Does High Energy Behaviour Depend on Quark Masses?

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Arguments based on the Renormalization Group invariance indicate the vanishing of hadron scattering amplitudes at infinite energies in massless confined QCD with mass gap in physical spectrum. It is shown also that if at least one quark is massive the RG arguments do not forbid increasing amplitudes.

As it is known for a long time the masslessness of fundamental–quark and gluon-fields in the non-Abelian gauge theory in no way precludes existence of physical excitations separated from the vacuum by a non-zero mass gap. This feature, called *dimensional transmutation*, goes back to the pioneering paper [1] and since then was being repeatedly discussed in various contexts (from relatively recent papers see, e.g. [2]). Moreover it is considered that QCD with three massless quarks reflects quite well the basic properties of hadron physics in the low-energy *light sector*. For instance, the nucleon mass practically does not change in the limit of massless quarks [3]. One can therefore believe that such a theory differs even less from the *genuine QCD* at very high energies when the role of mass is, to the great extent, negligible. For instance, the famous BFKL results [4] that give estimates for infinitely growing total cross-sections (due to the Pomeron intercept exceeding 1) were obtained in massless QCD.

1 Renormalization Group Argument

We are going to verify the possibility of infinitely growing cross- sections assuming existence of confined massless QCD [5] , i.e. of a still hypothetical but seemingly quite plausible theory with the physical state spectrum consisting of massive (due to *dimensional transmutation*) colourless hadrons and without quark and gluon asymptotic states. It means that the scattering amplitude as an analytic function does not possess any kind of zero-mass singularities related to elementary quark-gluon fields and has only singularities related to massive colourless hadrons. The only fundamental mass parameter Λ_{QCD} is *hidden* in the running coupling $\alpha_s(\mu^2)$. Now, the physical hadron masses M_i are related with the coupling constant by the formula that reflects the renormalization invariance (everywhere below this also includes the scheme invariance) of physical quantities

$$M_i^2 = c_i\mu^2\exp(-K(\alpha_s)),$$

$$dK(\alpha_s)/d\alpha_s = 1/\beta(\alpha_s),$$

where c_i are fixed numerical parameters, $\beta(\alpha_s)$ is the Gell-Mann-Low function and μ stands for renormalization scale. Actually it means that (see, e.g., [6])

$$M_i^2 = c_i\Lambda^2_{\text{QCD}}. \tag{1}$$

For definiteness, let us consider the scattering amplitude of two hadrons, T. This amplitude is a function of two independent Mandelstam variables, s and t, and also of a (generally infinite) set of hadron masses, marked by parameters c_i from Eq. (1):

$$T = T(s, t; \mu^2, \alpha_s; \{c_i\}).$$

Presence of quantities μ^2 and α_s amongst the arguments of T reflects the provenance of the amplitude from the fundamental Lagrangian and freedom of the choice of the normalization scale. In the same way as physical masses, the amplitude of a physical process must be renormalization invariant. The dependence on α_s is related to calculations based on the fundamental QCD Lagrangian while the very amplitude is, actually, a function of $s/\Lambda^2_{\mathrm{QCD}}$, t/s and $\{c_i\}$, i.e the amplitude depends only on Mandelstam variables and physical hadron masses which define singularities of the amplitude in s- and t- planes. Nonetheless the genetic relation with the underlying theory leads to a non-trivial conclusion. Let us take for simplicity the case of forward scattering ($t = 0$). Due to our assumptions the amplitude is analytic (or at least finite) in this point. Our main argument is that the renormalization invariance, though does not define, but quite strongly restricts the functional dependence of the dimensionless amplitude on its dimensionful parameters. It is easy to verify that, in our case, the renormalization invariance implies the following general form of the amplitude:

$$T(s, 0; \mu^2, \alpha_s; \{c_i\}) = \Phi((s/\mu^2)\exp(K(\alpha_s)); \{c_i\}),$$

where the concrete form of $\Phi(Z; \{c_i\})$ is to be fixed by the dynamics.

