Theoretical Overview on Soft Diffraction.

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Theoretical approaches to description of diffractive processes at high energies are discussed. It is pointed out that inelastic diffractive processes should be suppressed at small impact parameters. Important role of the pion exchange for analytic structure of the pomeron trajectory is emphasised. Models for large mass diffraction and recent calculations of survival probabilities are reviewed.

1 Introduction

Diffractive processes (elastic and inelastic) constitute a substantial part (about 1/2) of the total interaction cross sections of hadrons at high energies. Investigation of these processes provides an important information on mechanisms of high-energy interactions. There are important problems of QCD which can be studied in diffractive processes:
a) Nature of the pomeron in QCD.
b) Role of the s-channel unitarity and multi-pomeron exchanges.
c) Small-x problem and "saturation" of parton densities at $x \to 0$.
d) Violation of Regge and QCD-factorisations in diffractive processes.

Elastic scattering at high energies is a classical example of diffractive process. Absorption of the initial wave due to many inelastic channels leads by unitarity to elastic diffractive scattering.

Inelastic diffractive processes were first considered by E.L. Feinberg and I.Ya. Pomeranchuk [1] and elegant formulation in terms of the eigen-states was given by M.L. Good and W.D. Walker (GW) [2]. In this approach cross section for inelastic diffraction is related to a dispersion of eigen-amplitudes. Thus in a black disk limit inelastic diffraction is absent (exists at the edge of the disk only). This is an s-channel view on diffractive scattering. Note that GW-approach assumes a separation of diffractive and multiparticle states. This is not true in general for production of large mass states, which we will discuss in terms of the t-channel or Regge approach.

In the t-channel approach amplitudes of diffractive processes are described by an exchange of the pomeron, which has vacuum quantum numbers (positive signature and parity and C-parity , isospin I=0) (see for example review [3]. An increase with energy of the total interaction cross sections indicates that an intercept of the pomeron is larger than unity. An exchange by a Regge pole with $\Delta \equiv \alpha_P(0) - 1 > 0$ leads to a violation of the s-channel unitarity and for such "supercritical" pomeron multi-pomeron exchanges in the t-channel are very important. They restore unitarity and make theory consistent with Froissart bound.

2 Unitarity Effects in Gribov's Approach

A general method of calculation of multi-pomeron contributions to amplitudes of diffractive processes was formulated by V.N. Gribov [4]. In this approach a contribution of two-pomeron exchange (PP) to the process of elastic scattering can be expressed, using analyticity and unitarity for pomeron-particle scattering amplitudes, as the sum over all intermediate diffractive states in the s-channel.

In the same way amplitudes for nP-exchanges are expressed through all possible diffractive intermediate states. This result is based on general properties of Feynman diagrams and is valid in QCD. It allows to build approximate schemes for calculation of multi-pomeron contributions. The simplest approximation corresponds to an account of elastic intermediate states only. It leads to the eikonal approximation for scattering amplitudes. For elastic scattering amplitudes in the impact parameter space:

$$\mathrm{Im}\ f(s,b)\ = \frac{1}{2}\left(1\ -\ e^{-\Omega/2}\right), \tag{1}$$

where the eikonal $\Omega = -4i\delta_P(s,b)$ is the Fourier transform of the pomeron pole exchange.

Low mass diffractive states are often approximated by several resonance states. In this case the same method leads to Eq. (1) with Ω being a matrix, which elements correspond to transitions between different diffractive states. The simplest treatment is a diagonalisation of this matrix. Thus an account of the low mass diffraction in the Gribov's method is equivalent to the Good-Walker [2] approach to inelastic diffraction.

A simplest generalisation of the eikonal model, which takes into account inelastic diffractive intermediate states is so called "quasi-eikonal" model, where the amplitude $f(s,b)$ has the form

$$f(s,b) = \frac{i}{2C}\left[1 - \exp(2iC\delta_P(s,b))\right] \tag{2}$$

The function $\delta_P(s,b) \sim s^{\Delta}$ and it becomes large at very high energies. In this limit the scattering amplitude for elastic scattering $f(s,b) \to i/2$ in the eikonal model (scattering on a black disk) and $f(s,b) \to 1/2C$ in the quasi-eikonal model (scattering on a grey disc). This property of the quasi-eikonal model is closely related to the fact that one of the eigen-states for the diffractive matrix $\hat{\Omega}_P$ has in this model a zero eigen-value.

