

Analytic properties of DPE amplitudes  
or  
Collinear Factorization for Central  
Production



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# Main Topics

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- QCD factorization and its violation
- QCD factorization vs analyticity
- Analyticity and crossing for hard exclusive lepton hadron reactions
- DPE specifics and the role of Steinmann relations
- BABAR data: violation of factorization typical?
- Possible implications for Higgs production
- Conclusions



# QCD factorization

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- Hard (calculable Perturbatively ) vs Soft (Non-Perturbative but universal) parts
- Collinear – transverse Perturbative (DGLAP)+ longitudinal NP (PDF)
- Violations: (calculable) non-universality of soft; IR instability of hard
- Kt- longitudinal Perturbative (BFKL)+transverse NP(Unintegrated Gluon Distributions)



# Exclusive processes

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- Complicated – new P and NP ingredients
- Kt-Generalized UGD
- - difficulties in factorization proof
- Durham (standard) model
- Collinear – ERBL evolution + GPD's
- Possible implication of BABAR data – IR contribution even in the simplest case`

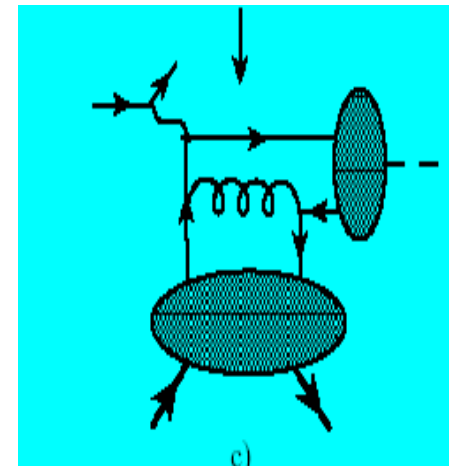
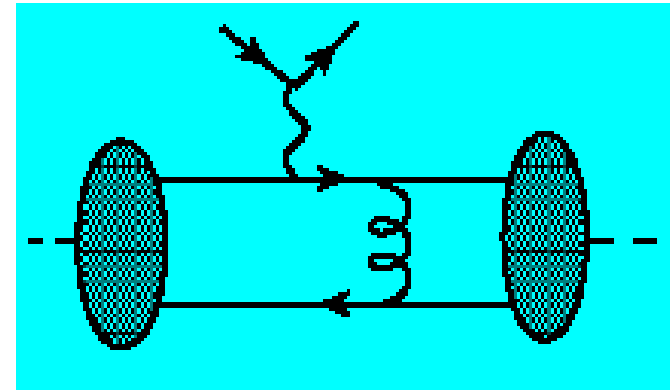
# Explore crossing (+modification of soft parts)

- Starting from  
(Pion) form  
factor- 2 DA's

$$F \propto \left( \int dx \frac{\phi(x)}{1-x} \right)^2$$

- 1 DA  $\rightarrow$  GPD :  
Exclusive meson  
production (F,S,...)

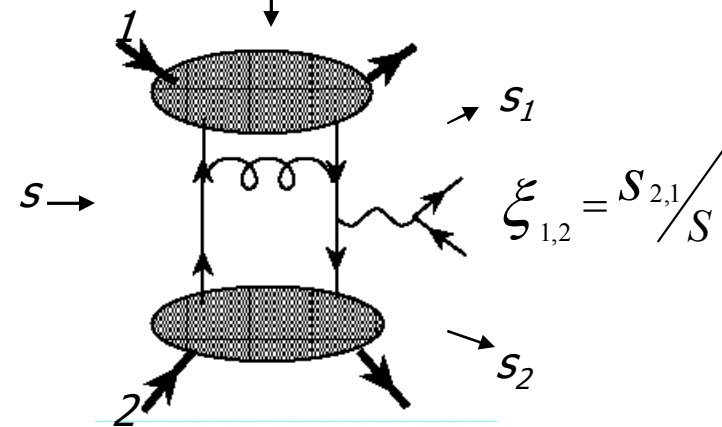
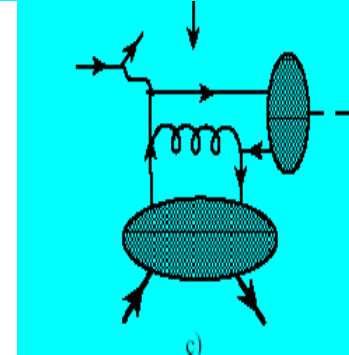
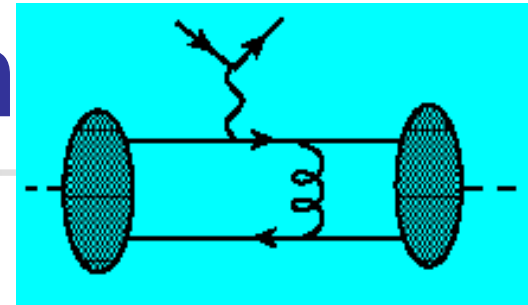
$$M \propto \int dx \frac{\phi(x)}{1-x} \int dx \frac{H(x, \xi)}{x - \xi + i\epsilon}$$



