Theoretical aspects of high energy elastic nucleon scattering

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1. Introduction

Elastic collisions of charged nucleons at high energies:

- hadronic interactions at *all* t , Coulomb scattering at *small* $|t|$ →
- experiment with high statistics → precise data → accurate analysis

pp at $p_{lab} = 24 \div 2900$ GeV/c

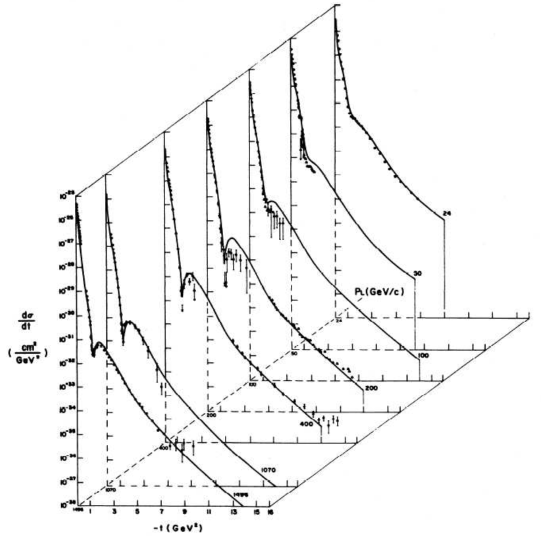
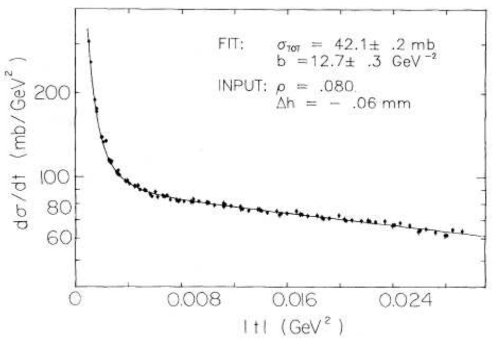


Fig. 3.1. Proton–proton elastic differential cross-sections as a function of momentum transfer and of incident laboratory momentum (from ref. [3.1]).



$$\frac{d\sigma}{dt} = \frac{\pi}{sp^2} |F^{C+N}(s,t)|^2$$

- influence of both interactions (spins neglected)

→ complete amplitude $F^{C+N}(s,t)$ (Bethe (1958))

$$F^{C+N}(s,t) = F^{C}(s,t)\, e^{\,i\,\alpha\,\Phi(s,t)} + F^{N}(s,t)$$

$F^{C}(s,t)$ … Coulomb (QED), $F^{N}(s,t)$ …hadronic (unknown)

$\alpha\Phi(s,t)$ … relative phase; $\alpha = 1/137.036$ … fine struct. const.

- unknown functions: $\Phi(s,t)$, $F^N(s,t)$

(i) relative phase $\alpha\Phi(s,t)$? (West - Yennie (1968), Locher (1967) … Feynman diagrams)

$$\alpha\Phi(s,t) = \mp\alpha\left[\ln\left(\frac{-t}{s}\right) - \int_{-4p^2}^{0} \frac{dt'}{|t-t'|}\left(1 - \frac{F^N(s,t')}{F^N(s,t)}\right)\right]$$

(ii) West–Yennie phase simplified: under two assumptions

- $|F^N(s,t)| = e^{Bt/2}$… $B = const$ … for all kinematically allowed t
- $\rho(s,t) = Re\, F^N(s,t) / Im\, F^N(s,t) = \rho(s)$ … for all kinematically allowed t
- high-energy approximations applied, form factors added by hand
  (V.K., M. Lokajíček, Phys. Lett. B 232 (1989) 263; ibid B 611 (2005) 102)

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha\Phi} + \frac{\sigma_{tot}}{4\pi} p\sqrt{s}(\rho + i)e^{Bt/2}$$

$$\alpha\Phi = \mp\alpha(\ln(-Bt/2) + \gamma) \qquad \gamma = 0.577215$$

- σ_{tot}, ρ, B … constant; dominant $Im\, F^N(s,t)$ at all t

- used: $|t| \leq .01\ GeV^2$ (assumptions not fulfilled by data)
- higher $|t|$ … Coulomb scattering neglected → only hadronic amplitude used → *two divers formulas used for $d\sigma/dt$* → deficiency!
- extending validity of simplified WY phase: t dependence of $B(s,t)$, $\rho(s,t)$ … not sufficient justification!

  hadronic amplitude:

$$F^N(s,t) = i|F^N(s,t)|e^{-i\zeta^N(s,t)}$$

however: *WY phase should be real !* … *for any t !*

$$\Im\left[\alpha\Psi(s,t)\right] \equiv 0 \quad \rightarrow \quad \boxed{\alpha I(t) = \alpha \int_{-4p^2}^{0} dt' \frac{\sin[\zeta^N(s,t) - \zeta^N(s,t')]}{|t'-t|} |F^N(s,t')| \equiv 0}$$

