Low-x Gluon Distribution from Discrete BFKL Pomerons

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Using a modification of the BFKL equation which generates discrete Regge pole solutions, we obtain a good fit to the low-x deep-inelastic data from HERA as well as an integrated gluon distribution which is everywhere positive.

In a recent paper [1], we obtained a good fit to the HERA deep-inelastic data at low-x using a discretized version of the BFKL pomeron [2], which is in line with the Regge picture of diffractive events (and hence deep-inelastic events at low-x) whereby the amplitude is dominated by an isolated Regge pole (the "pomeron").

The purely perturbative BFKL equation predicts a cut rather than a pole. However, in 1986, Lipatov [3] suggested the following modifications to the BFKL equation:

1. Accounting for the running of the coupling as a function of the transverse momentum, $\mathbf{k}$, of the exchanged gluons, which spans a large range as one moves away from the top or bottom of the "ladder".

2. Assuming that the non-perturbative (infrared) sector of QCD imposes a fixed phase, η, on the oscillatory eigenfunctions of the BFKL kernel at some low value, k_0, of gluon transverse momentum.

This leads to a discrete set of eigenfunctions, $f_i(\mathbf{k})$ with discrete eigenvalues, ω_i, which can be interpreted as isolated Regge poles., i.e. the scattering of a gluon with transverse momentum $\mathbf{k}$ off some target with CM energy $\sqrt{s}$, has an amplitude which can be written in the form

$$\mathcal{A}(\mathbf{k}, s) \;=\; \sum_i a_i f_i(\mathbf{k}) s^{\omega_i}$$

The eigenfunctions have an oscillating behaviour with a decreasing frequency up to a value of transverse momentum $\mathbf{k}_{crit}$, above which they decay exponentially with $\ln(k)$

A very good fit was obtained using only the first four such eigenfunctions. The only unknown quantity is the proton impact factor $\Phi_p(\mathbf{k})$, which encodes the coupling of the proton to the gluon-scattering amplitude. Since the eigenfunctions form a complete orthonormal set, this impact factor can be expanded in the form

$$\Phi_p(\mathbf{k}) \;=\; \sum_i b_i f_i(\mathbf{k}),$$

where the first four coefficients, b_i were fit to data.

Unfortunately, when we tried to reproduce the full impact factor from this fit, we obtained an un-integrated gluon density $\tilde{g}(x, k^2)$, which becomes negative over a sufficient range that the (integrated) gluon density

$$g(x, Q^2) \equiv \int^{Q^2} \tilde{g}(x, k^2) dk^2,$$

is also negative.

We therefore sought a solution in which the impact factor has a "sensible" form such as

$$\Phi_p(k) = Ak^2 e^{-bk^2}, \quad \text{or} \quad \frac{A}{(k^2 + \mu^2)^\alpha}.$$

This suggested that taking only the first four eigenfunctions was insufficient. Indeed, if we take n_0 eigenfunctions , where the first eigenvalue is ω_1 and the last is ω_{n_0} then we expect an error of order $x^{(\omega_1 - \omega_{n_0})}$. This is 30% for $x \sim 10^{-2}$ and 17% for $x \sim 10^{-3}$.

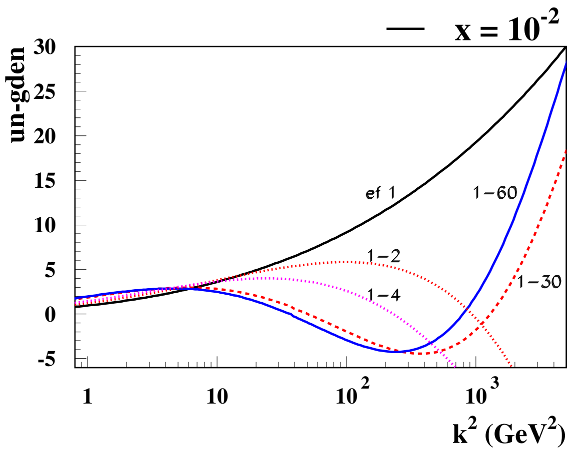

Figure 1: The effect on the un-integrated gluon density from increasing the number of eigenfunctions included.

We are now able to construct many more eigenfunctions, but we find that, although the un-integrated gluon density becomes positive at relatively large **k** when 30 eigenfunctions are used, the range of negative values still generates a physically unacceptable negative gluon density and that a further increase in the number of eigenfunctions taken only marginally improves this situation, as can be seen in Fig. 1.