# Next step: 2 DA's -> 2 GPD's- Double Diffraction

- Exclusive double diffractive DY process (quarks-exclusive DY)
- Direct calculation – NO factorization in physical region

$$M \propto \iint dx dy \frac{H(x, \xi_1) H(y, \xi_2)}{(x - \xi_1)(y - \xi_2) + i\epsilon}$$





# Analytic continuation

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- Try to calculate in the unphysical region

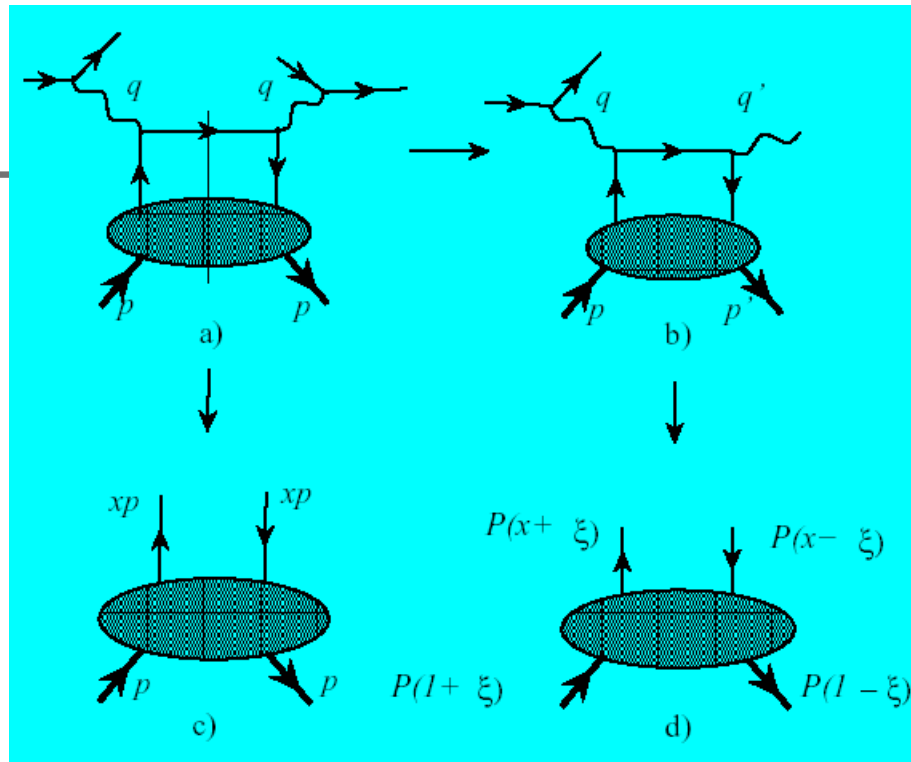
- $|\xi_{1,2} = S_{2,1}/S| > 1$

- How to continue?

