Spin correlations in the $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ systems generated in relativistic heavy ion collisions

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Introduction

- Spin correlations for the $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ pairs generated in one act of collision of hadrons or nuclei and respective angular correlations at the joint registration of hadronic decays of two hyperons give the important information about the character of multiple processes.

- The advantage of $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ systems is due to the fact that the *P*-odd decays $\Lambda \rightarrow p + \pi^-$ and $\overline{\Lambda} \rightarrow \overline{p} + \pi^+$ serve as effective analyzers of the spin states of Λ and $\overline{\Lambda}$ particles.

1. General structure of the spin density matrix of the pairs of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$

- The spin density matrix of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, just as the spin density matrix of two spin-½ particles in general, can be presented in the following form [1,2,3] :

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left[\, \hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\sigma}^{(1)}\mathbf{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\sigma}^{(2)}\mathbf{P}_2) + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\ \hat{\sigma}_i^{(1)} \otimes\ \hat{\sigma}_k^{(2)} \,\right]$$

in doing so, $tr_{(1,2)}\,\hat{\rho}^{(1,2)} = 1$.

Here $\hat{I}$ is the two-row unit matrix, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the vector Pauli operator (x, y, $z \rightarrow 1, 2, 3$),

$\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarization vectors of the first and second particle ($\mathbf{P}_1 = \langle\hat{\sigma}^{(1)}\rangle$, $\mathbf{P}_2 = \langle\hat{\sigma}^{(2)}\rangle$), $T_{ik} = \langle\, \hat{\sigma}_i^{(1)} \otimes\ \hat{\sigma}_k^{(2)}\rangle$ are the correlation tensor components . In the general case $T_{ik} \neq P_{1i}\ P_{2k}$. The tensor with components $C_{ik} = T_{ik} - P_{1i}\ P_{2k}$ describes the spin correlations of two particles .

The respective one-particle density matrices are as follows:

$$\hat{\rho}^{(1)} = \frac{1}{2}\left(\hat{I}^{(1)} + \mathbf{P}_1 \, \hat{\sigma}^{(1)} \right) \quad , \qquad \hat{\rho}^{(2)} = \frac{1}{2}\left(\hat{I}^{(2)} + \mathbf{P}_2 \, \hat{\sigma}^{(2)} \right) \quad .$$

The trace of the correlation tensor is $T = T_{xx} + T_{yy} + T_{zz} = \langle \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)} \rangle$.

The eigenvalues of the operator $\hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}$ equal $\lambda_t = 1$ for three triplet states (total spin $S = 1$) and $\lambda_s = -3$ for the singlet state (total spin $S = 0$) .

- Let us introduce the operators of projection onto the triplet states and onto the singlet state:

$$\hat{P}_t = \frac{3 + \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}}{4} \quad ; \quad \hat{P}_s = \frac{1 - \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}}{4} \quad .$$

The following matrix equalities are satisfied:

$$\hat{P}_t^{\,2} = \hat{P}_t \; ; \quad \hat{P}_s^{\,2} = \hat{P}_s \; ; \quad \hat{P}_t \, \hat{P}_s - \hat{P}_s \, \hat{P}_t = 0 \quad .$$

For all the purely triplet states the “trace” of the correlation tensor $T = 1$, whereas for the purely singlet state $T = -3$.

The two-particle spin density matrix $\hat{\rho}^{(1,2)}$ may be decomposed into the triplet, singlet and the “mixed” singlet-triplet parts:

$$\hat{\rho}^{(1,2)} = \hat{\rho}_t + \hat{\rho}_s + \hat{\rho}_{ts} + \hat{\rho}_{st} \quad , \quad \text{in doing so,}$$

$$\hat{\rho}_t = \hat{P}_t\, \hat{\rho}^{(1,2)} \hat{P}_t \;\; ; \;\; \hat{\rho}_s = \hat{P}_s\, \hat{\rho}^{(1,2)} \hat{P}_s \;\; ; \;\; \hat{\rho}_{ts} = \hat{P}_t\, \hat{\rho}^{(1,2)} \hat{P}_s \;\; ; \;\; \hat{\rho}_{st} = \hat{P}_s\, \hat{\rho}^{(1,2)} \hat{P}_t \, .$$

