Reflective elastic scattering at LHC

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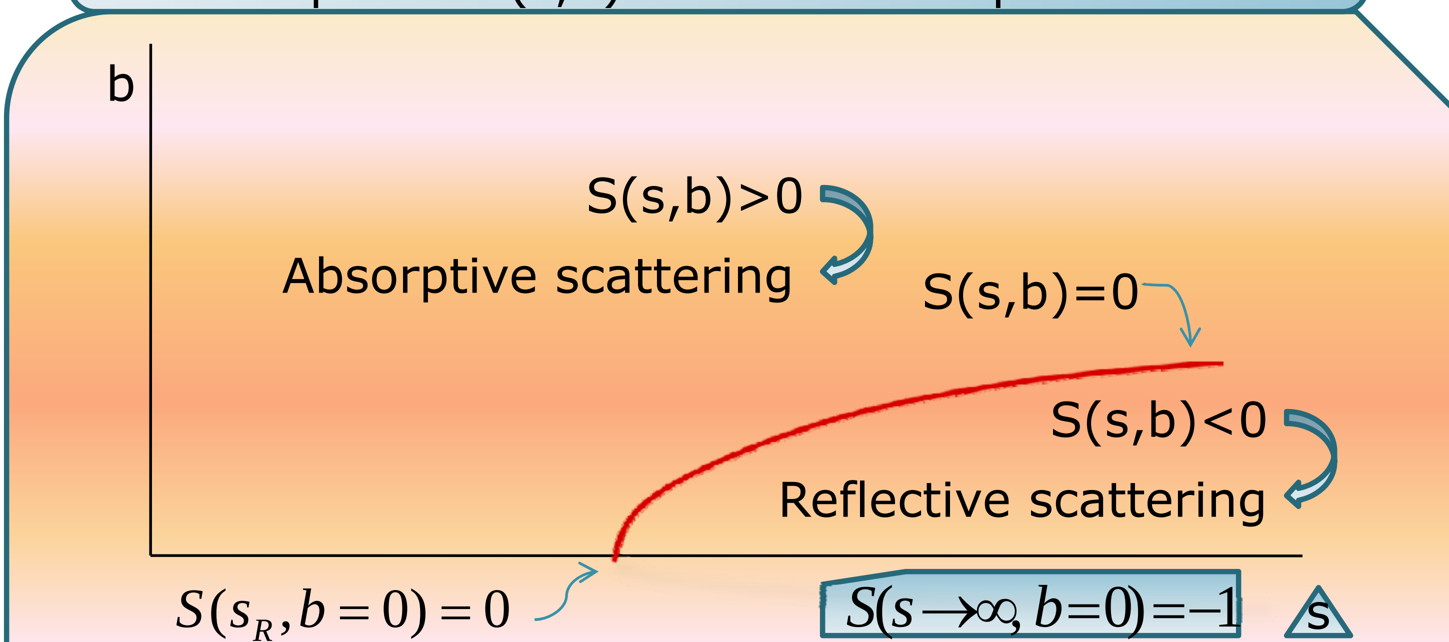
Elastic scattering, amplitude and S-matrix of this process:

$$S_l(s) = 1 + 2if_l(s) \Longrightarrow S(s,b) = 1 + 2if(s,b)$$

Impact parameter representation (unitarity):

$$\mathrm{Im} f(s,b) = |f(s,b)|^2 + \eta(s,b)$$

Structure of S(s,b) for the pure imaginary amplitude f(s,b) in the s and b plane:



