# Initial-State Interactions in Drell-Yan Processes at Hadron Collisions

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Single-spin asymmetry (SSA) phenomena in hadron physics are studied. The SSA in semi-inclusive DIS is understood by the final-state interactions from gluon exchange between the outgoing quark and the target spectator system. The asymmetry of the angular distribution in Drell-Yan processes is investigated by the SSA of the quark spin which is induced by the initial-state interactions.

## 1 Introduction

Since the observation of large transverse polarization of produced $\Lambda$ hyperons in the inclusive reactions $pp \to \Lambda^\uparrow X$ [1] and $p\,Be \to \Lambda^\uparrow X$ [2] in the middle of the 1970's, there have been many experimental and theoretical investigations aimed at understanding this striking polarization phenomenon [3, 4], which is called the single-spin asymmetry (SSA). The possibility of measuring the $\Lambda$ polarization at LHC was also studied [5]. SSAs in hadronic reactions have been among the most attractive phenomena to understand from basic principles in QCD. The problem has became more acute because of the observations in the semi-inclusive DIS by the HERMES [6, 7] collaboration of a strong correlation between the target proton spin $\vec{S}_p$ and the plane of the produced pion and virtual photon in semi-inclusive deep inelastic lepton scattering $\ell p^\uparrow \to \ell' \pi X$ at photon virtuality as large as $Q^2 = 6$ GeV$^2$. Large azimuthal single-spin asymmetries have also been seen in hadronic reactions such as $pp^\uparrow \to \pi X$ [8, 9, 10], where the target antiproton is polarized normal to the pion production plane.

It was found in Ref. [11] that the final-state interaction of quark and gluon induces the single-spin asymmetry in the semi-inclusive deep inelastic scattering at the twist-two level. In Ref. [11] Brodsky, Schmidt and I calculated the single-spin asymmetry in semi-inclusive electroproduction $\gamma^* p \to HX$ induced by final-state interactions in a model of a spin-$\frac{1}{2}$ proton with mass $M$ composed of charged spin-$\frac{1}{2}$ and spin-0 constituents with respective mass $m$ and $\lambda$, which is a QCD-motivated quark-scalar diquark model of a nucleon. The basic electroproduction reaction is then $\gamma^* p \to q(qq)_0$, as illustrated in Fig. 1. Then, this time-odd twist-two effect was interpreted as the Sivers effect [12] by finding that the final-state interaction can be treated as the source of the time-odd Sivers distribution function [13, 14, 15, 16, 17, 18]. It is also often referred to as "naively $T$-odd", because the appearance of this function does not imply a violation of time-reversal invariance, since they can arise through the final-state interactions. With these developments, the existence of the Sivers distribution function has gained a firm theoretical support. The Sivers distribution function $f_{1T}^\perp$ describes the difference between the momentum distributions of quarks inside the nucleon transversely polarized in opposite directions. There is another quark distribution function of the nucleon induced by the final-state

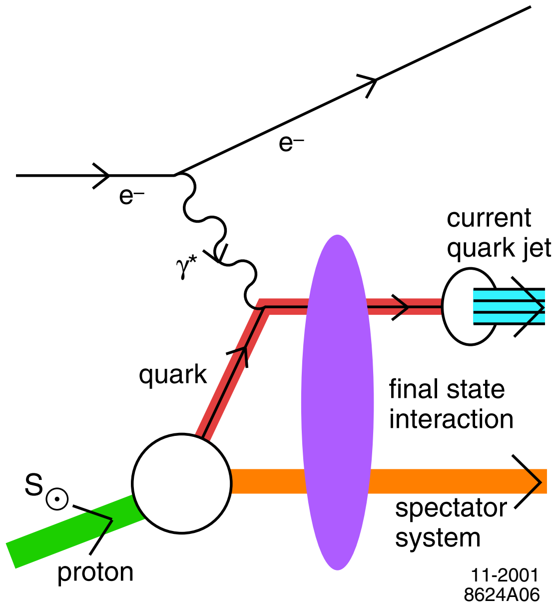


Figure 1: The final-state interaction in the semi-inclusive deep inelastic lepton scattering $\ell p^{\uparrow} \to \ell' \pi X$.

interaction of quark and gluon, which is called the Boer-Mulders distribution function $h_1^{\perp}$. $h_1^{\perp}$ describes the difference between the momentum distributions of the quarks transversely polarized in opposite directions inside the unpolarized nucleon [19]. The distribution functions $f_{1T}^{\perp}$ and $h_1^{\perp}$ are depicted in Figs. 2 and 3.