The relative fraction of the triplet states amounts to:

$$W_t = tr\, \hat{\rho}_t = tr\, (\hat{P}_t\, \hat{\rho}^{(1,2)} \hat{P}_t\,) = tr\, (\hat{P}_t\, \hat{\rho}^{(1,2)}) = \frac{3 + T}{4} ,$$

and the relative fraction of the singlet state amounts to:

$$W_s = tr\, \hat{\rho}_s = tr\, (\hat{P}_s\, \hat{\rho}^{(1,2)} \hat{P}_s\,) = tr\, (\hat{P}_s\, \hat{\rho}^{(1,2)}) = \frac{1 - T}{4} ;$$

in doing so,

$$W_t + W_s = 1 \; ; \quad T = W_t - 3W_s = 1 - 4W_s \quad .$$

- Due to the orthogonality of the projection matrices $\hat{P}_t$ and $\hat{P}_s$, the following relation holds:

$$tr\,\hat{\rho}_{ts} = tr\,\hat{\rho}_{st} = tr\,(\hat{\rho}^{(1,2)}\hat{P}_s\,\hat{P}_t) = 0$$

.

- If the first particle and second particle have different relativistic momenta, the polarization vectors $\mathbf{P}_1$, $\mathbf{P}_2$ and the correlation tensor components with "left" and "right" indices are specified, respectively, in the rest frames of the first and second particle -- in the unified coordinate axes of the c.m. frame of two particles .

- Let us remark that, although the spin state of a particle is specified in its rest frame, this state -- due to the kinematic effect of relativistic spin rotation – depends on the concrete frame, from which the transition to this rest frame is realized. For two particles with nonzero relative momentum, in the case when the laboratory momenta of the particles are not collinear, the angles of relativistic spin rotation at the transformation from the laboratory frame to the c.m. frame of the pair are different. Therefore, the values of polarization parameters (correlation tensor components) in the laboratory frame are, in general, different from the respective values of polarization parameters in the c.m. frame of the pair [4]. It is natural to determine the correlation tensor components and the relative fractions of singlet state and triplet states namely in the c.m. frame of two particles, in which their total momentum equals zero, and, thus, to perform the transition to the rest frames of each of the particles through their c.m. frame.

2. Spin correlations and angular correlations at joint registration of decays of two Λ particles into the channel $\Lambda \to p + \pi^-$

- Any decay with the space parity nonconservation may serve as an analyzer of spin state of the unstable particle [3].

The normalized angular distribution at the decay $\Lambda \to p + \pi^-$ takes the form:

$$\frac{dw(\mathbf{n})}{d\Omega_{\mathbf{n}}} = \frac{1}{4\pi}(1 + \alpha_\Lambda \mathbf{P}_\Lambda \mathbf{n}) .$$

Here $\mathbf{P}_\Lambda$ is the polarization vector of the Λ particle, $\mathbf{n}$ is the unit vector along the direction of proton momentum in the rest frame of the Λ particle, α_Λ is the coefficient of *P*-odd angular asymmetry (α_Λ = 0.642).

The decay $\Lambda \to p + \pi^-$ selects the projections of spin of the Λ particle onto the direction of proton momentum; the analyzing power equals $\xi = \alpha_\Lambda \mathbf{n}$.