This is a crude approximation, which takes into account a big difference in the interaction cross section of hadrons with different transverse sizes. Configurations of quarks inside hadrons with small transverse size r have total interaction cross sections $\sim r^2$, because hadrons in QCD interact as colour dipoles. There is a distribution of quarks and gluons inside colliding hadrons with different values of r so one can expect that there will be a slow approach to the black disk limit for elastic scattering amplitude as $s \to \infty$. In this limit the effective radius of interaction increases as $\ln s$. Thus total interaction cross sections for the supercritical Pomeron theory have Froissart type behaviour $\sigma^{(tot)} \sim \ln^2(s)$ as $s \to \infty$.

Unitarisation effects due to multi-pomeron exchanges most strongly influence amplitudes of inelastic diffractive processes. In the eikonal approximation a suppression of cross section for inelastic diffractive process S^2 is:

$$S^2\ =\ \frac{\int\ |\mathcal{M}(s,b)|^2\ e^{-\Omega(b)}\ d^2b}{\int\ |\mathcal{M}(s,b)|^2\ d^2b}, \tag{3}$$

This expression is easily generalised to the case of several diffractive channels. The same Eq. (2) is valid for each diagonal state and it is necessary to sum over all diagonal states (with corresponding weights).

The quantity Ω increases with energy as s^{Δ} and becomes large at very high energy. According to Eq. (2) cross sections of inelastic diffractive processes becomes negligible at small impact parameter and is concentrated at the edge of interaction region at $b > 1\,\mathrm{fm}$ and the radius of this region increases with energy as $\ln s$. Note that models based on perturbative QCD are not valid in this peripheral region. On the other hand account of $\pi\pi$-cut in the pomeron trajectory play an important role in this region (especially in view of smallness of $\alpha'_P(t=0)$). An imaginary part of the P-trajectory due to $\pi\pi$-cut has the form [5]

$$\mathrm{Im}\,\alpha_P(t) = \frac{b_P(t))(t-4\mu^2)^{\alpha_P(t)+1/2}}{16\pi C(\alpha_P(t))(4s_0)^{\alpha_P(t)}\sqrt{t}}, \tag{4}$$

where $C(j) = \frac{\pi\Gamma(2j+2)}{2^{2j+1}\Gamma(j+1)^2}$, $s_0 = 1\,\mathrm{GeV}^2$, μ is the pion mass and $B_P(t)$ is the pomeron residue in $\pi\pi$-scattering. Due to smallness of μ the pion cut has an important influence on the pomeron trajectory [6]. In particular it gives a contribution to the slope of the pomeron trajectory $\delta\alpha'_P(0) = \frac{\sigma^{tot}_{\pi\pi}}{16\pi^3}(\ln(\frac{m}{\mu})-1) \approx 0.05\,\mathrm{GeV}^{-2}$ for $m \approx 1\,\mathrm{GeV}$, which can be considered as a lower bound on the slope of the pomeron. The singularity in the P-trajectory leads also to a substantial curvature in the pomeron trajectory in the small-t region. It is essential for resolving problems with s-channel unitarity in inelastic diffractive processes at super-high energies [3]

The value of S^2 is not universal: it depends on behaviour of a matrix element $\mathcal{M}(s,b)$ on impact parameter b.

3 Interplay of Soft and Hard Diffraction

At very high energies diffractively produced system can contain hard subsystem. For example diffractive production of dijets, W, Z-bosons and heavy quarks. Especially interesting class of hard diffractive processes is exclusive central production of Higgs boson. It allows to study Higgs bosons in a very clean environment and gives a possibility to determine quantum numbers of Higgs. In hard diffraction the subprocess of a heavy state production can be calculated using QCD perturbation theory. The simplest inclusive diffractive process is a diffractive dissociation of a highly virtual photon. In this case the photon interacts with a quark and a study of these processes at HERA gave a possibility to determine distributions of quarks and gluons in the pomeron. These distributions and QCD factorisation can be used to predict cross sections of hard inclusive diffractive processes in hadronic interactions. However both QCD and Regge-factorisations in hard diffractive processes are violated due to multi-pomeron exchanges. They strongly modify predictions based on a single pomeron exchange. This is a manifestation of an interplay of soft and hard diffraction. CDF data [7] show that cross section of diffractive dijet production about an order of magnitude smaller than prediction based on QCD factorisation and partonic distributions extracted from HERA results. Dependence on β (distribution of partons in the pomeron) is also substantially different from the predicted one. Calculation of suppression in the two-channel eikonal model [8] allows to reproduce both the observed suppression and β-dependence.

It is interesting that the same suppression is observed for double gap (double pomeron exchange) events at Tevatron [9]. This observation is in accord with a dominance of eikonal-type rescatterings [10].

The problem of calculation of survival probability and its energy dependence is very important for prediction of cross section of double-pomeron production of Higgs bosons and I shall return to this problem after discussion of influence of large mass diffraction on a magnitude of suppression of hard diffractive processes.

