Soft Gluon Resummation for Gaps between Jets

Simone Marzani, Jeffrey Forshaw and James Keates

School of Physics & Astronomy, University of Manchester,
Oxford Road, Manchester, M13 9PL, United Kingdom

We investigate soft gluon resummation for the gaps-between-jets cross-section. After reviewing the theoretical framework that enables one to sum logarithms of the hard scale over the veto scale to all orders in perturbation theory, we then present a study of the phenomenological impact of Coulomb gluon contributions and super-leading logarithms on the gaps between jets cross-section at the LHC.

1 Jet Vetoing: Gaps between Jets

We study dijet production with a veto on the emission of a third jet in the inter-jet rapidity region, Y, harder than Q_0. We shall refer generically to the “gaps between jets” process, although the veto scale is chosen to be large, $Q_0 = 20$ GeV, so that we can rely on perturbation theory. Thus a “gap” is simply a region of limited hadronic activity.

Because of the magnitude of the dijet cross section, gaps between jets will be studied with early LHC data, hence computation of a reliable theoretical prediction has become an urgent task. It is an interesting and at the same time challenging calculation because this observable is sensitive to a remarkably diverse range of QCD phenomena. For instance, the limit of large rapidity separation corresponds to the limit of high partonic centre of mass energy and BFKL effects are expected to become important [1]. On the other hand one can study the limit of emptier gaps, becoming more sensitive to wide-angle soft gluon radiation. Furthermore, if one wants to investigate both of these limits simultaneously, then the non-forward BFKL equation enters the game [2]. In the following we limit ourselves to only wide-angle soft emissions only.

Accurate studies of these effects are important also in relation to other processes, in particular the production of a Higgs boson in association with two jets. This process can occur via gluon-gluon fusion and weak-boson fusion (WBF). QCD radiation in the inter-jet region is clearly different in the two cases and, in order to enhance the WBF channel, one can veto emission between the jets [3,4]. This situation is very closely related to gaps between jets since the Higgs carries no colour charge, and QCD soft logarithms can be resummed using the same technique [5].

2 Soft Gluons in Gaps between Jets

Given a hard scattering process, we can study how it is modified by the addition of soft radiation. If the observable is inclusive enough, then we have no effects because soft contributions cancel when real and virtual corrections are added together, as a result of the Bloch-Nordsieck theorem. However, if we restrict the real radiation to a corner of the phase space, as happens for the gap cross-section, we encounter a miscancellation and are left with a logarithm of the ratio of the hard scale and veto scale, Q/Q_0. The resummation of wide-angle soft radiation in the gaps between jets process was originally performed assuming that the real–virtual cancellation is perfect outside the gap, so that one needs only to consider virtual gluon corrections integrated over momenta for which real emissions are forbidden, i.e. over the "in gap" region of rapidity and with k_T above the veto scale Q_0 [6–8]. We shall refer to these contributions as global logarithms. The resummed squared matrix element can be written as:

$$|\mathcal{M}|^2 = \frac{1}{V_c}\langle m_0|e^{-\xi\mathbf{\Gamma}^\dagger}e^{-\xi\mathbf{\Gamma}}|m_0\rangle\,, \quad \xi = \frac{2}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\alpha_s(k_T)\,, \tag{1}$$

where V_c is an averaging factor for initial state colour. The vector $|m_0\rangle$ represents the Born amplitude and the operator $\mathbf{\Gamma}$ is the soft anomalous dimension:

$$\mathbf{\Gamma} = \frac{1}{2}Y\mathbf{t}_t^2 + i\pi\mathbf{t}_a\cdot\mathbf{t}_b + \frac{1}{4}\rho_{\rm jet}(Y,|\Delta y|)(\mathbf{t}_c^2+\mathbf{t}_d^2)\,, \tag{2}$$

where $\mathbf{t}_i$ is the colour charge of parton i and the function $\rho_{\rm jet}(Y,\Delta y)$ is related to the jet definition. The operator $\mathbf{t}_t^2$ represents the colour exchanged in the t-channel:

$$\mathbf{t}_t^2 = (\mathbf{t}_a+\mathbf{t}_c)^2 = \mathbf{t}_a^2+\mathbf{t}_c^2+2\,\mathbf{t}_a\cdot\mathbf{t}_c\,. \tag{3}$$

