Pomeron and Odderon: Saturation and Confinement, and Gauge/String Duality

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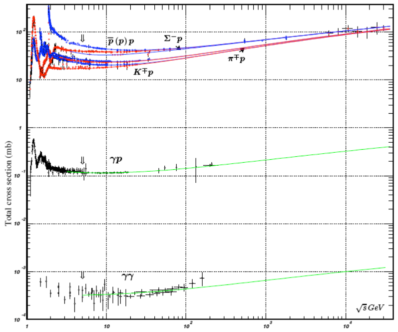
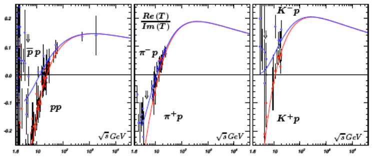
13th Workshop on Elastic and Diffractive Scattering

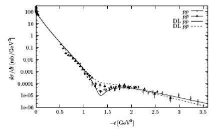
High Energy scattering after AdS/CFT

- R. Brower, J. Polchinski, M. Strassler, and C-I Tan, "The Pomeron and Gauge/String Duality", JHEP 0903:092,2009, (hep-th/0603115.)
- R. Brower, M. Strassler, and C-I Tan, "On the eikonal approximation in AdS space", JHEP 0903:050,2009, hep-th/0707.2408; "On The Pomeron at Large 't Hooft Coupling", JHEP 0903:092,2009, arXiv:0710.4378 [hep-th].
- R. Brower, M. Djuric, and C-I Tan, "Odderon and Gauge/String Duality", JHEP (in press), (arXiv:0812.0354.)

High Energy Near-Forward Scattering:

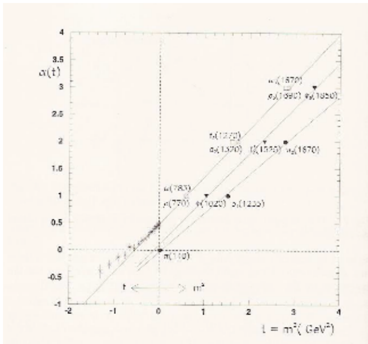






Regge Behavior

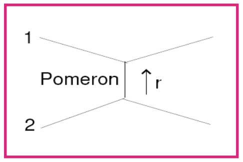
$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$



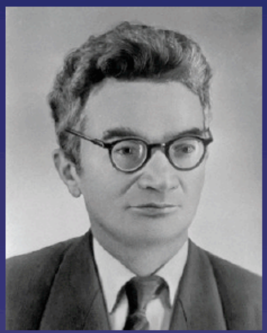
Total Cross Sections

$$\sigma_{totl} \sim s^{-1}\mathcal{A}(s,0) \sim s^{j_{effective}(0)-1}$$

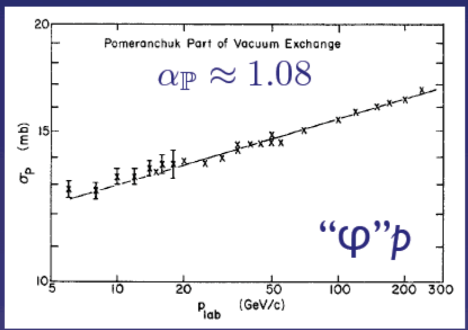
$$j_{effective}(0) = \alpha(0) > 1$$



Pomeron > Pomeranchukon > Pomeranchuk singularity



I. Ya. Pomeranchuk



Experiments Suggest:

- Exchanging C=+1 color-singlet "state" with

$$j^{(+)}_{effective} > 1$$

- Exchanging C=-1 color-singlet "state" with (??)

$$j^{(-)}_{effective} \simeq 1$$

- Weak-coupling: BFKL, etc.

- **Non-Perturbative QCD? Answer: AdS/CFT, or Gauge/String Duality.**

What is the (bare) Pomeron anyway?

Definition:

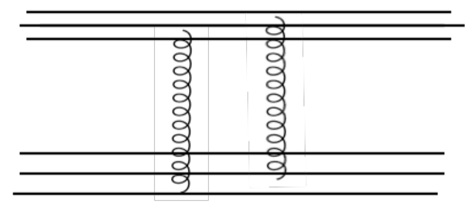
The Pomeron ´ the vacuum exchange contribution to scattering at high energies at leading order in $1/N_c$ expansion.

$$A(s,t) = g_s^2 A_1(s,t,\lambda) + g_s^4 A_2(s,t,\lambda) + \cdots$$

Where $\lambda = g^2 N_c$ & $g_s = 1/N_c$

Two gluon exchange (Low-Nussinov Pomeron!)

- $J_{cut} = 2\,(J-1) + 1 = 1$

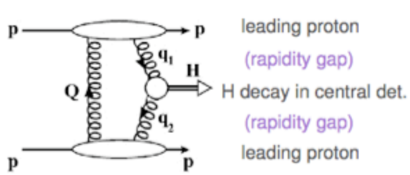


F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

Central exclusive Higgs: Motivation

[A.de Roeck et al, EPJC 25(02)391]

- Higgs boson central exclusive production (CEP): pp → p H p



m_H via "missing-mass" method:

$$m_H^2 = (p + p - p' - p')^2$$

$\Delta m_H = \mathcal{O}(1-2)\ \mathrm{GeV/c^2}$

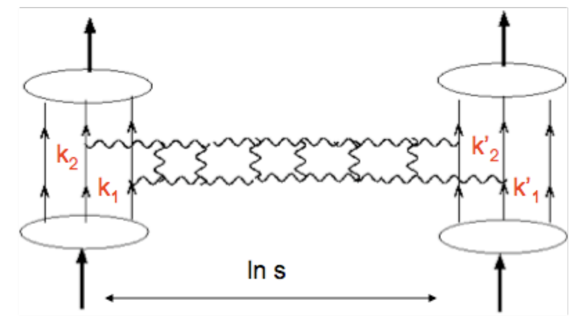
- Excellent physics motivations:
  - Quantum numbers: central system mostly scalar & CP-even ($J^{PC} \approx 0^{++}$ rule)
  - Reduced QCD (color-singlet bb) backgd. ($J_z \approx 0$ rule). H→ bbar accessible !
  - Good mass resol. (even invisible-H): from protons independ. of H decay products
  - CP violation in Higgs sector: directly measurable from protons φ asymmetry
  - Discovery channel (bbar, $\tau^+\tau^-$) in certain regions of MSSM

Martin, Khoze and Ryskin, "Diffractive Higgs production"

BFKL: Balitsky & Lipatov; Fadin, Kuraev, Lipatov'75

$t = -(k_1 + k_2)^2$ →

$\lambda = g^2 N_c \sim 0$



- ❑ Sum diagrams 1st order in $g^2 N_c$ & all orders $(g^2 N_c \log s)^n$
- ❑ BKFL equation for 2 "reggized" gluon ladder is L = 2 SL(2,C) spin chain to one loop order.
- ❑ Accidentally "planar" diagrams (e.g. $N_c = 1$) and conformal.

The QCD Pomeron

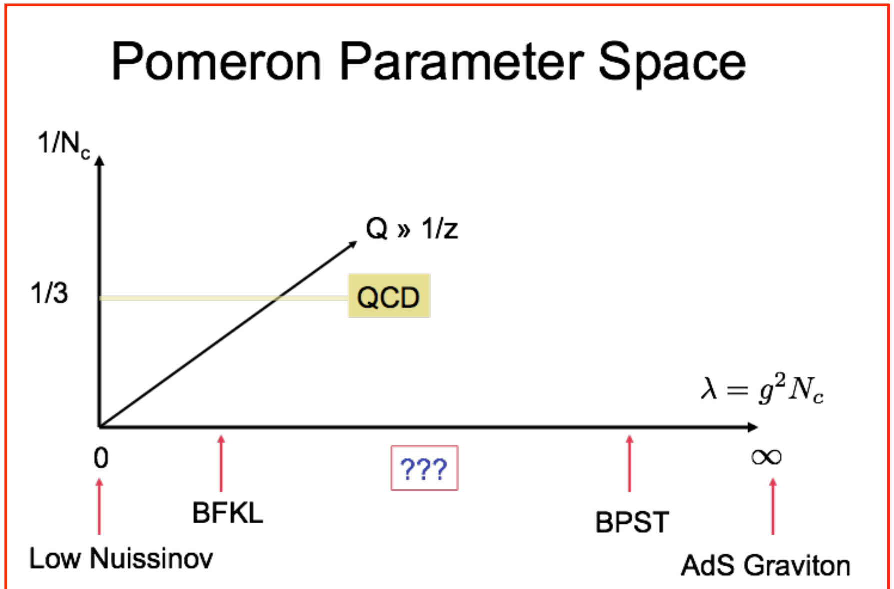
In gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

Pomeron can be associated with a Reggeized Massive Graviton.

Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

R. Brower, J. Polchinski, M. Strassler, and C-I Tan,
"The Pomeron and Gauge/String Duality", (hep-th/0603115.)

Gauge/String Duality: QCD at Strong Coupling



- C=+1: Pomeron <=> Graviton:

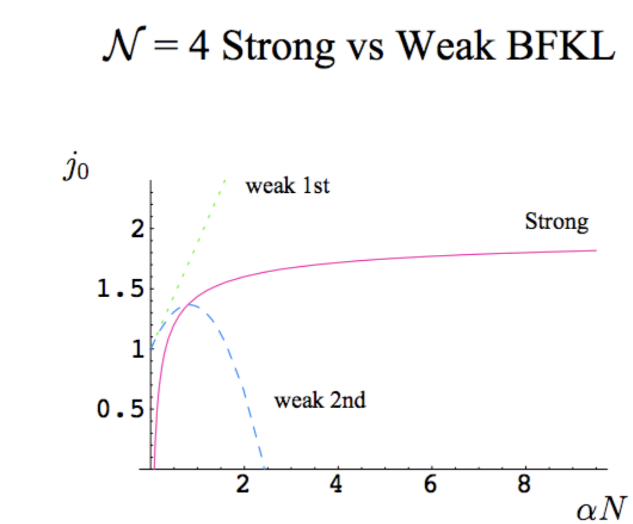
$$\alpha_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$$

$$(symmetric\, tensor : g_{\mu\nu})$$

- C=-1: Odderon <=> Kalb-Ramond

$$\alpha_0^{(-)} = 1 - m_{ads}^2/2\sqrt{\lambda} + O(1/\lambda)$$

$$(anti-symmetric\, tensor : b_{\mu\nu})$$



- New Questions: Unitarity, Saturation, Confinement, Froissart, etc.?

Froissart bound:

$$\sigma_{total}(p + p \to X) \leq (\pi/m_0^2)C(m_p/m_0)\log^2(s/s_0) + \cdots$$

Questions:

- Saturation? (equality) If so, can C be calculated?
- Events and/phase space responsible for $C(m_p/m_0) > 0$?
- Does AdS/CFT provide a generic mechanism for $C(m_p/m_0) > 0$?

Outline

- Gauge/String Duality
- Pomeron/Odderon as "Fluctuations" in AdS space
  - Graviton/Odderon in AdS becomes a fixed Regge Cut: (Conformal Invariance)
  - Pomeron/Odderon as a Reggeized Massive Graviton/Kalb-Ramond fields: (Confinement)
- Aspects of Analyticity, Unitarity and Confinement
- Conformal Invariance and Transverse Space,
- Phase of Eikonal, Saturation, Confinement.

II: Gauge/String Duality

QCD Pomeron as “metric fluctuations” in AdS

- Strong <==> Weak duality
- Geometry of AdS/CFT and Scale Invariance
- High Energy Scattering
- Confinement and Glueball Spectrum
- Pomeron as Reggeized Massive Graviton

IIa: Degrees of Freedom

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar\psi(x)\psi(x),\ \ \bar\psi(x)D_\mu\psi(x)$

$$S(x) = TrF^2_{\mu\nu}(x),\ \ O(x) = TrF^3(x)$$

$$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x),\ \ etc.$$

$$\mathcal{L}(x) = -TrF^2 + \bar\psi \not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g^{(0)}_{mn}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x),\ a(x),\ etc$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

IIb: $\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x) \mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string} \left[\phi_i(x,z)|_{z \sim 0} \to \phi_i(x) \right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

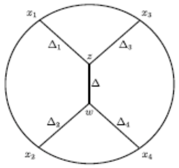
- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

Conformal Invariance and Pomeron Interaction from AdS/CFT



+ ....

