Optical Theorem and Elastic Nucleon Scattering

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In the theoretical analysis of high-energy elastic nucleon scattering one starts commonly from the description based on the validity of the optical theorem, which allows to derive the value of the total cross section directly from the experimentally measured t-dependence of the elastic differential cross section at the corresponding energy. It may be shown, however, that this theorem has been derived on the basis of one assumption that might be regarded perhaps as acceptable in the case of long-range (e.g. Coulomb) forces but must be denoted as quite unacceptable in the case of finite-range hadron forces. Consequently, the conclusions leading to the increase of the total cross section with energy at higher collision energies must be newly analyzed and reevaluated. It concerns also the value of the beam luminosity derived from elastic data. The necessity of new analysis concerns the derivation of the hadronic t-dependence at very low transverse momenta as the separation of Coulomb scattering may be also strongly influenced. It will be shown in conclusion that all mentioned problems might be solved on the basis of the ontological model proposed quite recently.

1 Introduction

It is commonly argued that it is possible to derive also the value of the total cross section directly from the measured t-dependence of the elastic differential cross section when the validity of the optical theorem is taken into account. And the derived values indicate that the total cross section should rise with energy. From these values the beam luminosity is also being derived.

However, we will show that in the derivation of the optical theorem some important assumption has been involved that may be perhaps acceptable for infinite-range Coulomb force, but can be hardly brought to harmony with the finite-range forces between nucleons. It means that neither the total cross section nor the beam luminosity may be derived from the measured differential elastic scattering cross section in the framework of the standard phenomenological theory without adding some assumption or additional property.

The validity of the optical theorem does not correspond to reality and all conclusions based on it should be reevaluated. However, it will be shown in the last additional section that some properties of ontological model might help in solving newly the given problems.

2 Optical Theorem and its Derivation

Let us start with the standard derivation approach. We shall follow the approach described in the book of Barone and Predazzi [1]. They have started from Fraunhofer diffraction and from Babinet's principle. The profile function of a hole has been denoted by $\Gamma(b)$ and that of an obstacle by $S(b)$. When the initial state is represented by a plane wave $\psi_{in} = e^{ikz}$, the final state may be expressed in the case of an obstacle as the superposition of the set of individual scattered states (see also [2]):

$$U(x, y.z) = -\frac{ik}{2\pi} U_0 \frac{e^{ikr}}{r} \int d^2\mathbf{b}\, S(\mathbf{b})\, e^{-i\mathbf{q}.\mathbf{b}}$$

and similarly for the hole if $S(b)$ is substituted by $\Gamma(b)$. According to the Huygens-Fresnel principle ($S(\mathbf{b}) + \Gamma(\mathbf{b}) = 1$) the same shapes of hole and obstacle combine again to the original plane wave. The whole approach being based on the assumption of small diffraction angles ($\sin\theta \cong \theta$).

The amplitude may be divided in principle into two parts: scattered and unscattered, where the scattered part is represented by the function of transferred momentum $\mathbf{q}$:

$$U_{out}(x, y, z)) \;=\; U_{unsc} \;+\; U_{scatt} = \; U_0 \left(\psi_{unsc} + f(\mathbf{q}) \frac{e^{ikr}}{r} \right)$$

$$\psi_{unsc} = \alpha e^{ikz}, \;\; |\alpha| < 1; \qquad \mathbf{q} = \mathbf{k}' - \mathbf{k}, \quad |\mathbf{k}'| = |\mathbf{k}| = k$$

The unscattered part is represented by a plane wave; of course, with some norm less than one. The modulus squared of $U(x, y, z)$ represents the final intensity and U_0 - incoming intensity.

The differential elastic cross section is given by the square of $f(\mathbf{q})$, and the total elastic cross section may be expressed in integral form:

$$\frac{d\sigma}{d\Omega} = |f(\mathbf{q})|^2, \qquad \sigma_{el} \cong \frac{1}{k^2} \int |f(\mathbf{q})|^2 d^2\mathbf{q} = \int d^2\mathbf{b} |\Gamma(\mathbf{b}|^2$$

where $\Gamma(\mathbf{b})$ represents the corresponding profile.

