Diffractive and Total pp Cross Sections at LHC

Konstantin Goulianos

The Rockefeller University, 1230 York Avenue, New York, NY 10065-9965, USA

The single-diffractive and total *pp* cross sections at the LHC are predicted in a phenomeno-logical approach that obeys all unitarity constraints. The approach is based on the renor-malization model of diffraction and a saturated Froissart bound for the total cross section yielding $\sigma_t = (\pi/s_o) \cdot \ln^2(s/s_F)$ for $s > s_F$, where the parameters s_o and s_F are experi-mentally determined from the $\sqrt{s}$-dependence of the single-diffractive cross section.

1   Single Diffraction

The measurements of the elastic [1] (σ_{el}), to-tal [2] (σ_t), and single-diffractive [3] (σ_{sd}) cross sections by the Collider Detector at Fer-milab (CDF), published in 1994, brought into sharp focus the unitarity problems inherent in the traditional Regge theory description of soft cross sections (see [4]). According to the theory, the cross sections at high energies are dominated by Pomeron ($I\!P$) exchange, and with a Pomeron trajectory of intercept $\alpha(0) = 1 + \epsilon$ the s-dependence is given by:

$$\left. \frac{d\sigma_{el}}{dt} \right|_{t=0} \sim \left(\frac{s}{s_o} \right)^{2\epsilon},$$

$$\sigma_t \sim \left(\frac{s}{s_o} \right)^{\epsilon},$$

$$\sigma_{sd} \sim \left(\frac{s}{s_o} \right)^{2\epsilon}.$$

Such behaviour would violate unitarity at high energies with the elastic and/or the single-diffractive cross section(s) becoming larger than the total cross section. Unitarity, of course, should be obeyed in nature, and the CDF measurements of σ_{sd} at $\sqrt{s} = 540$

Figure 1: Total $pp/\bar{p}p$ single diffraction dissoci-ation cross section data (*both* $\bar{p}$ and p sides) for a forward $\bar{p}$ or p momentum loss fraction $\xi < 0.05$ compared with Regge theory predictions based on the standard and the renormalized Pomeron flux (from Ref. [5]).

and 1800 GeV showed that σ_{sd} is suppressed at high energies relative to Regge predictions preserving unitarity. This result is spectacularly displayed in Fig. 1 from Ref. [5].

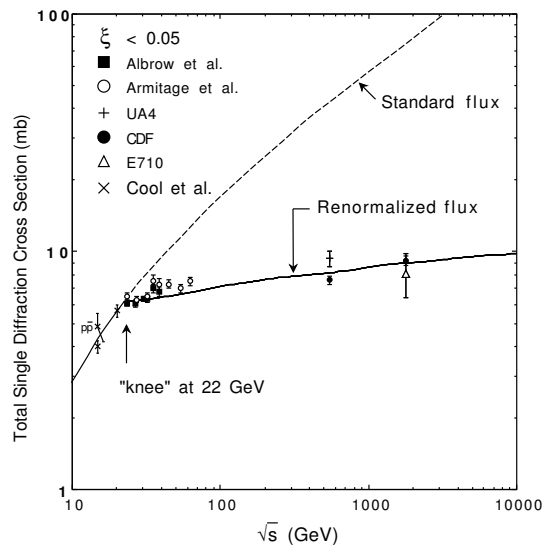The Pomeron exchange contribution to the cross sections can be written as:

$$\sigma_t(s) = \beta_{I\!Ppp}^2(0) \left(\frac{s}{s_\circ}\right)^{\alpha(0)-1} \Rightarrow \sigma_\circ \left(\frac{s}{s_\circ}\right)^\epsilon, \tag{1}$$

$$\frac{d\sigma_{el}(s,t)}{dt} = \frac{\beta_{I\!Ppp}^4(t)}{16\pi} \left(\frac{s}{s_\circ}\right)^{2\,[\alpha(t)-1]}, \tag{2}$$

$$\frac{d^2\sigma_{sd}(s,\xi,t)}{d\xi dt} = \underbrace{\frac{\beta_{I\!Ppp}^2(t)}{16\pi} \xi^{1-2\alpha(t)}}_{f_{I\!P/p}(\xi,t)} \underbrace{\beta_{I\!Ppp}(0)\, g(t) \left(\frac{s'}{s_\circ}\right)^{\alpha(0)-1}}_{\sigma^{I\!Pp}(s',t)}. \tag{3}$$