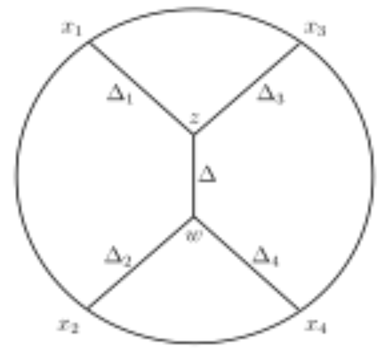
Technique: Summing generalized Witten Diagrams

Freedman et al., hep-th/9903196

Brower, Polchinski, Strassler, and Tan, hep-th/0003115

- Draw all "Witten-Feynman" Diagrams in AdS_5,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$



One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

- Strong Coupling Pomeron has $J = 2$
- Need to consider λ finite.
- For QCD, needs confinement to introduce a scale.

IIc: Geometry of AdS/CFT and Scale Invariance

What is the curved space?

Maldacena: UV (large r) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + ds_X^2$$

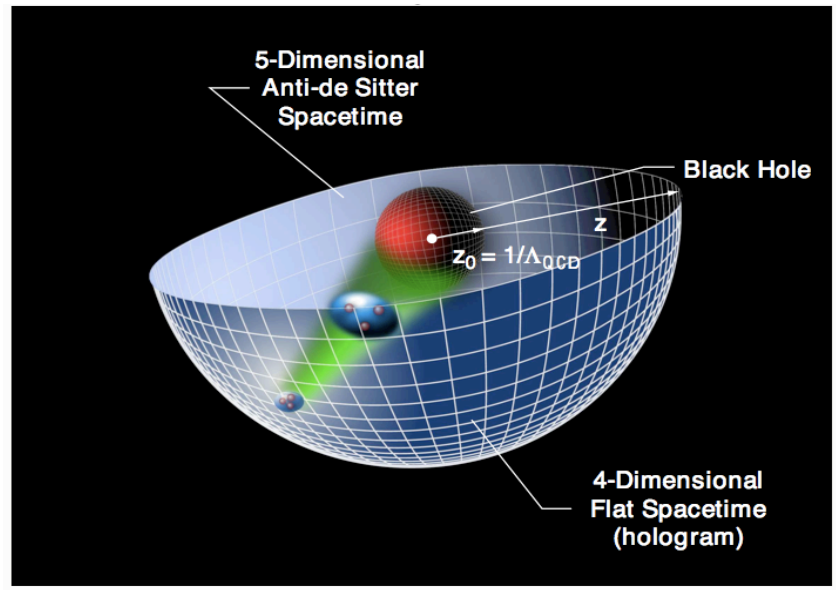
Captures QCD's approximate UV conformal invariance

$$x \to \zeta x \ , \ r \to \frac{r}{\zeta} \qquad (\text{recall } r \sim \mu)$$

Confinement: IR (small r) is cut off in some way

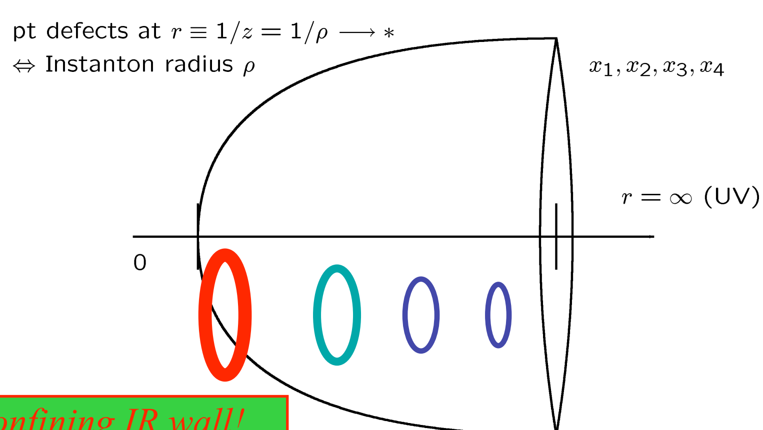
$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

For Pomeron: *string theory* on cut-off AdS_5 (X plays no role)



Cutoff AdS_5

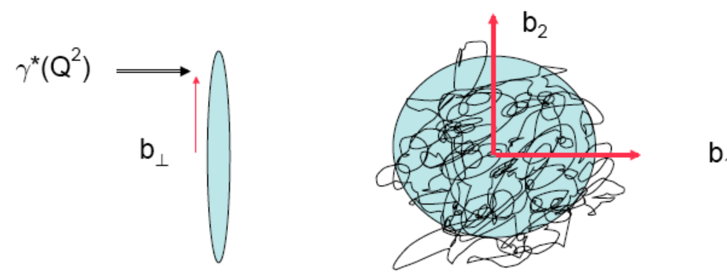
Large Sizes



Add Confining IR wall!

z=1/r,

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

2-d Longitudinal	$p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$
2-d Transverse space:	$x'_\perp - x_\perp = b_\perp$
1-d Resolution:	$z = 1/Q$ (or $z' = 1/Q'$)

II-d: Pomeron Propagator at Finite Coupling λ :

due to Diffusion in AdS (next section)

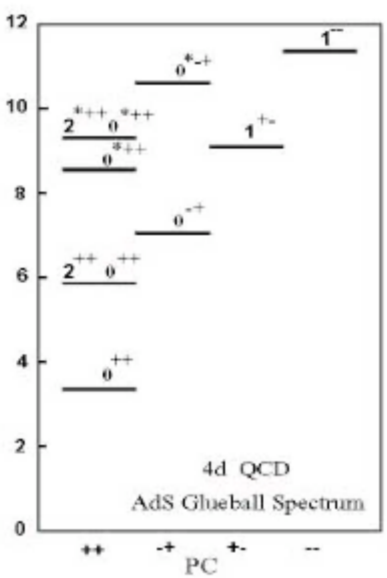
- Pomeron becomes cut at

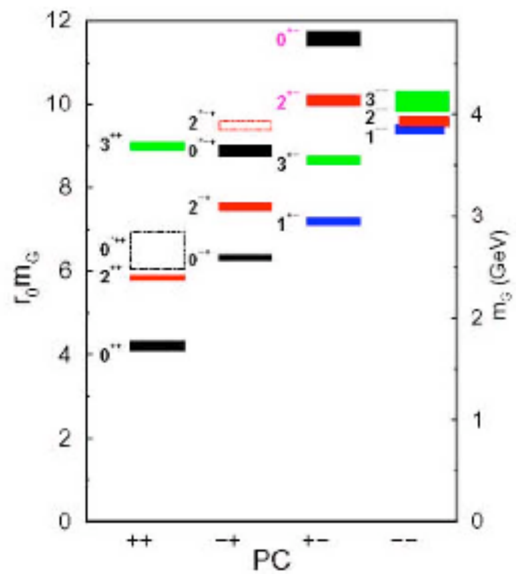
$$j_0 = 2 - 2/\sqrt{\lambda}$$

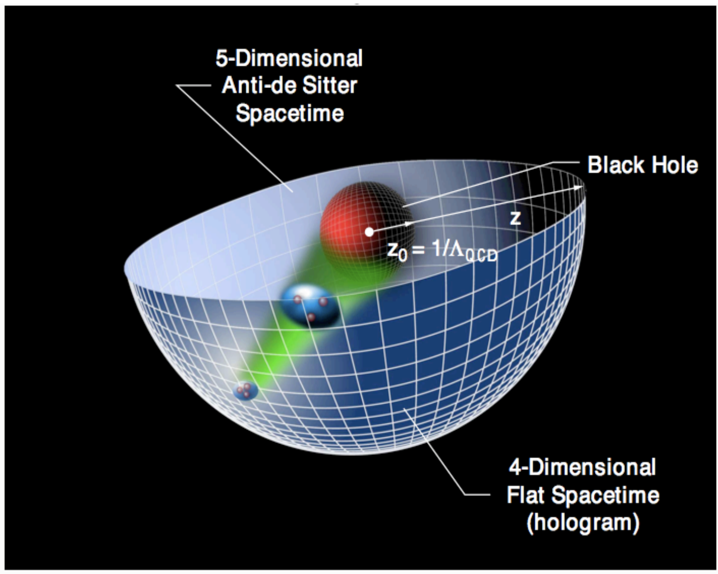
$$j_0 = 2 - 2/\sqrt{\lambda}$$

- Conformal: No scale and No Regge trajectory

II-e: Confinement Deformation: Glueball Spectrum







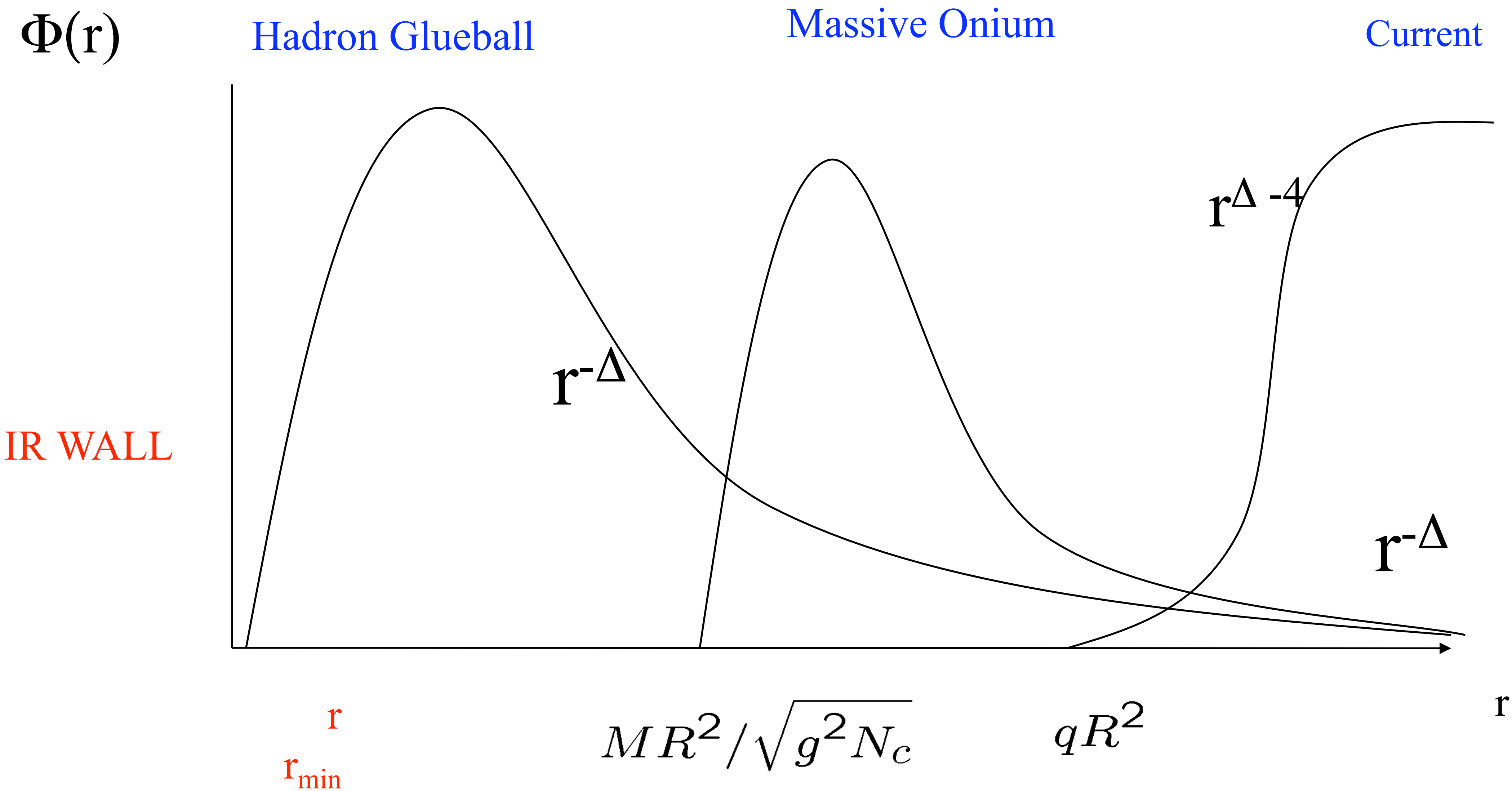
Four-Dimensional Mass:

