THE FROISSART BOUND FOR INELASTIC CROSS-SECTIONS

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Refs: - arXiv: 0812.0680v3 [hep-ph]
+ PROCEEDINGS OF LA LONDE CONF.
- arXiv: 0904.372v2 [hep-ph]
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I HISTORY

M. FROISSART, (1960)

$$\sigma_T < \text{const} (\log s)^2$$

$$F(s, \cos\theta = 1) < (Cs \log s)^2$$

FROM MANDELSTAM

A. MARTIN (1966)

PROOF FROM MASSIVE LOCAL FIELD THEORY + UNITARITY

L. LUKASZUK (†) + A. MARTIN (1968)

$$\sigma_T < \frac{\pi}{m_\pi^2} (\log s)^2$$

FOR DINO
$\frac{\hbar}{m_\pi c} = 1.4 \; 10^{-13}$ cm
$1\text{mb} = 10^{-27}$ cm^2

m_π = PION MASS

$\boxed{\frac{\pi}{m_\pi^2} = 60 \text{ mb} \quad \text{MUCH MUCH TOO LARGE}}$

(PETER LANDSHOFF etc....)

J. KUPSCH, S.M ROY, D. ATKINSON TRY TO IMPROVE

TO IMPROVE : FROISSART BOUND NON LOCAL. WORK WITH AVERAGE CROSS SECTION (COMMON, YNDURAIN †)

BEST AVERAGE

$$\bar{\sigma}_T(s) = \frac{1}{s}\int_s^{2s} \sigma_T(s')\,ds' \quad \text{(LA LONDE)}$$

SOLVES THE <u>SCALE</u> PROBLEM

PRINCIPLE OF THE PROOF

NOTATIONS

$$F(s, \cos\theta) = \frac{\sqrt{s}}{2k} \sum (2\ell+1) f_\ell P_\ell(\cos\theta)$$

$$A_s(s, \cos\theta) = \frac{\sqrt{s}}{2k} \sum (2\ell+1) \operatorname{Im} f_\ell P_\ell(\cos\theta)$$

$$= A(s,t) \qquad t = 2k^2(\cos\theta - 1)$$

$$\sigma_t = \frac{4\pi}{k^2} \sum (2\ell+1) \operatorname{Im} f_\ell$$

$$1 > \operatorname{Im} f_\ell \geq |f_\ell|^2 > 0$$

1) $F(s,t)$ ANALYTIC IN $t < 4m_\pi^2$

2) $\displaystyle\int_{s_0}^{\infty} \frac{A_s(s,t)\,ds}{s^3} < \infty$ (JIN-MARTIN, 1964)

2') $\longrightarrow |A_s(s,t)| < \frac{s^2}{\ln s}$ ON A SET ASYMPTOTIC DENSITY 1, IF A CONTINUOUS

$t < 4m_\pi^2$

HERE COMES THE PROBLEM OF THE <u>SCALE</u>

(AVOIDED BY USING THE AVERAGE σ_T)

FORGET THIS NOW!

3) MAXIMIZATION PROBLEM

FIND THE MAXIMUM OF

$$\sum (2\ell+1) \operatorname{Im} f_\ell \quad \text{if}$$

$$\sum (2\ell+1) \operatorname{Im} f_\ell \, P_\ell\left(1+\frac{t}{2k^2}\right) < \frac{s^2}{\ln s}$$

$$0 < t < 4m_\pi^2$$

ANSWER

$$\operatorname{Im} f_\ell = 1 \text{ for } \ell \leq L_T$$

$$\operatorname{Im} f_\ell = 0 \text{ for } \ell > L_T$$

$$\boxed{L_T = (k/\sqrt{t})(\ln s + c)}$$

$$\rightarrow \sigma_t < \frac{4\pi}{t}(\ln s)^2$$

$t = 4m_\pi^2 \rightarrow$ LUKASZUK–MARTIN

WORKS EVEN IN THE LIMIT CASE

IF ONE USES THE AVERAGE :

$$\overline{\sigma_T} < \frac{\pi}{m_\pi^2}\left[\ln s + c\right]^2$$

c KNOWN FROM LOW ENERGY PARAMETERS IN THE t CHANNEL

THERE IS $\boxed{\text{NO}}$ EXAMPLE KNOWN WHERE ANALYTICITY O.K.

$$\operatorname{Im} f_\ell \geq |f_\ell|^2$$

$\operatorname{Im} f_\ell = |f_\ell|^2$ IN ELASTIC REGION

(IMPORTANT, FROM GRIBOV)

TO-DAY: OTHER LINE OF ATTACK:

FIND BOUND ON

$$\boxed{\sigma_{\text{INELASTIC}}}$$

$$\sigma_{\text{INELASTIC}} = \frac{4\pi}{k^2} \sum (2l+1) \left[\operatorname{Im} f_l - |f_l|^2 \right]$$

$$\sigma_{\text{INELASTIC}} \leq \frac{4\pi}{k^2} \sum (2l+1) \left[\operatorname{Im} f_l - \operatorname{Im} f_l^2 \right]$$

IF IT IS THE SAME $L_{\max}$, TRIVIAL TO GET

$$\sigma_{\text{INELASTIC}} < \frac{\pi}{4 m_\pi^2} (\ln s)^2$$

SINCE $0 \leq \operatorname{Im} f_l - \operatorname{Im} f_l^2 \leq \frac{1}{4}$

HOWEVER IT IS NOT THE SAME MINIMIZATION PROBLEM.

FOR GIVEN

$$A(s,t) = \frac{\sqrt{s}}{2k} \sum (2l+1) \operatorname{Im} f_l \, P_l\left(1 + \frac{t}{2k^2}\right)$$

THE MAXIMUM OF

$$\sum (2l+1) \left(\operatorname{Im} f_l - \operatorname{Im} f_l^2\right) \text{ is}$$

GIVEN BY

$$\boxed{\begin{array}{l} \operatorname{Im} f_l = \frac{1}{2}\left[1 - \dfrac{P_l\left(1+\frac{t}{2k^2}\right)}{P_{L_I}\left(1+\frac{t}{2k^2}\right)}\right] \quad \text{for } l \leq L_I \\ \operatorname{Im} f_l = 0 \quad \text{for } L > L_I \end{array}}$$

YET, WE STILL FIND

$$L_I = \frac{k}{\sqrt{t}}\left(\ln s + c'\right)$$

while $L_T = \dfrac{k}{\sqrt{t}}\left(\ln s + c\right)$

$$c \neq c'$$

SO, WE STILL HAVE, FOR

$$t = 4 m_\pi^2$$

$$\boxed{\sigma_{\text{INELASTIC}} < \frac{\pi}{4 m_\pi^2}\left(\ln s\right)^2}$$

i.e. A BOUND $\boxed{\text{4 TIMES}}$ SMALLER THAN L-M.

{ OBTAINING A BOUND FOR AN AVERAGE σ_I IS IN PROGRESS, BUT TECHNICALLY NON TRIVIAL

END OF THE "RIGOUROUS" PART

IMPLICATIONS FOR THE TOTAL CROSS-SECTION

IN ALMOST ALL MODELS (CHENG-WU etc) EXCEPTION TROSHIN - TYURIN

$$\sigma_{elastic} \cancel{\leq \sigma_{inelastic}} < \frac{1}{2}\sigma_T$$

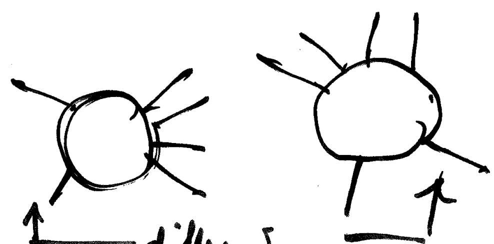
MOST CONVINCING ARGUMENT:

VAN HOVE'S "OVERLAP" FUNCTION)

$$O = \frac{\sqrt{s}}{2k} \sum (2l+1)\left(\mathrm{Im} f_l - |f_l|^2\right) P_l(\cos\theta)$$

OVERLAP =

$\sum$ all inelastic processes



different directions of 2 body state

VAN HOVE NEGLECTS THE REAL PART

$$\sigma = \frac{\sqrt{s}}{2k} \sum (2\ell+1)\, \sigma_\ell\, P_\ell(\cos\theta)$$

$$\sigma_\ell = \mathrm{Im} f_\ell - \mathrm{Im} f_\ell^{\,2}$$

$$\mathrm{Im} f_\ell = \frac{1 \pm \sqrt{1 - 4\sigma_\ell}}{2}$$

FOR LARGE ℓ ; CHOOSE $-$ SIGN

LVH : Keep $-$ SIGN FOR SMALL ℓ

WHY ? SEEN FROM ELASTIC SCATTERING, NO REASON

BUT σ_ℓ = EFFECT OF INELASTIC CHANNELS

DIFFICULT TO BELIEVE THAT σ_ℓ is

CONCLUSION

$$\boxed{\text{I BELIEVE THAT} \quad \sigma_T < \frac{\pi}{2 m_\pi^2} (\ln s)^2}$$

YOU ARE NOT FORCED TO AGREE