And if it is assumed that the unscattered part may be identified with $f(0)$ and if the interaction is normalised to unity one obtains the following expressions for absorption and total cross sections:

$$\sigma_{abs} = \int d^2\mathbf{b} (1 - |S(\mathbf{b})|^2)$$

$$\sigma_{tot} \;=\; \sigma_{el} + \sigma_{abs} = 2 \int d^2\mathbf{b}\, Re\Gamma(\mathbf{b})$$

It means that the total cross section should be represented by the imaginary part of scattering amplitude at point $\mathbf{q} = 0$:

$$\sigma_{tot} \;=\; \frac{4\pi}{k}\, Im\, f(\mathbf{q} = 0)$$

However, some additional assumptions have been used that cannot be applied to in the case of finite nucleon force, as it will be shown in the next section.

3 Nucleon Force and Optical Theorem

By identifying the unscattered state with one vector of scattered states the function $f(\mathbf{q})$ has been substituted in principle by capital $F(\mathbf{q})$, where one singular point has been added to the previous function:

$$U(x, y, z) \;=\; U_0 F(\mathbf{q}) \frac{e^{ikr}}{r}; \qquad F(0) = f(0) + |\psi_{unsc}|\,.$$

However, in all approaches two assumptions concerning the function $F(\mathbf{q})$ have been added; the given function has been taken as continuous, which has meant that the unscattered part has equalled zero, and as decreasing from $\mathbf{q} = 0$ without any actual reason.

Both these assumptions might hold for infinite range Coulomb force, but they can hardly correspond to finite nucleon force where also the whole fully unscattered part lies in the range of measured beam. In any case the optical theorem cannot be applied to in the case of nucleon scattering.

And we must ask: What is actual shape of nucleon scattering amplitude in the case of very small scattering angles? And what are other physical consequences? E.g., how it is with luminosity estimation?

4 Elastic Collisions at Very Small Angles

The situation may be demonstrated in Fig. 1 where two pairs of lines are pictured representing the amplitudes of elastic scattering (real parts being neglected). Each pair is represented once by complete (Coulomb and hadronic) amplitude and once by hadronic part. It has been assumed that approximately at $t = -0.02\,\mathrm{GeV}^2$ the contributions of both (Coulomb and hadronic) components are the same.

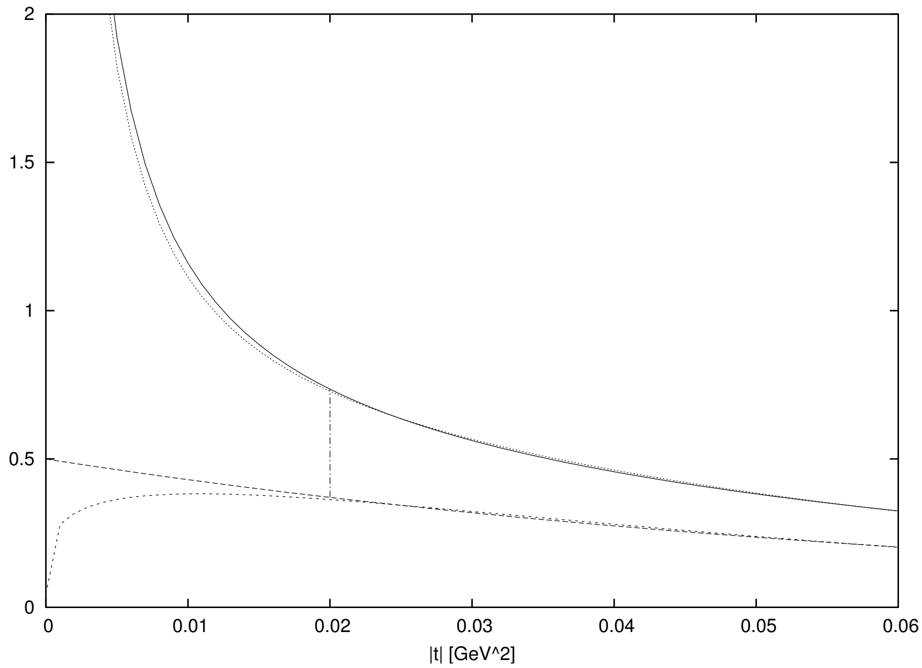

Figure 1: *Different hadronic amplitudes corresponding approximately to elastic data obtained at the energy of 53 GeV.*

The first pair of curves represents the standard approach at ISR energy; the curves corresponding approximately to experimental data obtained at energy of 53 GeV (see, e.g., [3]). The other pair shows then that practically the same complete elastic amplitude may be obtained even if the hadronic part behaves quite differently at very small values of $|t|$; the behaviour corresponding better to the situation at finite-range forces when the profile function $\Gamma(b)$ is equal to zero for all values of b greater than b_{max} ($\Gamma(b > b_{max}) = 0$) and then rises rather strongly with decreasing impact parameter b. The validity of the optical theorem cannot be required, of course, in such a case.