The imaginary part of Eq. (2) is due to Coulomb gluon exchange. These contributions play an important role in the proof of QCD factorisation and they are also responsible for super-leading logarithms [9, 10]. We notice that for processes with less than four coloured particles, such as deep-inelastic scattering or Drell-Yan processes, the imaginary part of the anomalous dimension does not contribute to the cross-section. For instance, if we consider three coloured particles, then colour conservation implies that $\mathbf{t}_a+\mathbf{t}_b+\mathbf{t}_c=0$, and consequently

$$i\pi\,\mathbf{t}_a\cdot\mathbf{t}_b = \frac{i\pi}{2}\left(\mathbf{t}_c^2-\mathbf{t}_a^2-\mathbf{t}_b^2\right)\,, \tag{4}$$

which contributes as a pure phase. Coulomb gluons do play a role in dijet production, but they are not implemented in angular-ordered parton showers. We shall evaluate the impact of these contributions on the cross-section in the next section.

It was later realised [11] that the above procedure is not enough to capture the full leading logarithmic behaviour. Real gluons emitted outside of the gap are forbidden to re-emit back into the gap and this gives rise to a new tower of logarithms, formally as important as the primary emission corrections, known now as non-global logarithms. The leading logarithmic accuracy is therefore achieved by considering all $2\to n$ processes, i.e. $n-2$ out-of-gap gluons, dressed with "in-gap" virtual corrections, and not only the virtual corrections to the $2\to 2$ scattering amplitudes. The colour structure quickly becomes intractable and, to date, calculations have been performed only in the large N_c limit [11–13].

A different approach was taken in [9, 10], where the specific case of only one gluon emitted outside the gap, dressed to all orders with virtual gluons but keeping the full N_c structure, was considered. That calculation had a very surprising outcome, namely the discovery of a new class of "super-leading" logarithms (SLL), formally more important than the "leading" single logarithms. Their origin can be traced to a failure of the DGLAP "plus-prescription", when the out-of-gap gluon becomes collinear to one of the incoming partons. Real and virtual contributions do not cancel as one would expect and one is left with an extra logarithm. This

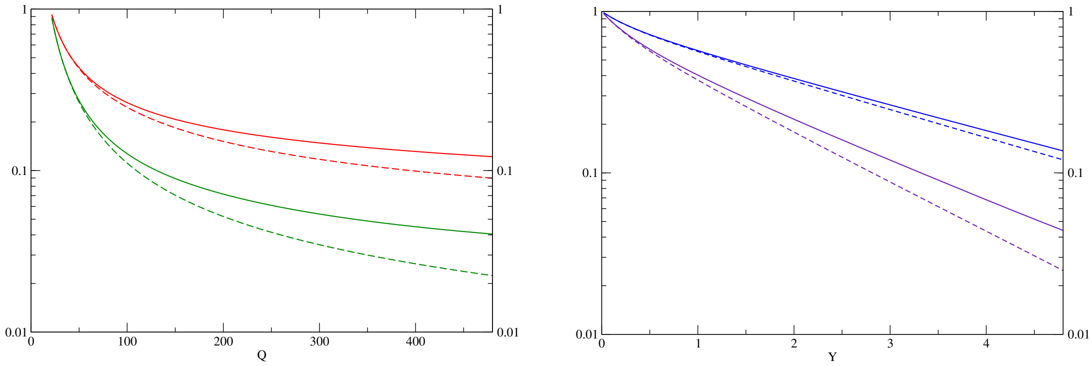

Figure 1: On the left we plot the gap fraction for $Y = 3$ (upper red curves) and $Y = 5$ (lower green curves) as a function of Q and on the right as a function of Y, for $Q = 100$ GeV (upper blue curves) and $Q = 500$ GeV (lower violet curves). The solid lines are the full resummation of global logarithms, while the dashed ones are obtained by omitting the $i\pi$ terms in the anomalous dimension. The veto scale is $Q_0 = 20$ GeV and the jet radius $R = 0.4$.

miscancellation first appears at the fourth order relative to the Born cross-section and it is caused by the imaginary part of loop integrals, induced by Coulomb gluons. The presence of SLL has been also confirmed by a fixed order calculation in [15]; in this approach SLL have been computed at $\mathcal{O}(\alpha_s^5)$ relative to Born, i.e. going beyond the one out-of-gap gluon approximation. The SLL contributions originating from one gluon outside the gap have been recently resummed to all orders [14]. The result takes the form:

$$|\mathcal{M}_1^{\rm SLL}|^2 = -\frac{2}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\rm out} dy\,\left(\Omega_R^{\rm coll}+\Omega_V^{\rm coll}\right)\,, \qquad (5)$$

where $\Omega_{R(V)}^{\rm coll}$ is the resummed real (virtual) contribution in the limit where the out-of-gap gluon becomes collinear to one of the incoming partons.