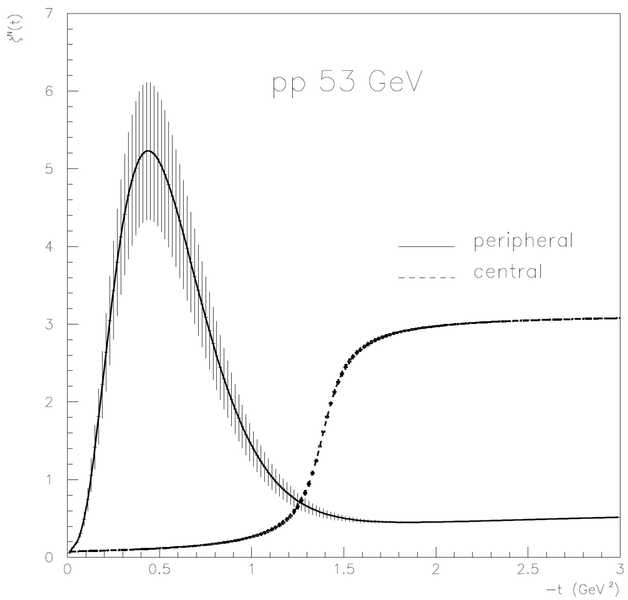
(nonlinear singular Cauchy type integral equation of the first kind of non-continuous function with parameter – mathematically unsolved problem yet)

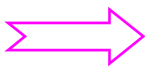
- solution: $\zeta^N(s,t) \equiv \zeta^N(s,t')$

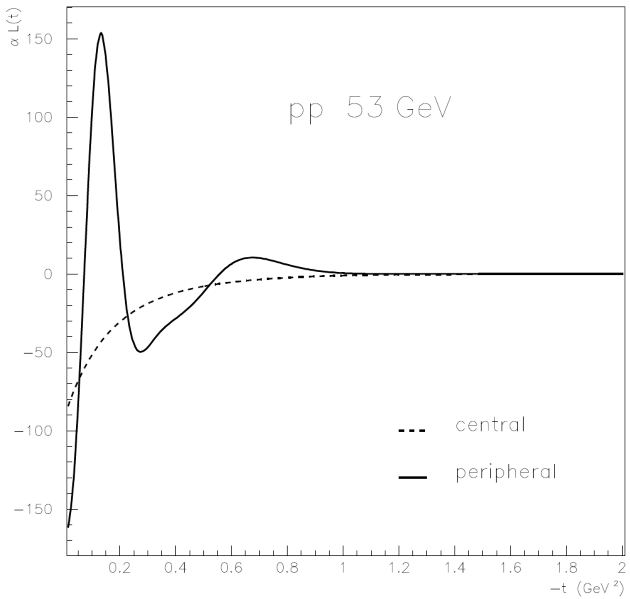
uniqueness of solution: (V.K., M. Lokajíček, I. Vrkoč, Phys. Lett. B656 (2007) 182)

$$\rho(s,t) \equiv \frac{\Re F^N(s,t)}{\Im F^N(s,t)} \equiv \rho(s) \equiv const$$ provided $|\xi^N(s,t)| < \pi$

- central (standard model): $|\xi^N(s,t)| < \pi$ ..... (analytical proof)
- peripheral $|\xi^N(s,t)| < 2\pi$ ..... (numerical 'proof')







$\alpha\Psi(s,t)$ is complex → WY formula should be abandoned !

2. General eikonal model approach

- mathematically rigorous use of Fourier-Bessel transformation →

eikonal representation of scatt. amplitude valid at any s and t

(Adachi et al. (1965-1976), Islam (1968), (1976))

$$F(s, q^2 = -t) = \frac{s}{4\pi i} \int_{\Omega_b} d^2 b e^{i\vec{q}\vec{b}} \left[e^{2i\delta(s,b)} - 1 \right]$$

additivity of potentials → additivity of eikonals (Franco (1973), Cahn (1982))

$$\delta^{C+N}(s,b) = \delta^{C}(s,b) + \delta^{N}(s,b)$$

$\delta^C(s,b)$ … Coulomb eikonal

$\delta^N(s,b)$ … hadronic eikonal

$\delta^{C+N}(s,b)$ … total eikonal

- complete elastic amplitude (not calculating $\alpha\Phi(s,t)$)

$$F^{C+N}(s, t = -q^2) = \frac{s}{4\pi i} \int_{\Omega_b} d^2 b e^{i\vec{q}\vec{b}} \left[e^{2i(\delta^C(s,b) + \delta^N(s,b))} - 1 \right] \rightarrow$$

$$F^{C+N}(s,t) = F^C(s,t) + F^N(s,t) + \frac{s}{4\pi i} \int_{\Omega_b} d^2b\, e^{i\vec{q}\vec{b}} \left[e^{2i\delta^C(s,b)} - 1 \right] \left[e^{2i\delta^N(s,b)} - 1 \right]$$

$$F^{C+N}(s,t) = F^C(s,t) + F^N(s,t) + \frac{i}{\pi s} \int_{\Omega_{q'}} d^2q' F^C(s,q'^2) F^N(s,[\vec{q} - \vec{q}']^2)$$

