $\langle \phi_1^0 \rangle$, M_2 from $\langle \chi_2 \rangle$, and M_3 from $\langle {\chi_2}^\dagger \rangle$.

$m_1 = 0$ at tree level, because there is no Higgs doublet with $L = -4$, and $M_1 = 0$ at tree level, because there is no Higgs singlet with $L = \pm 6$.

In one loop, M_1 will be induced, thus giving ν_2' an inverse seesaw mass $= M_1 m_2^2 / M_3^2$. Once ν_2 is massive, ν_1 also gets a two-loop radiative mass from the exchange of two W's [Babu/Ma(1988)].

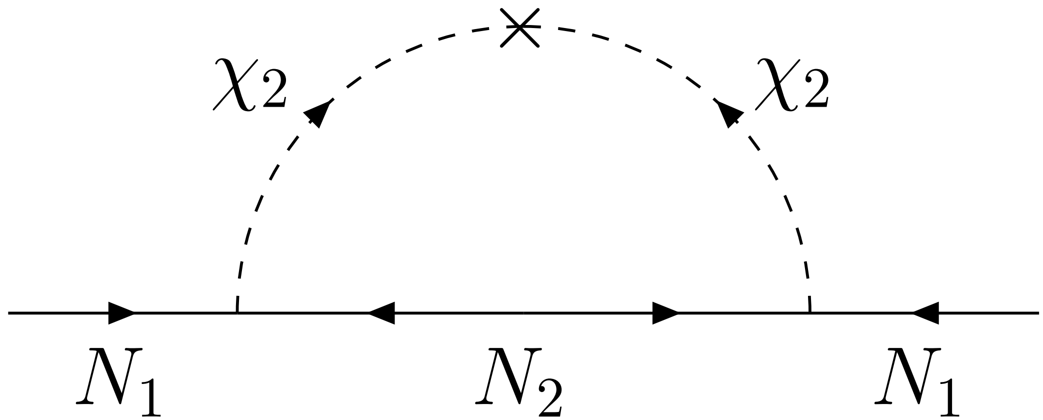


Figure 1: One-loop generation of M_1.