We do not know it but we do know that, at any rate, if the coupling α_s goes to zero the amplitude goes there as well. As

$$K(\alpha_s) \sim 1/\beta_0\alpha_s + O(\log(1/\alpha_s)) \ \text{ at } \ \alpha_s \to 0,$$

it is equivalent to the statement that

$$\Phi(Z; \{c_i\}) \to 0 \ \text{ at } \ Z \to \infty.$$

Recalling that $Z = (s/\mu^2)\exp(K(\alpha_s))$, we see that the limit $Z \to \infty$ can be realised with $s \to \infty$ at μ and α_s fixed as well. In its turn it means that, in our theory, **the forward scattering amplitude vanishes with infinite energy growth**:

$$T(s, 0) \to 0 \ \text{ at } \ s \to \infty.$$

For example, the total cross-sections asymptotically decrease to zero:

$$\sigma_{\mathrm{tot}} \to 0 \ \ at \ \ s \to \infty.$$

Moreover,

$$\sigma_{\mathrm{tot}} < \mathrm{const}/s \ \ at \ \ s \to \infty.$$

Partial cross-sections have to drop even faster than the total one to compensate the growth of the number of open channels. It is not difficult to be convinced that the same conclusion takes place for the case of scattering at fixed non-zero angle (t/s fixed). As to fixed t we can rely on the results by Cornille and Martin [7] according to which if the even-signature amplitude dominates forward scattering then $|T(s,t)| \leq |T(s,0)|$, so in this case the amplitude asymptotically tends to zero at high energies and fixed momentum transfers as well. We have to note that the conclusion depends critically on the asymptotic freedom. If the number of fermion flavors were larger than 16 then our arguments would lead to quite an innocuous result that the amplitude vanishes at $s = 0$.

2 Discussion

From a purely theoretical viewpoint general principles of quantum field theory do not forbid such a phenomenon. For instance, if to assume the absence of oscillations, one can obtain the following high-energy lower bound [8]:

$$|T(s,0)| > \text{const}/(\log s)^{1/2}.$$

The result obtained above could mean the presence of oscillations in energy.

More liberal lower bound is [8]:

$$\sigma_{\text{tot}} > \text{const}/s^6(\log s)^2.$$

As to the fixed angle scattering the result seems to be fully consistent with the famous *quark counting rule* [9].

From the point of view of existing experimental data on total and differential small-angle cross-sections the result obtained here seems to be absolutely incredible: the total cross-sections grow at energies up to tens TeV, if to add the cosmic rays data. Differential cross sections of, say, pp-scattering at $t = 0$ which are proportional to $|T(s,0)|^2/s^2$ also grow. Certainly, the result deals with *infinite* energies but even so, the present-day data if to believe in the result we discuss - would mean that there exists some gigantic energy scale (no less than several tens TeV) from which the decreasing starts. However, the theory in question has, as was said, one fundamental scale Λ_{QCD} that, in any case, does not surpass several hundreds MeV. It is very difficult to imagine, in the framework of this theory, a mechanism of generation of such a huge scale. This could be considered as an indication of invalidity of the theory for, at least, high-energy diffractive scattering. At the same time it does not contradict to the well known - both theoretically and experimentally - decrease of hadronic amplitudes at fixed angles.

3 Massive Case

If the fundamental QCD Lagrangian contains fermion mass terms then instead of one single RG invariant mass scale

$$\Lambda_1^2 = \mu^2 \exp(-K(\alpha_s))$$

we have another one (for one massive flavour):

$$\Lambda_2^2 = m^2 \exp(L(\alpha_s))$$

where

$$dL(\alpha_s)/d\alpha_s = \gamma_m(\alpha_s)/\beta(\alpha_s)$$

and $\gamma_m(\alpha_s) = -d(\ln m^2)/d\ln\mu^2$ is the *mass anomalous dimension* defined as in [6]. Here, taking into account the general scheme invariance of physical quantities, we use for simplicity the minimal renormalization scheme. Now the argument used above does not pass through. In fact, the amplitude now has the following general form

$$T(s,0) = F(s/\Lambda_1^2, \Lambda_2^2/\Lambda_1^2).$$

At infinite energy $T \to F(\infty, \Lambda_2^2/\Lambda_1^2)$, while at $\alpha_s \to 0$, $T > F(\infty, \infty) = 0$ because $\Lambda_2^2 \sim (\frac{1}{\alpha_s})^{\frac{\gamma_m}{\beta_0}}$ at $\alpha_s \to 0$. So the (massive) free-field limit is generally different from the high-energy limit and we cannot come to any definite conclusion concerning the latter.

The general conclusion of this paper is that massless QCD is not a good underlying approximation for high-energy diffractive scattering while it seems to be admissible for hard processes or for the low- energy sector of light hadrons.

The crucial importance of quark non-zero-masses for the rise of the total cross-sections seems a bit counterintuitive.

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