⇑

convolution integral

• describes simultaneous actions of both Coulomb and hadronic interactions; new complex function (convolution integral) is added

– at difference to WY amplitude; equation valid at any *s* and *t*

(Cahn (1982) for small $|t|$; used in: V.K., M. Lokajíček, D. Krupa: Phys. Rev. D35 (1987) 1719; D41 (1990) 1687;D46 (1992) 4087)

$$F^{C+N}_{CAHN}(s,t) = \pm \frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t) \left\{ 1 \mp i\alpha \int_{t_{min}}^{0} dt' \ln \frac{t'}{t} \left[f_1(t') f_2(t') \frac{F^N(s,t')}{F^N(s,0)} \right] \right.$$

- complete scattering amplitude valid up to terms linear in α for any *t* (V. K., M. Lokajíček, Z. Phys. C63 (1994) 619)

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t)\left[1 \mp i\alpha G(s,t)\right]$$

$$G(s,t) = \int_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}\left[f_1(t') f_2(t')\right] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\}$$

$$I(t,t') = \int_0^{2\pi} d\Phi'' \frac{f_1(t'') f_2(t'')}{t''} \qquad t_{min} = -s + 4m^2$$

$$t'' = t + t' + 2\sqrt{tt'}\cos\Phi''$$

$$\sigma_{tot} = \frac{4\pi}{p\sqrt{s}}|F^N(s,0)| \quad \rho(s,t) \equiv \frac{\Re F^N(s,t)}{\Im F^N(s,t)}$$

$$B(s,t) = \frac{d}{dt}\left[\ln\frac{d\sigma^N}{dt}\right] = \frac{2}{|F^N(s,t)|}\frac{d}{dt}|F^N(s,t)|$$

B(s,t) & *ρ(s,t)* … model dependent quantities (separation of Coulomb and hadronic scattering)

- analytically calculated *I(t,t')* (for form factors of Borkowski et al (1975))

3. Elastic hadronic amplitude $F^N(s,t)$

- no reliable theory → phenomenological models

(i) dominance of imaginary part – standard approach

- standard approach: dominant $Im\, F^N(s,t)$ in broad interval of $|t|$ → phase increases with increasing $|t|$ and $Im\, F^N(s,t_{diff})=0$; $Re\, F^N(s,t)$ relatively increases with $|t|$ to obtain non-zero $d\sigma/dt$ at t_{diff} ;
- diffractive minimum → minimum of sum of the squares of both the real and imaginary parts → no argument for $ImF^N(s,t_{diff})=0$!!!
- dominance only at small $|t|$ (V.K., M. Lokajíček: Mod. Phys. Lett. 11 (1996) 2241)
- other characteristics?

(ii) mean-squares of impact parameter in nucleon collisions

- total, elastic and inelastic mean-squares of impact parameters characterize the range of forces responsible for individual interactions

$$\langle b^2(s)\rangle_h = \int_0^\infty bdb\, b^2\, h(s,b) \Big/ \int_0^\infty bdb\, h(s,b)$$

- elastic (Henyey, Pumplin (1976), V.K., M. Lokajíček Jn., M. Lokajíček Sn., Czech. J. Phys. B 31 (1981) 1334; V.K., M. Lokajíček, D. Krupa, Phys. Lett. B 544 (2002) 132))

$$< b^2(s) >_{el} = 4 \frac{\int_{t_{min}}^{0} dt\ |t|\ (\frac{d}{dt}|F^N(s,t)|)^2}{\int_{t_{min}}^{0} dt\ |F^N(s,t)|^2} + 4 \frac{\int_{t_{min}}^{0} dt\ |t|\ |F^N(s,t)|^2 (\frac{d}{dt}\zeta^N(s,t))^2}{\int_{t_{min}}^{0} dt\ |F^N(s,t)|^2}$$

- total mean-square

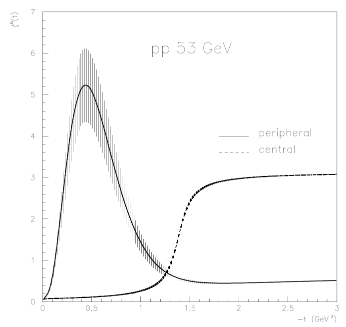
$$\langle b^2(s)\rangle_{tot} = 2B(s,0)$$