$$E^2 = (p_1{}^2 + p_2{}^2 + p_3{}^2) + M^2$$

5-Dim Massless Mode:

$$0 = E^2 - (p_1{}^2 + p_2{}^2 + p_3{}^2 + p_r{}^2)$$

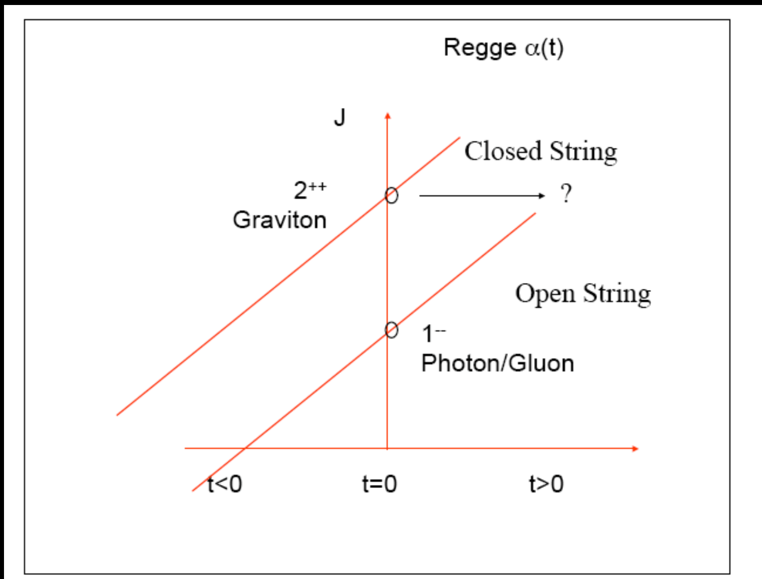
Approx. Scale Invariance and the 5th dimension



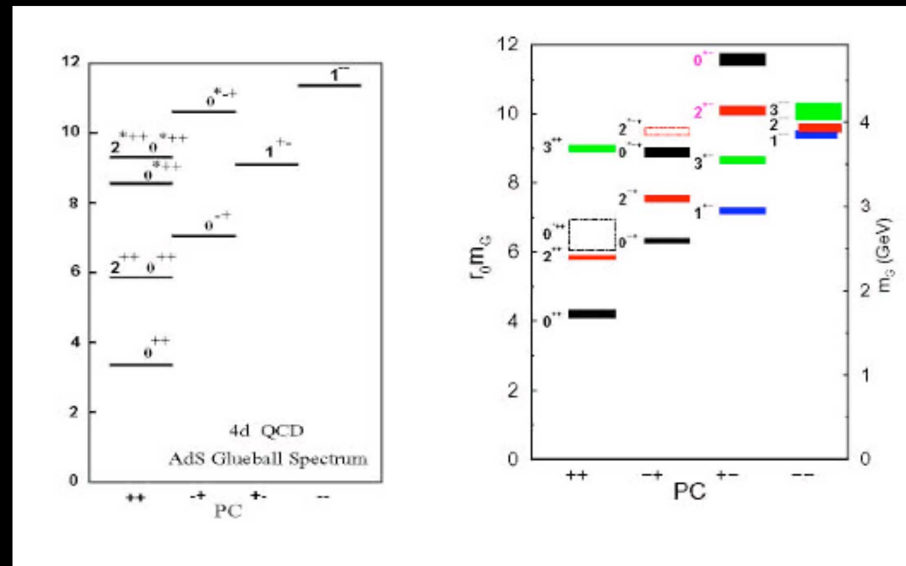
==> Hard Scattering (Polchinski-Strassler)

QCD Pomeron <===> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

Fixed cut in J-plane:

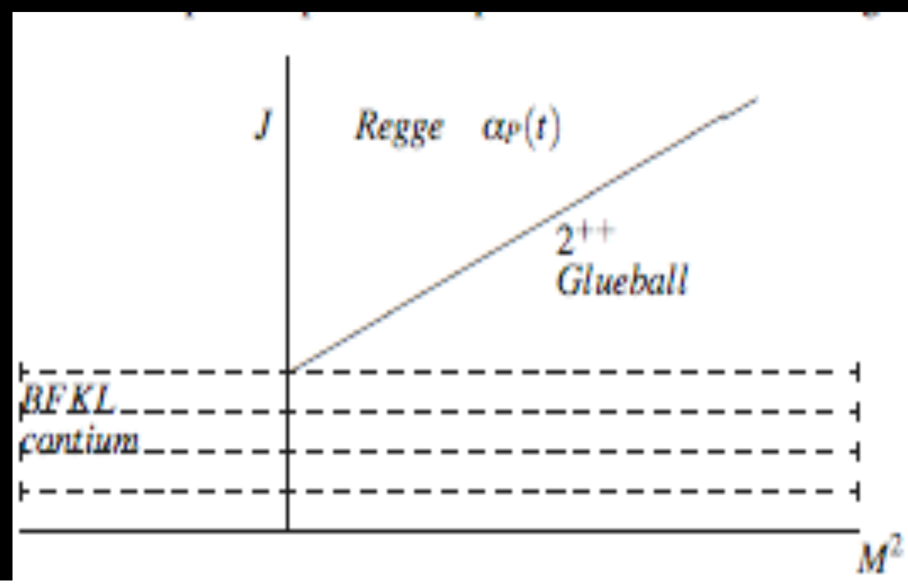
Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

Strong coupling:

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Pomeron in AdS Geometry



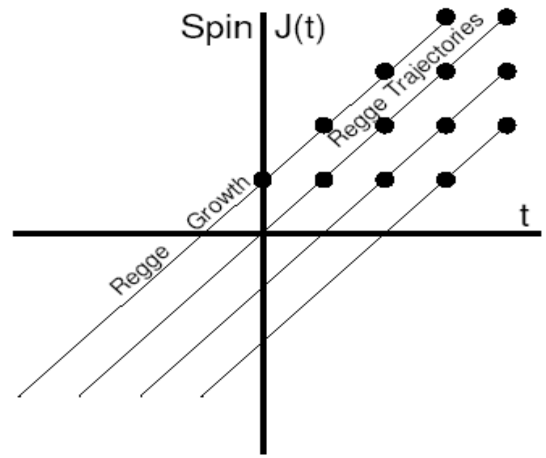
III: Pomeron as Diffusion in AdS

Flat Space String Scattering -- Regge Behavior

$$\mathrm{Im}\mathcal{A} \sim \sum_i s^{J_i(t)}$$

$$J(t) = \alpha(t) = \alpha_0 + \alpha' t$$

$$t \leftrightarrow \nabla_b^2$$



$$G(s;\vec{b},\vec{b}') \longleftrightarrow \langle\, \vec{b} \,|\, s^{2+\alpha'\nabla_b^2/2} \,|\, \vec{b}' \,\rangle$$

$$\sim s^{\alpha_0} \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

$\Leftarrow$ Diffusion in Impact Space

Diffusion in AdS

AdS, C=+1:

$$\alpha'\tilde{t} \to \alpha'\Delta_P \equiv \frac{\alpha' R^2}{r^2}\nabla_b^2 + \alpha'\Delta_{\perp P}$$

$$s^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} s^j G(j)$$

with

$$G(j) = \frac{1}{j - 2 - \alpha'\Delta_P/2}$$

Effective Schrodinger Equation:

$$(j - 2 - \alpha'\Delta_P/2)G(j; z, z', t) = \delta(z - z')$$

Fixed cut in J-plane:

Weak coupli
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

Strong couplin

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

$At \ \ t = 0 \ \ and \ \ z = e^{-u}$

$$\left[-\partial_u^2 + 4 + 2\sqrt{\lambda}(j-2)\right] = e^u \delta(u - u')$$

Comparison of strong vs weak coupling kernel at t=0

Strong Coupling:

$$\mathcal{K}(r,r',s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}} e^{-(\ln r - \ln r')^2/4\mathcal{D}\ln s}$$

Diffusion in "warped co-ordinate"

$$j_0 = 2 - \frac{2}{\sqrt{g^2 N}} + O(1/g^2 N)$$

$$\mathcal{D} = \frac{1}{2\sqrt{g^2 N}} + O(1/g^2 N) \quad .$$

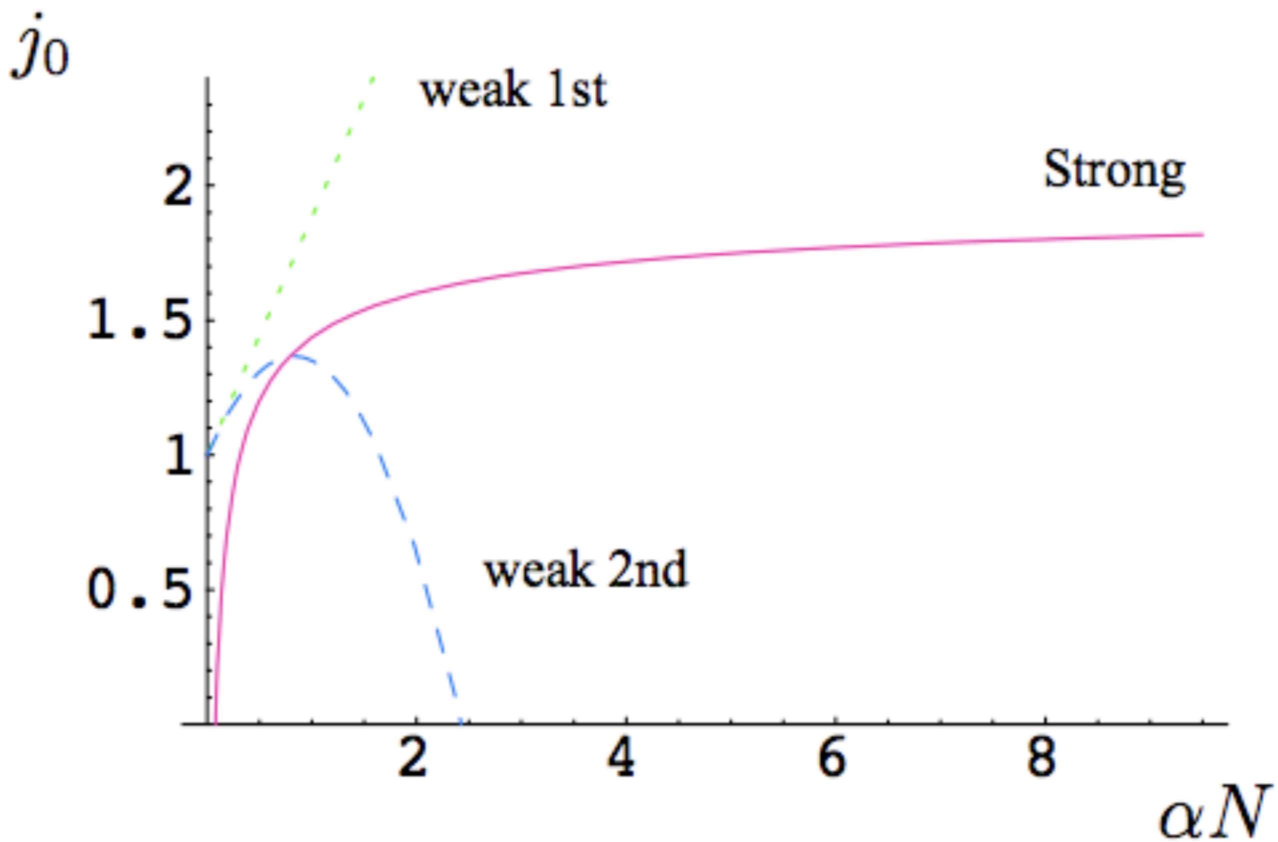
Weak Coupling:

$$K(s,k_\perp,k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-[(\ln k'_\perp - \ln k_\perp)^2/4\mathcal{D}\ln s]}$$