- Now let us consider the double angular distribution of flight directions for protons formed in the decays of two Λ particles into the channel $\Lambda \rightarrow p + \pi^-$, normalized by unity (the analyzing powers are $\xi_1 = \alpha_\Lambda \mathbf{n}_1$, $\xi_2 = \alpha_\Lambda \mathbf{n}_2$) . It is described by the following formula [2,3]:

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} = \frac{1}{16\,\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_1\,\mathbf{n}_1 + \alpha_\Lambda \mathbf{P}_2\,\mathbf{n}_2 + \alpha_\Lambda^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\, n_{1i}\, n_{2k}\right]$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are polarization vectors of the first and second Λ particle, T_{ik} are the correlation tensor components, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the respective rest frames of the first and second Λ particle, defined in the common (unified) coordinate axes of the c.m. frame of the pair ($i, k = \{1, 2, 3\} = \{ x, y, z \}$) .

- Using the method of moments, the components of polarization vectors and correlation tensor may be determined as a result of averaging combinations of trigonometric functions of angles of proton flight over the double angular distribution [2,3]:

$$P_{1i} = \frac{3}{\alpha_\Lambda}\langle n_{1i} \rangle, \quad P_{2k} = \frac{3}{\alpha_\Lambda}\langle n_{2k} \rangle, \quad T_{ik} = \frac{9}{\alpha_\Lambda^2}\langle n_{1i}\, n_{2k} \rangle$$

Here

$$\langle \ldots . \rangle \equiv \int (\ldots .) \left(\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} \right) d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2} \quad ;$$

$n_{1x} = \sin\theta_1 \cos\varphi_1$; $n_{1y} = \sin\theta_1 \sin\varphi_1$; $n_{1z} = \cos\theta_1$;
$n_{2x} = \sin\theta_2 \cos\varphi_2$; $n_{2y} = \sin\theta_2 \cos\varphi_2$; $n_{2z} = \cos\theta_2$,

where θ_1 and φ_1 , θ_2 and φ_2 are the polar and azimuthal angles of emission of protons in the rest frames of the first and second Λ particle, respectively – with respect to the unified system of coordinate axes of c.m. frame of pair; $d\Omega_{\mathbf{n}_1} = \sin\theta_1 \, d\theta_1 \, d\varphi_1$ and $d\Omega_{\mathbf{n}_2} = \sin\theta_2 \, d\theta_2 \, d\varphi_2$ are the elements of solid angles of proton emission .

- The double angular distribution may be integrated over all angles except the angle θ between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$:

  $\cos\theta = \mathbf{n}_1 \, \mathbf{n}_2 = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)$.

  At this integration, the solid angle element $d\Omega_{\mathbf{n}_2}$ can be defined, without losing generality, in the coordinate frame with the axis z being parallel to the vector $\mathbf{n}_1$, and the solid angle element $d\Omega_{\mathbf{n}_1}$ is defined in the coordinate frame where the polarization parameters are specified:

$$d\Omega_{\mathbf{n}_2} = \sin\theta \, d\theta \, d\varphi \qquad d\Omega_{\mathbf{n}_1} = \sin\theta_1 \, d\theta_1 \, d\varphi_1$$

here φ is the azimuthal angle of rotation of the vector $\mathbf{n}_2$ around the vector $\mathbf{n}_1$.

- So, the angular correlation between the proton momenta at the decays of two Λ particles is expressed as follows:

$$dw(\cos\theta) = \left(\int \frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} d\varphi \, d\Omega_{\mathbf{n}_1} \right) \sin\theta \, d\theta \ .$$

In doing so,

$$\overline{\mathbf{n}_1} = \frac{1}{8\pi^2} \int \mathbf{n}_1 \, d\varphi \, d\Omega_{\mathbf{n}_1} = 0 \ , \qquad \overline{\mathbf{n}_2} = \frac{1}{8\pi^2} \int \mathbf{n}_2 \, d\varphi \, d\Omega_{\mathbf{n}_1} = 0 \ ,$$

$$\overline{n_{1i} \, n_{2k}} = \frac{1}{8\pi^2} \int n_{1i} \, n_{2k} \, d\varphi \, d\Omega_{\mathbf{n}_1} = \frac{1}{3} \cos\theta \, \delta_{ik} \ .$$

The angular correlation, being described by the formula [2,3,5,6]

$$dw(\cos\theta) = \frac{1}{2} \left(1 + \frac{1}{3} \alpha_\Lambda^2 \, T \cos\theta \right) \sin\theta \, d\theta \ ,$$

is determined only by the ``trace" of the correlation tensor $T = W_t - 3W_s$, and it does not depend on the polarization vectors (single-particle states may be unpolarized).