- inelastic mean square

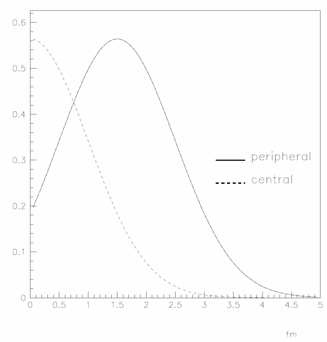
$$\langle b^2(s)\rangle_{inel} = \frac{\sigma_{tot}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{tot} - \frac{\sigma_{el}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{el}$$

if: $\zeta^{N\prime}(s,t) >> 0$ at small $|t|$ $\rightarrow$ peripheral (larger elast. RMS)

$\zeta^N(s,t) \sim const$ at small $|t|$ $\rightarrow$ central (small elast. RMS)



FB $\Rightarrow$



- central: large transparency of proton in 'head-on collisions' (Miettinen (1974) … 'puzzle' (Giacomelli, Jacob (1979)); artefact; can be removed if elast. hadronic scattering is peripheral (V.K., M. Lokajíček, M. Lokajíček Jr. (1981))

4. Profiles for pp scattering at 53 GeV

(Adachi, Kotani (1965-1976) … 16 papers, Islam (1968, 1976))

$$h_{el}(s,b) = h_1(s,b) + h_2(s,b)$$
$$= \frac{1}{4p\sqrt{s}} \int_{t_{min}}^{0} dt\ F^N(s,t)\ J_0(b\sqrt{-t}) + \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{min}} dt\ \lambda(s,t)\ J_0(b\sqrt{-t})$$

- required by consistent definition of FB transformation; also for inelastic overlap function → oscillations at higher *b* values
- unitarity condition

$$\Im h_1(s,b) = |h_1(s,b)|^2 + g_1(s,b) + K(s,b) \qquad (*)$$

- oscillations can be removed by adding real function *c(s,b)* to (*)

(V.K., M. Lokajíček, D. Krupa, Phys. Lett. B 544 (2002) 132)

$$h_{tot}(s,b) = \Im h_1(s,b) + c(s,b), \quad g_{inel}(s,b) = g_1(s,b) + K(s,b) + c(s,b)$$

$$h_{tot}(s,b) = |h_1(s,b)|^2 + g_{inel}(s,b)$$

result: ↑ central ↑ peripheral ↑ central

- why peripheral elastic profile?
  - DIS: large $|t| \leftrightarrow$ small distances; Diffraction: small $|t| \leftrightarrow$ large distances; scattering of light ions on He^4 ... Glauber formalism (Franco, Yin (1985)) , …

- function $c(s,b)$
  - all profiles non-negative; shape of elastic preserved (peripheral)
  - all dynamical characteristics (i.e., σ_{tot}, σ_{el} , σ_{inel} and all RMS) preserved (*)
  - total and inelastic profiles … Gaussian shapes (monotony decreasing) (**)
  - from (*): $\int_0^\infty bdb\, c(s,b) = 0$ (σ_{tot},…); $\int_0^\infty b^3 db\, c(s,b) = 0$ (RMS)
  - from (**): $h_{tot} = a e^{\mu b^2}$
  - again from (*):

$$\frac{\sigma_{tot}}{8\pi} = \int_0^\infty bdb\, a e^{\mu b^2} = \frac{a}{2\mu}$$

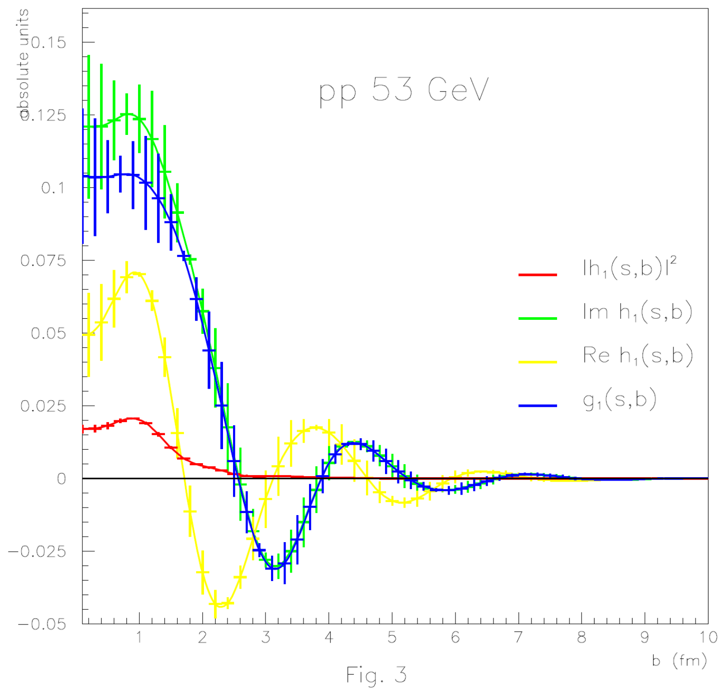
$$\frac{<b^2_{tot}>}{8\pi} = \int_0^\infty b^3\, db\, a e^{\mu b^2} = \frac{a}{2\mu^2}$$

