One-Particle Inclusive Distribution in the Unitarized Pomeron Models

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It is shown that starting from the pomeron contribution with intercept $\alpha_P(0) > 1$ one can obtain in a quasi-eikonal approach inclusive cross section which is similar to contribution of triple pole (at $t = 0$) pomeron. Generalizing this analogy we consider tripole and dipole pomeron contributions to inclusive cross section. They lead to $\langle n\rangle \propto \ln^3 s$ (tripole) or $\langle n\rangle \propto \ln^2 s$ (dipole) and describe well the data on charged hadron distributions in $\bar{p}p$. Predictions of one particle p_t and rapidity distributions for LHC energies are given.

The model of simple pole pomeron with intercept $\alpha_P(0) = 1 + \varepsilon,\quad \varepsilon > 0$ [1] gives a simple and compact parametrization for many high-energy soft processes (elastic and deep inelastic scattering, diffraction and others), it describes well many experimental data at high energies.

On the other side at $s \to \infty$ contribution of such pomeron violates unitarity explicitly. The model leads to total cross section of hadron interaction $\sigma_t(s) \propto (s/s_0)^{\varepsilon}$ ($s_0 = 1$ GeV^2) in contradiction to the Froissart bound $\sigma_t \le (\pi/m_\pi^2)\ln^2(s/s_0)$.

Thus the model is only a phenomenological tool and must be improved in order to restore unitarity. There are a few known ways to avoid at least a rough violation of unitarity bound. The most simple method for that is to sum multipomeron diagrams of Fig. 1.

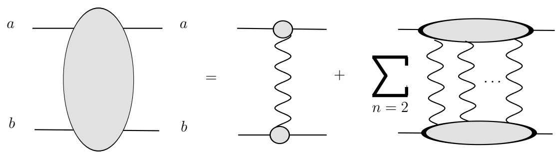

Figure 1: Multipomeron contributions to elastic scattering amplitude.

Starting with one pomeron exchange written in the form

$$a(s,t) = \eta_P(t)\tilde{g}_{ab}(t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)} = -g_a(t)g_b(t)\left(-i\frac{s}{s_0}\right)^{\alpha_P(t)},$$

where $s_0 = 1\mathrm{GeV}^2$ and $\eta_P(t) = \frac{1+\exp(-i\pi\alpha_P(t))}{-\sin(\pi\alpha_P(t))}$, then going to impact parameter representation

$$h(s,b) = \frac{1}{8\pi s}\int_0^{\infty} dq\, q\, J_0(qb)\, a(s,-q^2)$$

one can obtain under some simplifying assumption (see below)

$$H(s,b) = -\frac{1}{2i}\sum_{n=1}^{\infty}\frac{G_a(n)G_b(n)}{n!}[-2ih(s,b)]^n.$$

The amplitude $H(s,b)$ takes this form if we assume that two-hadrons-n-pomeron amplitude is proportional to the product of two-hadron-pomeron vertices (a pole approximation for intermediate states) as shown in Fig. 2.

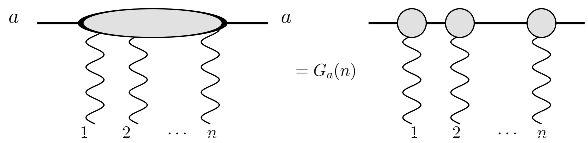

Figure 2: Amplitude of interaction of two hadrons with n pomerons in a pole approximation but with phenomenological factor $G_a(n)$.

Moreover, assuming either $G(n) = C^n$ or $G(n) = C^n\sqrt{n!}$ we obtain two well known schemes of pomeron unitarization: quasi-eikonal [2] or quasi-U-matrix models [3, 4].

$$H(s,b) = \begin{cases} \frac{1}{2iC_aC_b}\left(1 - e^{-2iC_aC_bh(s,b)}\right), & \text{if} \quad G_{a,b}(n) = C_{a,b}^n \\ \frac{h(s,b)}{1+2iC_aC_bh(s,b)}, & \text{if} \quad G_{a,b}(n) = C_{a,b}^n\sqrt{n!} \end{cases}$$

If $\alpha_P(t) = 1 + \varepsilon + \alpha_P^{'}\, t$ and $g_{a,b}(t) = \exp(B_{a,b}\, t)$ one can find that at $s \to \infty$ in both models

$$\sigma_t^{ab}(s) \approx 8\pi\varepsilon R^2(s)\ln(s/s_0) \approx 8\pi\varepsilon\alpha_P^{'}\ln^2(s/s_0),$$

where $R^2(s) = B_a + B_b + \alpha_P^{'}\ln(s/s_0)$.