The two terms in the diffractive cross section, Eq. (3), are the *Pomeron flux*, $f_{I\!P/p}(\xi,t)$, presumed to be emitted by the diffractively scattered proton, and the $I\!P$-p total cross section, $\sigma^{I\!Pp}(s',t)$. The parameters appearing in Eq. (3) are identified as follows:

- $\alpha(t) = \alpha(0) + \alpha' t = (1+\epsilon) + \alpha' t = (1+0.08) + 0.25\,t$ is the Pomeron trajectory;
- $\beta_{I\!Ppp}(t)$ is the coupling of the Pomeron to the proton, $\beta_{I\!Ppp}^2(t) = \sigma_\circ \cdot e^{b_\circ t}$, where $\sigma_\circ \equiv \beta_{I\!Ppp}^2(0)$ and $e^{b_\circ t}$ is the form factor of the diffractively escaping proton, $F_p^2(t) = e^{b_\circ \cdot t}$;
- $g(t)$ is the triple-Pomeron ($I\!P I\!P I\!P$) coupling, which was found experimentally to be independent of t [6];
- $s' \equiv M^2$ is the $I\!P$-p c.m.s. s-value, with M the mass of the diffractively excited proton;
- $\xi \approx M^2/s$ is the momentum fraction of the incident proton carried by the Pomeron;
- $s_\circ$ is an energy scale parameter.

In Ref. [5], the unitarity problem arising from the $s^{2\epsilon}$ dependence of the single-diffractive cross section was addressed by interpreting the Pomeron flux factor as the probability of forming a diffractive rapidity gap and *renormalizing* the integrated probability over all phase space in ξ and t to unity if it exceeded unity. Technically, the renormalization was accomplished by dividing the differential diffractive cross section by the flux integral above the $\sqrt{s}$ value of the $\bar{p}p$ collision energy at which the flux became unity.

The renormalization procedure solved an outstanding energy scale problem in diffraction. From Eq. (1), one sees that $\beta_{I\!Ppp}^2(0) \sim s_o^\epsilon$, and therefore in the diffractive cross section given in Eq. (3) the Pomeron flux contains a scale factor s_o^ϵ while the Pomeron-proton cross section contains a factor $s_o^{\epsilon/2} \cdot g(t)$. Consequently, neither s_o nor $g(t)$ can be independently determined from the measurement of the differential or total diffractive cross sections, but only the product $g(t) \cdot s_o^{\epsilon/2}$. This is of such importance that it deserves being framed:

$$\boxed{f_{I\!P/p}(\xi,t) \sim s_o^\epsilon} \qquad \boxed{\sigma^{I\!Pp}(s',t) \sim s_o^{-\epsilon/2} \cdot g(t)} \quad \Rightarrow \quad \boxed{\sigma_{sd} \text{ determines: } \boldsymbol{g(t) \cdot s_o^{\epsilon/2}}}$$

In Ref. [5], this entanglement was resolved by using $s_o = 1\,\text{GeV}^2$, a value determined from the results displayed in Fig. 1. It was argued that the *knee* in the cross section observed at $\sqrt{s} = 22\,\text{GeV}$ occurs at the energy at which the Pomeron flux integral becomes unity. Since this integral depends on s and s_o, determining the s-value at which the integral is unity yields s_o. The value of s_o was found to be $s_o = 1\,\text{GeV}^2$, and that of the triple-Pomeron coupling $g(t) = 0.69\,\text{mb}^{\frac{1}{2}} = 1.1\,\text{GeV}^{-1}$. It was also mentioned in the paper that the uncertainty in s_o in

terms of the uncertainty in the position of the *knee* is $\delta s_o/s_o = -\delta s/s = -4(\delta\sqrt{s}/\sqrt{s})$, and thus a *reasonable* 10% uncertainty in the $\sqrt{s}$-position of the *knee* would result in a 40% uncertainty in the value of s_o.