So, finally we have:

$$dw(\cos\theta) = \frac{1}{2} \left(1 - \alpha_\Lambda^2 \, (W_s - \frac{W_t}{3}) \, \cos\theta \right) \sin\theta \, d\theta \ ,$$

W_s and W_t are relative fractions of the singlet state and triplet states, respectively .

3. Correlations at the joint registration of the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$

- Due to *CP* invariance, the coefficients of *P*-odd angular asymmetry for the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$ have equal absolute values and opposite signs: $\alpha_{\overline{\Lambda}} = -\alpha_{\Lambda} = -0.642$. The double angular distribution for this case is as follows [2,3]:

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} = \frac{1}{16\pi^2}\left[1 + \alpha_{\Lambda}\mathbf{P}_{\Lambda}\,\mathbf{n}_1 - \alpha_{\Lambda}\mathbf{P}_{\overline{\Lambda}}\,\mathbf{n}_2 - \alpha_{\Lambda}^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\, n_{1i}\, n_{2k}\right]$$

(here $-\alpha_{\Lambda} = +\alpha_{\overline{\Lambda}}$ and $-\alpha_{\Lambda}^2 = +\alpha_{\Lambda}\alpha_{\overline{\Lambda}}$) .

- Thus, the angular correlation between the proton and antiproton momenta in the rest frames of the Λ and $\overline{\Lambda}$ particles is described by the expression :

$$dw(\cos\theta) = \frac{1}{2}\left(1 - \frac{1}{3}\alpha_{\Lambda}^2\, T\cos\theta\right)\sin\theta\, d\theta = \frac{1}{2}\left(1 + \alpha_{\Lambda}^2\,(W_s - \frac{W_t}{3})\cos\theta\right)\sin\theta\, d\theta\,,$$

- where θ is the angle between the proton and antiproton momenta .

4. Model of one-particle sources

- For describing the momentum-energy correlations and related spin correlations of identical particles, generated in processes with high multiplicity, the model of one-particle sources (“constituents”) is widely applied [7].
- In the framework of this model, the sources emitting particles do not overlap in space and time, -- it is supposed that the sizes of the sources themselves are small as compared with the distances between them. In accordance with this, each source is characterized by the 4-coordinate $x_i = \{ \mathbf{r}_i , t_i \}$. In doing so, the spatial region occupied by all the sources is very small as compared with the sizes of detectors measuring the particle momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, and the duration of the generation process is very small as compared with the time parameters of detectors.
- According to the model of one-particle sources (Kopylov, Podgoretsky) [7], particles are emitted by the sources independently and incoherently. Thus, at the early stage of particle formation (at the stage of hadronization of quarks and gluons) the momentum-energy and spin correlations are absent .

Correlations for identical particles, produced in the same event of collision, arise on account of the effects of quantum statistics (Bose statistics for particles with integer spin and Fermi statistics for particles with half-integer spin) and final-state interaction [8]. At present the model of sources is successfully used as well for the description of pair momentum-energy correlations of non-identical particles, conditioned exclusively by the final-state interaction .

- It is essential that the pair momentum-energy correlations depend upon the momentum difference in the c.m. frame of the pair and,-- as upon the parameters of the process,-- upon the space--time characteristics of the region of multiple generation of particles, corresponding to the so-called "freeze-out" . In accordance with the model of sources, the correlations reach the maximum at relative momenta being small as compared with the inverse space--time dimensions of the generation region, whereas in the limit of large relative momenta they disappear . This fact has served as a basis for elaboration of the correlation method (the so-called correlation femtoscopy), allowing one to investigate experimentally the space--time development of the processes of multiple generation of leptons, photons and hadrons .