4 Large Mass Diffraction and Interaction of Pomerons

So far we have considered the low mass excitations in diffractive intermediate states. A mass of diffractively excited state at large s can be large. The only condition for diffraction dissociation is $M^2 \ll s$. For large masses of excited states $M^2 \approx (1-x)s$ and rapidity gap $\Delta y \approx \ln(s/M^2) \equiv \xi'$. The large mass behaviour of the pomeron-particle amplitudes is described by the triple-pomeron and multi-pomeron diagrams.

The cross section for inclusive single diffraction dissociation in the Regge pole model can be written in the following form

$$\frac{d^2\sigma}{d\xi_2 dt} = \frac{(g_{11}(t))^2}{16\pi} \left|G_p(\xi',t)\right|^2 \sigma_{P2}^{(tot)}(\xi_2,t) \tag{5}$$

where $\xi_2 \equiv \ln(M^2/s_0)$ and $G_p(\xi',t) = \eta(\alpha_p(t)) \exp[(\alpha_p(t)-1)\xi']$ is the pomeron Green function. The quantity $\sigma_{P2}^{(tot)}(\xi_2,t)$ can be considered as the pomeron-particle total interaction cross section [3]. At large M^2 this cross section in Regge-model has the same behaviour as usual cross sections

$$\sigma_{P2}^{(tot)}(M^2,t) = \sum_k g_{22}^k(0)\, r_{PP}^{\alpha_k}(t) \left(\frac{M^2}{s_0}\right)^{\alpha_k(0)-1} \tag{6}$$

where the $r_{PP}^{\alpha_k}(t)$ is the triple-reggeon vertex , which describes coupling of two pomerons to reggeon α_k.

In this kinematic region $s \gg M^2 \gg m^2$ the inclusive diffractive cross section is described by the triple–Regge diagrams and has the form

$$f^1 = \sum_k G_k(t)(1-x)^{\alpha_k(0)-2\alpha_P(t)} \left(\frac{s}{s_0}\right)^{\alpha_k(0)-1} \tag{7}$$

The pomeron-proton total cross section and triple-Regge vertices r_{PP}^P, r_{PP}^f have been determined from analysis of experimental data on diffractive production of particles in hadronic collisions (see review [3]). Account of multi-pomeron rescattering for triple-reggeon diagram in analysis of large mass diffraction dissociation leads to a substantial change (increase) in values of triple-reggeon couplings (for recent analysis see [12, 13]).

It is clear that for very large masses it is not enough to consider the triple-pomeron contribution only as it violates unitarity for the pomeron-particle scattering amplitude. An important theoretical question is: what is the structure of the vertices for n pomeron to m pomeron transitions. The simplest approximation is to assume an eikonal-type structure for the pomeron-particle amplitudes at large M^2:

$$g_{mn} = cg^{m+n} \tag{8}$$

where c and g are some functions of t. This behaviour of vertices follows from multi-peripheral model and is natural from the t-channel unitarity point of view. It was used in [11] (KPTM) to sum all diagrams with interactions of pomerons. This model leads to a good description of total, elastic and single diffraction dissociation cross sections (σ_{SD}) in $pp(p\bar{p})$ interactions [11] with $\Delta \approx 0.2$. It is worth to note that without multi-pomeron effects σ_{SD} has too fast increase with energy and exceeds experimentally observed cross section by a factor ~ 10.

In recent analysis of single and double diffraction dissociation [13] a simplified version of KPTM was used, where besides eikonal rescatterings between colliding protons eikonalization of each leg of the triple-reggeon diagrams was taken into account. More complicated t-channel iterations, which become important at extremely high energies, were neglected. This model gives a very good description of data on diffractive production from energies of fixed targets up to Tevatron.

An important question is how to apply Abramovsky, Gribov, Kancheli (AGK) cutting rules [14] in presence of multi-pomeron vertices. This is necessary in order to describe processes of multi-particle production in presence of interactions between pomerons. Strictly speaking AGK cutting rules were not proved for $n \to m$ pomeron transitions. For simplest application of AGK rules the problem was considered by S. Ostapchenko [15, 16] and generalisation of the Quark-Gluon Strings Model (QGSM) [17], was formulated in a Monte Carlo version.