- Solved for hard SPE –type amplitudes (LO - OT, EDS@Blois'05; Anikin, OT; Polyakov, Semenov-Tian-Shansky; Muller, Kumericky; NLO – Diehl, Ivanov)

# Factorization for DIS and DVCS

DVCS



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^1 dx \frac{H(x)}{x - x_B + i\epsilon}$$

- Extra dependence on  $\xi$

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, \xi)}{x - \xi + i\epsilon}$$





# Unphysical regions

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- DIS : Analytical function – polynomial in  $X_B$  if  $1 \leq |X_B|$

$$H(x_B) = - \int_{-1}^1 dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

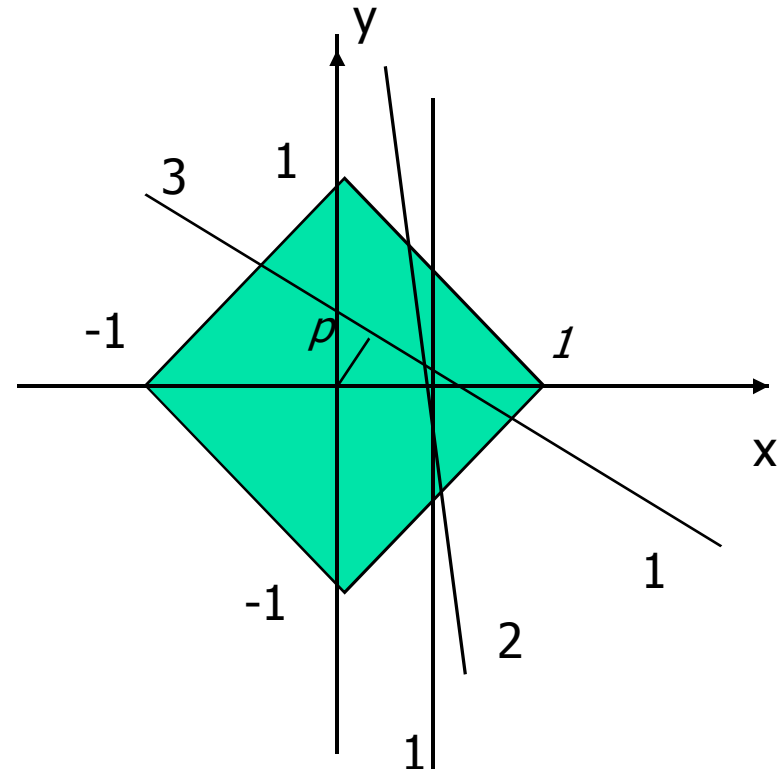
- DVCS – additional problem of analytical continuation of  $H(x, \xi)$

- Solved by using of Double Distributions (Radon transform)

$$H(z, \xi) = \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)$$

# Double distributions and their integration

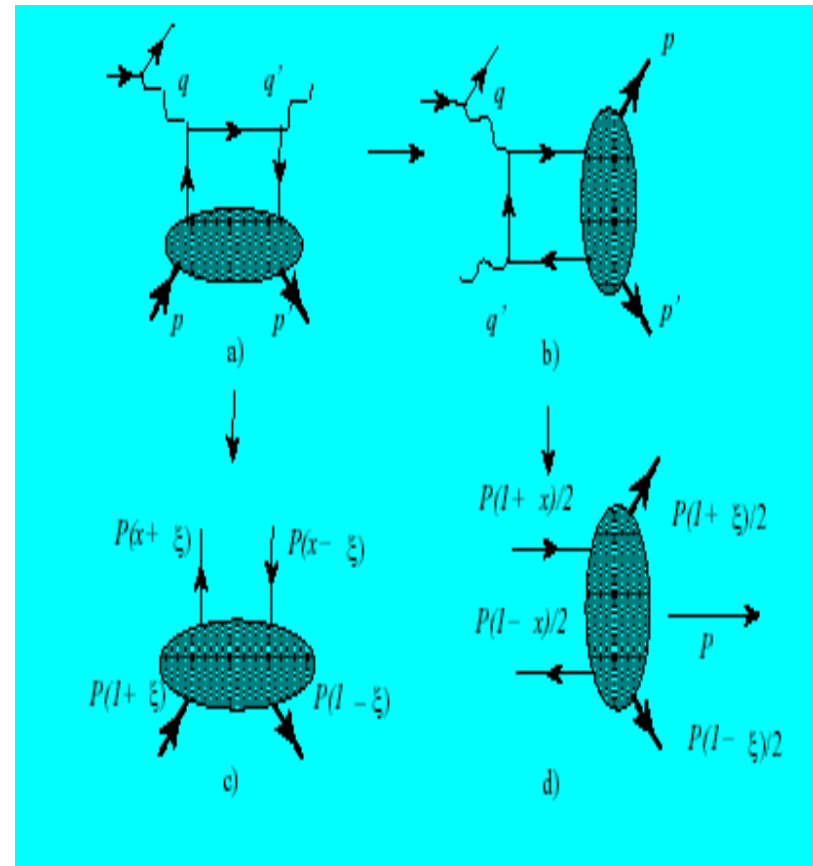
- Slope of the integration line-skewedness
- Kinematics of DIS:  $\xi = 0$   
("forward") - vertical line (1)
- Kinematics of DVCS:  $\xi < 1$   
- line 2
- Line 3:  $\xi > 1$  (unphysical region - required also to restore DD by inverse Radon transform - tomography) ?!