- Since the momentum-energy correlations are substantial only in a sufficiently narrow range of small relative momenta, in most of real events the particle density in phase space is enough small, so that it would be possible to consider the pair correlations only, disregarding the triple correlations (moreover, the correlations of higher orders) as well as neglecting their influence upon the pair correlations .

- In the model under consideration, the sources have a very broad momentum spectrum as compared with the relative momenta being characteristic for pair correlations; thus, the emission probabilities for each of the one-particle sources change insignificantly under the variation of 4-momenta p_1 and p_2 within the correlation effect (that is the so-called "smoothness condition").

- The method of correlation femtoscopy, based on the source model, have been used enough successfully for studying the correlations in processes of collision of elementary particles . But, by its essence, this approach is the most adequate one for processes of multiple generation of particles in collisions of heavy nuclei .

5. Spin correlations at the generation of ΛΛ pairs in multiple processes

a) The Fermi-statistics effect leads not only to the momentum-energy ΛΛ correlations at small relative momenta (correlation femtoscopy), but to the spin correlations as well .

The following relation holds, in consequence of the symmetrization or antisymmetrization of the total wave function of any identical particles with nonzero spin (bosons as well as fermions) [9] :

$$(-1)^{S+L} = 1$$

.

Here S is the total spin and L is the orbital momentum in the c.m. frame . At the momentum difference $q = p_1 - p_2 \to 0$ the states with nonzero orbital momenta "die out" , and only states with $L = 0$ and even total spin S survive .

Since the Λ-particle spin is equal to ½ , at $q \to 0$ the ΛΛ pair is generated only in the singlet state with $S = 0$.

Meantime, at the 4-momentum difference $q \neq 0$ there are also triplet states generated together with the singlet state .

- Within the accepted model of one-particle sources emitting unpolarized particles, the triplet states with spin projections +1, 0 and –1 are produced with equal probabilities. If correlations are neglected, the singlet state is generated with the same probability, -- the relative "weights" are $\widetilde{W}_t = 3/4$, $\widetilde{W}_s = 1/4$.

  Taking into account the Fermi statistics and s -wave final-state interaction, which is essential at close momenta (at orbital momenta $L \neq 0$ the contribution of final-state interaction is suppressed), the fractions of triplet states and the singlet state become proportional to the quantities [8,10]:

$$W_t(q) = \frac{3}{4}(1 - \langle \cos qx \rangle), \quad W_s(q) = \frac{1}{4}(1 + \langle \cos qx \rangle + 2B_{\text{int}}(q)) \ ;$$

here $q = p_1 - p_2$ is the difference of 4-momenta, $x = x_1 - x_2$ is the difference of 4-coordinates of two sources .

In the above formula $\langle \cos qx \rangle = \int W(x) \cos qx \, d^4x$ is the Fermi-statistics contribution; here $W(x)$ is the distribution of difference of 4-coordinates of two sources; $B_{\text{int}}(q)$ is the contribution of s -wave final-state interaction of two Λ particles.

In doing so,

$$R(q) = W_t(q) + W_s(q) = 1 - \frac{1}{2}\langle \cos qx \rangle + \frac{1}{2} B_{\text{int}}(q)$$

is the correlation function describing the momentum-energy correlations of two Λ particles with close momenta .

- The correlation function $R(q)$ represents the ratio of the two-particle spectrum to the non-correlated background, which is constructed usually as a product of one-particle spectra from different events at the same values of momenta. In terms of inclusive cross sections we have [10]:

$$\frac{d^6\sigma}{d^3\mathbf{p}_1\, d^3\mathbf{p}_2} = \frac{R(q)}{\sigma_{tot}} \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} \left(\frac{d^3\sigma}{d^3\mathbf{p}_1}\right)\left(\frac{d^3\sigma}{d^3\mathbf{p}_2}\right) \quad ,$$

where n is the multiplicity and σ_{tot} is the total interaction cross-section

(for Poisson distribution of multiplicity we have $\langle n(n-1)\rangle / \langle n\rangle^2 = 1$).