3 LHC Phenomenology

In this section we perform two different studies. Firstly we consider the resummation of global logarithms and we study the importance of Coulomb gluon contributions, comparing the resummed results to the ones obtained with a parton shower approach. We then turn our attention to super-leading logarithms and we evaluate their phenomenological relevance. In both studies we consider $\sqrt{S} = 14$ TeV, $Q_0 = 20$ GeV, jet radius $R = 0.4$ and we use the MSTW 2008 LO parton distributions [16].

3.1 Comparison to Parton Shower

Soft logarithmic corrections are implemented in Herwig++ via angular ordering of successive emissions. Such an approach cannot capture the contributions coming from the imaginary part of the loop integrals. We evaluate the importance of these contributions in Fig. 1. On the left

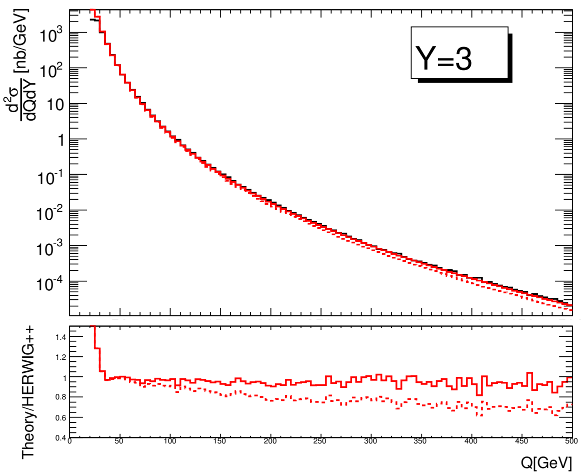

Figure 2: The gap cross-section obtained using HERWIG++ (black histogram) is compared to the one from resummation (red curves). As before the solid line is the full result, while the dashed line is obtained by omitting the Coulomb gluon contributions. At the bottom we plot the ratio between the results obtained from the resummation and the one from HERWIG++.

we plot the gap cross-section, normalised to the Born cross-section (i.e. the gap fraction), as a function of Q at two different values of Y and, on the right, as a function of Y at two different values of Q. The solid lines represent the results of the resummation of global logarithms; the dashed lines are obtained by omitting the $i\pi$ terms in the soft anomalous dimension, Eq. 2. As a consequence, the gap fraction is reduced by 7% at $Q = 100$ GeV and $Y = 3$ and by as much as 50% at $Q = 500$ GeV and $Y = 5$. Large corrections from this source herald the breakdown of the parton shower approach. In Fig. 2 we compare the gap cross-section obtained after resummation to that obtained using HERWIG++ [17–20] after parton showering (Q is taken to be the mean p_T of the two leading jets). The broad agreement is encouraging and indicates that effects such as energy conservation, which is included in the Monte Carlo, are not too disruptive to the resummed calculation. Nevertheless, the histogram ought to be compared to the dotted curve rather than the solid one, because HERWIG++ does not include the Coulomb gluon contributions. In the bottom part of the figure the ratio of the resummation prediction over the Monte Carlo is plotted. The resummation approach and the parton shower differ in several aspects: some non-global logarithms are included in the Monte Carlo and the shower is performed in the large N_c limit. Of course the resummation would benefit from matching to the NLO calculation and this should be done before comparing to data.