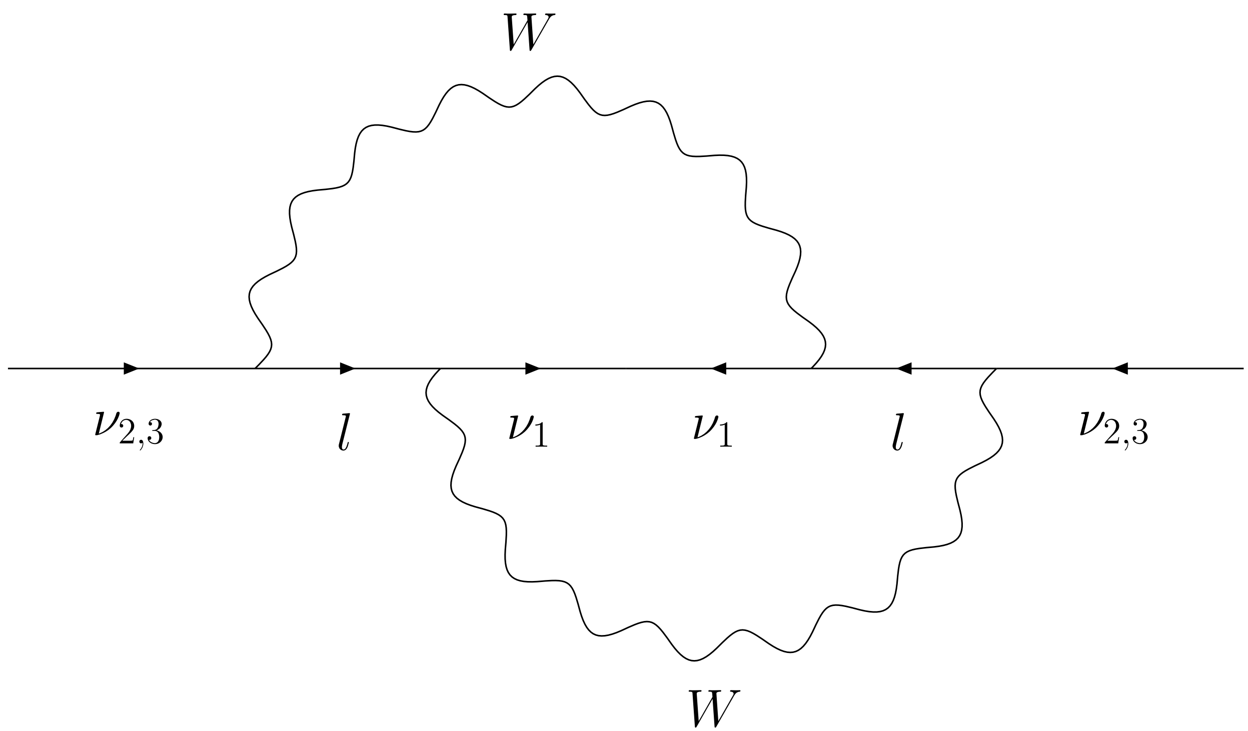


Figure 1: Two-W generation of neutrino mass.

Radiative Seesaw and Dark Matter: Scotogenic Neutrino Mass

Ma(2006): Add to the Standard Model (SM) a second scalar doublet (η^+, η^0) and 3 neutral singlet Majorana fermions $N_{1,2,3}$ which are odd under an exactly conserved Z_2, with all SM particles even. Hence $\nu N \phi^0$ is forbidden and $\nu N \eta^0$ is allowed, but $\langle \eta^0 \rangle = 0$. Neutrino mass is generated in one loop, i.e. scotogenic, being caused by darkness. Here, η^0_R is a dark-matter candidate, later studied by [Barbieri/Hall/Rychkov(2006)]. They call η the inert Higgs doublet. I call it the dark scalar doublet.

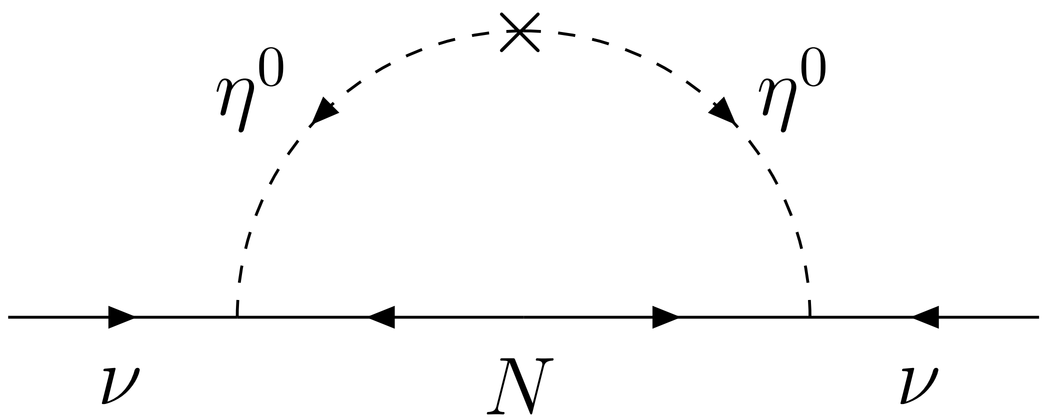
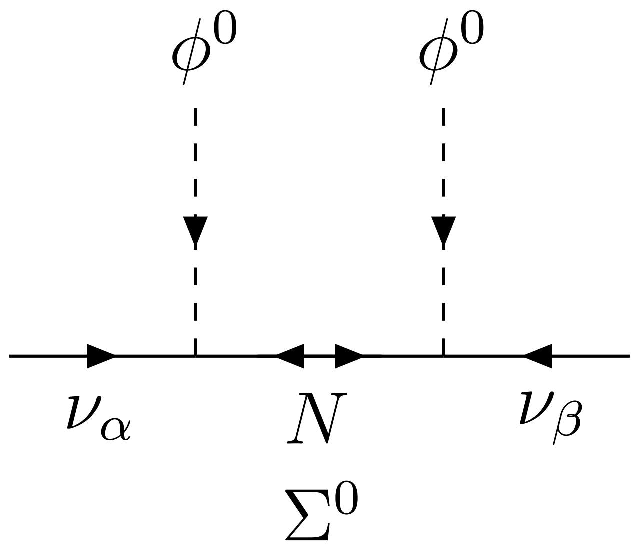
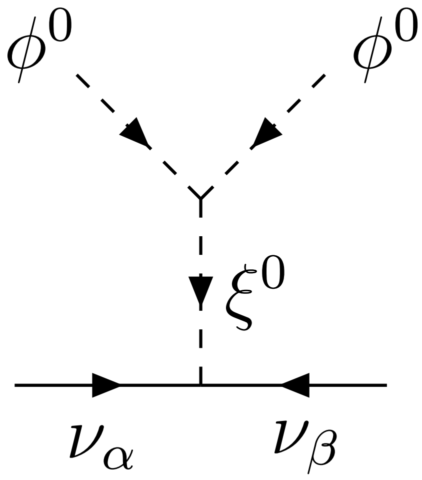


Figure 1: One-loop m_ν from Z_2 dark matter.