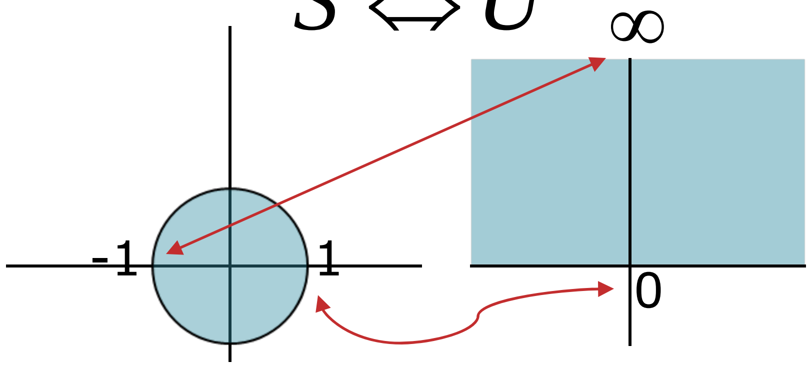
S-matrix representation

$$S(s,b) = \frac{1+iU(s,b)}{1-iU(s,b)}$$

One-to-one transform

$$\eta(s,b) = \frac{\mathrm{Im}U(s,b)}{|1-iU(s,b)|^2}$$

$$S \Leftrightarrow U$$



$$\lim_{s\to\infty} S(s,b)|_{b=0} = -1$$

Various models for U(s,b) (Regge-type, geometrical, quark…): exponential decrease with impact parameter and power-like increase with energy.

Light reflection off a dense medium; the phase of reflected light is changed by 180^{o}. Appearance of a reflecting ability of scatterer due to increase of its density beyond some critical value.