3.2 Super-Leading Logarithms

In the following we study the phenomenology of super-leading logarithms. In [15], the coefficients of the SLL have been computed order-by-order in perturbation theory up to $\mathcal{O}\left(\alpha_s^5\right)$, with respect to the Born cross-section. At this order in the perturbative expansion one must consider the contributions coming from one and two gluons outside the rapidity gap. We start by computing the impact of the super-leading contributions on the hadronic cross-section order-by-order in perturbation theory. To this end we define

$$K = \frac{\sigma^{(0)} + \sigma^{\mathrm{SLL}}}{\sigma^{(0)}}\,, \tag{6}$$

where $\sigma^{(0)}$ contains the resummed global logarithms and σ^{SLL} is the super-logarithmic contributions. We plot these K-factors in Fig. 3 as a function of Q, for $Y = 3$ (left) and for $Y = 5$ (right). The curves on the left (right) are obtained by considering one out-of-gap gluon at $\mathcal{O}(\alpha_s^4)$ (dotted) and $\mathcal{O}(\alpha_s^5)$ (dashed). The two-gluons outside the gap contribution is included in the dash-dotted curves. We also plot these K-factors for two values of Q as a function of the rapidity separation Y in Fig. 4. The plot on the left is for $Q = 100$ GeV, while the one on the right is for $Q = 500$ GeV. The different line styles are as in Fig. 3. From these plots it is clear that, for jets with rapidity separation $Y \geq 3$ and transverse momentum bigger than 200 GeV, the inclusion of fixed order SLL contributions leads to sizeable but unstable K-factors: higher order contributions are important and their resummation is necessary. Currently the resummation of SLL contributions has been carried out only for the case of one gluon outside the gap. Figs. 3 and 4 offer some hope that the impact of the two-or-more gluons outside the gap contribution may be modest, since the difference between the K-factors which include the two-out-of-gap gluon contributions (dash-dotted curves) and the ones that do not (dashed curves) is not large. The resummed results are added (as the solid lines) to the plots in Fig. 3 and Fig. 4. Generally the effects of the SLL are modest, reaching as much as 15% only for jets with $Q > 500$ GeV and rapidity separations $Y > 5$. Remember that we have fixed the value of the veto scale $Q_0 = 20$ GeV and that the impact will be more pronounced if the veto scale is lowered.

4 Conclusions and Outlook

We have discussed two phenomenological studies concerning the gaps between jets cross section at the LHC. In particular there are significant contributions arising from the exchange of Coulomb gluons, especially at large Q/Q_0 and/or large Y, which are not implemented in the parton shower Monte Carlos. However before comparing to data, there is a need to improve the resummed results by matching to the fixed order calculation. Matching to NLO is work in progress. These observations will have an impact on jet vetoing in Higgs-plus-two-jet studies at the LHC.

We have also studied the super-leading logarithms that occur because gluon emissions that are collinear to one of the incoming hard partons are forbidden from radiating back into the vetoed region. Even if their phenomenological relevance is generally modest, they deserve further study because they are deeply connected to the fundamental ideas behind QCD factorization.

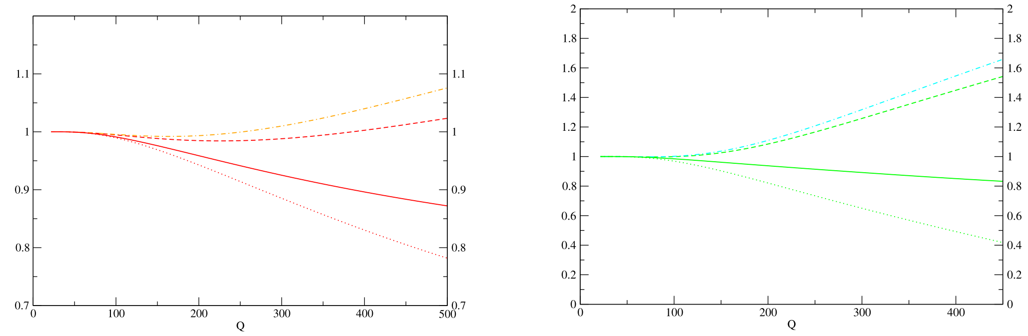

Figure 3: Plots of the K-factors as a function of Q, for $Y = 3$ on the left and for $Y = 5$ on the right. The curves are obtained by considering the one out-of-gap gluon cross-section at $\mathcal{O}(\alpha_s^4)$ (dotted) and $\mathcal{O}(\alpha_s^5)$ (dashed). The two-gluons outside of the gap contribution is included in the dash-dotted curves. The solid line corresponds to the resummation of the one out-of-gap gluon contributions.