ϕ^0
ϕ^0
ν_α
N
Σ^0
ν_β


ϕ^0
ϕ^0
ξ^0
ν_α
ν_β

The mass splitting between η_R^0 and η_I^0 comes from the Z_2 allowed term $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)],$$

where $f(x) = -\ln x/(1-x)$.

Let $m_R^2 - m_I^2 = 2\lambda_5 v^2 << m_0^2 = (m_R^2 + m_I^2)/2$, then

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} I(M_i^2/m_0^2),$$

$$I(x) = \frac{\lambda_5 v^2}{8\pi^2} \left(\frac{x}{1-x} \right) \left[1 + \frac{x \ln x}{1-x} \right].$$

For $x_i >> 1$, i.e. N_i very heavy,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_5 v^2}{8\pi^2} \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} [\ln x_i - 1]$$

instead of the canonical seesaw $v^2 \sum_i h_{\alpha i} h_{\beta i}/M_i$.
In leptogenesis, the lightest M_i may then be much below the Davidson-Ibarra bound of about 10^9 GeV, thus avoiding a potential conflict of gravitino overproduction and thermal leptogenesis if supersymmetry is considered.

Supersymmetric $SU(5)$ Completion:

Gauge-coupling unification in the MSSM is preserved by complete $SU(5)$ multiplets.

Ma(2007): Add $\underline{5} = h(3,1,-1/3) + (\eta_2^+, \eta_2^0)(1,2,1/2)$,

$\underline{5}^* = h^c(3^*,1,1/3) + (\eta_1^0, \eta_1^-)(1,2,-1/2)$,

and singlet superfields N and χ.

Under $Z_2^{(1)} \times Z_2^{(2)}$, let

$(u,d), u^c, d^c, (\nu, e), e^c \sim (-,+)$;

$(\phi_1^0, \phi_1^-), (\phi_2^+, \phi_2^0) \sim (+,+)$;

$h, h^c, (\eta_1^0, \eta_1^-), (\eta_2^+, \eta_2^0) \sim (+,-)$;

$N \sim (-,-)$; $\chi \sim (+,-)$.

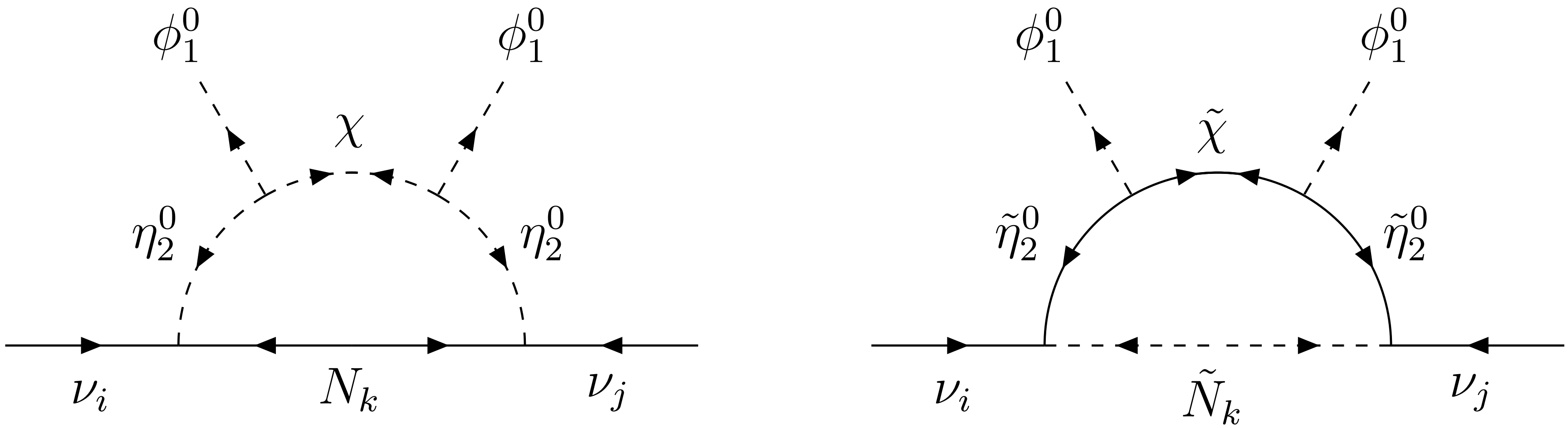


Figure 1: One-loop supersymmetric neutrino mass.