It has been shown that initial-state interactions contribute to the $\cos 2\phi$ distribution in unpolarized Drell-Yan lepton pair production $p\,p$ and $p\,\overline{p} \to \ell^+\ell^- X$, without suppression [17, 20, 21]. The asymmetry is expressed as a product of chiral-odd distributions $h_1^{\perp}(x_1, \boldsymbol{p}_\perp^2) \times \overline{h}_1^{\perp}(x_2, \boldsymbol{k}_\perp^2)$, where the quark-transversity function $h_1^{\perp}(x, \boldsymbol{p}_\perp^2)$ is the transverse momentum dependent, light-cone momentum distribution of transversely polarized quarks in an unpolarized proton. This (naive) $T$-odd and chiral-odd distribution function and the resulting $\cos 2\phi$ asymmetry were computed explicitly in a quark-scalar diquark model for the proton with initial-state gluon interaction in Ref. [17]. In this model the function $h_1^{\perp}(x, \boldsymbol{p}_\perp^2)$ equals the $T$-odd (chiral-even) Sivers effect function $f_{1T}^{\perp}(x, \boldsymbol{p}_\perp^2)$. This suggests that the single-spin asymmetries in the SIDIS and the Drell-Yan process are closely related to the $\cos 2\phi$ asymmetry of the unpolarized Drell-Yan process, since all can arise from the same underlying mechanism. This provides new insight regarding the role of quark and gluon orbital angular momentum as well as that of initial- and final-state gluon exchange interactions in hard QCD processes.

The light-cone wave functions are useful for studying the hadronic processes by treating the non-perturbative effects in a relativistically covariant way [22, 23, 24, 25, 26]. Here we calculate the Sivers and Boer-Mulders distribution functions by using their light-cone wave function representations. Then, we calculate the SSA in semi-inclusive DIS and the asymmetry of the angular distribution in the Drell-Yan process using the Sivers and the Boer-Mulders

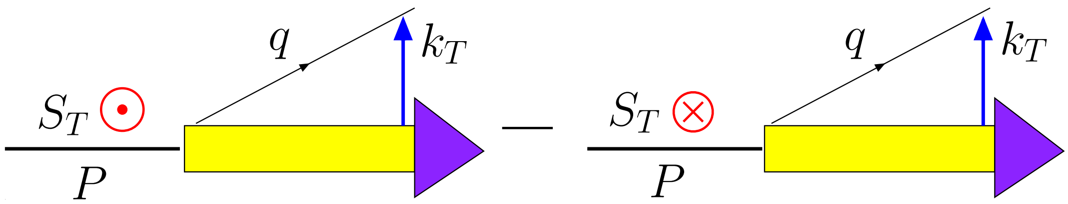


Figure 2: Schematic depiction of the Sivers distribution function $f_{1T}^{\perp}$. The spin vector $S_T$ of the nucleon points out of and into the page, respectively, and $k_T$ is the transverse momentum of the extracted quark.

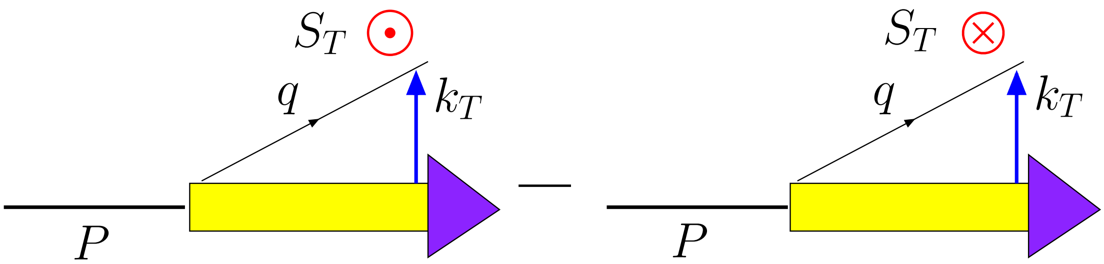


Figure 3: Schematic depiction of the Boer-Mulders distribution function $h_1^{\perp}$. The spin vector $S_T$ of the quark points out of and into the page, respectively, and $k_T$ is the transverse momentum of the extracted quark.

distribution functions.