It means that the values of hadronic scattering may be strongly overvalued in the region around $t = 0$ in the standard approach. However, it is evident on the other side that in such a case also the value of luminosity L derived from elastic scattering values:

$$\frac{d\sigma_{el}(t)}{dt} = L|F^{C+N}(t)|^2,$$

might be very different, its contemporary values being significantly undervalued.

5 Conclusion

It follows from the preceding analysis that the contemporary phenomenological theories of elastic collisions do not allow to establish the total cross sections without some additional experiments or without more detailed analysis based on a model in which elastic and inelastic processes would be mutually correlated on realistic physical grounds. There is also the question how it is with the interpretation of nucleon elastic data for very small values of $\mathbf{q}$. Are they really decreasing from the value at $\mathbf{q} = 0$ or may they exhibit different behaviour being fully hidden under the effect of Coulomb force?

The problem is related very closely to the determination of corresponding total cross section and to establishing luminosity value from elastic scattering data. All these values may be hardly derived from elastic data if one must start from phenomenological models of elastic scattering only.

Important problem must be seen in the fact that the t-dependence of elastic differential cross section is the only available experimental set of data from which the mere modulus of complex amplitude function may be established while the phase remains practically undetermined. Quite different impact-parameter characteristics may be then derived according to its choice; see, e.g., [4, 5]. It means that the behaviours ontologically very different may be derived on the basis of standard phenomenological models.

The invalidity of optical theorem brings, therefore, rather inconclusive outlook as to the consequences following from the analysis of experimental elastic data. And one should ask if it is possible to find a procedure how to make use of elastic data in a more effective way. Such an approach seems to follow from applying an ontological model to processes running in particle collisions, which will be shortly described in the last additional section.

6 Additional Section: Ontological Model

In this additional section we should like to present a more promising view. We have shown quite recently that the elastic data may be well interpreted on the basis of ontological approach that allows to provide a correlation between elastic and inelastic processes. A nucleon consisting of many constituents, quarks and partons, has been assumed to be a matter object existing in a series of states differing by external dimensions.

The model has been explained and demonstrated with the help of elastic data obtained at energy 53 GeV; see [6]. It has been shown that this data in the interval $t \in (-4., 0.)\,\mathrm{GeV}^2$ may be truly interpreted as the superposition of three collision types between two states that exhibit maximal dimensions. All available elastic data at the given energy (in the interval $t \in (-14., 0.)\,\mathrm{GeV}^2$ may be then well interpreted as the superposition of six collision kinds between three nucleon states of maximal dimensions.

In addition to the already mentioned basic assumption (i.e., nucleon consisting of different internal states) it has been assumed, too, that the probability of each elastic event at any value of impact parameter may be expressed as the product of two probabilities:

$$P_{el}(b) \;=\; P_{tot}(b)\,.\,P_{rat}(b);$$

the first factor representing total collision probability and the other one representing the ratio of elastic processes to all collision processes.

And it may be regarded as quite natural from realistic point of view to add the third assumption that these partial probabilities may be represented by two oppositely monotone functions of impact parameter b. And it has been possible to establish both the monotone functions for all involved collision kinds by performing the alternative fit of earlier ISR results.

Here are the corresponding preliminary results. The maximum dimensions of three largest states (involved in the given collision process) should be 1.64, 1.42, 0.88 fm; these states should exist in individual nucleons with following frequencies: $\approx$ 57, 31, 11 %.

Having known the b-dependent probabilities of total and elastic collisions for all collision kinds it has been possible to determine also the approximate values of corresponding cross sections:

$$\sigma_{tot} \cong 36\,\mathrm{mb}, \quad \sigma_{el} = 7.3\,\mathrm{mb}.$$

While the elastic cross section has had practically the same value as standardly introduced the total cross section has differed rather significantly from the value obtained on the basis of the optical theorem validity. This earlier value seems to have been overvalued approximately by 15%. It means, of course, that also the luminosity value taken for ISR collider has been probably undervalued by 15%.

The probability distribution of elastic processes in impact parameter plane have exhibited, of course, clear peripheral behaviour, which has corresponded to similar results obtained on the basis of eikonal approach [3].

References

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