Low-x Gluon Distribution from Discrete BFKL Pomerons

J.Ellis, H. Kowalski, L.N.Lipatov, G.Watt, D.A. Ross

CERN, DESY, St. Petersburg, UCL, Southampton

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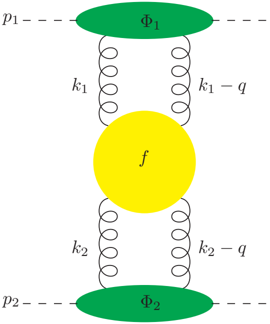
Goal

- To fit low-x data from HERA to discretized eigenfucntions of BFKL kernel.
  (Discrete eigenfunctions simulate description of data by a Pomeron as a Regge pole),
- Extract low-x gluon distribution for applications to jet production and diffractive events at LHC.

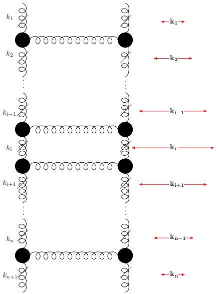
BFKL Approach

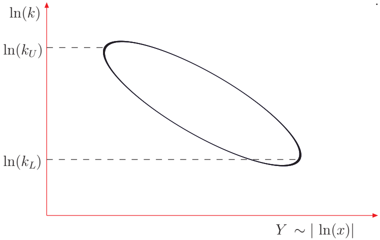
Consider the amplitude for a process involving the exchange of a colour singlet "object" (quantum numbers of vacuum - QCD "pomeron")



$\mathbf{\Phi_1}$, $\mathbf{\Phi_2}$ are process dependent "impact factors", but f is the process independent "BFKL kernel" depending on s and the gluon transverse momenta $\mathbf{k_1}$, $\mathbf{k_2}$.

$$\alpha_s \int d\mathbf{k}' \mathcal{K}(\mathbf{k},\mathbf{k}')\phi_\omega(\mathbf{k}') = \omega\phi_\omega(k)$$



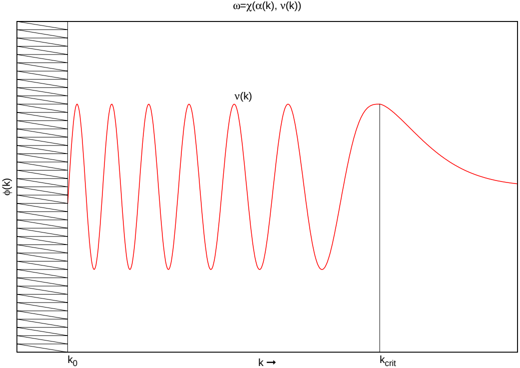


Gluon transverse momentum at top and bottom of ladder is controlled by process-dependent impact factors.

But as one moves away from these fixed (peaked) transverse momenta, the range of relevant transverse momenta increases.

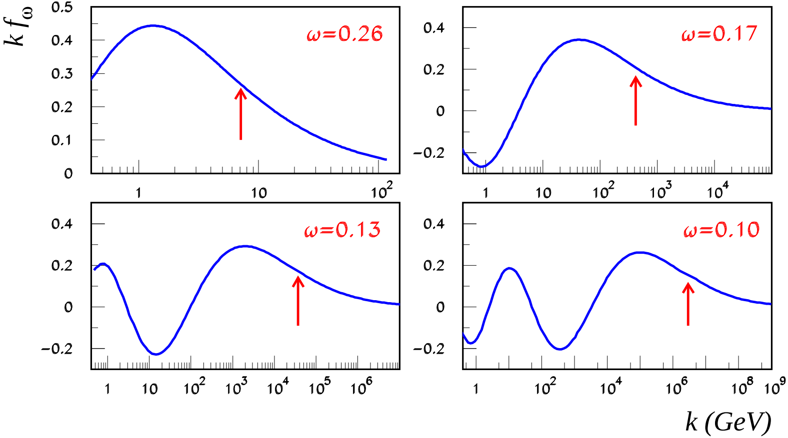
Need to account for the running of the coupling.

