Optical theorem and elastic nucleon scattering

M. V. Lokajíček, V. Kundrát
Institute of Physics, AVČR, v.v.i., 18221 Prague

1. Introduction

Optical theorem:

Elastic collisions (t-dependence)

$\Longrightarrow$ total cross section

(increase with energy!)

However -

Important difference:

infinite-range Coulomb force

vs.

finite-range nucleon force

(assumption - unphysical)

2. Optical theorem and its derivation

V.Barone, E.Predazzi:

High-Energy Particle Diffraction; Springer 2002

Fraunhofer diffraction $(kR^2 << D)$:

$\Gamma(\mathbf{b})$ - profile function of hole

Babinet's principle:

hole and obstacle ($\mathbf{S}(\mathbf{b})$)

$$U(x, y.z) = -\frac{ik}{2\pi} U_0 \frac{e^{ikr}}{r} \int d^2\mathbf{b}\, S(\mathbf{b})\, e^{-i\mathbf{q}.\mathbf{b}}$$

Huygens-Fresnel principle -

$(S(\mathbf{b}) + \Gamma(\mathbf{b}) = 1) \longrightarrow$ plane wave

Complete amplitude:

$$U_{out}(x, y, z)) = U_{unsc} + U_{scatt}$$

$$= U_0 \left(\psi_{unsc} + f(\mathbf{q}) \frac{e^{ikr}}{r} \right),$$

$$\psi_{unsc} = \alpha e^{ikz}, \quad |\alpha| < 1$$

$$\mathbf{q} = \mathbf{k}' - \mathbf{k}, \quad |\mathbf{k}'| = |\mathbf{k}| = k$$

$$U(x,y,z) = U_0 \left(\psi_{unsc} + f(\mathbf{q}) \frac{e^{ikr}}{r} \right)$$

$$f(\mathbf{q}) = \frac{ik}{2\pi} \int d^2\mathbf{b}\, \Gamma(\mathbf{b})\, e^{-i\,(\mathbf{q}.\mathbf{b})}$$

$\Gamma(\mathbf{b})$ **- profile function**

differential cross section - $\frac{d\sigma}{d\Omega} = |f(\mathbf{q})|^2$

integrated elastic cross section -

(small scattered angles + Parseval's theorem)

$$\sigma_{el} \cong \frac{1}{k^2} \int |f(\mathbf{q})|^2 d^2\mathbf{q} = \int d^2\mathbf{b} |\Gamma(\mathbf{b}|^2$$
$$= \int d^2\mathbf{b} |1 - S(\mathbf{b})|^2$$

Further assumptions:

$\psi_{unsc} \equiv f(0)$ **and unitarity**

absorption cross section -

$$\sigma_{abs} = \int d^2\mathbf{b}(1 - |S(\mathbf{b})|^2)$$
$$= \int d^2\mathbf{b}\,(2Re\Gamma(\mathbf{b}) - |\Gamma(\mathbf{b}|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_{abs} = 2 \int d^2\mathbf{b}\, Re\Gamma(\mathbf{b})$$

$\Longrightarrow$ **total cross section -**

$$\sigma_{tot} = \frac{4\pi}{k} Im\, f(\mathbf{q} = 0)$$

3. Hadron force and optical theorem

$$U(x,y,z) \;=\; U_0 \left(\psi_{unsc} + f(\mathbf{q}) \frac{e^{ikr}}{r} \right)$$

$$U(x,y,z) \;=\; U_0 F(\mathbf{q}) \frac{e^{ikr}}{r}$$

$$F(0) = f(0) + |\psi_{unsc}|$$ **(singular point)**

However:

$F(\mathbf{q})$ **- as continuous**

(and decreasing)

$\Longrightarrow \quad F(0) = f(0)$ **or** $(\psi_{unsc} \;=\; 0)$

Acceptable:

- **for Coulomb force**
- **not for finite nucleon force:**

$\psi_{unsc} \neq 0$ **(in measured beam)**

Optical theorem: **<u>not generally valid</u>**

Questions: **actual shape of** $f(\mathbf{q})$?

especially - value of $f(0)$?

luminosity estimation?