In treatment of diagrams with interactions between pomerons it is necessary to take into account that the notion of the pomeron exchange is meaningful for large rapidity gaps only (usual choice $y > y_0$, with $y_o = \ln(10) = 2.3$). Thus a cutoff at small rapidities for each pomeron line should be introduced. It leads to a natural limitation to the number n of the t-channel iterations of pomeron exchanges (or number of gaps) at each initial energy : $n < \ln(s/s_0)/y_0$ with $s_0 = 1\,\mathrm{GeV}^2$. This threshold effect was taken into account in Ref. [11] and should be accounted for in all realistic calculations with pomeron interactions. It plays an important role in calculations of survival probabilities (see below).

The Durham group (KMR) has made recently a new fit of data on cross sections of diffractive processes [18, 19]. All $n \to m$ pomeron transition were taken into account in the framework of a partonic model, which lead to the behaviour $g_{mn} \sim nmg^{n+m}$, which is somewhat different from the one discussed above in the eikonal approximation. Summation of diagrams was performed by numerical solution of a system of highly nonlinear equations for amplitudes. To account for semi-hard and hard interactions three types of pomeron poles were introduced. Formulae for cross sections of different inelastic diffractive processes were obtained using some probabilistic arguments (and not cutting rules as in the standard approach). In this model it is possible to obtain a reasonable description of total cross section for pp-interaction, its elastic cross section in the diffraction cone region and cross sections of single and double diffraction. In the version of the model, which takes into account transverse degrees of freedom [19], for intercepts of pomerons $\Delta = \alpha_P(0) - 1$ values close to 0.3 were obtained.

A different approach was used by the Tel-Aviv group (GLM) [20]. Arguments, based on a small value of the pomeron slope, were used to justify applicability of perturbative QCD (PQCD) for diffractive processes. Motivated by PQCD the authors used the triple-pomeron interaction only with maximal number of pomeron loops. The last assumption may be reasonable for interaction of very small dipoles, but is difficult to justify for interaction of protons. The diagrams without pomeron loops (for example the diagram with a single triple-pomeron vertex) should be taken into account for self-consistency. It is known [21, 22] that the one dimensional approximation, used in calculations of GLM, leads asymptotically to decreasing total cross sections. GLM propose to use their model in a limited energy range. I have emphasised above that inelastic diffractive processes are concentrated at large impact parameters and that nonperturbative effects (for example two-pion cut in the pomeron trajectory) are important in this region. The fit of GLM to total pp-interaction cross section, differential cross section of elastic scattering and SD and DD-integrated cross sections [20] lead to the value $\Delta = 0.33$ for the intercept of the pomeron. Note that the threshold effects, discussed above, have not been taken into account both by GLM and KMR groups

A general feature of models, which take into account interactions between pomerons ("enhanced" diagrams), is a slower increase with energy of total cross sections. For example predictions of both KMR and GLM models for the total cross section of pp interaction at LHC energy are close to 90 mb, which is substantially smaller than in models without these interactions. Same effect exists in the model of Ref. [11], though the corresponding cross section is closer to 100 mb. Values of the pomeron intercept is substantially higher than in the eikonal-type models.

There is an interesting problem of influence of pomeron interactions on survival probabilities for hard processes [8, 23, 24]. The largest difference in KMR and GLM models is in predictions for survival probabilities due to enhanced diagrams. For DPE Higgs production at LHC an account of threshold effects is very important and calculation of KMR in a simplified model, but with account of these effects [25], show that for central Higgs production an extra suppression due to pomeron interactions is insignificant. On the other hand in GLM model a modification of survival probabilities due to enhanced diagrams is very strong: at LHC there is a decrease by a factor ≈ 16. For DPE processes at Tevatron GLM model predicts a decrease of survival probability by an extra factor 3.5. It is not clear how these factors depend on the mass of the produced hard system. A comparison of CDF data on diffractive dijet production [7] with prediction based on QCD factorisation and survival factor of two channel eikonal model show that extra suppression due to enhanced diagrams does not exceed 50%. Analogous restriction follows from CDF data on DPE dijet production [9, 26].

Thus up to energies of Tevatron interaction between pomerons play a minor role in hard diffractive processes. This is to a large extent related to the phase-space limitations. For soft diffraction enhanced diagrams are important and lead to a change of parameters of the "bare" pomeron in reggeon theory. At LHC effects of enhanced diagrams will be observable in hard diffractive processes. Their influence on survival probabilities can be studied, in particular, in diffractive production of jets (with not too large masses).

My main conclusions are:
unitarity effects due to multi-pomeron interactions are very important for diffractive processes at very high energies. They lead to a violation of both Regge and QCD-factorisations for hard diffraction.

Inelastic diffraction is peripheral in impact parameter space and account of $\pi\pi$-cut in the pomeron trajectory is necessary for proper description of amplitudes.

Experimental investigation of diffractive processes at LHC will give an important information on QCD–dynamics at high energies.

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