## 2 Model Calculation of Sivers and Boer-Mulders Functions with Scalar Diquark Model

The final-state interactions in semi-inclusive deep inelastic scattering are commonly treated as a part of the proton distribution function [13, 17]. If we adopt the same treatment for the light-cone wave functions, we can consider that the final-state interactions for the scalar diquark model depicted in Fig. 4 induce the spin-dependent complex phases to the light-cone wave functions:

$$\left\{ \begin{array}{l} \psi^{\uparrow}_{+\frac{1}{2}}(x,\vec{k}_\perp) = \frac{(m+xM)}{x}\left(1+ia_1\right)\varphi\ , \\ \psi^{\uparrow}_{-\frac{1}{2}}(x,\vec{k}_\perp) = -\frac{(+k^1+ik^2)}{x}\left(1+ia_2\right)\varphi\ , \end{array} \right. \tag{1}$$

$$\left\{ \begin{array}{l} \psi^{\downarrow}_{+\frac{1}{2}}(x,\vec{k}_\perp) = -\frac{(-k^1+ik^2)}{x}\left(1+ia_2\right)\varphi\ , \\ \psi^{\downarrow}_{-\frac{1}{2}}(x,\vec{k}_\perp) = \frac{(m+xM)}{x}\left(1+ia_1\right)\varphi\ , \end{array} \right. \tag{2}$$

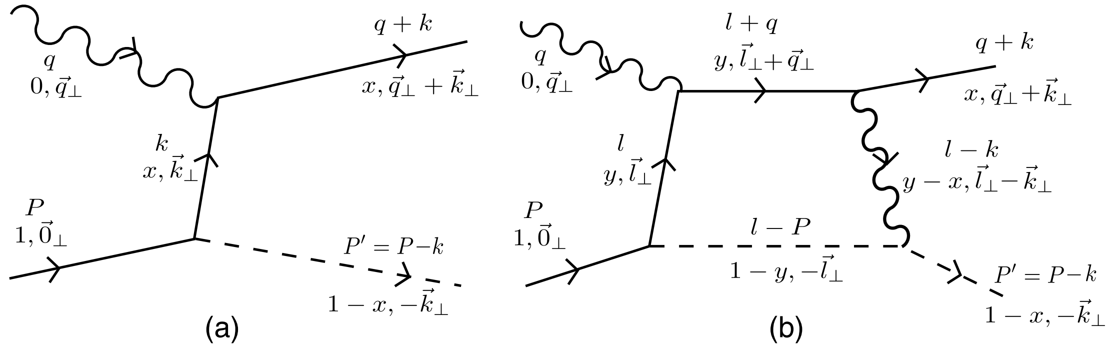


Figure 4: (a) Tree level diagram and (b) diagram with final-state interaction.

where $\varphi = \varphi(x, \vec{k}_\perp) = -g\, x\sqrt{1-x}/(\vec{k}_\perp^2 + B)$ with the nucleon-quark-diquark coupling constant $g$ and $B = -x(1-x)M^2 + (1-x)m^2 + x\lambda^2$, and $a_1$ and $a_2$ are given by

$$a_{1,2} = \frac{e_1 e_2}{8\pi}\ (\vec{k}_\perp^2 + B)\ g_{1,2} \tag{3}$$

with [11]

$$g_1 = \int_0^1 d\alpha\ \frac{-1}{\alpha(1-\alpha)\vec{k}_\perp^2 + \alpha\lambda_g^2 + (1-\alpha)B}\ , \qquad g_2 = \int_0^1 d\alpha\ \frac{-\alpha}{\alpha(1-\alpha)\vec{k}_\perp^2 + \alpha\lambda_g^2 + (1-\alpha)B}\ . \tag{4}$$

In the above, $e_1$ and $e_2$ are the quark and diquark charge, and $M$, $m$, $\lambda$ and $\lambda_g$ are the nucleon, quark, diquark and gluon mass, respectively. We take $\lambda_g = 0$ at the end of the calculation. Our analysis can be generalized to the corresponding calculation in QCD. The final-state interaction from gluon exchange has the strength $\frac{e_1 e_2}{4\pi} \to C_F \alpha_s(\mu^2)$.