Assume that at some low-$k = k_0$, IR (non-perturbative) features of QCD fix the phase, η of eigenfunctions.
(Lipatov '86)



Leads to a discrete spectrum of Regge poles

$$\sum_i f_{\omega_i}(k^2) s^{\omega_i}$$



$k\,f_\omega$
ω=0.26
ω=0.17
ω=0.13
ω=0.10
k (GeV)

Unintegrated Gluon Density

$$x\bar{g}(x,k) \;=\; \sum_n \int d\mathbf{k}'\Phi_p(\mathbf{k}')\phi_n^*(\mathbf{k}')\phi_n(\mathbf{k})\left(x\frac{\mathbf{k}'}{\mathbf{k}}\right)^{-\omega_n}$$

Don't need to know Φ_p exactly, but since ϕ_n form a complete orthonormal set

$$\Phi_p \;=\; \sum a_n\phi_n(\mathbf{k})\,\mathbf{k}^{\omega_n}$$

Leads to

$$x\bar{g}(x,k) \;=\; \sum_n a_n\phi_n(\mathbf{k})k^{\omega_n}x^{-\omega_n}$$

Fit a_n to data at low-x (but above the saturation scale in Q^2) , but do NOT need the complete proton impact factor.

Towards a Realistic Proton Impact Factor

$$\Phi_P(\mathbf{k}) \;=\; \sum_n a_n \, \phi_n(\mathbf{k}) \, k^{\omega_n}$$

Our fit determines a_n

But the photon impact factor predicted becomes negative for large k

which implies that the gluon density becomes negative.

Expect Φ_P to have some "sensible" form such as

$$\Phi_p(\mathbf{k}) \;\sim\; \frac{1}{(\mathbf{k}^2+\mu^2)^\alpha} \quad \text{or} \quad \left(\mathbf{k}^2\right)^\alpha e^{-b\mathbf{k}^2}$$

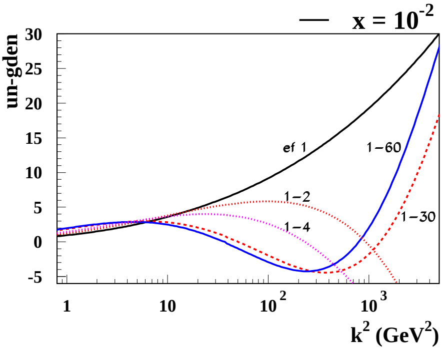
Such an ansatz is not compatible with our values for a_n

This suggests that the fit using only 4 eigenfunctions was too good !!

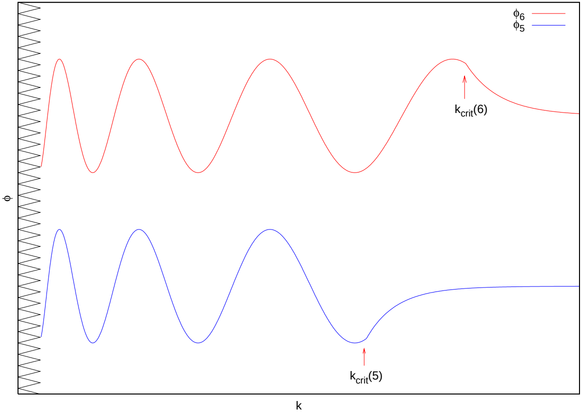
Truncating the infinite series at $n = n_0$ introduces errors of order

$$\begin{aligned} x^{\omega_1 - \omega_{n_0}} &\sim 0.3 \text{ for } x = 0.01 \\ &\sim 0.17 \text{ for } x = 0.001 \end{aligned}$$

Can we solve this problem by taking more eigenfunctions?



Taking many eigenfunctions still leaves unintegrated gluon density negative over a sufficient range to lead to negative integrated gluon density.



Except at very high k eigenfunctions are almost identical.

We cannot fit a “sensible” proton impact factor by summing over more eigenfunctions.

Revisit Ansatz of a constant phase in the infrared region

$$\omega_n = \int \mathcal{K}(\alpha_s(k), k, k') \phi_n(k') dk'$$

In perturbation theory

$$\omega_n = \int \alpha_s(k) \mathcal{K}_0(k, k') \phi_n(k') dk' + \cdots$$

If ω_n is small, as $\alpha_s(k)$ gets large, the eigenfunctions ϕ_n must behave such that