4. Scattering at very small angles (all at $\approx 53 GeV$)

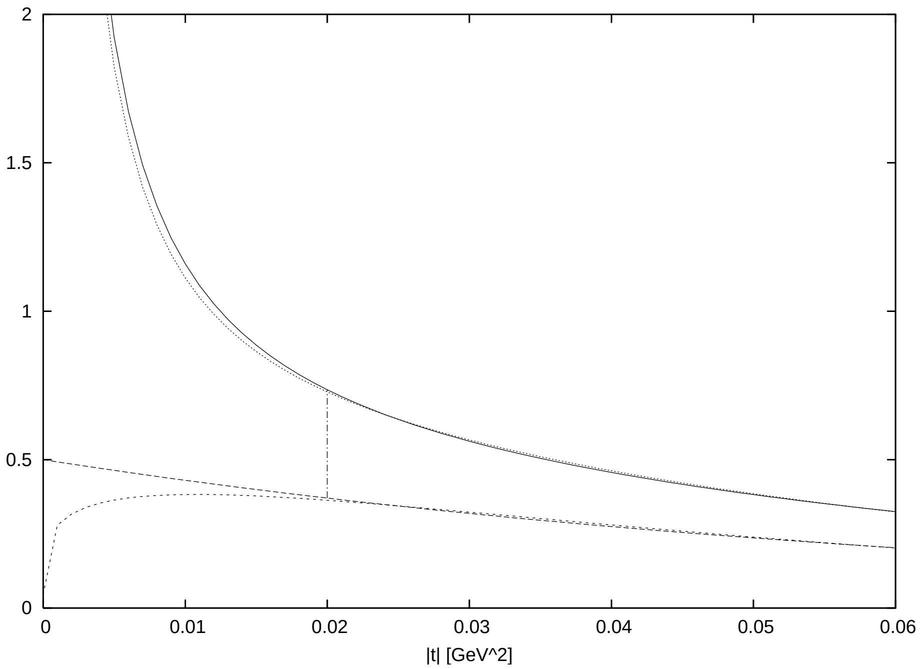
Figure:

Two pairs of lines - (complete ampl. + hadronic parts)

first pair (vertical line):
standard approach

second pair:
nucleon amplitude rises for small $|t|$
$[\Gamma(b > b_{max}) = 0]$

Luminosity : $\frac{d\sigma_{el}(t)}{dt} = L|F^{C+N}(t)|^2$
derived at $|t| < 0.02$ - undervalued?



5. Conclusion

see: /arXiv:0906.3961

Standard approage:

no optical theorem

(for hadron forces)

Phenomenological models:

cross sections and luminosity ??

(practically no possibility)

pessimistic outlook?!

However:

ontological approach?

M.L.: Hidden-variable theory vs. Copenhagen QM;
/arxiv:0905.0140,
will be published in Concepts of Physics

(see also: Conference Proceedings, No. 1018,
American Institute of Physics, 2008, pp. 40–5)

Appendix

Ontological model:

Two assumptions:

- each proton in different internal states (divers maximal dimensions - b_{max});

- for each internal state and any impact parameter: $P_{el}(b) = P_{tot}(b) \, . \, P_{rat}(b)$

P_{rat} **- ratio of elastic to total probab.,**

$P_{tot}, \; P_{rat}$**: opposite monotony in** $(0, b_{max})$

Preliminary results (53 GeV):

good (comparable) fits - $|t| \in (0, 14) GeV^2$

3 largest internal states: (6 comb.)

b-max: $1.61, \quad 1.4, \quad 0.88$ **fm** (≥ 0.4)

frequency: $\approx 57., \quad 37., \quad 5.$ **%**

$$\sigma_{tot} = \int_0^{b_{max}} db P_{tot}(b) \cong 34mb, \; \sigma_{el} = 7.3mb$$

Luminosity undervaluated: $\approx 20\%$

solution - peripheral

(comp.: similar eikonal result)