$a = 0.3245; \quad \mu = 0.9465$

pp at 53 GeV

Quantity		Original values	New values
σ_{tot}	[mb]	42.864	42.872
σ_{el}	[mb]	7.479	7.479
$\sqrt{< b^2 >_{tot}}$	[fm]	1.0	1.028
$\sqrt{< b^2 >_{el}}$	[fm]	1.803	1.803
$\sqrt{< b^2 >_{inel}}$	[fm]	0.772	0.772
$\int_0^\infty bdb\, c(s,b)$	[fm^2]	-	0.029
$\int_0^\infty b^3 db\, c(s,b)$	[fm^4]	-	0.097

resulting profiles



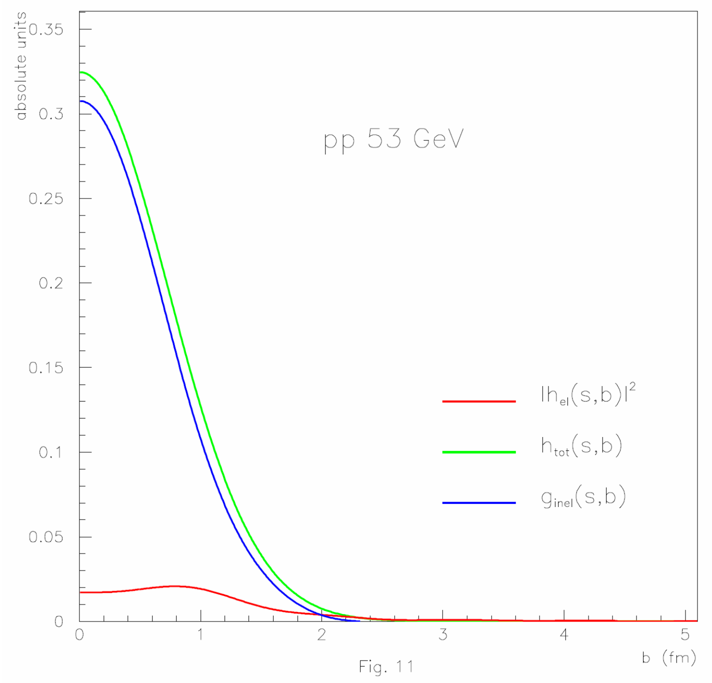


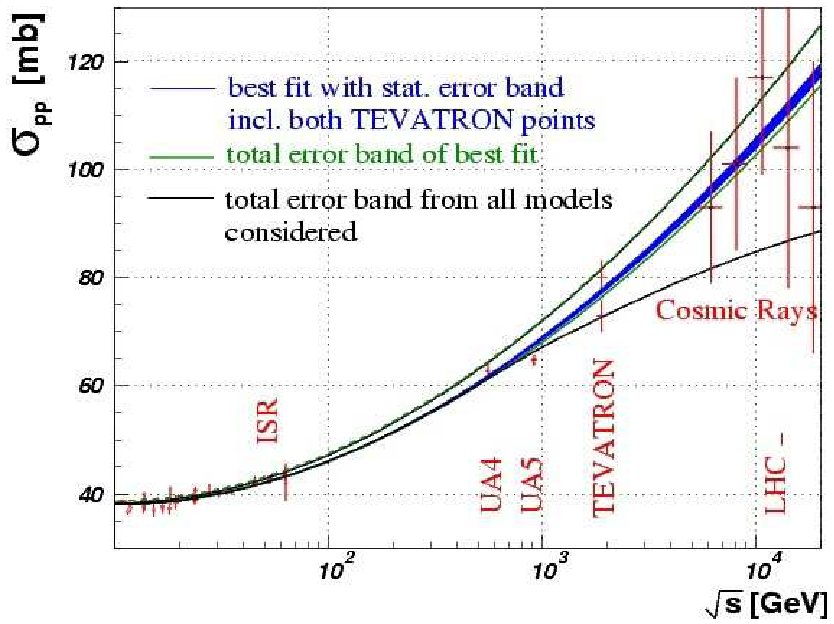
Fig. 3: original oscillating profiles (statistical errors)

Fig. 11: final shape of profiles; full lines:

red … peripheral elastic profile
green … central total profile
yellow … central inelastic profile

“original” values of total, elastic and inelastic rms and of cross sections conserved

5. Model predictions for pp elastic scattering at LHC



COMPETE Collab.: fits to all available data; predictions based on DR technique

PRL 89 201801 (2002)

$$\sigma_{tot} = 111.5 \pm 1.2 \begin{array}{l} +4.1 \\ -2.1 \end{array} \text{ mb}$$

TOTEM aim: ~ 1% error of σ_{tot}

model predictions: 90 ÷ 130 mb

σ_{tot} = *125 ± 25 mb* based on QCD

(Landshoff, arXiv:0709.0395 [hep-ph])

analyzed models:

- M.M. Islam, R.J. Luddy, A.V. Prokudin: Phys. Lett. B605 (2005) 115
- V.A. Petrov, E. Predazzi, A.V. Prokudin: Eur. Phys. J. C28 (2003) 525
- C. Bourrely, J. Soffer, T.T. Wu: Eur. Phys. J. C28 (2003) 97 … (BSW)
- M.M. Block, E.M. Gregores, F. Halzen, G. Pancheri: Phys. Rev. D60 (1999) 0504024

results of model analysis (eikonal total amplitude)… pp at 14 TeV

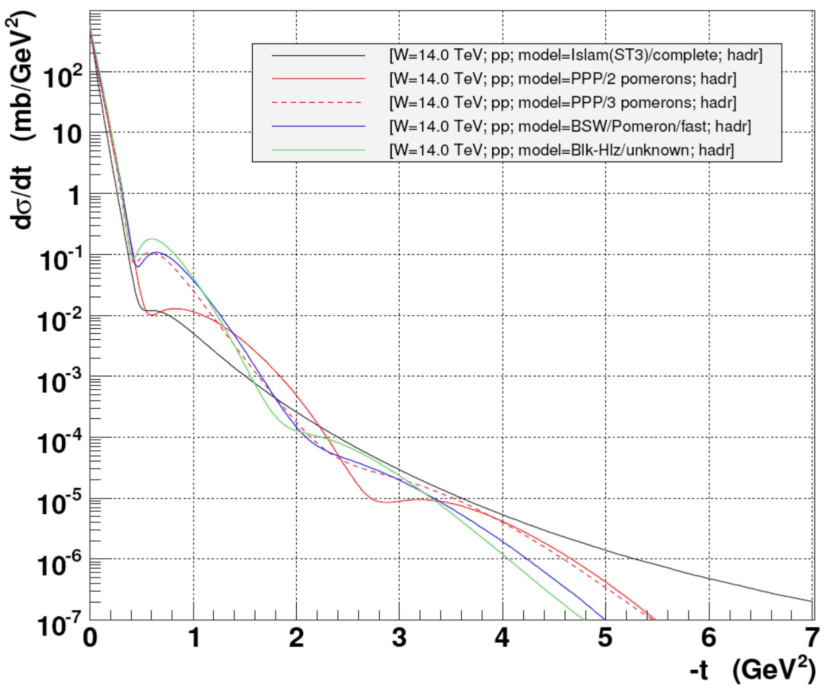
model	σ_{tot} [mb]	σ_{el} [mb]	$B(0)$ [GeV^{-2}]	ρ	$<b_{tot}^2>^{1/2}$ [fm]	$< b_{el}^2>^{1/2}$ [fm]	$<b_{inel}^2>^{1/2}$ [fm]
Islam	109.17	21.99	31.43	0.123	1.552	1.048	1.659
Petrov et al. 2P	94.97	23.94	19.34	0.0968	1.227	0.875	1.324
Petrov et al. 3P	108.22	29.70	20.53	0.111	1.263	0.901	1.375
BSW	103.64	28.51	20.19	0.121	1.249	0.876	1.399
Block Halzen	106.74	30.66	19.35	0.114	1.223	0.883	1.336
upper bounds						~ 2.39	~ 1.47

$$\langle b^2(s)\rangle_{el} \ \leq \ 2B(s,0)\frac{\sigma_{tot}(s)}{\sigma_{el}(s)} \qquad \langle b^2(s)\rangle_{inel} \ \leq \ 2B(s,0)\frac{\sigma_{tot}(s)}{\sigma_{inel}(s)}$$

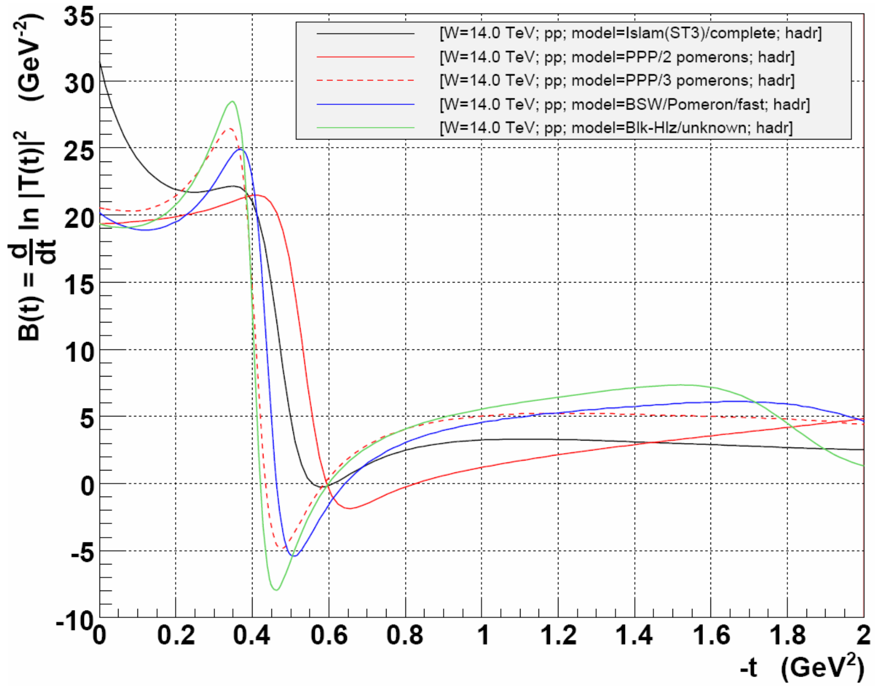
- elastic rms are lesser than inelastic ones → hadron elastic nucleon collisions are more central than the inelastic nucleon collisions → 'puzzle' or paradox ?