This result gives a ground for another method of constructing amplitude. One can consider just from the beginning more complicated singularities of partial amplitudes than usual simple angular momentum poles. Because the factorization of residues is valid not only for simple j-poles but also for any isolated j-singularity [5] one can consider, for instance, double pole (dipole pomeron) [6] or triple pole (at $t \neq 0$ because of analyticity it must be a pair of hard branch points collided to a triple pole at $t = 0$) instead of simple pole. In these models $\sigma_t(s) \propto \ln(s/s_0)$ (dipole) or $\sigma_t(s) \propto \ln^2(s/s_0)$ (tripole) at $s \to \infty$. Both models lead to a very good description of the hadron total cross sections as well as of the differential elastic cross sections, deep inelastic scattering and vector meson photoproduction.

Therefore it is interesting to see how many-particle processes are described in these unitarized pomeron models. We consider here one particle distribution in rapidity and pseudorapidity.

1 Multipomeron Exchanges in the Model with $\alpha_{\mathbf{P}}(\mathbf{0}) > 1$

Due to the generalized Optical Theorem the differential cross section of one particle inclusive production ($a + b \to c + X$) in the central kinematic region where

$$
\begin{array}{rcll}
s & = & (p_a + p_b)^2 \to \infty, & \\
t & = & (p_a - p_c)^2, & |t| \to \infty, \\
u & = & (p_b - p_c)^2, & |u| \to \infty, \\
M^2 & = & (p_a + p_b - p_c)^2, & s/M^2 \to 1, \\
tu/s & = & m_t^2 = m_c^2 + p_l^2 &
\end{array}
$$

is related with the diagram of Fig. 3.

More exactly, at large energy and for the simple pomeron pole with $\alpha_P(0) - 1 = \varepsilon$

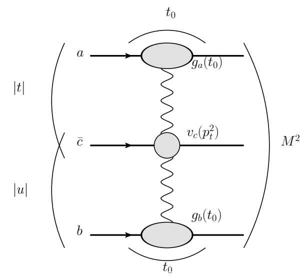

Figure 3: Pomeron contribution to inclusive production in the central region.

$$
\begin{aligned}
E\frac{d^3\sigma}{d^3p} = E\frac{d^3\sigma}{dp_l d^2p_t} &= 8\pi \, \mathrm{Disc}_{M^2}\mathcal{M}(a + b + \bar{c} \to a + b + \bar{c}) \\
&= g_a(0)\left(\frac{|t|}{s_0}\right)^{\varepsilon} v_c(p_t^2)\left(\frac{|u|}{s_0}\right)^{\varepsilon} g_b(0).
\end{aligned}
$$

where $E, p_l, \vec{p}_t$ are energy and momenta of the inclusive hadron c, $g_{a,b}(t_0 = 0)$ are the coupling vertices aPa, bPa, $s_0 = 1\mathrm{GeV}^2$.

It is more convenient for what follows to use another set of variables, (p_t, y) or (p_t, η)

$$
y = \frac{1}{2}\ln\frac{E + p_l}{E - p_l}, \quad |y| \le y_0 = 1/2\ln(s/m_t^2), \qquad \eta = -\ln(\tan\vartheta/2),
$$

where ϑ is the scattering angle of hadron c in the centre-of-mass system. With the rapidity variable the cross section in the one pole approximation is read as

$$
E\frac{d^3\sigma}{d^3p} = g_a(0)e^{\varepsilon(y_0 - y)}v_c(p_t^2)e^{\varepsilon(y_0 + y)}g_b(0) = g_a(0)v_c(p_t^2)g_b(0)e^{2\varepsilon y_0}. \tag{1}
$$

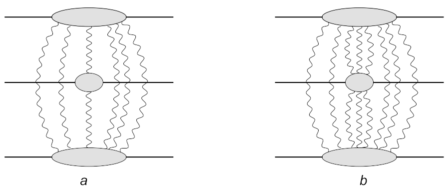

Figure 4: Multipomeron exchange diagrams for one particle inclusive production, (a) - diagrams calculated in [8], (b) - diagrams calculated in [10, 11].