Using the wave functions (1) and (2) in the light-cone wave function representations of Sivers and Boer-Mulders functions presented in Ref. [27], we obtain [11, 17, 27, 28]

$$f_1(x, \vec{k}_\perp) = \frac{1}{16\pi^3}\ \Big[(M + \frac{m}{x})^2 + \frac{\vec{k}_\perp^2}{x^2}\Big]\varphi^2\ , \tag{5}$$

$$f_{1T}^\perp(x, \vec{k}_\perp) = \frac{1}{16\pi^3}\ 2\ \frac{M}{x}\ (M + \frac{m}{x})\ \varphi^2\ \frac{e_1 e_2}{8\pi}\ (\vec{k}_\perp^2 + B)\ \frac{1}{\vec{k}_\perp^2}\ \ln\frac{(\vec{k}_\perp^2 + B)}{B}\ , \tag{6}$$

$$h_1^\perp(x, \vec{k}_\perp) = \frac{1}{16\pi^3}\ 2\ \frac{M}{x}\ (M + \frac{m}{x})\ \varphi^2\ \frac{e_1 e_2}{8\pi}\ (\vec{k}_\perp^2 + B)\ \frac{1}{\vec{k}_\perp^2}\ \ln\frac{(\vec{k}_\perp^2 + B)}{B}\ . \tag{7}$$

## 3 Semi-Inclusive DIS

The SSA in the semi-inclusive DIS (SIDIS) was calculated in Refs. [11, 17] with the light-cone wave functions given in the previous section. The formula for the SSA in the SIDIS is given by $\mathcal{P}_y = -(r_\perp^1/M)\,(f_{1T}^\perp(x, \boldsymbol{r}_\perp)/f_1(x, \boldsymbol{r}_\perp))$ [11, 17], which gives the results presented in Fig. 5 [11].

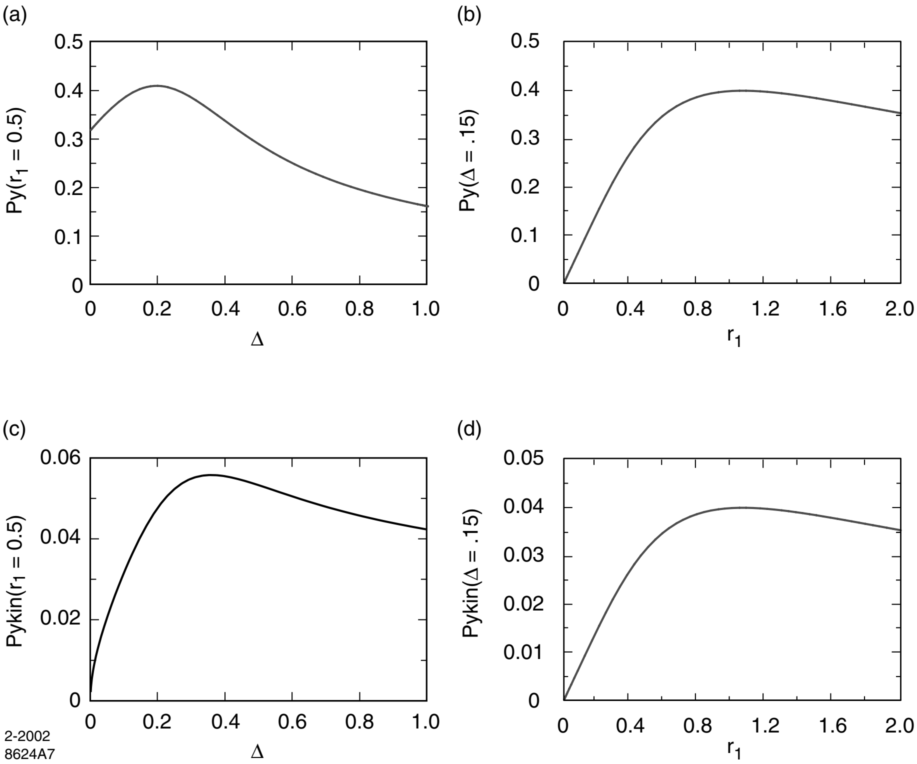


Figure 5: Model predictions for the single spin asymmetry of the proton in electroproduction resulting from gluon exchange in the final state as a function of $\Delta = x_{bj}$ and quark transverse momentum $r_\perp$. The parameters are given in the text of Ref. [11].