pp 14 TeV LHC

$\frac{d\sigma}{dt}$ at higher $|t|$ values

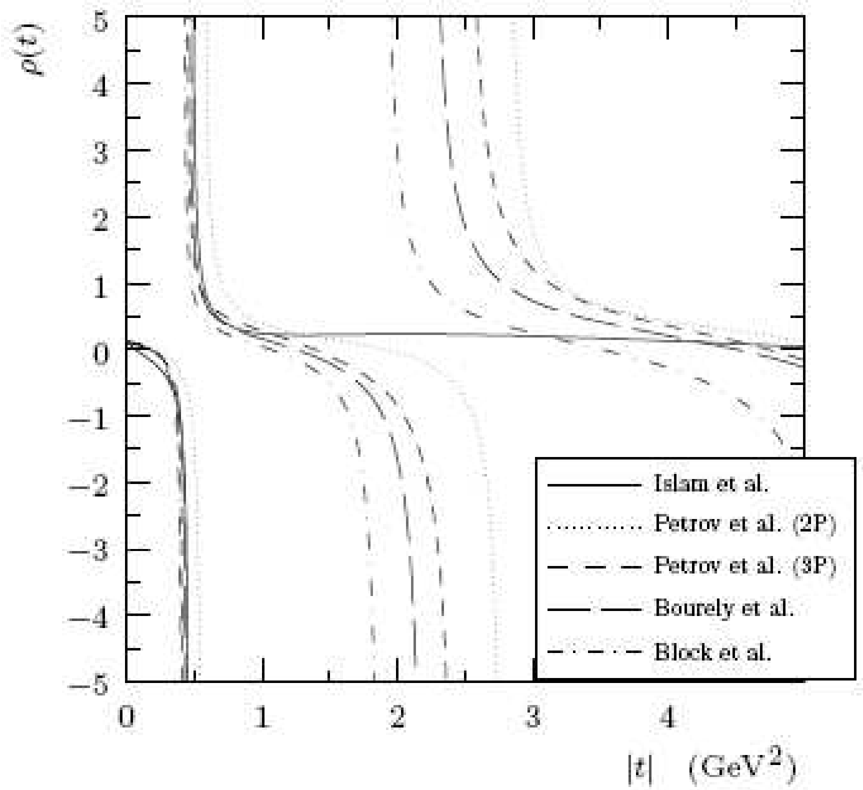


diffractive slope $B(t)$

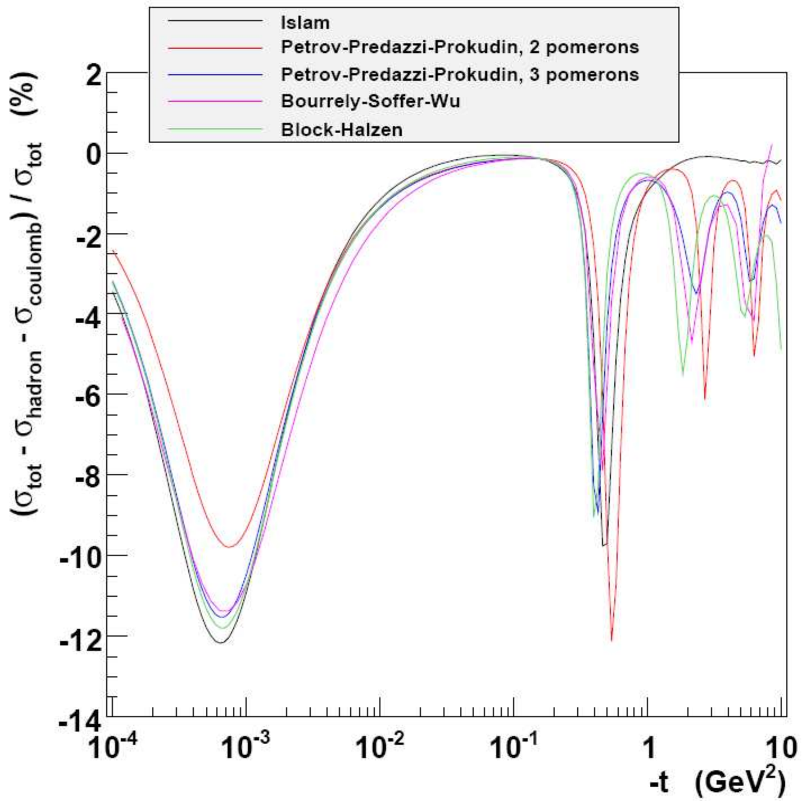


pp 14 TeV LHC

$\rho(t)$ model dependence



interference term

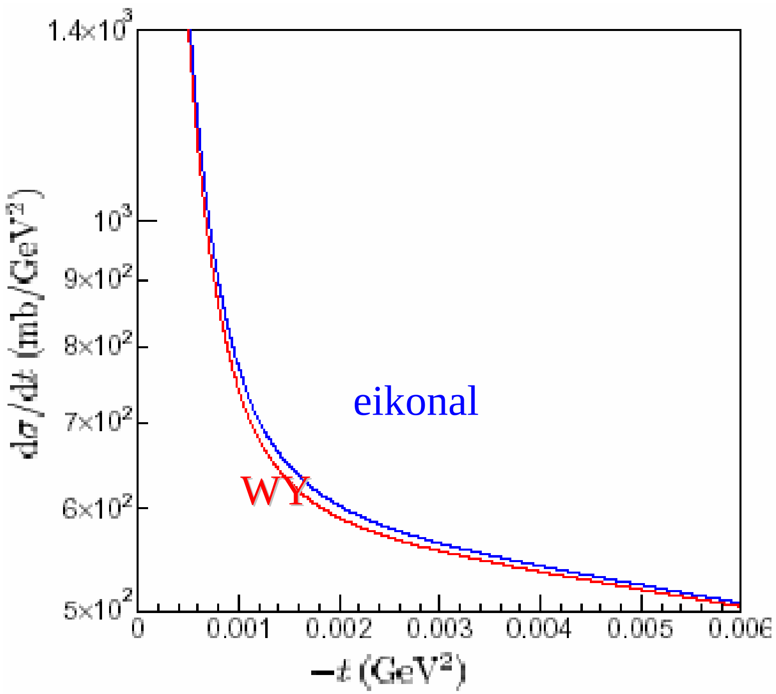


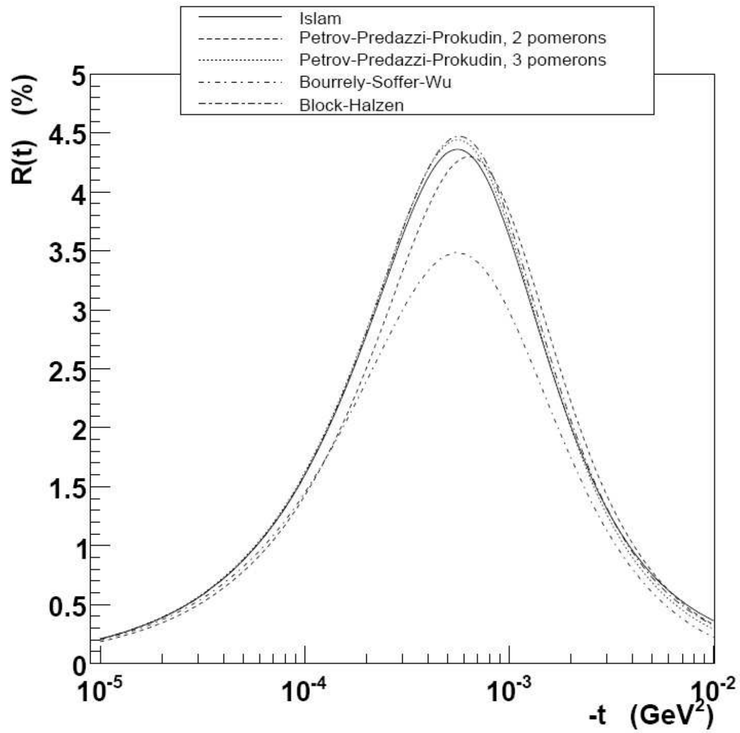
6. Luminosity estimation at 14 TeV

$$\frac{1}{\mathcal{L}}\frac{dN_{el}}{dt} = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2$$

$$R(t) = \frac{|F^{C+N}_{eik}(s,t)|^2 - |F^{C+N}_{WY}(s,t)|^2}{|F^{C+N}_{eik}(s,t)|^2} * 100.$$

- different complete amplitudes $\rightarrow$ different luminosity values





- luminosity determined with large systematic error of 4÷5 % systematic error if WY complete amplitude used

7. Conclusion

- earlier description of elastic nucleon scattering: West-Yennie approach not reliable; central interpretation of elastic collisions unrealistic
- our approach to elastic scattering: exact solutions at finite energies, current models offer central behavior;

  however: values of RMS determined from $F^N(s,t)$ prefer peripheral impact parameter profiles

  luminosity estimation: possible systematic error of $\approx$ 5 %
- TOTEM experiment: convenient tool for selection of description