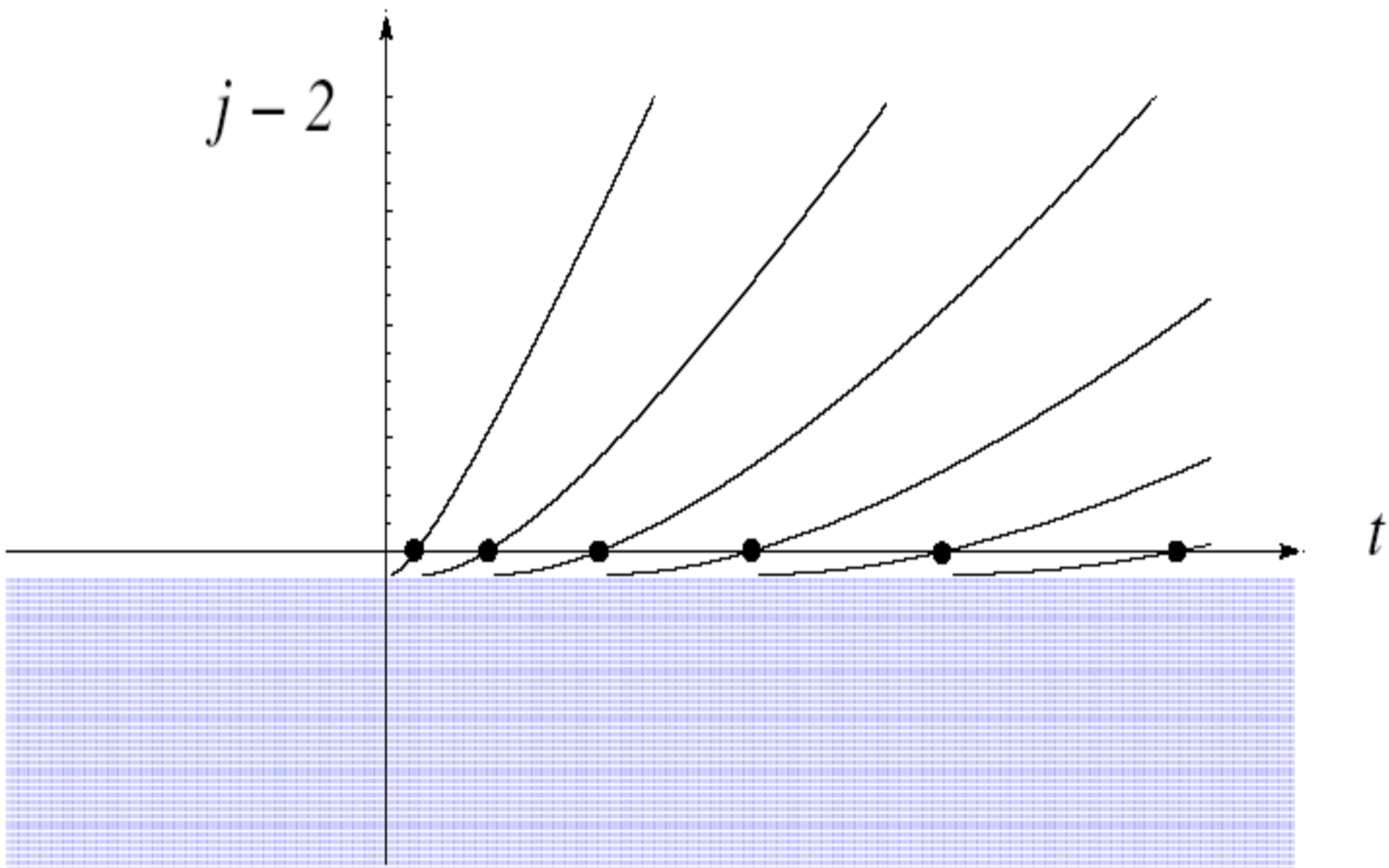
$$j_0 = 1 + \ln(2) g^2 N/\pi^2$$

$$\mathcal{D} = \frac{14\zeta(3)}{\pi} g^2 N/4\pi^2 .$$

$\mathcal{N} = 4$ Strong vs Weak BFKL

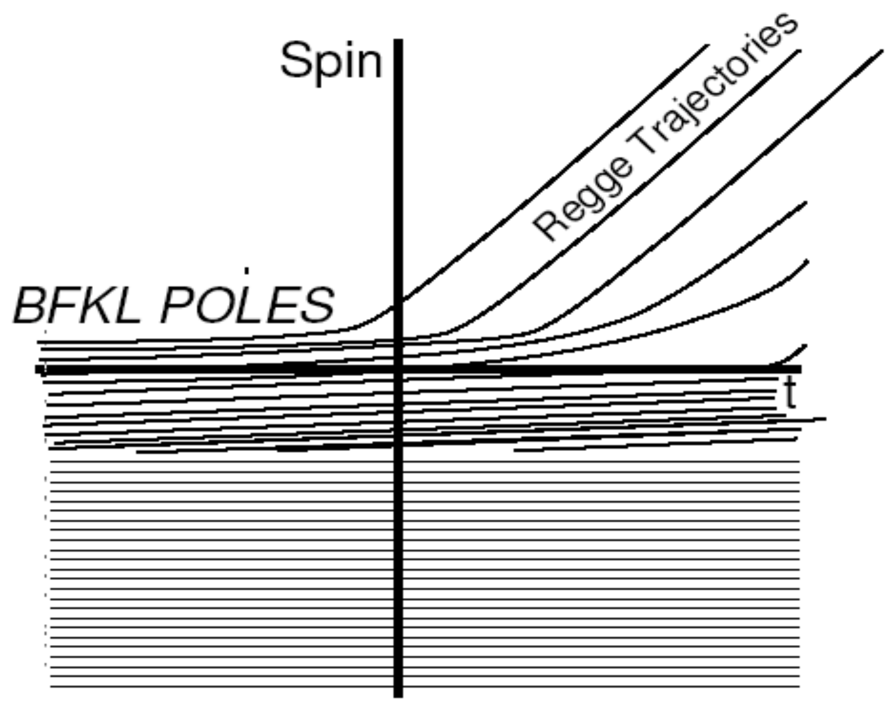


Hardwall Regge Spectrum and Cut



Pomeron in QCD

Running UV, Confining IR (large N)



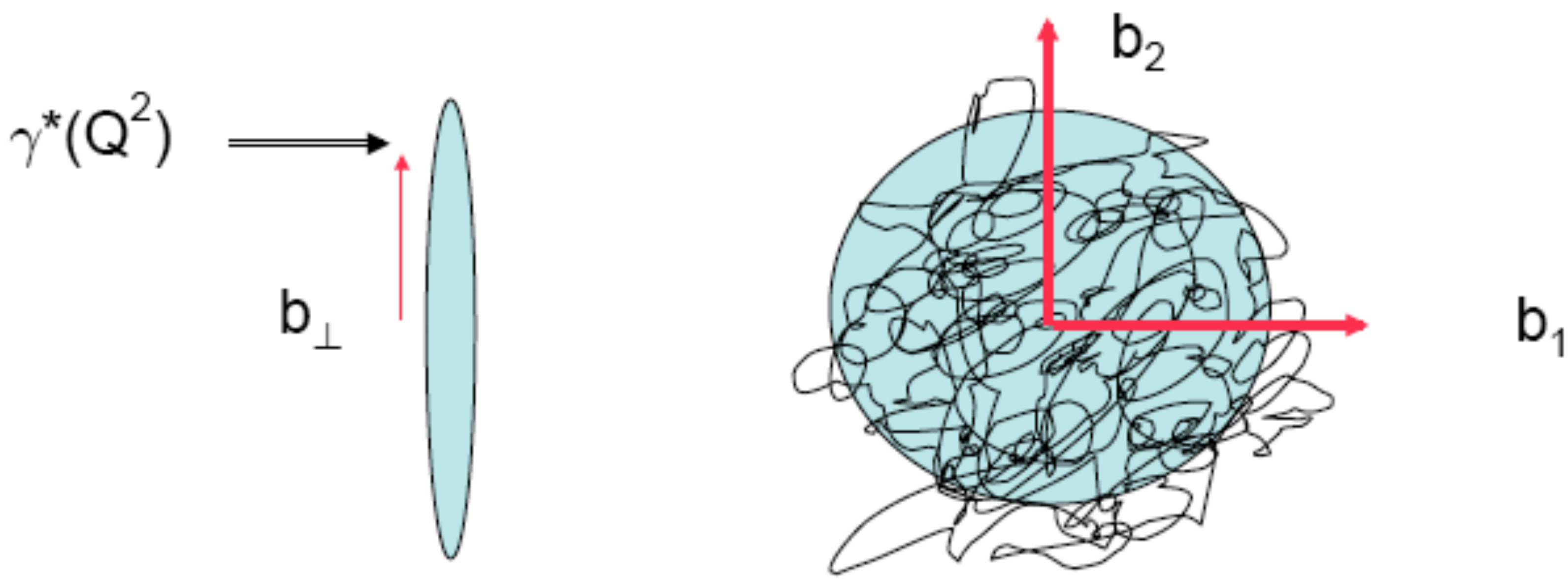
The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

Emergence of 5-dim AdS-Space

Let $z = 1/r$, $0 < z < z_0$, where $z_0 \sim 1/\Lambda_{qcd}$

"Fifth" co-ordinate is size z / z' of proj/target



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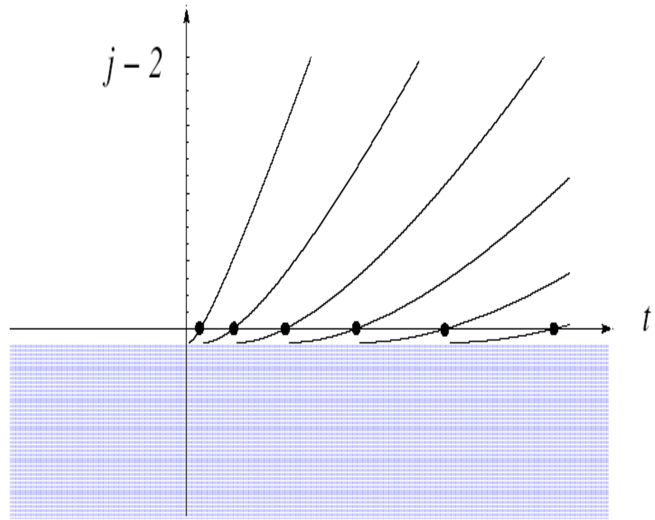
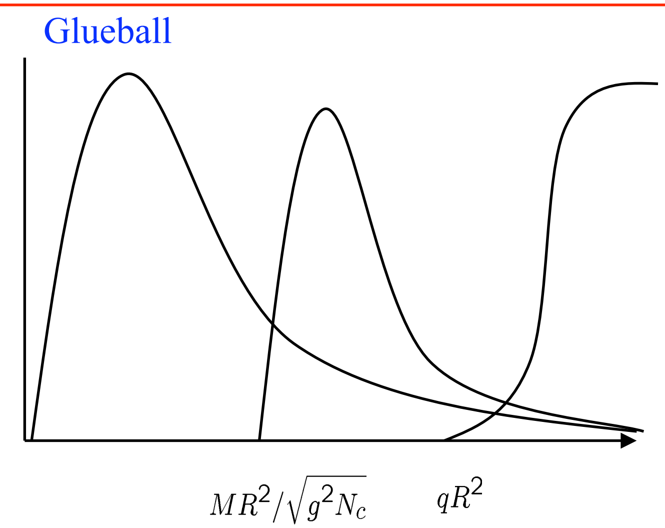
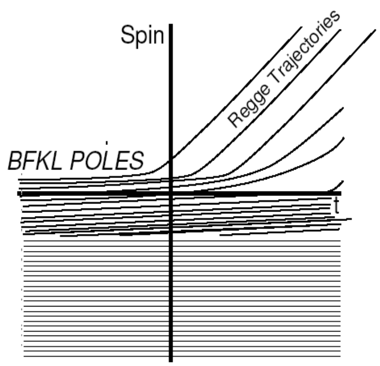
Summary: Pomeron in QCD