We now understand why this is the case. A detailed explanation will appear in a forthcoming publication [4]. Within the context of a fixed phase for the oscillations at low transverse momentum, an adjacent eigenfunction has a larger k_{crit} by approximately one half wavelength,

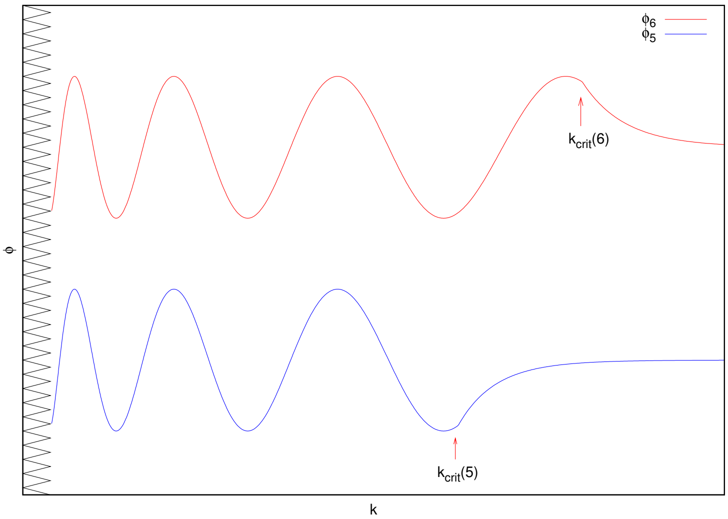

Figure 2: Sketch of eigenfunctions numbers 5 and 6.

whereas the frequency of the oscillations in the relevant range of k is almost identical. This is demonstrated in Fig. 2.

In Fig. 3 we plot the initial frequencies (i.e. the frequencies for $k \ll k_{crit}$) for the first 30 eigenfunctions and note that they accumulate at a value $\nu_{max} \sim 0.7$. This means that we can only expect to expand an impact factor as a sum of these eigenfunctions provided the impact factor has non-negligible support up to a value of transverse momentum k_{max} where

$$\nu_{max} \ln\left(\frac{k_{max}}{k_0}\right) \gg \pi.$$

The minimum value of k_0 that can be taken without encountering serious perturbative instabilities is $k_0 \sim 0.3\,\mathrm{GeV}$, which leads to a k_{max} far larger than the expected value for a proton impact factor which should be $\mathcal{O}(\Lambda_{QCD})$.

Put another way, this means that an impact factor with support for $k \leq \Lambda_{QCD}$ is *not* compatible with a fixed phase at $k_0 \sim 0.3\,\mathrm{GeV}$. This in turn implies that the second assumption of [3] needs to be revisited.

At leading order, we can write the BFKL equation with running coupling in the form

$$\int \mathcal{K}_0(\mathbf{k}, \mathbf{k}') f_i(\mathbf{k}') d^2\mathbf{k}' \;=\; \frac{\omega_i}{\overline{\alpha}_s(k^2)} f_i(\mathbf{k}).$$

In the infrared limit, as $\overline{\alpha}_s$ increases, the RHS goes to zero and it was therefore argued in Ref. [3] that the infrared limit of the eigenfunctions, $f_i(\mathbf{k})$, would be independent of ω and hence possess a universal phase.

In practice, however, with an infrared cutoff $k_0 \sim 0.3\,\mathrm{GeV}$, the ratio $\omega_i/\overline{\alpha}_s(k^2)$ is *not* negligibly small and so we might expect the infrared phase, η, to have a dependence on ω.

Within perturbation theory (recalling that the eigenvalues ω can be expanded in powers of $\overline{\alpha}_s$ starting at first order), such a dependence is expected to be analytic, so that one would

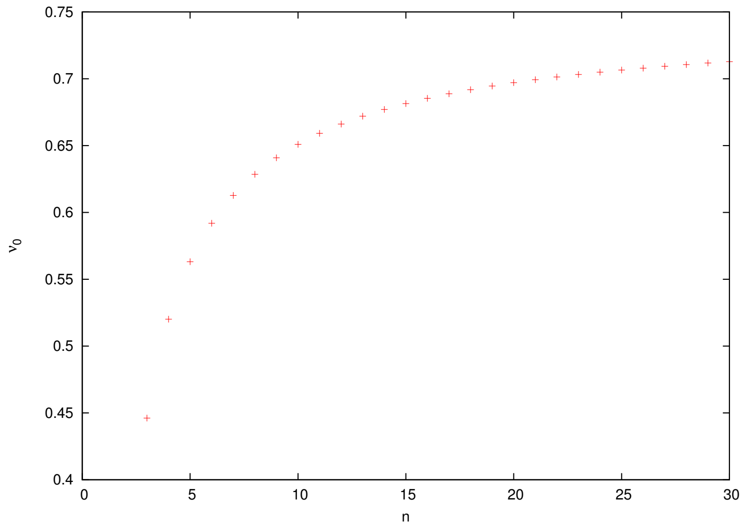

Figure 3: The oscillation frequencies for gluon transverse momentum $k \ll k_{crit}$ for the first 30 eigenfunctions.

expect an improved fit (with small ω) with a phase $\eta(\omega)$ of the form

$$\eta(\omega) \;=\; \eta_0 + \eta' \omega$$

Unfortunately, we were neither able to obtain a satisfactory fit using this ansatz for η, nor to rectify the problem of a negative gluon density. The best fit has a χ^2/DoF of 3.0.