- b) The spin density matrix of two Λ particles with close momenta at the emission of unpolarized Λ particles has the following structure:

$$\hat{\rho}^{(1,2)} = \frac{W_s(q)\,\hat{\rho}_s + W_t(q)\,\hat{\rho}_t}{W_s(q)+W_t(q)} = \frac{1}{R(q)}\left[\left(\frac{1}{4}\left(1+\langle\cos qx\rangle + 2B_{\rm int}(q)\right)\right)\hat{\rho}_s + \frac{3}{4}\left(1-\langle\cos qx\rangle\right)\hat{\rho}_t\right]$$

Here

$$\hat{\rho}_s = \frac{1}{4}\left(\hat{I}^{(1)}\otimes\hat{I}^{(2)} - \hat{\sigma}^{(1)}\otimes\hat{\sigma}^{(2)}\right)$$

is the density matrix of the singlet state, and

$$\hat{\rho}_t = \frac{1}{4}\left(\hat{I}^{(1)}\otimes\hat{I}^{(2)} + \frac{1}{3}\hat{\sigma}^{(1)}\otimes\hat{\sigma}^{(2)}\right)$$

is the density matrix of the unpolarized triplet state, averaged over the spin projections $\lambda = +1, 0, -1$:

$$\hat{\rho}_t = \frac{1}{3}\left(\hat{\rho}_{t+1} + \hat{\rho}_{t0} + \hat{\rho}_{t-1}\right); \qquad \hat{\rho}_s + 3\hat{\rho}_t = \hat{I}^{(1)}\otimes\hat{I}^{(2)} \quad .$$

- It is easy to see that the above formula for $\hat{\rho}^{(1,2)}$ can be rewritten in the form:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left(\hat{I}^{(1)}\otimes\hat{I}^{(2)} - \frac{\langle\cos qx\rangle + B_{\rm int}(q)}{2\,R(q)}\,\hat{\sigma}^{(1)}\otimes\hat{\sigma}^{(2)}\right) \quad .$$

- The correlation tensor components [2]

$$T_{ik} = C_{ik} = -\frac{\langle \cos qx \rangle + B_{\rm int}(q)}{2 - \langle \cos qx \rangle + B_{\rm int}(q)}\,\delta_{ik}$$

depend upon the momentum difference as well as upon the space-time parameters of the generation region;

the trace of the correlation tensor amounts to

$$T = \sum_i T_{ii} = -3\,\frac{\langle \cos qx \rangle + B_{\rm int}(q)}{2 - \langle \cos qx \rangle + B_{\rm int}(q)} \quad .$$

Thus, on account of the effects of quantum statistics and final-state interaction, at small relative momenta two identical particles, initially unpolarized ($\mathbf{P}_1 = \mathbf{P}_2 = 0$) and non-correlated by spins, remain unpolarized as well but their spins become correlated .

At $q \to 0$ we obtain: $\langle \cos qx \rangle \to 1, \quad T_{ik} \to -\delta_{ik}$ (singlet state).

On the other hand, in the limit of large q :

$$\langle \cos qx \rangle \to 0,\ B_{\rm int}(q) \to 0,\ R(q) \to 1,\ T_{ik} \to 0,$$

i.e. both the momentum-energy and spin correlations vanish .

c) Let two one-particle sources emit Λ particles with equal polarization vectors $\tilde{\mathbf{P}}_1 = \tilde{\mathbf{P}}_2 = \tilde{\mathbf{P}}$.