Text

Running UV, Confining IR (large N)



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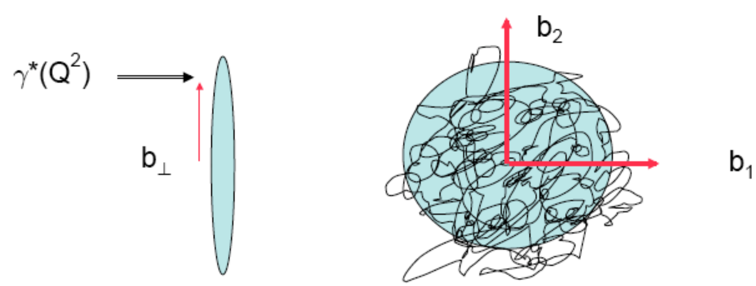

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Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

Unification

- *Soft Pomeron*: Diffusion in Impact space,

Hard Pomeron: Diffusion in Virtuality,

- *Heterotic Pomeron* -- G. M. Levin and CIT (ISMD--1993)

- After nearly15 years, *Unification* through AdS/CFT Correspondence via AdS5

- **Pomeron is the Graviton in Curved Space (AdS)**

IV: Odderon in AdS

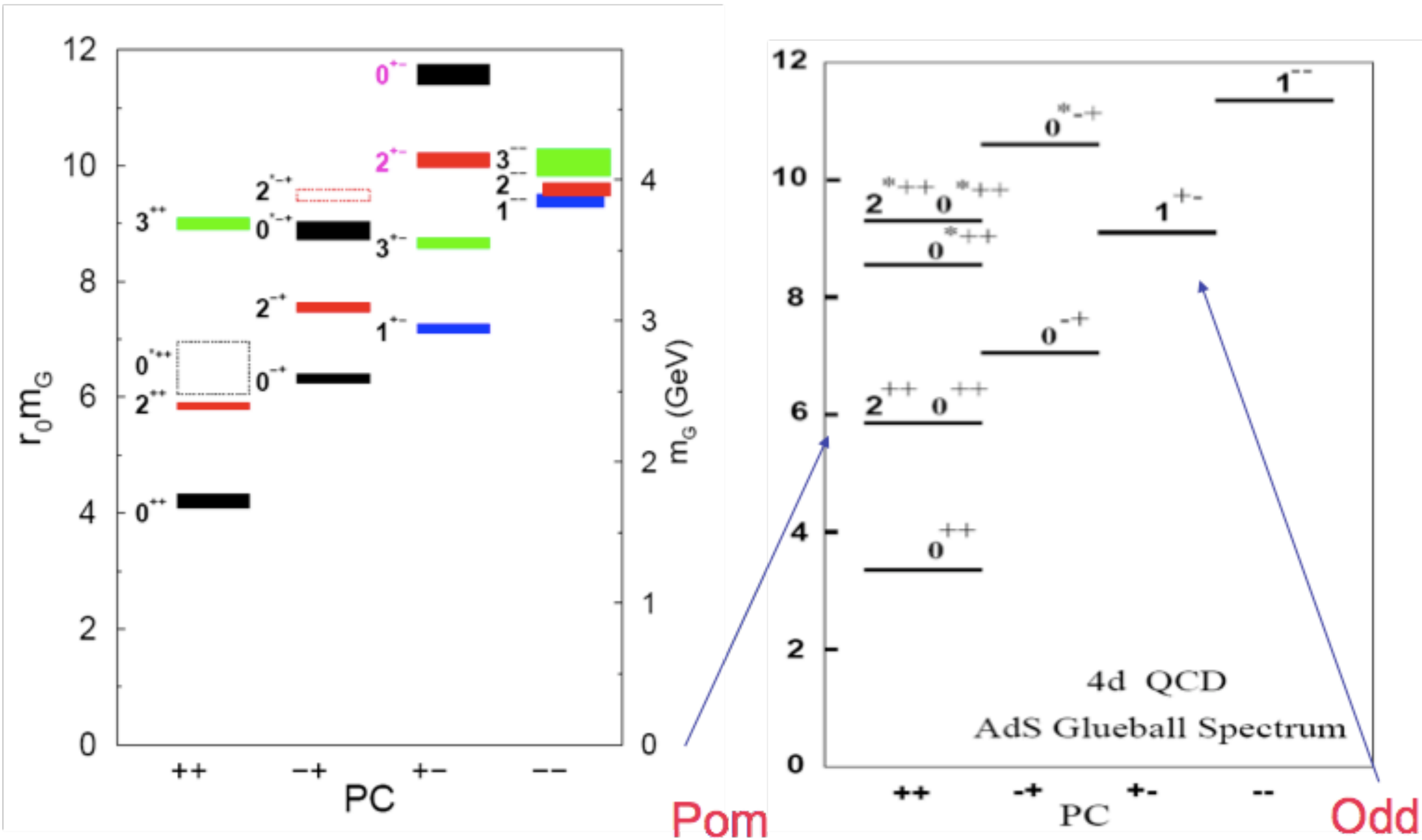
Massless modes of a closed string theory:

metric tensor, $G_{mn} = g^0_{mn} + h_{mn}$

Kolb-Ramond anti-sym. tensor, $b_{mn} = -b_{nm}$

dilaton, etc. $\phi, \chi, \cdots$

Confinement gives a discrete spectrum of Glueballs: Lattice Data vs AdS IIA Gravity dual Gauge ($\alpha' = 0$)



flat-space expectation

$$F_{\bar{c}b\to\bar{a}d} \equiv \bar{F} = F^+ + F^- \qquad [\sigma_T(\bar{a}b) + \sigma_T(ab)] \sim (2/s)\,\mathrm{Im}\,F^+$$

$$F_{ab\to cd} \equiv F = F^+ - F^- \qquad [\sigma_T(\bar{a}b) - \sigma_T(ab)] \sim (2/s)\,\mathrm{Im}\,F^-$$

$$\mathcal{T}_{10}^{(+)}(s,t) \to f^{(+)}(\alpha' t)\left[\frac{(-\alpha' s)^{2+\alpha' t/2} + (\alpha' s)^{2+\alpha' t/2}}{\sin \pi(2+\alpha' t/2)}\right]$$

$$\mathcal{T}_{10}^{(-)}(s,t) \to f^{(-)}(\alpha' t)\left[\frac{(-\alpha' s)^{1+\alpha' t/2} - (\alpha' s)^{1+\alpha' t/2}}{\sin \pi(1+\alpha' t/2)}\right]$$

$$\alpha_+(t) = 2 + \alpha' t/2$$

$$\alpha_-(t) = 1 + \alpha' t/2$$

Massless Modes at t=0

Massless Modes in Flat-Space String Theory

$$|I, J; k\rangle = a^\dagger_{1,I} \tilde{a}^\dagger_{1,J} |NS\rangle_L |NS\rangle_R |k\rangle$$

$$|h\rangle = \sum_{I,J} h^{IJ} |I, J; k\rangle \quad , \quad |B\rangle = \sum_{I,J} B^{IJ} |I, J; k\rangle \quad , \quad |\phi\rangle = \sum_{I,J} \eta^{IJ} |I, J; k \perp\rangle$$

fluctuations of the metric G_{MN}

anti-symmetric Kalb-Ramond background B_{MN}

dilaton, ϕ

Diffusion in AdS

Flat Space: $t \to \nabla_b^2$

$$\tau = \log(\alpha' s) \qquad \langle \vec{b} \,|\, (\alpha' s)^{\alpha_\pm(0)+\alpha' t/2} \,|\, \vec{b}' \rangle \to (\alpha' s)^{\alpha_\pm(0)} \frac{e^{-(\vec{b}-\vec{b}')^2/(2\alpha'^2\tau)}}{\tau^{(D-2)/2}}$$

AdS5, C=+1:

$$\alpha' \tilde{t} \to \alpha' \Delta_P \equiv \frac{\alpha' R^2}{r^2} \nabla_b^2 + \alpha' \Delta_{\perp P}$$

$$\tilde{s}^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 2 - \alpha' \Delta_P/2}$$

AdS5, C=-1:

$$\tilde{s}^{1+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} \tilde{s}^j \, G^{(-)}(j) = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 1 - \alpha' \Delta_O/2}$$

Gauge/String Duality: Conformal Limit

- C=+1: Pomeron <===> Graviton

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda) \ .$$

- C=-1: Odderon <===> Kalb-Ramond Field

$$j_0^{(-)} = 1 - m_{AdS}^2/2\sqrt{\lambda} + O(1/\lambda) \ .$$

	Weak Coupling	Strong Coupling
$C = +1$	$j_0^{(+)} = 1 + (\ln 2)\ \lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C = -1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\ \lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Table 1: Pomeron and Odderon intercepts at weak and strong coupling.

V. Gauge/String Duality Beyond Pomeron

- Sum over all Pomeron graph (string perturbative, $1/N^2$)
- Eikonal summation in AdS_3
- Constraints from Conformal Invariance, Unitarity, Analyticity, Confinement, etc.
- "non-perturbative" (e.g., blackhole production)

- # Eikonal Sum: derived both via Cheng-Wu or by Shock-wave method

$$A_{2\to 2}(s,t) \simeq -2is \int d^2b\, e^{-ib^{\perp}q_{\perp}} \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^{\perp},z,z')} - 1\right]$$

transverse AdS_3 space !!

$$P_{13}(z) = (z/R)^2 \sqrt{g(z)} \Phi_1(z) \Phi_3(z) \qquad P_{24}(z) = (z'/R)^2 \sqrt{g(z')} \Phi_2(z') \Phi_4(z')$$

$$\chi(s, x^{\perp} - x'^{\perp}, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s, x^{\perp} - x'^{\perp}, z, z')$$