# 4 Drell-Yan Process

The unpolarized Drell-Yan process cross section has been measured in pion-nucleon scattering: $\pi^- N \to \mu^+ \mu^- X$, with $N$ deuterium or tungsten and a $\pi^-$-beam with energy of 140, 194, 286 GeV [29] and 252 GeV [30]. Conventionally the differential cross section is written as

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin^2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right) , \tag{8}$$

where the angles $\theta$ and $\phi$ are defined, for example, in Fig. 1 of Ref. [17]. These angular dependencies can all be generated by perturbative QCD corrections, where for instance initial quarks radiate off high energy gluons into the final state. Such a perturbative QCD calculation at next-to-leading order leads to $\lambda \approx 1, \mu \approx 0, \nu \approx 0$ at very small transverse momentum of the lepton pair. More generally, the Lam-Tung relation $1 - \lambda - 2\nu = 0$ [31] is expected to hold at order $\alpha_s$ and the relation is hardly modified by next-to-leading order ($\alpha_s^2$) perturbative QCD corrections [32]. However, this relation is not satisfied by the experimental data [29, 30]. The Drell-Yan data shows remarkably large values of $\nu$, reaching values of about 30% at transverse momenta of the lepton pair between 2 and 3 GeV (for $Q^2 = m^2_{\gamma^*} = (4-12\,\mathrm{GeV})^2$ and extracted in the Collins-Soper frame [33] to be discussed below). These large values of $\nu$ are not compatible with $\lambda \approx 1$ as also seen in the data.

The asymmetry given by $\nu$ in Eq. (8) is proportional to the product of chiral-odd distributions $h_1^\perp(x_1, \boldsymbol{p}_\perp^2) \times \overline{h}_1^\perp(x_2, \boldsymbol{k}_\perp^2)$ [20]. The parameter $\nu$ was estimated in Refs. [17, 20, 21], and here we present in Fig. 6 the result of Ref. [21], which was obtained with the Gaussian transverse momentum dependence.

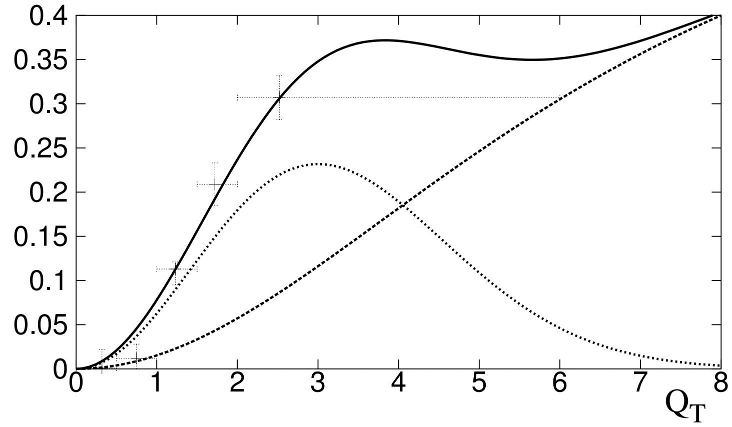


Figure 6: Possible contributions to $\nu$ as function of $Q_T$ compared to DY data of NA10 (for $Q =$ 8 GeV), which was presented in Ref. [21]. Thick dotted curve: contribution from perturbative one-gluon radiation. Thin dotted curve: contribution from a nonzero $h_1^\perp$. Solid curve: their sum.

# 5 Conclusion

Single-spin asymmetries in hadron reactions have been mysterious since the discovery of large transverse polarization of $\Lambda$ hyperons. The SSA in semi-inclusive DIS is understood by the final-state interactions from gluon exchange between the outgoing quark and the target spectator system. The asymmetry of the angular distribution in the Drell-Yan process given by the $\cos 2\phi$ distribution is investigated by the SSA of the quark spin which is induced by the initial-state interactions. This approach could explain the asymmetry of the angular distribution measured by the NA10 collaboration. It would be interesting to study this asymmetry in the Drell-Yan process further at hadron collider experiments like RHIC and LHC.