Then, at the stage of emission by sources, correlations are absent . Due to the effects of Fermi statistics and *s* –wave final-state interaction, the polarization parameters are renormalized :

$$\mathbf{P}_1 = \mathbf{P}_2 = \frac{1}{R(q)} (1 - \langle \cos qx \rangle)\ \tilde{\mathbf{P}} \quad ;$$

$$T_{ik} = \frac{1}{R(q)} \left[(1 - \langle \cos qx \rangle)\, \tilde{P}_i\, \tilde{P}_k - \frac{1 - \tilde{P}^2}{2} (\langle \cos qx \rangle + B_{\text{int}}(q))\, \delta_{ik} \right] ,$$

where

$$R(q) = 1 - \frac{1 + \tilde{P}^2}{2} \langle \cos qx \rangle + \frac{1 - \tilde{P}^2}{2} B_{\text{int}}(q)$$

is the function describing the momentum-energy correlations . Irrespective of the primary polarization $\tilde{P}$, at the momentum difference $q \to 0$ only the singlet state of the $\Lambda\Lambda$ pair is realized and the renormalized polarization vectors $\mathbf{P}_1 = \mathbf{P}_2$ tend to zero . The *s*- wave final-state interaction amplifies the predominant role of the singlet state. If $\tilde{P} = 1$, then in the limit $q \to 0$ the generation of $\Lambda\Lambda$ pairs is forbidden – in full accordance with the Pauli principle .

d) Lorentz-transformation of 4-momenta and 4-coordinates of sources to the c.m. frame of the $\Lambda\Lambda$ pair.

In the c.m. frame we have: q = {0, 2**k** }, where **k** is the momentum of one of the particles. In doing so, the momentum **k** is connected with the relative momentum **q** in the laboratory frame by the Lorentz transformation [10]:

$$\mathbf{k} = \frac{1}{2}\left[\mathbf{q} + (\gamma - 1)\frac{(\mathbf{qv})\,\mathbf{v}}{|\mathbf{v}|^2} - \gamma\,\mathbf{v} q_0 \right] \quad ;$$

here $\mathbf{v} = (\mathbf{p}_1 + \mathbf{p}_2) / (\varepsilon_1 + \varepsilon_2)$ is the velocity of the $\Lambda\Lambda$ pair in the laboratory frame, $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor, $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ and $q_0 = \varepsilon_1 - \varepsilon_2$.

The Lorentz transformations of 4-coordinates are given by the expressions :

$$\mathbf{r}^* = \mathbf{r} + (\gamma - 1)\frac{(\mathbf{rv})\,\mathbf{v}}{|\mathbf{v}|^2} - \gamma\,\mathbf{v}\,t \;, \qquad t^* = \gamma\,(t - \mathbf{vr}) \;,$$

where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ and $t = t_1 - t_2$.

- The interference term connected with identity (quantum statistics), is determined by the expression:

$$\langle \cos qx \rangle = \langle \cos 2\mathbf{kr}^* \rangle = \int W_{\mathbf{v}}(\mathbf{r}^*) \cos(2\mathbf{kr}^*)\, d^3\mathbf{r}^* \quad ,$$

where $$W_{\mathbf{v}}(\mathbf{r}^*) = \int W(x)\, dt^* = \int W(\mathbf{r}^*, t^*)\, dt^*$$

is the distribution of coordinate difference between two sources in the c.m. frame of the $\Lambda\Lambda$ pair .

- Meantime, the contribution of *s*-wave final-state interaction is expressed as follows (at the sizes of the generation region in the c.m. frame, exceeding the effective radius of interaction of two Λ particles):

$$B_{\text{int}}(q) = B^{(\Lambda\Lambda)}(\mathbf{k}, \mathbf{v}) = \int W_{\mathbf{v}}(\mathbf{r}^*)\, b(\mathbf{k}, \mathbf{r}^*)\, d^3\mathbf{r}^* \quad ,$$

where the function $b(\mathbf{k}, \mathbf{r}^*)$ has the structure [2,8,10]:

$$b(\mathbf{k}, \mathbf{r}^*) = |f^{(\Lambda\Lambda)}(k)|^2 \frac{1}{(r^*)^2} + 2\,\text{Re}\left(f^{(\Lambda\Lambda)}(k) \frac{e^{ikr^*} \cos \mathbf{kr}^*}{r^*} \right)$$

$$- 2\pi |f^{(\Lambda\Lambda)}(k)|^2 d_0^{(\Lambda\Lambda)} \delta^3(\mathbf{r}^*) \quad .$$