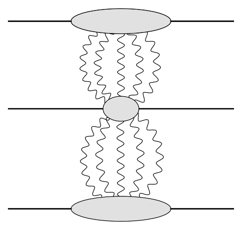

Figure 5: The dominating contribution to central inclusive production at $s \to \infty$.

In Ref. [8] the contribution to inclusive cross section of the diagrams given in Fig. 4a can be calculated. It was shown that due to Abramovsky-Gribov-Kancheli rules [9] only input diagram with one pomeron exchange contributes, rest sum of diagrams vanishes. Thus the cross section again is given by Eq. (1).

Making use of the sum rule

$$
\int \frac{d^3p}{E} E\frac{d^3\sigma(ab \to cX)}{d^3p} = \langle n_c\rangle \sigma_t^{ab}(s) \tag{2}
$$

one can calculate the mean multiplicity of hadrons as well as taking into account that $\sigma_t \approx \sigma_0 \ln^2(s/s_0)$ to find at $s \to \infty$

$$\frac{dn_c}{dy} = \frac{1}{\sigma_t(s)} 8\pi g_a(0)\, \tilde{V}_c g_b(0)\, (s/s_0)^{\varepsilon} \propto \frac{(s/s_0)^{\varepsilon}}{\ln^2(s/s_0)},$$

where $\tilde{V}_c$ is the integral of $v_c(p_t^2)$ over p_t^2. Thus, integrating over y we obtain a power growth for mean multiplicity, $\langle n_c \rangle \propto (s/s_0)^{\varepsilon}/\ln(s/s_0)$. Let us notice that dn_c/dy does not depend on y, however it is not supported by the experimental data.

More diagrams must be added to calculate the inclusive cross section under interest. These diagrams are shown in Fig. 4b and were calculated in [10, 11]. Likewise the case of diagrams Fig. 4a the sum of all contributions with the reggeons between hadrons a and b vanishes.

As result at $s \to \infty$ the inclusive cross section in the central region is dominated by the contribution of the diagram in Fig. 5 and can be written in a general form as

$$E\frac{d^3\sigma}{d^3p} = g_a(0)\mathcal{F}(y_0 - y)v_c(p_t^2)\mathcal{F}(y_0 + y)g_b(0), \quad \text{where} \quad \mathcal{F}(y_0 \pm y) = (y_0 \pm y)^2.$$

It is necessary to note that this result exactly coincides with those which can be obtained if we assume from the beginning that pomeron at $t = 0$ is a triple j-pole.

This fact and similar ones valid for the elastic amplitude allow us to make more general assumptions and consider the diagram of Fig. 3 with pomerons of arbitrary hardness at $t = 0$.

If the pomeron contribution to the partial amplitude (of elastic scattering) at $t = 0$ is proportional to $1/(j-1)^{\nu+1}$ then

$$\mathcal{F}(y_0 \pm y) = (y_0 \pm y)^{\nu} \qquad \text{and} \qquad \frac{dn}{dy} \propto (y_0 - y)^{\nu}(y_0 + y)^{\nu}.$$

We would like to remark that such a behaviour of dn/dy (at $\nu > 0$) is in a qualitative agreement with high energy experimental data, which show a rise dn/dy at y_0 and a parabolic form. Taking into account that such pomeron leads to $\sigma_t(s) \propto \ln^{\nu}(s/s_0)$ one can find

$$\frac{dn}{dy}(y = 0) \propto \ln^{\nu}(s/s_0) \quad \text{and} \quad \langle n \rangle \propto \ln^{1+\nu}(s/s_0).$$

It is known that excellent description of mean hadron multiplicity is achieved within a logarithmic energy dependence with $\nu = 2$ or $\nu = 3$. All mentioned properties of unitarized pomeron models concerning one particle inclusive distribution are rather attractive, but they should be checked out quantitatively with the data. We do that in the next section.