$$\begin{aligned}
 f(x, y) &= -\frac{1}{2\pi^2} \int_0^\infty \frac{dp}{p^2} \int_0^{2\pi} d\phi |\cos\phi| (H(p/\cos\phi + x + yt\cos\phi, t\sin\phi) - H(x + yt\cos\phi, t\sin\phi)) = \\
 &= -\frac{1}{2\pi^2} \int_{-\infty}^\infty \frac{dz}{z^2} \int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))
 \end{aligned}$$

# Crossing for DVCS and GPD

- DVCS  $\rightarrow$  hadron pair production in the collisions of real and virtual photons
- GPD  $\rightarrow$  Generalized Distribution Amplitudes (Diehl, Gousset, Pire, OT)





# Holographic property

- Factorization Formula

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- Analyticity ->  
Imaginary part ->  
Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, \xi)}{x - \xi + i\epsilon}$$

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, x)}{x - \xi + i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^1 dx \frac{H(x, x) - H(x, \xi)}{x - \xi + i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^1 H(x, \xi) dx (x - \xi)^{n-1} = \text{const}$$



# Cuts for DPE

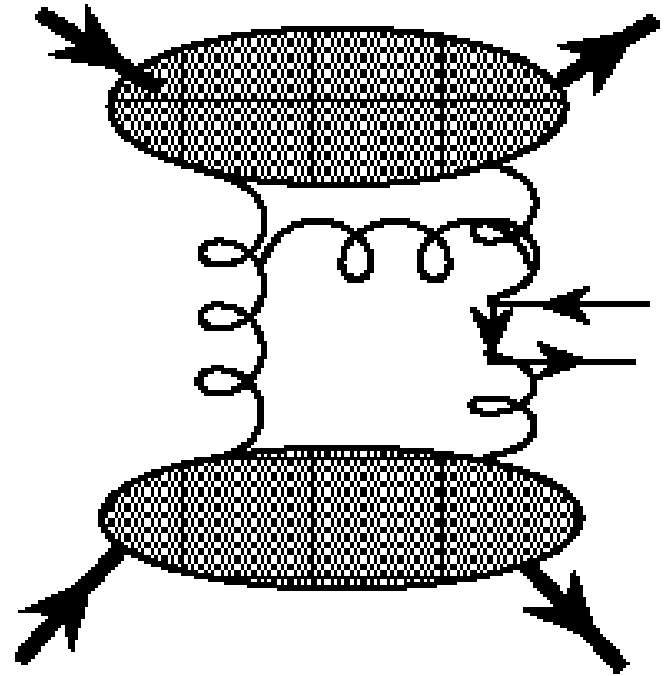
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- Scaling – dependence on Ratios  $\xi_{1,2} = S_{2,1}/S$
- No unique prescription to accommodate 2 cuts. Same for simultaneous sign change - cancellation
- For DPE – another variable – (invariant) mass of the produced system
- Double cuts in overlapping channels – absent for STABLE particles (Steinmann relations)
- For DY and HIGGS\double cut allowed!