Peripheral inelastic overlap function at large s and small impact parameters

$$\eta(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{inel}}{db^2}$$

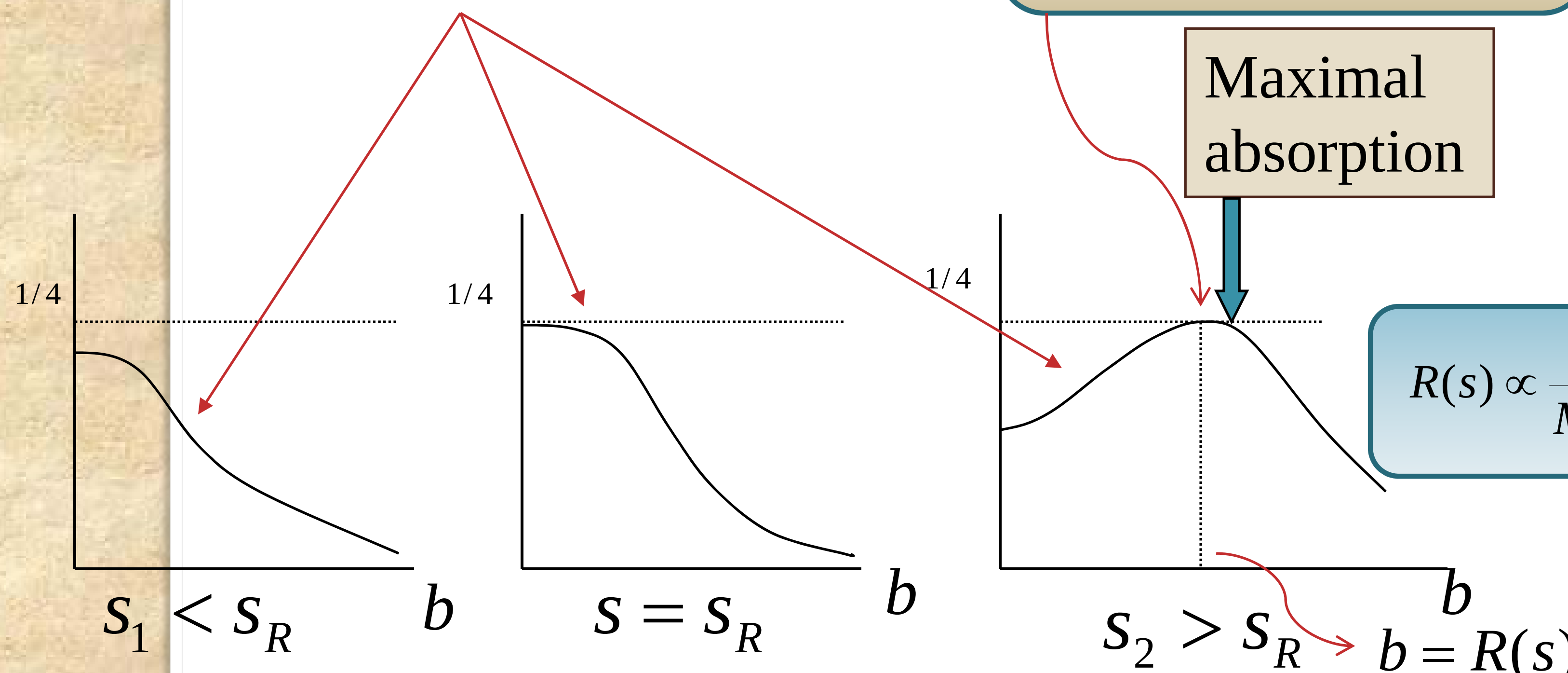
$$U(s, b = R(s)) = 1$$

$$\Downarrow$$

$$\eta(s, b = R(s)) = 1/4$$

Maximal absorption

$$R(s) \propto \frac{1}{M}\ln s$$

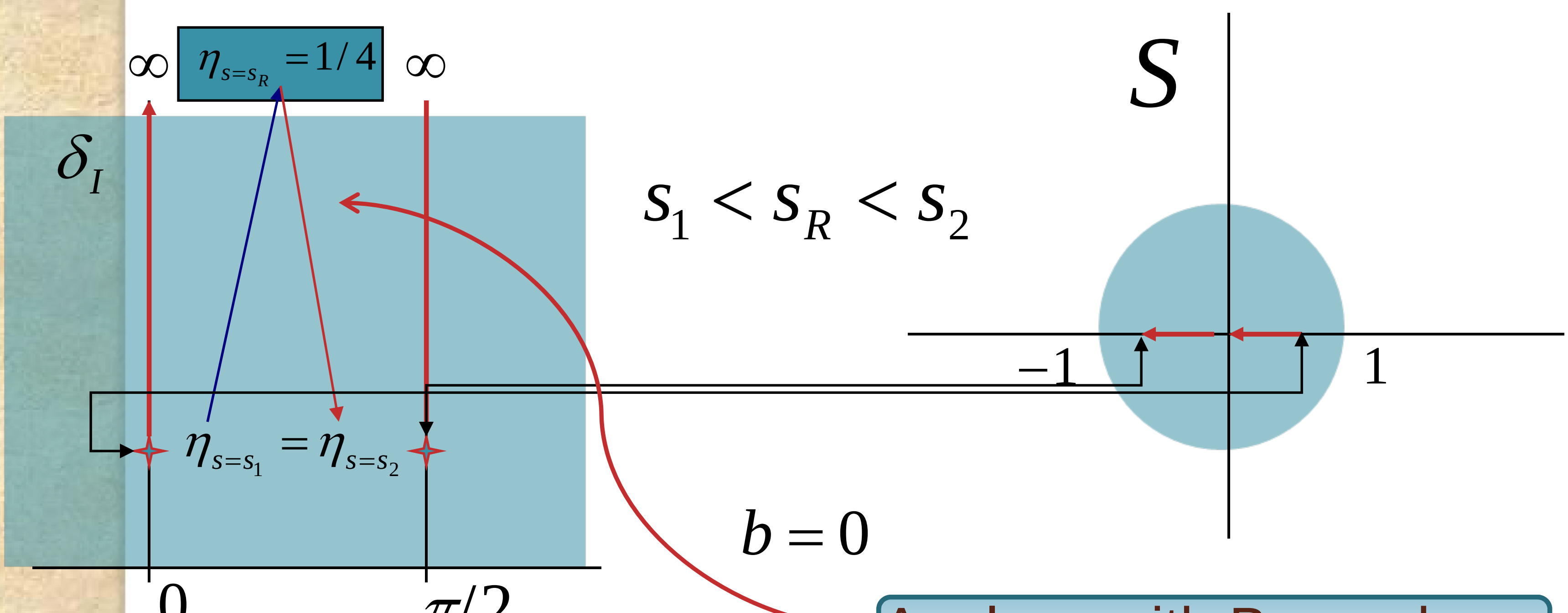


Energy evolution

$$\sigma_{el}(s) \propto \sigma_{tot}(s) \propto \ln^2 s$$

$$\sigma_{inel}(s) \propto \ln s$$

Inelastic scattering gives subleading contribution to the total cross-section at asymptotical energies