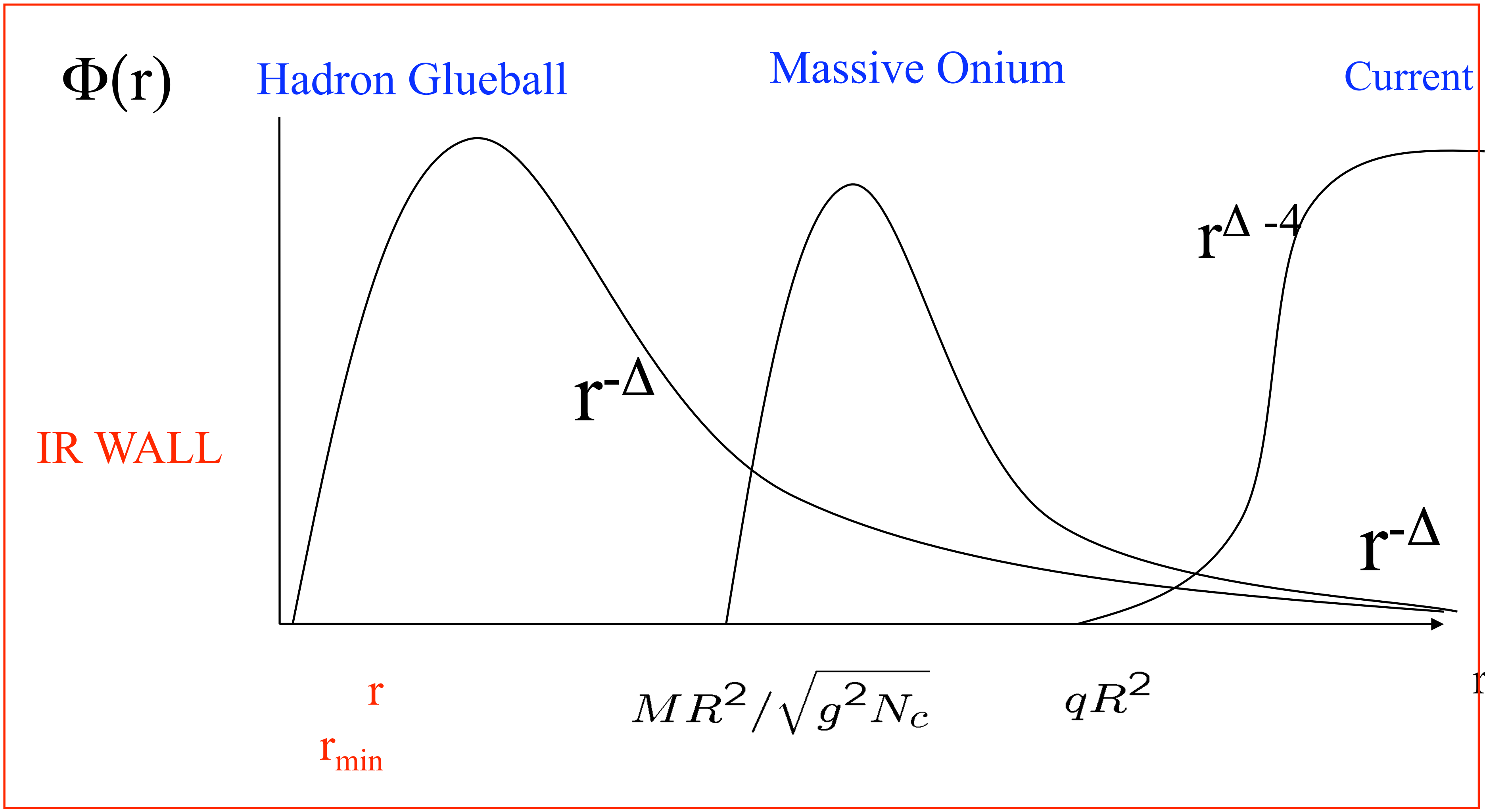
- Saturation:

$$\chi(s, x^{\perp} - x'^{\perp}, z, z') \;=\; O(1)$$

- Universality:

- **Universality:**

By choosing wave functions, Φ, can treat DIS, Onium-Onium, Proton-Proton, etc., on equal footing.



Saturation: $\chi(s, x^{\perp} - x'^{\perp}, z, z') = O(1)$

- Phase space:

$$s \leftrightarrow 1/x$$

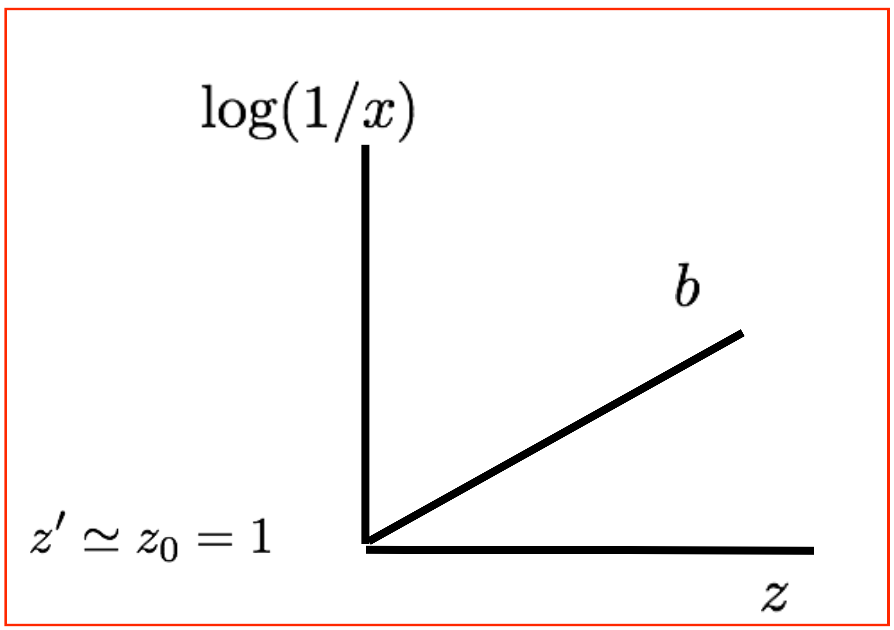
$$x_{\perp} \leftrightarrow impact\ space$$

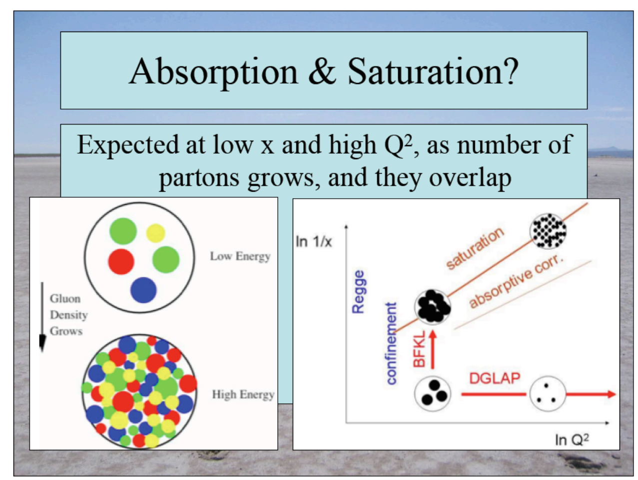
$$z \leftrightarrow 1/Q^2 \leftrightarrow virtuality$$

- Conformal Invariance:

$$\chi(s, x^{\perp} - x'^{\perp}, z.z') \rightarrow G(s, v)$$

$$v = \frac{(x^{\perp} - x'^{\perp})^2 + (z - z')^2}{2zz'}$$





Unitarity:

- Local Scattering in AdS_3 of "String Bits" or "Partons"

$$A_{2\to 2}(s,t) \simeq \int d^2b\, e^{-ib^\perp q_\perp} \int dz dz' P_{13}(z) P_{24}(z') \widetilde{A}(s, b^\perp, z, z')$$

$$\widetilde{A}(s, b^\perp, z, z') = -2is \left[e^{i\chi(s,b^\perp,z,z')} - 1\right]$$

$$\mathrm{Im}\, \widetilde{A}(s, b^\perp, z, z') \geq (1/4s) |\widetilde{A}(s, b^\perp, z, z')|^2 \,.$$

- "Parton-Hadron Duality": Equiv to Multi-Channel eikonal for hadrons in 2-dim Impact Space

$$A_{n_4,n_3\leftarrow n_2,n_1}(s,t) = -2is \int d^2b e^{-ibq_\perp} \left[e^{i\widehat{\chi}(s,b)} - 1\right]_{n_4,n_3;n_2,n_1}$$

$$\chi_{n_4 n_3; n_2 n_1}(s,b) = \int dz\, dz'\, P_{n_3 n_1}(z) P_{n_4 n_2}(z') \chi(s,b,z,z')$$

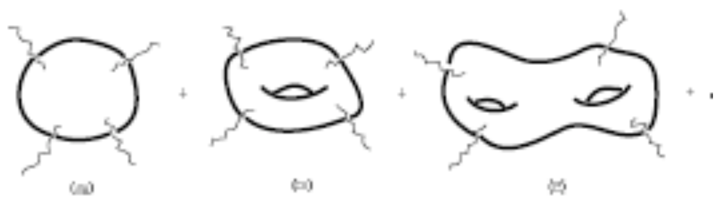
- For eikonal real, quasi-elastic scattering only, and no scattering into "long-string" states, (i.e., no soft multiperipheral jets.)

- With J ~ 2, eikonal predominantly real:

$$\frac{Re\chi}{Im\chi} = tan(j_0 - 1)\pi/2$$

$$\begin{array}{lcll} |\,\mathrm{Re}[\chi]\,| & \leq & |\,\mathrm{Im}[\chi]\,|\,, & 1 \leq J_0 \leq 1.5 \\ |\,\mathrm{Re}[\chi]\,| & \geq & |\,\mathrm{Im}[\chi]\,|\,, & 1.5 \leq J_0 \leq 2 \end{array}$$

- Inelastic Production



- Generalized Cutting Rules

$$\cos(j_0\pi)|\chi|^2 = \left[1 - 2\sin^2(j_0\pi/2) - 2\sin^2(j_0\pi/2) + 2\sin^2(j_0\pi/2)\right] |\chi|^2$$

$$\begin{array}{lccccccccc} j_0 = 1.0: & -1 & = & 1 & - & 2 & - & 2 & + & 2 \\ j_0 = 1.5: & 0 & = & 1 & - & 1 & - & 1 & + & 1 \\ j_0 = 2.0: & 1 & = & 1 & - & 0 & - & 0 & + & 0 \end{array}$$

- Real World: <u>$j_0 \sim 1.5$</u> and <u>$\lambda \sim O(1)$</u>

Analyticity:

- Amplitude is crossing even.

$$\mathcal{K}(s, b^{\perp}, z, z') = -(zz'/R^4)G_3(j_0, v) \times \widehat{s}^{j_0} \int_{-\infty}^{j_0} \frac{dj}{\pi} \frac{(1+e^{-i\pi j})}{\sin \pi j} \widehat{s}^{(j-j_0)} \sin\left[\xi(v)\sqrt{2\sqrt{\lambda}(j_0-j)}\right]$$

$$\cosh \xi = v + 1 \qquad e^{\xi} = 1 + v + \sqrt{v(2+v)}$$

- With λ large, Amplitude has a Large Real Part. Purely real at $\lambda \to \infty$.
- Need to know both Re [K] and Im [K] for all s>0.
- Im [K] can be found more easily. Re [K] can be found by Derivative Dispersion Relation.

- Im [K] can be evaluated analytically, exhibiting Diffusion in AdS_3, with diffusion time, $\tau \sim \log s$.

$$\mathrm{Im}[\mathcal{K}] \;=\; (zz'/R^4)G_3(j_0,v)(\sqrt{\lambda}/2\pi)^{1/2}\xi\; e^{j_0\tau}\,\frac{e^{-\sqrt{\lambda}\xi^2/2\tau}}{\tau^{3/2}}$$

- With λ large, derivative dispersion relation simplifies,

$$\partial_\tau[e^{-2\tau}\mathrm{Re}[\mathcal{K}]] = -(2/\pi)e^{-2\tau}\mathrm{Im}[\mathcal{K}]$$

- Re [K] can again be expressed simply as

$$\begin{aligned}\mathrm{Re}[\mathcal{K}] \;&\to\; (\sqrt{\lambda}/\pi)\mathrm{Im}[\mathcal{K}] \sim e^{j_0\tau}\,\frac{e^{-\sqrt{\lambda}\xi^2/2\tau}}{\tau^{3/2}}\,, \qquad \text{if} \quad \log\widetilde{s} > (\sqrt{\lambda}/2)\,\xi \\ &\to\; \frac{2}{\pi}\widehat{s}^2\left(\frac{zz'}{R^4}\right)G_3(2,v) + O(e^{j_0\tau})\,, \qquad \text{if} \quad \log\widetilde{s} < (\sqrt{\lambda}/2)\,\xi\end{aligned}$$

Scattering in Conformal Limit:

Use the condition: $\chi(s, x^\perp - x'^\perp, z, z') = O(1)$

Elastic Ring: $b_{\rm diff} \sim \sqrt{zz'}\,(zz's/N^2)^{1/6}$

No Froissart $\sigma_{total} \sim s^{1/3}$

Inner Absorptive Disc:

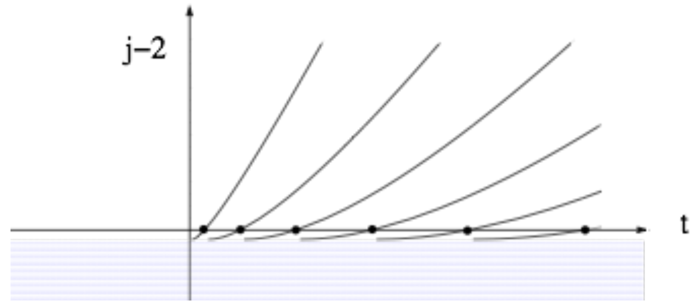
$b_{\rm black} \sim \sqrt{zz'}\ \frac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N}$ $b_{\rm black} \sim \sqrt{zz'}\left(\frac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2\sqrt{\lambda}}(j_0-1)}$

Inner Core: “black hole” production ?