However, since we are probing the non-perturbative behaviour of QCD, we are entitled to drop the requirement that η should be an analytic function of ω and try, for example, an ω dependence of the form

$$\eta(\omega) \;=\; \eta_0 + \eta' \sqrt{\omega}\,.$$

We found that this can generate a gluon distribution which is positive everywhere, as shown in Fig. 4, and produce a fit to HERA data with a χ^2/DoF of 1.1.

In Fig. 5 we show the best fits for the linear (dotted line) and non-linear (solid line) fits to the Zeus low-x data, using an impact factor of the form

$$\Phi_p(\mathbf{k}) \;=\; A k^2 e^{-bk^2}$$

The three free parameters used (apart from the overall normalization - which serves as a fourth parameter) are

	Linear	Non-linear
b [GeV^{-2}]	2.0	2.0
η_0	-0.74π	-0.74π
η'	2.8π	1.4π

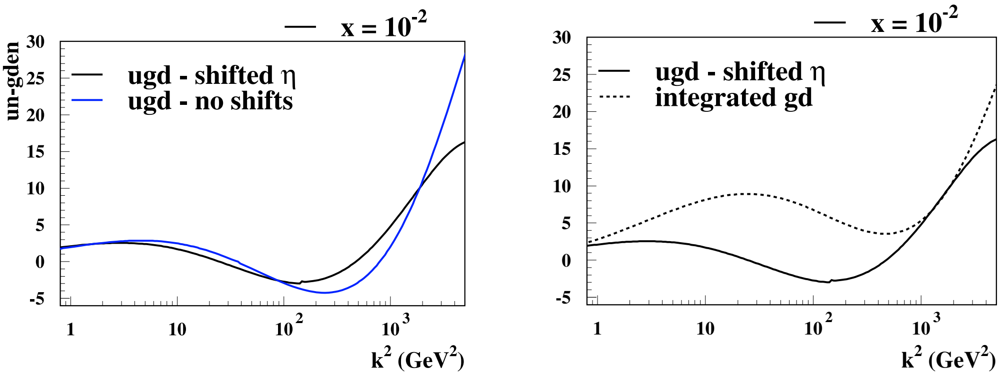

Figure 4: The left-hand graph shows the un-integrated gluon density with a fixed infrared phase and with a non-linear ω-dependent phase. The right-hand graph shows the un-integrated (solid line) with the non-linear ω-dependent infrared phase, and the corresponding (integrated) gluon density (dotted line).

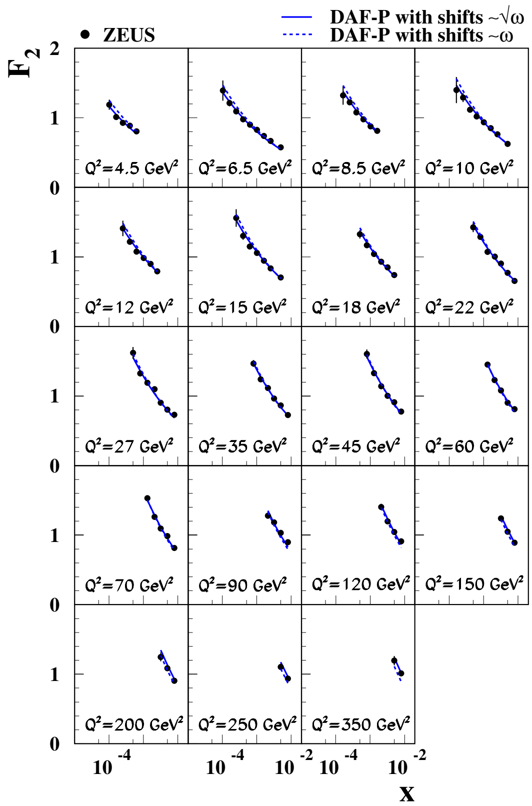

Figure 5: Fit to Zeus [5] low-x data with linear ω-dependence (dotted line) and non-linear ω-dependence (solid line)

We therefore have a low-x gluon density which is everywhere positive, fits the HERA data, and is consistent with the modified BFKL equation provided one allows a non-analytic dependence of the infrared phase, η, on the eigenvalue, ω, thereby reflecting the non-perturbative nature of the infrared effects.

This gluon density can now be tested by applying it to the prediction of cross-sections (such as jet production) at LHC which are dominated by the low-x gluon distribution.

References

[1] J. Ellis, H. Kowalski, and D.A. Ross Phys. Lett. **B668** 51 (2008).

[2] Y.Y. Balitski and L.N. Lipatov, Sov. J. Nucl. Phys.**28** 82 (1978); V.S. Fadin, E.A. Kuraev, and L.N. Lipatov, Sov. Phys. JETP **44** 443 (1976)

[3] L.N. Lipatov, Sov. Phys. JETP **63** 904 (1986)

[4] J. Ellis, H. Kowalski, L.N. Lipatov, D.A. Ross and G. Watt, in preparation.

[5] J. Breitweg et. al (Zeus collaboration), Phys. Lett. **B487** 53 (2000); S. Chekanov et. al. (Zeus collaboration), Eur. Phys. J. **C21** 443 (2001)