$$\xi_{1,2} = S_{2,1}/S + i0$$

# Implications for Higgs

- Already at the crossing level  
– GPD  $\rightarrow$  GDA
- Higgs production amplitude (GPD) -
- Higgs EXCLUSIVE DECAY amplitude to 2 hadron pairs (MC)
- More general property than collinear factorization framework
- Collinear factorization – quantification of the effect of analytical continuation

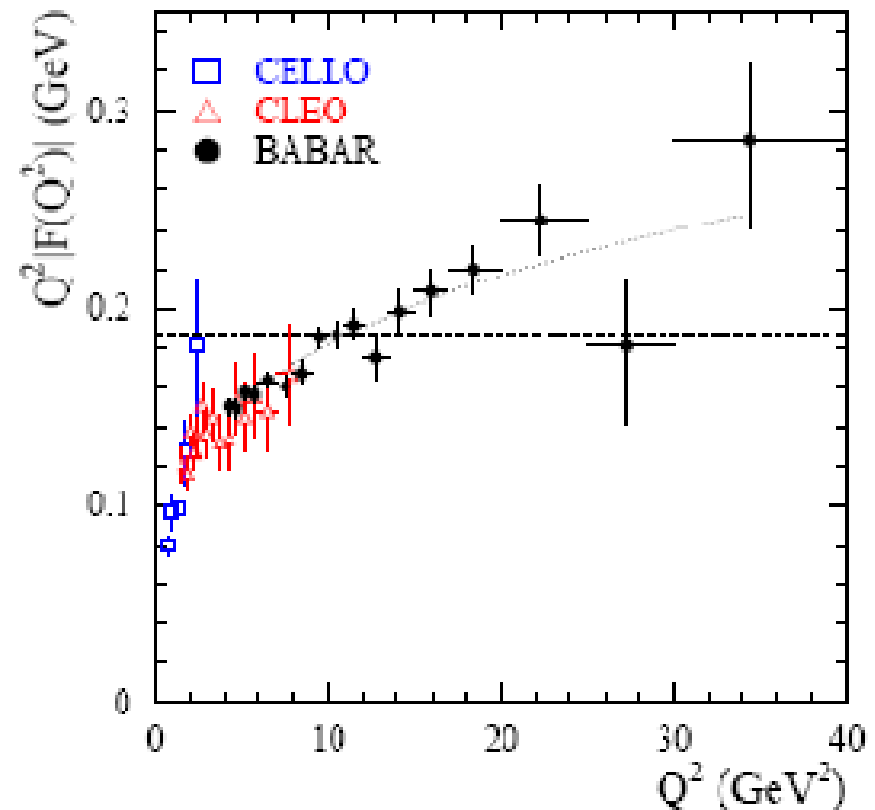


# Hot topic for (p)QCD: BABAR data

data **0905.4778** [hep-ex]

- Pion-photon transition FF

- Where is attractor?!



# Status of factorization and BABAR data

- Growth of pion photon transition FF- possible evidence for flat pion distribution (Radyushkin, Polyakov) Growth due to IR contribution

$$F \propto \left( \int dx \frac{\phi(x)}{1-x} \right)^2$$

- For pion FF – Brodsky-Lepage ratio
- Analogous ratio of VM amplitude squared to exclusive DY one





# Back to physical region

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- If IR divergence is typical (FF's, transverse vector meson production) –original integral is not much worse!

$$M \square \iint dx dy \frac{H(x, \xi_1) H(y, \xi_2)}{(x - \xi_1)(y - \xi_2) + i\varepsilon}$$

- Infinitesimal  $\rightarrow$  finite width (Higgs – gluon “mass”/non-local condensate width) in the denominator
- Counterpart of Sudakov in Duhram Model?



# CONCLUSIONS

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- $FF \rightarrow VMP \rightarrow DPE$     $DA \rightarrow GPD$
- No direct factorization in physical region. If pion DA is indeed flat – typical situation.
- Crossing analogs of GPD  $\rightarrow$  GDA: Relation of Higgs exclusive production and decay
- Double cuts - for unstable particle – imaginary part for hard DPE-type amplitudes
- Durham (standard) model: to be continued