Unitarity, Confinement and Froissart Bound

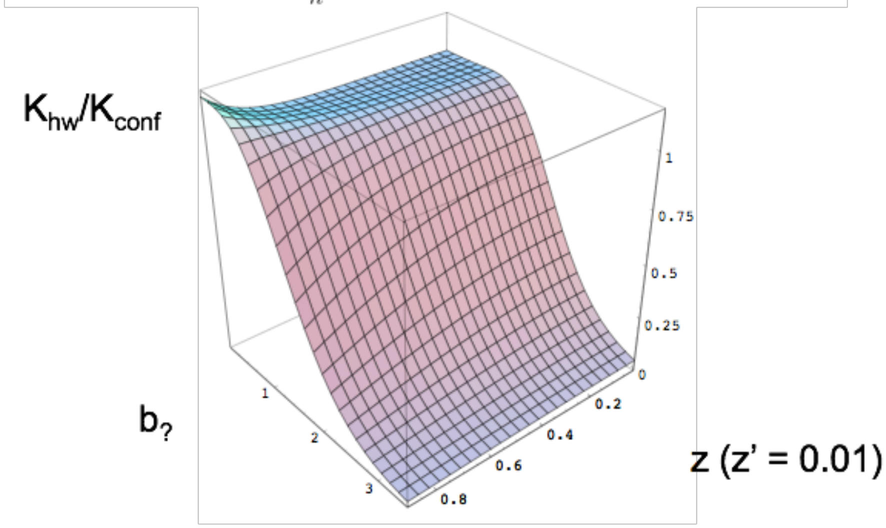
With Confinement:

discrete spectrum



Kernel for hardwall at z =1

$$K_{hw}(x_\perp, zz') \sim \frac{k_5^2 s^2}{zz'} \sum_n \frac{2}{J_2^2(m_n)} J_2(m_n z) K_0(m_n|x_\perp|) J_2(m_n z')$$



$$lim_{\Lambda\to 0} K_{hw}(x_\perp/\Lambda, z/\Lambda, z'/\Lambda) \sim \frac{\kappa_5^2 s^2}{zz'} \sum_n \frac{2}{y+\sqrt{y^2-1}} 4\pi\sqrt{y^2-1}$$

Mass of the lightest tensor Glueball provides scale

$$e^{-m_0 b}/\sqrt{m_0 b}$$

Elastic Ring:

$$b_{\rm diff} \simeq \frac{1}{m_0} \log(s/N^2\Lambda^2) + \dots$$

Absorptive Disc:

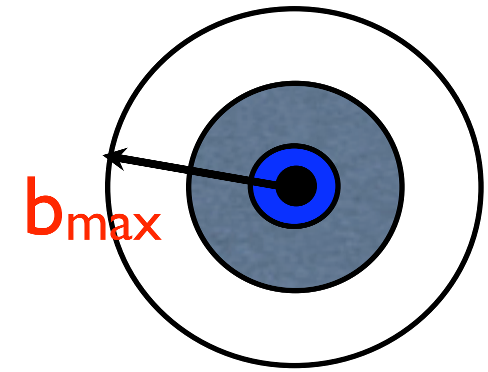
Inner Core:

Saturation of Froissart Bound

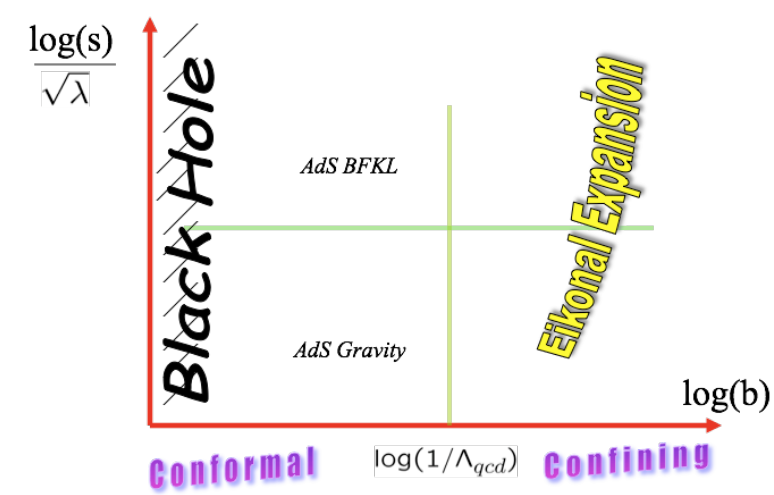
- The Confinement deformation gives an exponential cutoff for $b > b_{max} \sim c \log(s/s_0)$,
- Coefficient $c \sim 1/m_0$, m_0 being the mass of lightest tensor glueball.
- There is a shell of "conformal region" of width $\Delta b \sim \log(s/s_0)$

Froissart is respected and saturated.

Disk picture



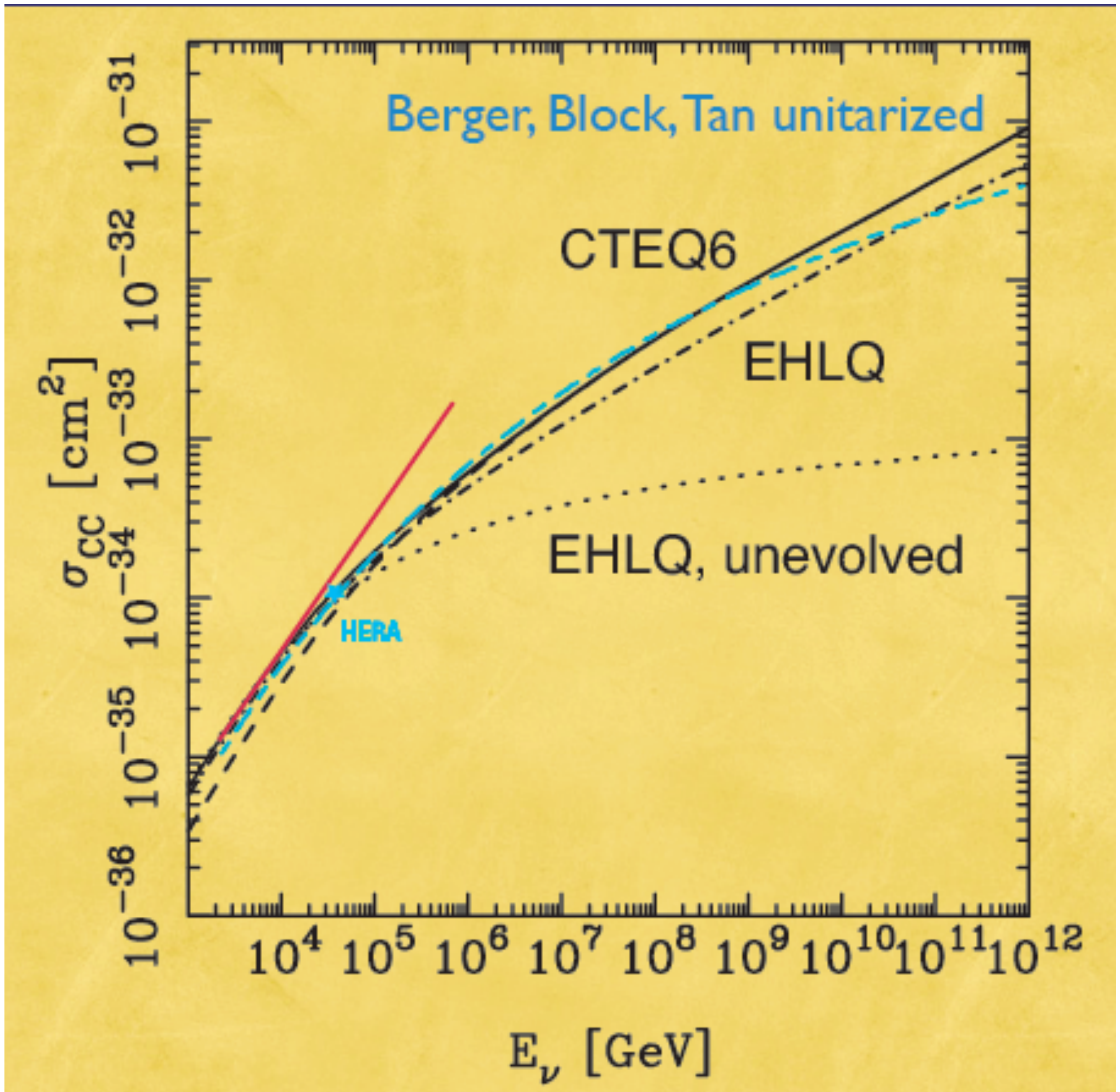
b_{max} determined by confinement.



Applications beyond the LHC

QCD influence on UHE ν detection

Importance of wee-x parton distributions



VI. Summary and Outlook

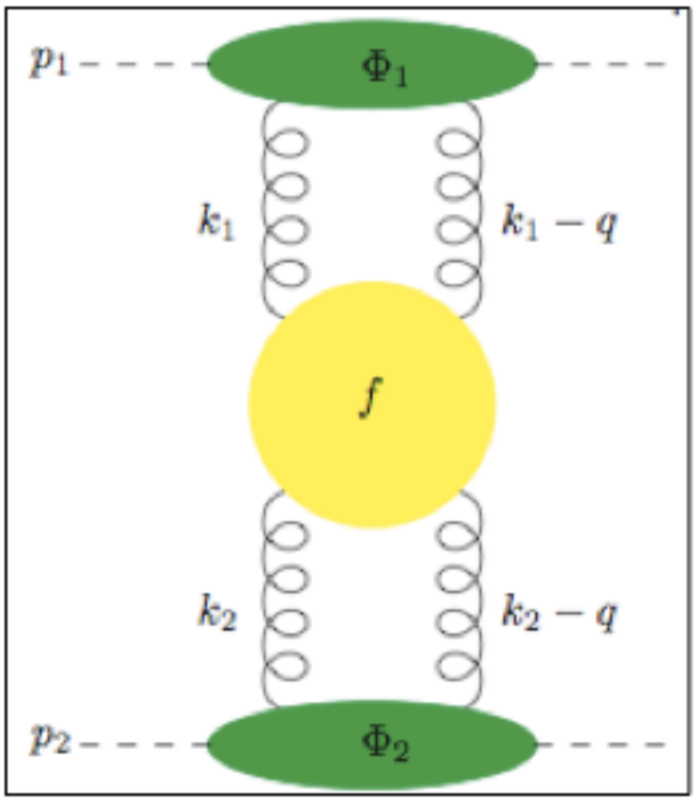
- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences, Diffractive Higgs production at LHC (in progress).