With all the parameters in Eq. (3) experimentally determined, the differential and total single diffractive cross sections at the LHC can be predicted. In Ref. [5], using a linear logarithmic expression $A + B\ln s$ (s in GeV^2) in the range $22 < \sqrt{s} < 10\,000$ GeV, the following parameterization was obtained for the total single diffractive cross section:

$$\sigma_{sd}^{p\bar{p}}\big|_{\xi<0.05} \approx (4.3 + 0.3\ln s)\,\text{mb} \quad (22 < \sqrt{s} < 10\,000\,\text{GeV}). \tag{4}$$

By extrapolating to LHC energies, this formula predicts $\sigma_{sd}^{pp}|_{\xi<0.05} = 10.0$ mb at $\sqrt{s} = 14$ TeV. An uncertainty of $\leq 10\%$ is estimated for the cross section in Eq. (4) given the 5% uncertainty of the CDF measurements and that resulting from the Pomeron trajectory parameters.

The underlying basis of the renormalization concept is revealed by a change of variables from ξ to M^2 using the relationship $\xi = M^2/s$. This leads to a diffractive cross section:

$$\frac{d^2\sigma_{sd}(s, M^2, t)}{dM^2 dt} = \left[\frac{\sigma_\circ}{16\pi}\sigma_\circ^{I\!Pp}\right]\frac{s^{2\epsilon}}{N(s, s_o)}\frac{e^{bt}}{(M^2)^{1+\epsilon}}, \tag{5}$$

where $b = b_0 + 2\alpha'\ln\frac{s}{M^2}$ is the slope parameter of the t-distribution and $N(s, s_o)$ the integrated Pomeron flux. The latter is obtained from a straight-forward integration:

$$\text{definition}: \; f_{I\!P/p}(\xi, t) \Rightarrow N^{-1}(s, s_o)\cdot f_{I\!P/p}(\xi, t),$$

$$N(s, s_o) \equiv \int\limits_{\xi(min)}^{\xi(max)} d\xi \int\limits_{t=0}^{-\infty} dt\, f_{I\!P/p}(\xi, t) \;\overset{s\to\infty}{\sim}\; s_o^\epsilon \cdot s^{2\epsilon}/\ln s. \tag{6}$$

The asymptotic form for $s \to \infty$ is given here to illustrate that division by the integrated flux in Eq. (5) replaces the $\boldsymbol{s^{2\epsilon}}$ term by a $\mathbf{\ln s}$ dependence preserving unitarity:

$$\frac{d^2\sigma_{sd}(s, M^2, t)}{dM^2\, dt} \;\overset{s\to\infty}{\sim}\; \ln s\,\frac{e^{bt}}{(M^2)^{1+\epsilon}}. \tag{7}$$

In view of the above, the renormalization concept can be phenomenologically understood within both multi-Pomeron exchange and QCD inspired models. In either case, the s^ϵ ($s^{2\epsilon}$) factor in σ_t (σ_{sd}) arises from overlapping rapidity gaps. Renormalization eliminates this type of *double counting* while preserving the (ξ, t) or (M^2, t) dependence of the cross section.

Integrating Eq. (7) over M^2 and t yields a constant total single diffractive cross section:

$$\sigma_{sd} \;\overset{s\to\infty}{\sim}\; 2\,\sigma_\circ^{I\!Pp}\,\exp\left[\frac{\epsilon\, b_0}{2\alpha'}\right] = \sigma_{sd}^\infty \Rightarrow (16.8 \pm 0.5)\,\text{mb (see text)}. \tag{8}$$

In a recent paper [8], it is suggested that since renormalization eliminates the overlaps caused by *wee-parton* exchanges, σ_{sd}^∞ must be set equal to the value of σ_o of $\sigma_t = \sigma_o e^\epsilon$. The global fit of Ref. [7] yields $\sigma_o = (16.8 \pm 0.5)$ mb, where the uncertainty is obtained from the uncertainty in the value of $\epsilon = 0.104 \pm 0.002$ quoted in the paper using the correlation between the errors in σ_o and ϵ for fixed σ_t, which results in $\delta\sigma_o = \sigma_o \cdot \ln s \cdot \delta\epsilon$.

Since renormalization converts $s^{2\epsilon} \Rightarrow \ln s$ in the differential diffractive cross section, the M^2 distribution would be expected to have no substantial explicit s-dependence. This prediction is confirmed by the data, as shown in Fig. 2. The straight line through the data points is not a fit but is shown here to guide the eye. A fit would have to take into account the dependence of the slope parameter b on ξ, and this should be done by comparing the data with a Monte Carlo simulation. However, the difference that would be obtained using such a comparison is estimated to be small, and the M^2-scaling behaviour exhibited in Fig. 2 does indeed correctly convey the message that renormalization removes the overlaps, which would cause the cross section to follow the disconnected *standard flux* dotted lines shown in the figure for the different collision energies.