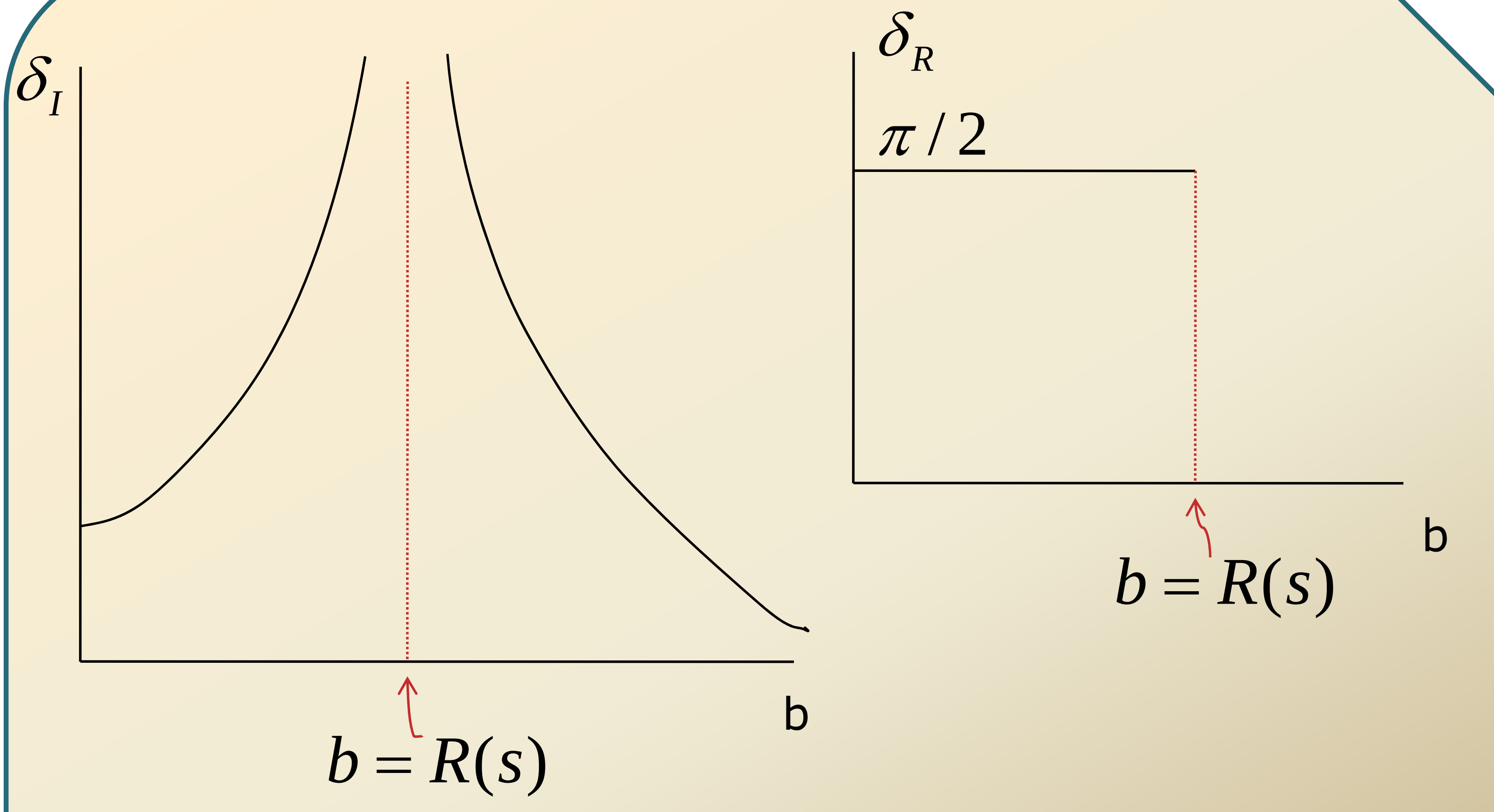
Analogy with Berry phase

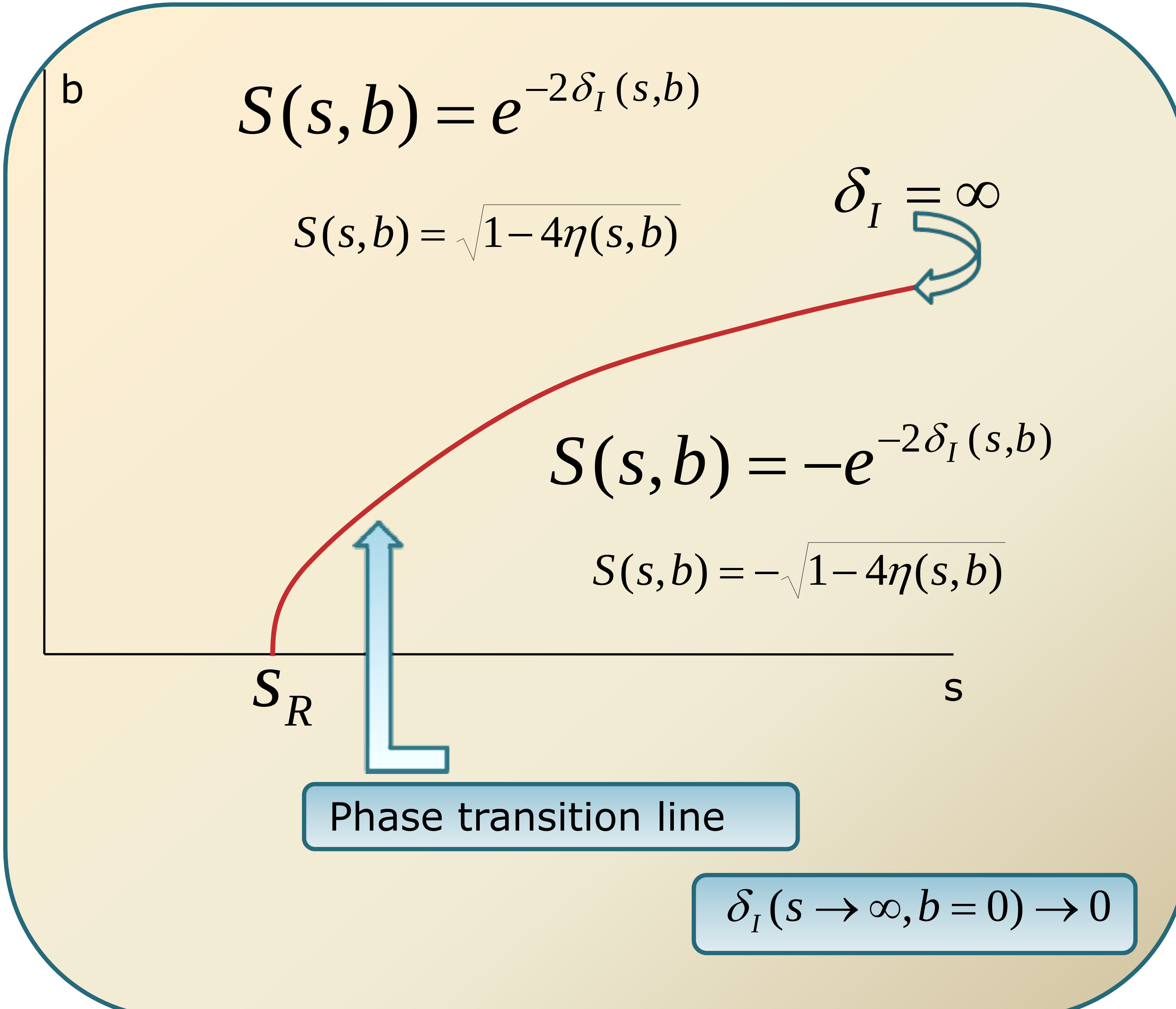
Phases (critical behavior) $S(s,b) = e^{2i\delta(s,b)}$

$$\delta(s,b) = \frac{1}{2i} \ln \frac{1-U(s,b)}{1+U(s,b)}$$

$$\delta(s,b) = \delta_R(s,b) + i\delta_I(s,b)$$

$$\delta_R(s,b) = \frac{\pi}{2}\theta(R(s)-b)$$





b
$S(s,b) = e^{-2\delta_I(s,b)}$
$S(s,b) = \sqrt{1-4\eta(s,b)}$
$\delta_I = \infty$
$S(s,b) = -e^{-2\delta_I(s,b)}$
$S(s,b) = -\sqrt{1-4\eta(s,b)}$
s_R
s
Phase transition line
$\delta_I(s \to \infty, b = 0) \to 0$