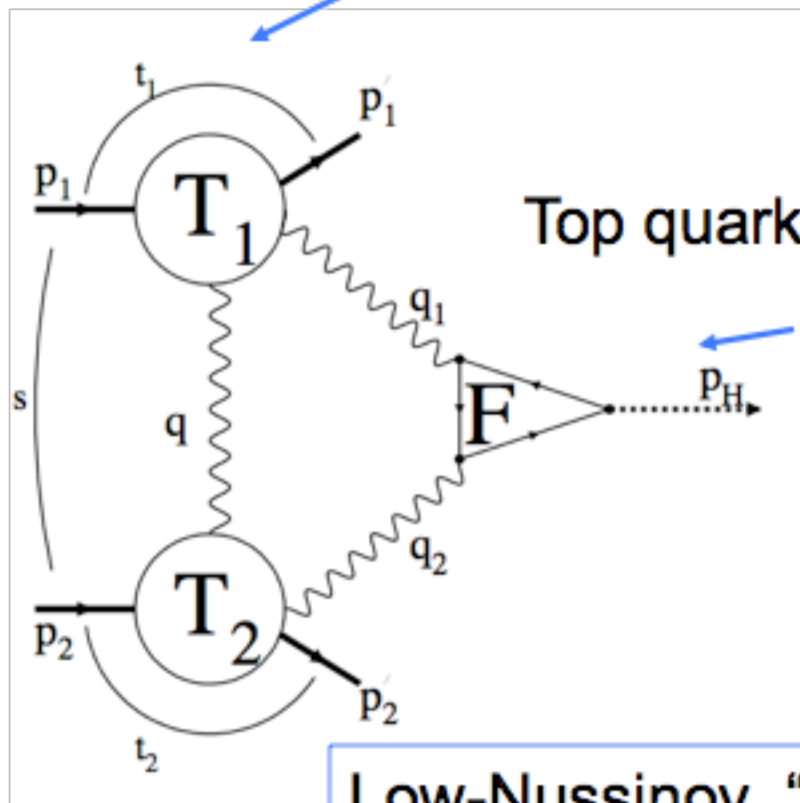
Diffractive Higgs Production (Building Blocks)

BFKL pp scattering

Martin, Khoze and Ryskin,
"Diffractive Higgs production"

Impact factor

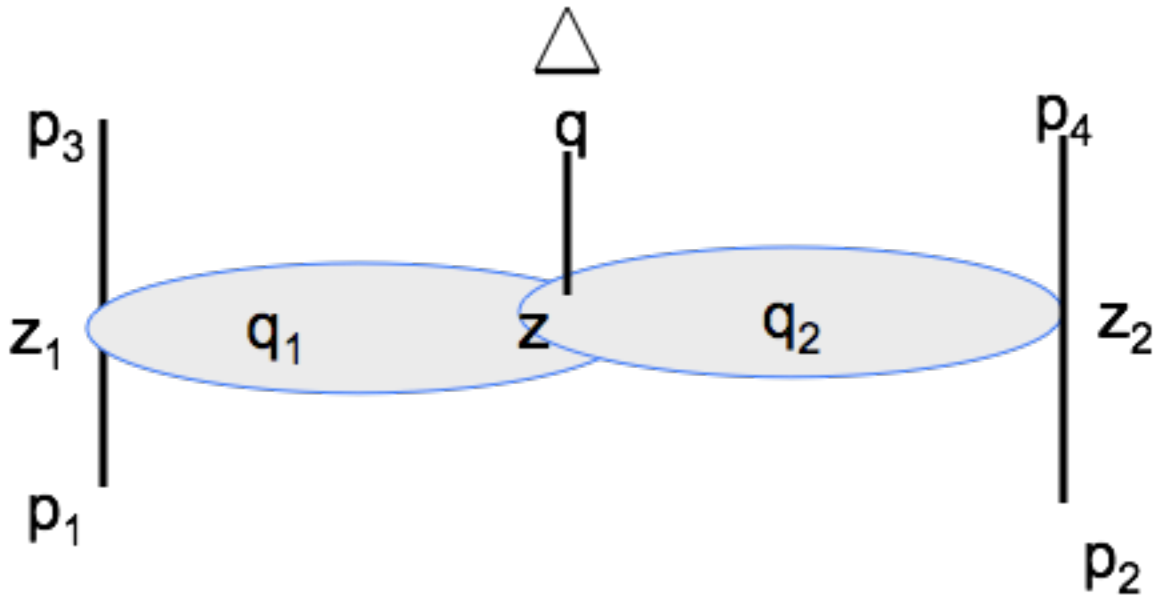




Top quark loop

Low-Nussinov "Two Gluon Pomeron"

Double Regge (Pomeron) exchange



❑ New issues:

- Pomeron-Pomeron Glueball vertex: $V(q_1^{\perp}, q_2^{\perp}, q^{\perp}, z, z')$
- Top quark loop: $F^2(x)$ source at $z = 0$;
- Bulk to boundary prop from Pomeron-Pomeron vertex to $F^2(x)$

References:

- R. Brower, J. Polchinski, M. Strassler, and C-I Tan, "The Pomeron and Gauge/String Duality", hep-th/0603115.
- R. Brower, M. Strassler, and C-I Tan, hep-th/0707.2408.
- R. Brower, M. Strassler, and C-I Tan, hep-th/0710.4378.
- R. Brower, M. Djuric, and C-I Tan, arXiv:0812.0354.
- Other related work, e.g., L. Cornalba, et al., (hep-th/0710.5480),
- Y. Hatta, E. Iancu, and A. H. Mueller, (hep-th/0710.2148),
- E. Levin, et al. (arXiv:0811.3586) and (arXiv:0902.3122).
- Many others.

Pomeron and Odderon

$$TrP_\sigma[e^{i\oint d\sigma x'_\mu(\sigma)A_\mu(x)} \pm e^{-i\oint d\sigma x'_\mu(\sigma)A_\mu(x)}]$$

$C = +1$

$$\langle \mathcal{P}_{\mu\nu}(x,y)\mathcal{P}_{\mu'\nu'}(x',y')\rangle \qquad \mathcal{P}_{\mu\nu}(x,y) = tr(A_\mu(x)A_\nu(y)) = (1/2)\delta_{ab}A^a_\mu(x)A^b_\nu(y)$$

$C = -1$

$$\langle \mathcal{O}_{\mu\nu\rho}(x,y,z)\mathcal{O}_{\mu'\nu'\rho'}(x',y',z')\rangle \qquad \mathcal{O}_{\mu\nu\rho}(x,y,z) \;=\; tr\left(\{A_\mu(x),A_\nu(y)\}\,A_\rho(z)\right) = (1/2)d_{abc}A^a_\mu(x)A^b_\nu(y)A^c_\rho(z)$$

AdS/CFT Dictionary

$$\langle e^{\int d^4x\phi_i(x)\mathcal{O}_i(x)}\rangle = \mathcal{Z}_{string}\left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x)\right]$$

$$S = \int d^4x \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x(C_0F\wedge F + C_2\wedge F + C_4)\rangle$$

Remarks on AdS3 Propagator:

$$G_3(j; x^\perp - x'^\perp, z, z') \sim \langle x^\perp, z \,|\frac{1}{2\sqrt{\lambda}(j-2) + H_{+,-}}|\, x'^\perp, z' \rangle$$

- Conformal Invariance, a function of a single AdS3 invariant.

$$v = \frac{(x_\perp - x'_\perp)^2 + (z - z')^2}{2zz'}$$

- Large λ ⇒ j ~ 2.
- λ infinite, s large and fixed ⇒ j=2, and Graviton exchange
- λ and s infinite, $\log s = O(\sqrt{\lambda})$ ⇒ Pomeron exchange, in order to resolve "fine structure", with

$$j \simeq j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Strong Coupling Pomeron Propagator-- Conformal Limit

- AdS-3 propagator:

$$\mathcal{K}(j, x_\perp - x'_\perp, z, z') = \frac{1}{4\pi z z'} \frac{\left[y + \sqrt{y^2 - 1}\right]^{(2-\Delta_+(j))}}{\sqrt{y^2 - 1}} ,$$

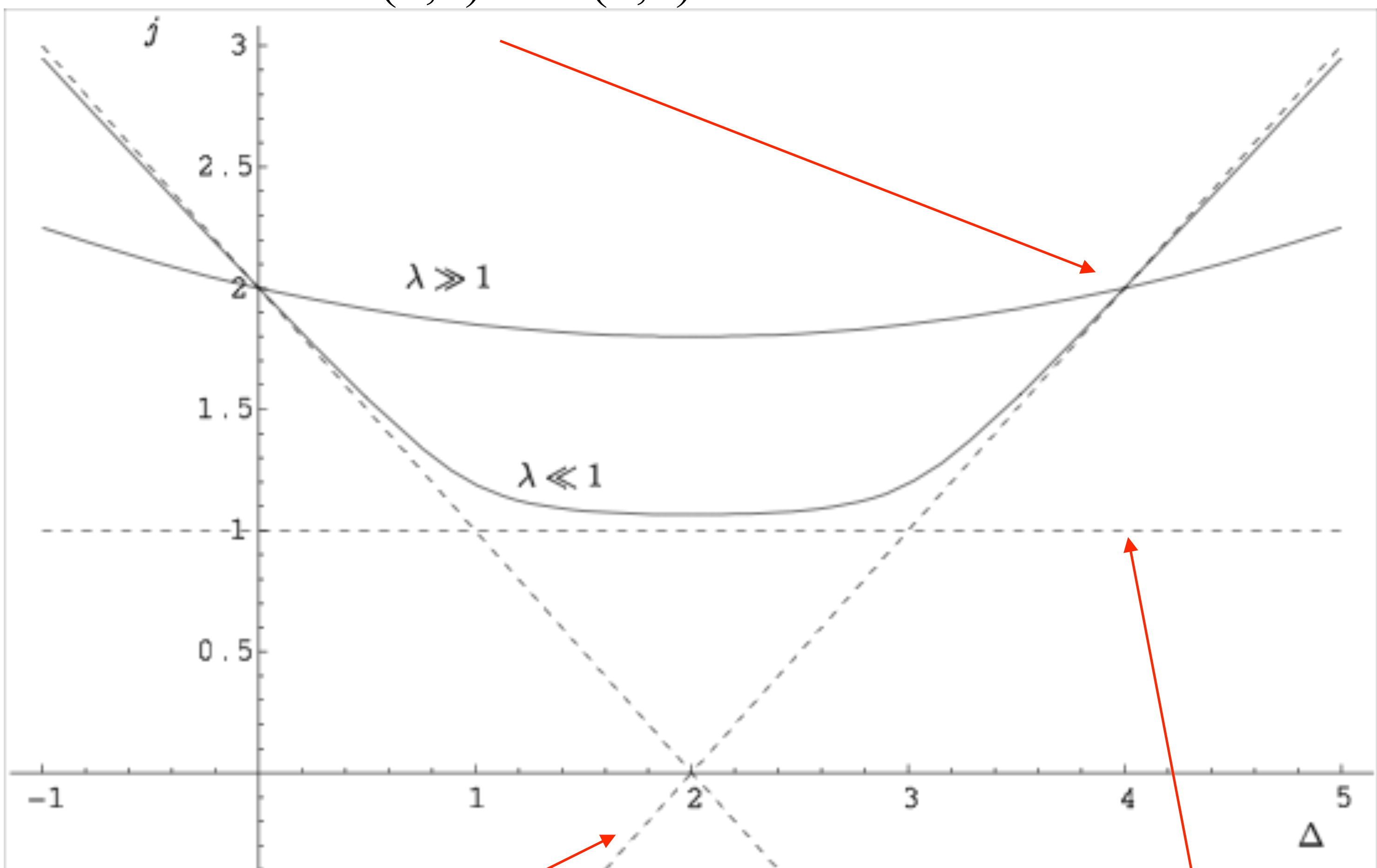
$$y \pm 1 = \frac{(z \mp z')^2 + (x_\perp - x'_\perp)^2}{2zz'}$$

- BFKL kernel:

$$\Phi_{n,\nu}(b_1 - b_0, b_2 - b_0) = \left[\frac{b_1 - b_2}{(b_1 - b_0)(b_2 - b_0)}\right]^{i\nu + (1+n)/2} \left[\frac{\bar{b}_1 - \bar{b}_2}{(\bar{b}_1 - \bar{b}_0)(\bar{b}_2 - \bar{b}_0)}\right]^{i\nu + (1-n)/2}$$

Spin-Dimension Curve

(4,2) and (0,2) have zero anomalous dimension



λ = 0 Anomalous Dim=0

λ = 0, BFKL

inversion symmetry: Δ ➔ 4 - Δ

J vs DGLAPP Curves

$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j - j_0^{(\pm)})}$$