The renormalization technique used here can be applied to all hadronic diffractive processes, soft and hard alike, which in terms of

Figure 2: Cross sections $d^2\sigma_{sd}/dM^2dt$ for $p + p(\bar{p}) \rightarrow p(\bar{p}) + X$ at $t = -0.05\,(\text{GeV/c})^2$ and $\sqrt{s} = 14,\ 20,\ 546$ and 1800 GeV. Standard (renormalized) flux predictions are shown as dashed (solid) lines (from Ref. [9]).

the final state event topology can be classified into three main categories: forward gap, central gap, and multi-gap diffraction. Moreover, it can be applied to photoproduction and Deep Inelastic Scattering diffractive processes, predicting the factorization breaking observed at the edges of the available phase space, as outlined in the talk on *Factorization Breaking in Diffraction* presented at this conference [10].

2   Total Cross Section

2.1   The SUPERBALL Model

Theoretical models predicting the total cross section at the LHC must satisfy all unitarity constraints. Available accelerator and cosmic ray data are routinely used to tune the parameters of the models before extrapolating to LHC energies. This process is usually cumbersome, as it involves fitting data which in some cases are not mutually compatible. Using all relevant published data often leads to fits with a χ^2/d.o.f. *pulled* by the *outliers* in the measurements, where *outliers* are data points in clear disagreement with adjacent points from other measurements. Here, we present a model in which these problems are minimized by an inherently unitarized approach based on a saturated Froissart bound above a value of $s = s_F$:

$$\textbf{saturated Froissart bound}:\ \sigma_t(s > s_F) = \sigma_t(s_F) + \frac{\pi}{s_o} \cdot \ln^2 \frac{s}{s_F}. \qquad (9)$$

The saturation occurs in the *wee*-parton exchange governed by the value of the scale parameter s_o that appears in the diffractive cross section in Eq. (3). This parameter is therefore interpreted as the mass-squared of an *object* that is exchanged, and when inserted into the Froissart formula
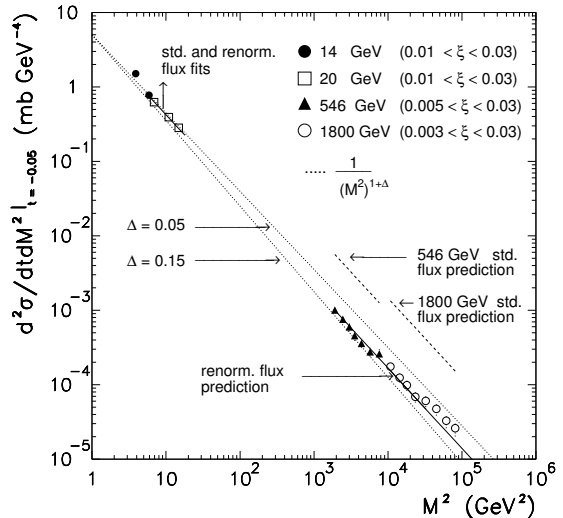
in place of the traditionally used m_π^2 should saturate the bound above the saturation energy $\sqrt{s}_F = 22$ GeV obtained in Sec. 1 from Fig. 1. As this exchanged *object* resembles a glue-ball, we will refer to this normalization procedure as the **SUPER**GLUE**BALL** or SUPERBALL model.

2.2 The Total Cross Section at LHC

Predicting the total cross section at LHC using Eq. (9) would require knowledge of $\sigma(s_F)$ at $\sqrt{s}_F = 22$ GeV. However, the cross section at this energy has substantial Reggeon exchange contributions and also contributions from the interference between the nuclear and Coulomb amplitudes. A complete description would have to take all this into consideration, using the Regge theory amplitudes to describe the Reggeon exchanges, and dispersion relations to obtain the real part of the amplitude from the measured total cross sections up to Tevatron energies. Here, we apply a strategy that bypasses all these complications.

Strategy:

- Use the Froissart formula as a *saturated* cross section rather than as a bound above s_F:

$$\boxed{\sigma_t(s > s_F) = \sigma_t(s_F) + \frac{\pi}{m^2} \cdot \ln^2 \frac{s}{s_F}}$$

- This formula should be valid above the *knee* in σ_{sd} vs. $\sqrt{s}$ at $\sqrt{s}_F = 22$ GeV (Fig. 1) and therefore valid at $\sqrt{s} = 1800$ GeV.