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# References


[1] A. Lesnik *et al.*, Phys. Rev. Lett. **35** 770 (1975).

[2] G. Bunce *et al.*, Phys. Rev. Lett. **36** 1113 (1976).

[3] For a review of experimental data, see, *e.g.*, K. Heller, in the Proceedings of Spin'96, edited by C.W. de Jager *et al.* (World Scientific, 1997); or A. D. Panagiotou, Int. J. Mod. Phys. **A5** 1197 (1990).

[4] For a review of theoretical models, see, *e.g.*, J. Felix, Mod. Phys. Lett. **A14** 827 (1999).

[5] D. Boer, C.J. Bomhof, D.S. Hwang, and P.J. Mulders, Phys. Lett. **B659** 127 (2008).

[6] HERMES Collaboration, A. Airapetian et al., Phys. Rev. Lett. **84** 4047 (2000).

[7] A. Airapetian *et al.* [HERMES Collaboration], Phys. Rev. Lett. **94** 012002 (2005).

[8] FNAL-E704 Collaboration, D.L. Adams et al., Phys. Lett. **B264** 462 (1991).

[9] STAR Collaboration, J. Adams et al., Phys. Rev. Lett. **92** 171801 (2004).

[10] STAR Collaboration, B.I. Abelev et al., Phys. Rev. Lett. **101** 222001 (2004).

[11] S.J. Brodsky, D.S. Hwang, and I. Schmidt, Phys. Lett. **B530** 99 (2002).

[12] D. Sivers, Phys. Rev. **D41** 83 (1990); Phys. Rev. **D43** 261 (1991).

[13] J.C. Collins, Phys. Lett. **B536** 43 (2002).

[14] S.J. Brodsky, D.S. Hwang and I. Schmidt, Nucl. Phys. **B642** 344 (2002).

[15] X. Ji and F. Yuan, Phys. Lett. **B543** 66 (2002).

[16] A. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656** 165 (2003).

[17] D. Boer, S.J. Brodsky, and D.S. Hwang, Phys. Rev. **D67** 054003 (2003).

[18] D. Boer, P.J. Mulders, and F. Pijlman, Nucl. Phys. **B667** 201 (2003).

[19] D. Boer and P. J. Mulders, Phys. Rev. **D57** 5780 (1998).

[20] D. Boer, Phys. Rev. **D60** 014012 (1999).

[21] D. Boer, in the Proceedings of "Transversity 2005", Como, Italy, 7-10 Sep. 2005; hep-ph/0511025.

[22] G.P. Lepage and S.J. Brodsky, Phys. Rev. **D22** 2157 (1980).

[23] S.J. Brodsky, H.C. Pauli, and S.S. Pinsky, Phys. Rep. **301** 299 (1998).

[24] S.J. Brodsky and S.D. Drell, Phys. Rev. **D22** 2236 (1980).

[25] S.J. Brodsky and D.S. Hwang, Nucl. Phys. **B543** 239 (1999).

[26] S.J. Brodsky, D.S. Hwang, B.-Q. Ma, and I. Schmidt, Nucl. Phys. **B593** 311 (2001).

[27] D.S. Hwang, in the Proceedings of "Light Cone 2008", Mulhouse, France, 7-11 July 2008; PoS LC2008: 033, 2008.

[28] M. Burkardt and D.S. Hwang, Phys. Rev. **D69** 074032 (2004).

[29] NA10 Collaboration, S. Falciano *et al.*, Z. Phys. **C31** 513 (1986);
NA10 Collaboration, M. Guanziroli *et al.*, Z. Phys. **C37** 545 (1988).

[30] J.S. Conway *et al.*, Phys. Rev. **D39** 92 (1989).

[31] C.S. Lam and W.K. Tung, Phys. Rev. **D21** 2712 (1980).

[32] A. Brandenburg, O. Nachtmann and E. Mirkes, Z. Phys. **C60** 697 (1993).

[33] J.C. Collins and D.E. Soper, Phys. Rev. **D16** 2219 (1977).