The first $Z_2^{(1)}$ yields the usual R parity of the MSSM. The second $Z_2^{(2)}$ allows scotogenic neutrino mass through the terms $(\nu\eta_2^0 - e\eta_2^+)N$, $\phi_1^0\eta_2^0\chi$, NN, and $\chi\chi$.

This extension of the DSDM is safe for proton decay because the would-be mediators h, h^c are odd under $Z_2^{(2)}$.

It also predicts the strong production of $h\bar{h}$ at the Large Hadron collider, with $h \to de^-\eta_2^+$ or $de^+\eta_2^-$.

Thus a possible unique signature of this model is the appearance of same-sign dileptons plus quark jets plus missing energy.

At least two out of the following three particles are dark-matter candidates:

(1) the usual lightest neutralino of the MSSM with $(R, Z_2^{(2)}) = (-, +)$,

(2) the lightest exotic neutral particle with $(+, -)$,

(3) and that with $(-, -)$.

The dark matter of the Universe may not be all the same, as most people have taken for granted! For a general discussion, see Cao/Ma/Wudka/Yuan, arXiv:0711.3881.

Krauss/Nasri/Trodden(2003):

Neutrino mass may be obtained in 3 loops, with neutral singlet fermions N and charged scalar S_2^+ having odd Z_2. This was the first proposal that N could be dark matter.

However, since lNS_2^+ is the only interaction involving N, it has to be rather big to have the correct annihilation cross section for the presumed dark-matter relic density.

Flavor-changing radiative decays such as $\mu \to e\gamma$ are then too big, without extreme fine tuning.

In the one-loop scotogenic case, this constraint is relaxed, because η_R^0 is available for dark matter.

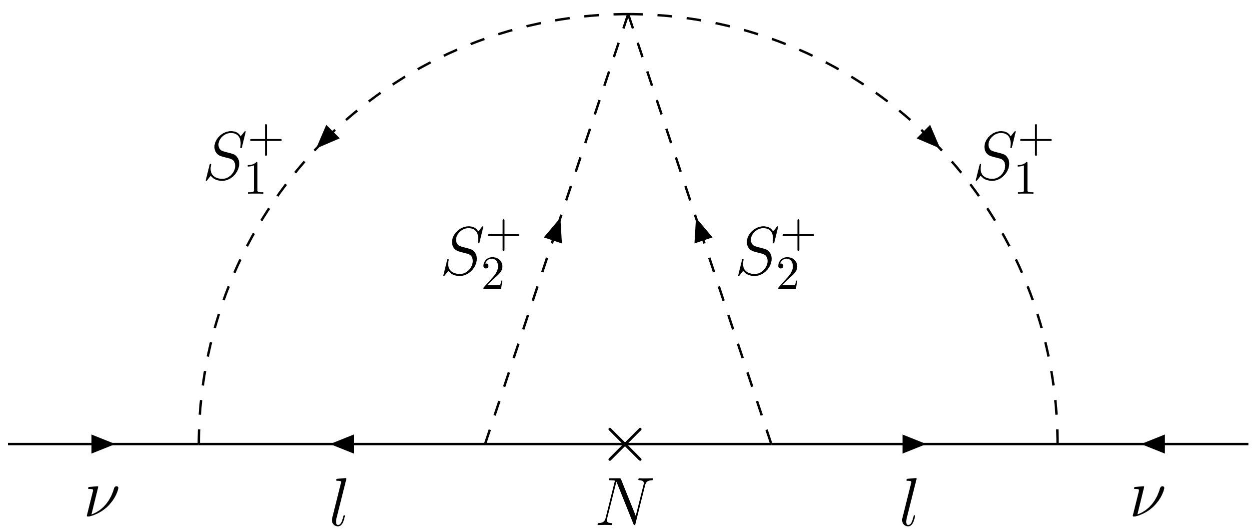


Figure 1: Three-loop neutrino mass (Krauss/Nasri/Trodden).