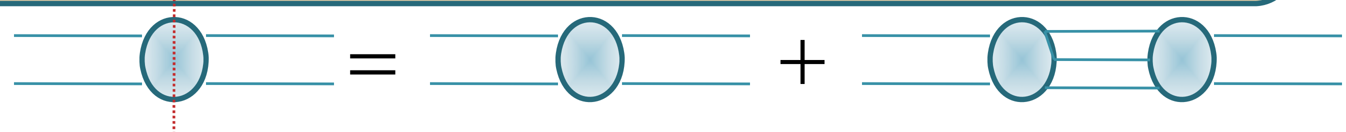
Physical scattering picture (beyond the black disk) evolves with energy by simultaneous increase of the reflective ability and decrease of the absorptive ability at the impact parameters $b < R(s)$

$$1 - |S(s,b)|^2$$

$$|S(s,b)|^2 \qquad \delta_R(s,b) = \pi/2$$

The generic geometric picture at fixed energy (beyond the black disk limit) can be described as a scattering off the partially reflective and partially absorptive disk surrounded by the black ring which becomes grey at larger values of the impact parameter.

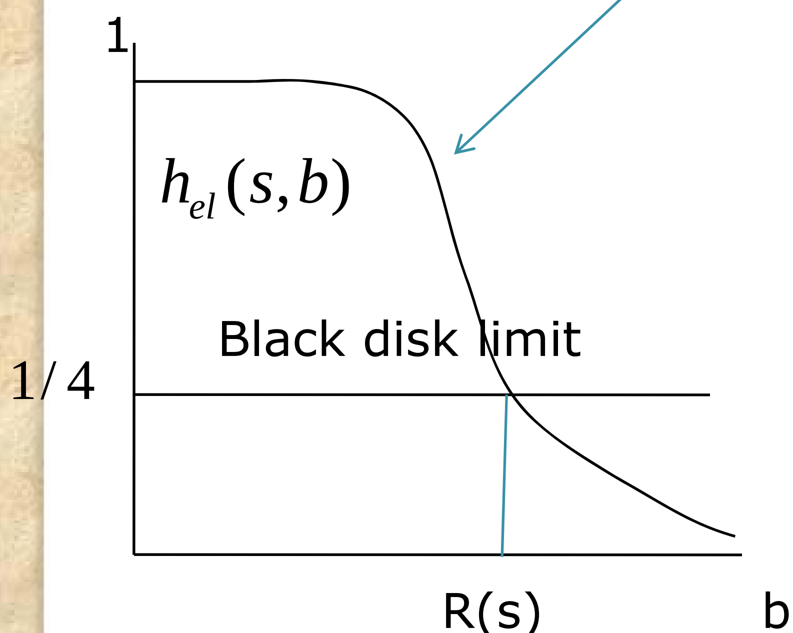
Reflective scattering and diffractive pattern in differential cross-section

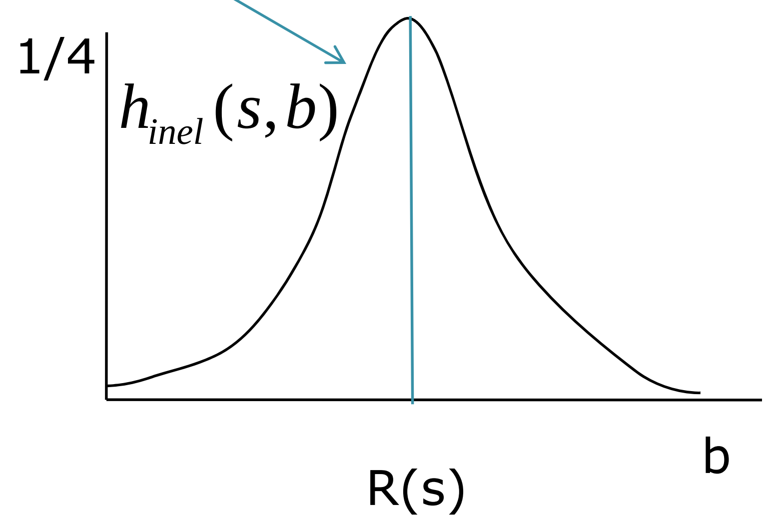


$$\operatorname{Im} F(s,t) = H_{el}(s,t) + H_{inel}(s,t)$$

$$H_{el,inel}(s,t) = \frac{s}{\pi^2}\int_0^\infty b\,db\,h_{el,inel}(s,b) J_0(b\sqrt{-t})$$

$$\sigma_{el,inel}(s) \propto \frac{1}{s} H_{el,inel}(s,t=0)$$





Forward elastic scattering

$$H_{el}(s,t) \propto \frac{RJ_1(R\sqrt{-t})}{\sqrt{-t}} \qquad H_{inel}(s,t) \propto RJ_0(R\sqrt{-t})$$