$$\int \mathcal{K}_0(k, k') \phi_n(k') dk' = 0$$

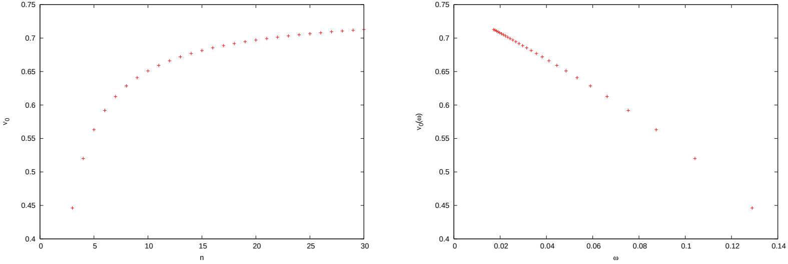
independent of ω_n

At low k this has a solution

$$\phi_n(\mathbf{k}) \sim \sin(\nu_0 \ln k + \eta)$$

But for the leading eigenfunction $\omega_0 \sim 0.3$
$\omega_0/\alpha_s(k)$ is NOT so small even when $\alpha_s(k) \sim 1$
We find an ω-dependence of ν,

$$\nu_0 \to \nu(\omega) \ (= \nu_0, \text{ as } \omega \to 0)$$



We would similarly expect an ω-dependence in the infrared phase, η

$$\eta(\omega) \stackrel{\omega\to 0}{\to} \eta$$

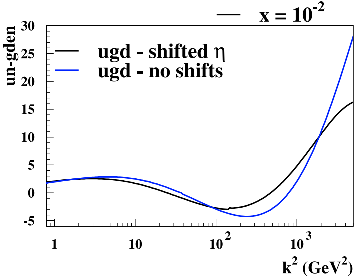
In perturbative QCD we might expect η to be a power series in ω
(since ω is a power series in α_s).
Try

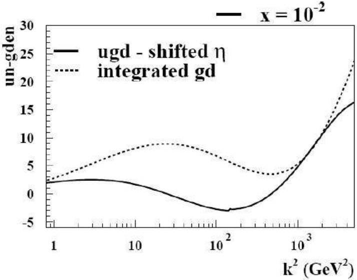
$$\eta = \eta_0 + \eta' \omega$$

This does not lead to a good fit to HERA data, and we still find that the integrated gluon density still goes negative

Try

$$\eta \;=\; \eta_0 + \eta' \sqrt{\omega}$$





Although the unintegrated gluon density is negative in a certain region of k, the integrated gluon density is positive everywhere

Impact Factor:

$$\Phi = Ak^2 e^{-bk^2}, \quad b = 2 \text{ Gev}^{-2}$$

1. Linear Shift:

$$\eta(\omega) = \eta_0 + \eta' \frac{\omega}{\omega_1}$$

$$\eta_0 = -0.74\pi, \quad \eta' = 0.76\pi$$

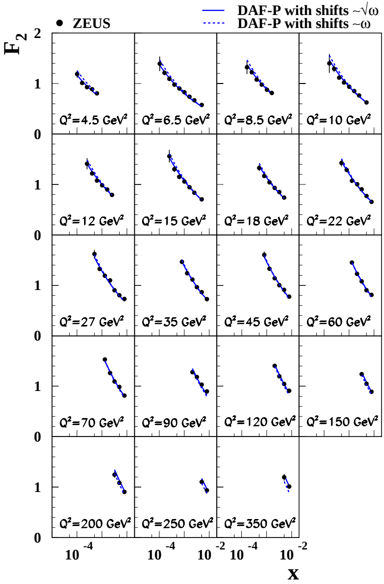
$$\chi^2 \sim 3 \text{ per DOF}$$

2. Non-linear Shift:

$$\eta(\omega) = \eta_0 + \eta' \sqrt{\frac{\omega}{\omega_1}}$$

$$\eta_0 = -0.74\pi, \quad \eta' = 0.76\pi$$

$$\chi^2 \sim 1.1 \text{ per DOF}$$



A dependence of the infrared phase, $\eta \propto \sqrt{\omega}$ cannot be understood within the context of perturbation theory

The improved fit using such a non-polynomial dependence on ω is a signal that the problem of setting the phase in the infrared region is dominated by non-perturbative effects.

Conclusion

Using discretized BFKL eigenfunctions, which accounts for running coupling, but with a non-polynomial dependence on ω of the infrared phase, we have a description of the gluon density at low-x, which is positive everywhere and which fits the HERA data very well.
There are four free parameters.
This gluon density can now be exploited (and tested) to predict events (e.g. jets) at LHC dominated by the low-x gluon distribution.