2 Comparison of the Unitarized Pomeron Models with the Data

Experimental data. Our aim is not the detailed description of all data, we would like to demonstrate only a possibility of the considered models to reproduce the main features of the high energy data. Evidently, at lower energy we need to add more Regge contributions increasing the number of the fitting parameters. To avoid extra number of contributions and parameters we consider the data on $E\,d^3\sigma/d^3p$ at $\sqrt{s} = 200,\ 540,\ 630,\ 900,\ 1800$ GeV (240 points) and on $dn/d\eta$ normalized to σ_{in} (48 points) [12].

Even for the high energies chosen there is a nontrivial dependence of cross sections on p_t (Fig. 6), their slope is changing with energy. The dependence on p_t in the pomeron contribution is coming only from the vertex function $v_c(p_n^2)$, therefore one has to conclude that the slope effect can be explained in the model only due to sub-asymptotic contributions. Besides this, an exponential increasing $E\,d^3\sigma/d^3p$ at small transverse momenta $p_t < 1$ GeV is changed for a power-like behaviour at higher p_t (larger than 1 GeV).

Another set of data, namely, $dn/d\eta$ is more interesting for our aim. It can be obtained from $Ed^3\sigma/d^3p$ by integration over p_t and with a replacing y for η. To perform the integration one has to know the vertex functions $v_c(p_t^2)$ which are not determined within any Regge model, we parameterize it in a some form to reproduce existing experimental data. The explicit form of p_t-dependence is not crucial for models under interest. It plays only subsidiary role in obtaining $d\sigma/d\eta$ or $dn/d\eta$. The dependence of the differential cross section on y is more important for a verification of our models.

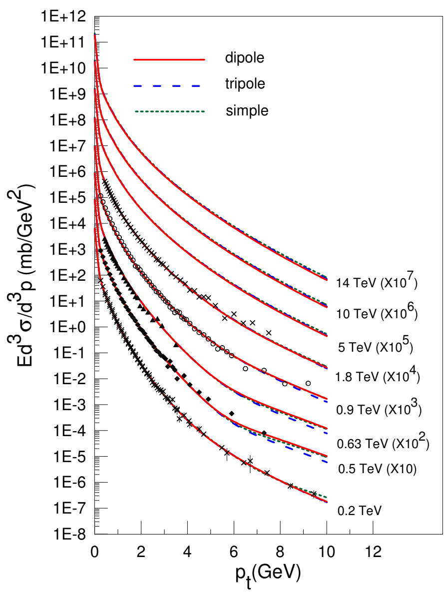

Figure 6: p_t-dependence of inclusive cross sections at high energies.

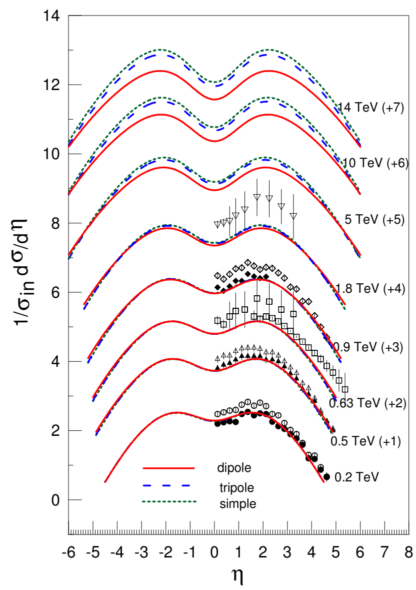

Figure 7: Density of the produced hadrons.

Pomeron models. At $\sqrt{s} \le 200$ GeV in the dipole pomeron model we take into account in the diagram in Fig. 3 the dipole (d) and simple (p) poles, both with $\alpha(0) = 1$, and f-reggeon for the upper and lower parts of the diagram. In the simple pole model instead of the dipole we have considered simple pole with $\alpha(0) = 1 + \varepsilon$. In the tripole pomeron case a triple pole has to be added. However, to avoid too many parameters we consider here a simplified model for tripole pomeron. The general form of the cross section for all considered models is the following:

$$E\frac{d^3\sigma}{d^3p} = \begin{array}{l} g_{11}v_{11}(p_t)z_u z_d + g_{12}v_{12}(p_t)(z_u+z_d) + g_{22}v_{22}(p_t) + \\ g_{13}v_{13}(p_t)[z_u e^{\varepsilon_f(y_0+y)} + z_d e^{\varepsilon_f(y_0-y)}] + \\ g_{23}v_{23}(p_t)[e^{\varepsilon_f(y_0-y)} + e^{\varepsilon_f(y_0+y)}] + g_{33}v_{33}(p_t)e^{2\varepsilon_f y_0} \end{array}$$

where g_{ab} with $a, b = 1, 2, 3$ are constants, $\varepsilon_f = \alpha_f(0) - 1$, $v_{ab}(p_t)$ are vertex functions

$$v_{ab}(p_t) = (1 + p_t^2/p_0^2)^{-\mu_{ab}}[e^{-Bp_t} + c(1 + p_t^2/p_1^2)^{-\mu}].$$

For the dipole pomeron model

$$z_u = (y_0 - y), \qquad z_d = (y_0 + y),$$

for the tripole pomeron model

$$z_u = \beta(y_0 - y)^2 + (y_0 - y), \qquad z_d = \beta(y_0 + y)^2 + (y_0 + y),$$

and for the simple pomeron model

$$z_u = e^{\varepsilon(y_0-y)}, \quad z_d = e^{\varepsilon(y_0+y)}.$$

The data fit. The description of the data is demonstrated in Figs. 6 and 7. Data are taken from [12]. The red solid line – dipole pomeron model, blue long dashed line – tripole pomeron model, green doted line – simple pomeron pole with $\alpha(0) > 1$. Predictions for three LHC energies are also shown. In Fig. 7 the solid symbols correspond to the data normalized to σ_{in}, open symbols correspond to data normalized to σ_{NSD} (not used in the fit procedure). Parameters of the models as well as χ^2 will be given in a more complete paper [13].

One can see that the theoretical curves in the three models are very close to each other, at least for energies where data exist. The fit gives χ^2/n.d.f. = 3.01 (dipole), 2.98 (tripole) and 2.96 (simple pole). It is not a surprise because the parameters of the tripole and simple models in fact mimic at not very high energy the dipole pomeron model. In the tripole model parameter β is equal to 0.03, thus the terms containing $(y - y_0)^2$ are not important at the achieved energy, $\sqrt{s} \leq 1800\,\text{GeV}$. A similar situation occurs in the simple pomeron model where a strong cancellation among the S- and P-terms occurs.

$$g_{ss}(s/s_0)^{\varepsilon} + g_{pp} \approx g_{ss} + g_{pp} + g_{ss}\varepsilon\ln(s/s_0)$$

However, a difference between the models' predictions is increasing with energy. It can be seen clearly in Figs. 8 and 9 which demonstrate the behaviour of $dn(\eta = 0)/d\eta$, $\langle n\rangle$ in energy.

3 Conclusion

We have shown that the high energy experimental data on one-particle inclusive distribution can be described well in the models of unitarized pomeron, which do not violate unitarity restrictions. They predict a small difference for differential cross sections, and mean multiplicities at LHC energies, giving $dn/d\eta(y = 0) \propto \ln^4 s(\ln^3 s)$ and $\langle n\rangle \propto \ln^3 s(\ln^2 s)$ for the dipole (tripole) pomeron model, correspondingly.

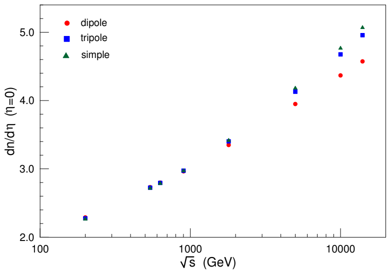

Figure 8: Density of the produced hadrons at $\eta = 0$ as function of energy.

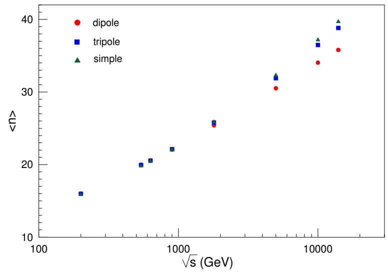

Figure 9: Mean multiplicity of the produced hadrons as function of energy, calculated in the interval $-3.5 \le \eta \le 3.5$.

Acknowledgement

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