$$-t = 0: \Longrightarrow \quad \sigma_{el}(s) \propto R^2(s) \qquad \sigma_{inel}(s) \propto R\ \ (s)$$

$$\langle b^2 \rangle_{tot}(s) = \frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \langle b^2 \rangle_{el}(s) + \frac{\sigma_{inel}(s)}{\sigma_{tot}(s)} \langle b^2 \rangle_{inel}(s)$$

$$\langle b^2 \rangle_{el,inel}(s) \propto R^2(s)$$

Slope of diffraction cone:

$$B(s) = \frac{d}{dt}(\ln \frac{d\sigma}{dt})|_{t=0}$$

$$B(s) \propto \left\langle b^2 \right\rangle_{tot}$$

$$B(s) = B_{el}(s) + B_{inel}(s)$$

$$B_{el}(s) \propto R^2(s) \quad B_{inel}(s) \propto R(s)$$

Subleading contribution of inelastic channels

Non-forward elastic scattering

$$-t \neq 0: \Longrightarrow \quad H_{el} \propto R^{1/2}(s) \quad H_{inel} \propto R^{1/2}(s)$$

Dips and bumps in $\dfrac{d\sigma}{dt}$

Enhancement of large –t region by factor $\sqrt{-t}$

Diffraction picture with dips and bumps in the differential cross-section will be kept in the case when the reflective scattering dominates.

Reflective scattering leads to power-like dependence

$$\frac{d\sigma}{dt} \propto \left(\frac{1}{s}\right)^N$$

when –t/s-fixed and s→∞ (fixed angles).

Absence of diffraction cone (and Orear type behavior) in the backward scattering, e.g. for πN -scattering, power-like dependence on –u in the backward hemisphere.

Phenomenology (based on chiral quark model for U-matrix)

Total, elastic and inelastic cross-sections;

Differential cross-section;

Knee in cosmic rays energy spectrum and other phenomena in this field;

Estimate for the gap survival probability is 0.2.

Large values of $\sigma_{tot}(s) \cong 230$ mb

and ratio $\sigma_{el}(s)/\sigma_{tot}(s) \cong 2/3$

at the LHC energies (14 TeV), while $\sigma_{inel}(s) \cong 77$ mb.

Standard background estimations remain valid.

Note the strong model dependence of numerical estimates.

Consequences for the LHC heavy-ion studies

At very high energies a certain part of central hadron collisions would look like collision of hard spheres. It will affect initial condition in nuclear reactions (with total number N of hadrons in the initial state) reducing the volume V of the available space which the hadrons are able to occupy in the initial stage of nuclear collision:

$$V \Rightarrow V - p_R(s) V_R(s) \frac{N}{2}$$

3D volume of reflective scattering

Averaged over volume V_R probability of reflective scattering

$$p_R(s) = \frac{1}{V_R(s)} \int_{V_R(s)} |S(s,r)|^2 d^3x$$

$$V_R(s) \cong (4\pi/3) R^3(s)$$

$$n_R(T,\mu) = \frac{n(T,\mu)}{1+\alpha(s)n(T,\mu)}$$

$$\alpha(s) = p_R(s)V_R(s)/2$$

$$p_R(s) \approx |S(s,b=0)|^2$$

$$n_R(T,\mu) \propto 1/\alpha(s) \propto M^3/\ln^3 s$$

Limiting dependence for the hadron density appears due to the presence of reflective scattering. It corresponds to saturation of unitarity (and the Froissart-Martin bound for the total cross-section).

Unitarity in models for $F(s,t)$?

$$\operatorname{Im} f(s,b) \geq |f(s,b)|^2$$

$F(s,t) \Longrightarrow f(s,b)$

?

Additional check is needed

Inconsistency of maximal odderon with unitarity saturation

Concluding remarks

Reflective scattering is just a hypothesis as well as unitarity saturation.

Appearance of the reflective scattering at very high energies is not promptly dictated by any specific dynamical model, it is a result of S-matrix unitarity saturation related to the total cross section growth.

The unitarity saturation can be related in its turn to the principle of the maximal strength of strong interactions proposed by Chew and Frautchi.

Presence or absence of reflective scattering can be checked at the LHC.