- Use $m^2 = s_o$ in the Froissart formula multiplied by $1/0.389$ to convert it to mb^{-1}.

- Note that contributions from Reggeon exchanges at $\sqrt{s} = 1800$ GeV are negligible, as can be verified from the global fit of Ref. [7].

- Obtain the total cross section at the LHC:

$$\boxed{\sigma_t^{\mathrm{LHC}} = \sigma_t^{\mathrm{CDF}} + \frac{\pi}{s_o} \cdot \left(\ln^2 \frac{s^{\mathrm{LHC}}}{s_F} - \ln^2 \frac{s^{\mathrm{CDF}}}{s_F} \right)}$$

For a numerical evaluation of σ^{LHC} we use as input the CDF cross section at $\sqrt{s} = 1800$ GeV, $\sigma_t^{\mathrm{CDF}} = (80.03 \pm 2.24)$ mb, the Froissart saturation energy $\sqrt{s}_F = 22$ GeV, and the parameter s_o. In Sec. 1, it it was mentioned that a value of $s_o = (1.0 \pm 0.4)$ GeV^2 was extracted from the s-dependence of the single-diffractive cross section. The extraction of s_o from the data assumed $\epsilon = 0.115 \pm 0.008$, which was the average of the CDF measurements at 540 and 1800 GeV. There is, however, a very strong correlation between the values of ϵ and s_o through the relationship displayed in Eq. (6). Using a more accurate value of ϵ extracted in [7], $\epsilon = 0.104 \pm 0.002$, yields $s_o^{\mathrm{CMG}} = 3.7$ GeV^2. The resulting prediction for the total cross section at the LHC at $\sqrt{s} = 14\,000$ GeV is:

$$\sigma_{14\,000\,\mathrm{GeV}}^{\mathrm{LHC}} = (80 \pm 3)\,\mathrm{mb} + (29 \pm 12)\,\mathrm{mb} = (109 \pm 12)\,\mathrm{mb}.$$

This result is in good agreement with the value of $\boxed{\sigma_t^{\mathrm{CMG}} = (114 \pm 5)\,\mathrm{mb}}$ obtained by the global fit of Ref. [7] using an *eikonal* approach, where the uncertainty is estimated from that in the value of the parameter ϵ given in the paper.

The agreement between $\sigma_t^{\text{superball}} = (109 \pm 12)\,\text{mb}$ and $\sigma_t^{\text{CMG}} = (114 \pm 5)\,\text{mb}$ is remarkable, but there are two items to bear in mind: (a) a value of $s_o = 1\,\text{GeV}^2$ was used in the CMG eikonalized evaluation of the cross section, since the result of Ref. [5] was already known by the authors of Ref. [7]; (b) the sensitivity of the present result on the value of ϵ cannot be overemphasized, and as is the case with the determination of s_F, it represents a limiting factor on the accuracy that can be achieved in the prediction of σ^{LHC}.

3 Summary and Conclusion

The single-diffractive and total pp cross sections at the LHC are predicted in a phenomenological approach that obeys all unitarity constraints. The approach is based on the renormalization model of hadronic diffraction, which corrects the double-counting caused by overlapping diffractive rapidity gaps while preserving the dependence of the differential cross section on the fractional momentum loss, ξ, and 4-momentum transfer squared, t, of the diffracted proton. The renormalization procedure replaces the $s^{2\epsilon}$ dependence of the *differential* diffractive cross section with an $\ln s$ dependence and leads to an asymptotically constant single-diffractive cross section as $s \to \infty$ of $\sigma_{sd}^{\infty} = (16.8 \pm 0.5)\,\text{mb}$.

The total cross section at the LHC is estimated using a *saturated* Froissart bound expression $\sigma_t(s > s_F) = \sigma_t(s_F) + \pi/s_o \cdot \ln^2(s/s_F)$, where the parameters s_o and s_F are experimentally determined from the dependence of the single-diffractive cross section on $\sqrt{s}$. Encoring σ_t to the CDF measured value at $\sqrt{s} = 1800\,\text{GeV}$, where Reggeon exchange contributions are negligible, serves to normalize the formula yielding $\sigma_t^{\text{LHC}} = (109 \pm 12)\,\text{mb}$, which is in good agreement with the global fit prediction of $\sigma_t^{\text{CMG}} = (114 \pm 5)\,\text{mb}$ of Ref. [7].

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