Aoki/Kanemura/Seto(2009):

Instead of two charged singlets, they propose two Higgs doublets and a neutral singlet η^0 $(40 - 65$ GeV), so that the latter can be dark matter, instead of N (3 TeV).

This model allows for electroweak baryogenesis, coming from a first-order phase transition in the Higgs potential.

At the LHC, the η^0 is hard to produce and detect because it is a singlet, whereas η^0_R and η^0_I of the one-loop scotogenic model are produced by the Z boson, with the subsequent decay $\eta^0_I \to \eta^0_R l^+ l^-$ as a possible signature. [Cao/Ma/Rajasekaran(2007)]

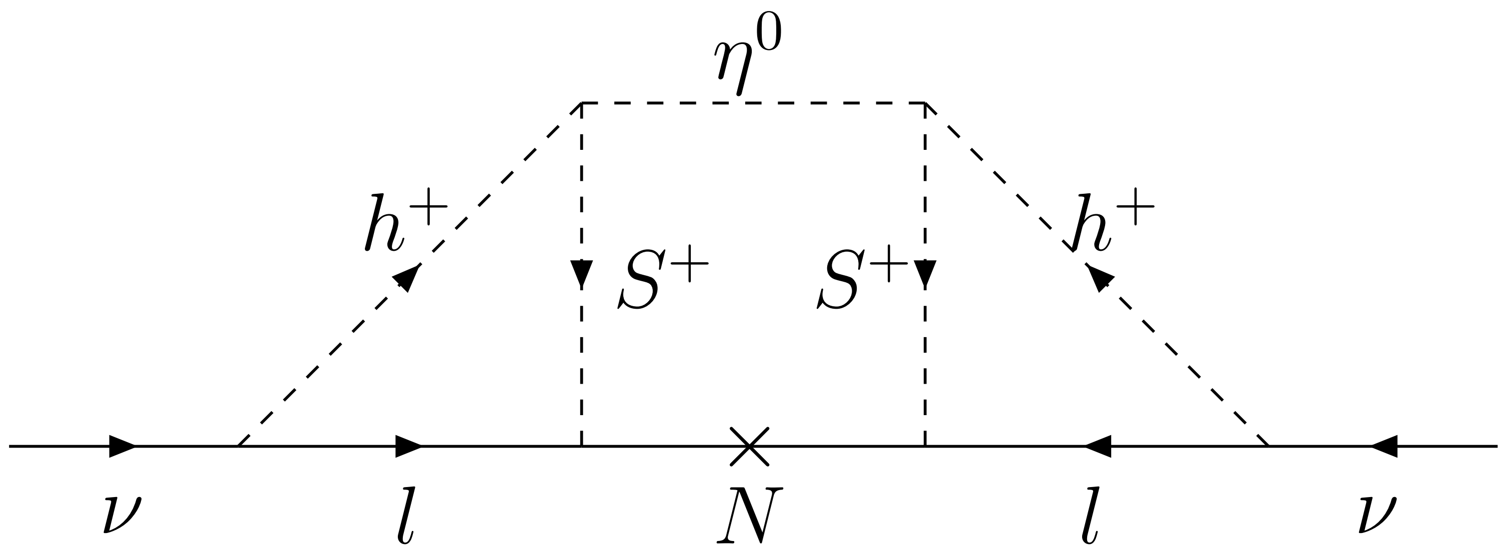


Figure 1: Three-loop neutrino mass (Aoki/Kanemura/Seto).

Concluding Remarks

- With the application of the non-Abelian discrete symmetry A_4, a plausible theoretical understanding of the tribimaximal form of the neutrino mixing matrix has been achieved.

- Seesaw variants at the TeV scale may allow this mechanism (in its inverse or linear manifestation) to be observable through the nonunitarity of the 3×3 neutrino mixing matrix, as well as flavor changing leptonic interactions.

- Dark matter may be the origin of radiative neutrino mass. This scotogenic mechanism may be implemented in a number of different models, and be observable also at the TeV scale. A complete supersymmetric $SU(5)$ version also exists with gauge-coupling unification.
- Other recent topics in neutrino theory, such as Type III seesaw, using a Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$, and small Dirac and pseudo-Dirac neutrino masses, are not covered in this talk, but are being actively pursued.
- Neutrino theory marches on, but what we really need are corroborating data!