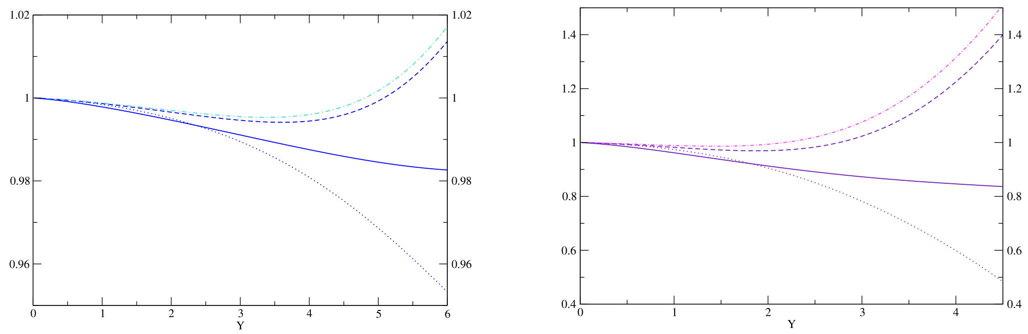

Figure 4: Plots of the K-factors as a function of Y, for $Q = 100$ GeV on the left and for $Q = 500$ GeV on the right. The curves are obtained by considering the one out-of-gap gluon cross-section at $\mathcal{O}(\alpha_s^4)$ (dotted) and $\mathcal{O}(\alpha_s^5)$ (dashed). The two-gluons outside the gap contribution is included in the dash-dotted curves. The solid line corresponds to the resummation of the one out-of-gap gluon contributions. Notice that for small values of Y, the resummed curves dip below the $\mathcal{O}\left(\alpha_s^4\right)$ curves. This behaviour is completely explained by the running of the coupling. In the fixed oder case we have $\alpha_s = \alpha_s(Q)$, while in the resummed case the running coupling is evaluated at $\alpha_s = \alpha_s(k_T)$, i.e. it sits inside the transverse momentum integration.

References

[1] A. H. Mueller and H. Navelet, Nucl. Phys. B **282**, 727 (1987).

[2] A. H. Mueller and W. K. Tang, Phys. Lett. B **284** (1992) 123.

[3] V. D. Barger, R. J. N. Phillips and D. Zeppenfeld, Phys. Lett. B **346** (1995) 106 [arXiv:hep-ph/9412276].

[4] N. Kauer, T. Plehn, D. L. Rainwater and D. Zeppenfeld, Phys. Lett. B **503** (2001) 113 [arXiv:hep-ph/0012351].

[5] J. R. Forshaw and M. Sjödahl, JHEP **0709** (2007) 119 [arXiv:0705.1504 [hep-ph]].

[6] N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. B **531** (1998) 365 [arXiv:hep-ph/9803241].

[7] G. Oderda and G. Sterman, Phys. Rev. Lett. **81** (1998) 3591 [arXiv:hep-ph/9806530].

[8] G. Oderda, Phys. Rev. D **61** (2000) 014004 [arXiv:hep-ph/9903240].

[9] J. R. Forshaw, A. Kyrieleis and M. H. Seymour, JHEP **0608** (2006) 059 [arXiv:hep-ph/0604094].

[10] J. R. Forshaw, A. Kyrieleis and M. H. Seymour, JHEP **0809** (2008) 128 [arXiv:0808.1269 [hep-ph]].

[11] M. Dasgupta and G. P. Salam, Phys. Lett. B **512** (2001) 323 [arXiv:hep-ph/0104277].

[12] R. B. Appleby and M. H. Seymour, JHEP **0212** (2002) 063 [arXiv:hep-ph/0211426].

[13] A. Banfi, G. Marchesini and G. Smye, JHEP **0208** (2002) 006 [arXiv:hep-ph/0206076].

[14] J. Forshaw, J. Keates and S. Marzani, arXiv:0905.1350 [hep-ph].

[15] J. Keates and M. H. Seymour, JHEP **0904** (2009) 040 [arXiv:0902.0477 [hep-ph]].

[16] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, arXiv:0901.0002 [hep-ph].

[17] L. Lönnblad, Comput. Phys. Commun. **118** (1999) 213.

[18] M. Bahr *et al.*, *Herwig++ Physics and Manual*, arXiv:0803.0883 [hep-ph].

[19] R. Kleiss et al, CERN 89-08, vol. 3, pp 129-131.

[20] S. Gieseke, P. Stephens and B. Webber,JHEP **0312** (2003) 045.