Here $k = |\mathbf{k}|$, $r^* = |\mathbf{r}^*|$, $f^{(\Lambda\Lambda)}(k)$ is the amplitude of low-energy $\Lambda\Lambda$ scattering. In the framework of the effective radius theory [9,11]

$$f^{(\Lambda\Lambda)}(k) = a_0^{(\Lambda\Lambda)}\left(1 + \frac{1}{2} d_0^{(\Lambda\Lambda)} a_0^{(\Lambda\Lambda)} k^2 - i\, k a_0^{(\Lambda\Lambda)}\right)^{-1},$$

where $(-a_0^{(\Lambda\Lambda)})$ is the scattering length and $d_0^{(\Lambda\Lambda)}$ is the effective radius:

$$d_0^{(\Lambda\Lambda)} = \frac{1}{k}\frac{d}{dk}\left(\mathrm{Re}\,\frac{1}{f^{(\Lambda\Lambda)}(k)}\right).$$

The integral for $B_{\mathrm{int}}(q)$, with the above expression for $b(\mathbf{k}, \mathbf{r}^*)$ inside, approximately takes into account the difference of the true wave function of two interacting Λ particles with momenta $\mathbf{k}$ and $-\mathbf{k}$ at small distances from the asymptotic wave function of continuous spectrum [8,12].

Information about the parameters of $\Lambda\Lambda$ scattering is contained in the works studying double hypernuclei and pair correlations in the reactions with formation of two Λ particles. Analysis of experimental data leads to the conclusion that the length of $\Lambda\Lambda$ scattering is comparable by magnitude (~ (–20) fm) with the length of neutron-neutron scattering [13].

6. Spin correlations at the generation of $\Lambda\overline{\Lambda}$ pairs in multiple processes

- In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\overline{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state. At small relative momenta, the *s* -wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$) , in the case of non-identical particles ($\Lambda\overline{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

  If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum-energy correlations has the following structure (in the c.m. frame of the $\Lambda\overline{\Lambda}$ pair) :

$$R(\mathbf{k},\mathbf{v}) = 1 + \frac{3}{4} B_t^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) + \frac{1}{4} B_s^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) \quad .$$

- The spin density matrix of the $\Lambda\overline{\Lambda}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\overline{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v})}{4\,R\,(\mathbf{k},\mathbf{v})}\ \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)}$$

and the components of the correlation tensor are as follows:

$$T_{ik} = \frac{B_t^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v})}{4 + 3B_t^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) + B_s^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v})}\ \delta_{ik}$$

here the contributions of final-state triplet and singlet $\Lambda\overline{\Lambda}$ interaction are determined by the expression (analogously to the case of $\Lambda\Lambda$ interaction [2,8] with the replacement $\cos \mathbf{kr}^* \to e^{i\mathbf{kr}^*}$ [10]):

$$B_{s(t)}^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) = |\, f_{s(t)}^{(\Lambda\overline{\Lambda})}(k)|^2 \left\langle \frac{1}{(r^*)^2} \right\rangle + 2\,\mathrm{Re}\left(f_{s(t)}^{(\Lambda\overline{\Lambda})}(k) \left\langle \frac{e^{ikr^*} e^{i\mathbf{kr}^*}}{r^*} \right\rangle \right)$$

$$-\frac{2\pi}{k} |\, f_{s(t)}^{(\Lambda\overline{\Lambda})}(k)|^2 \frac{d}{dk}\left(\mathrm{Re}\frac{1}{f_{s(t)}^{(\Lambda\overline{\Lambda})}(k)} \right) W_{\mathbf{v}}(0),$$

where $f_{s(t)}^{(\Lambda\overline{\Lambda})}(k)$ is the amplitude of low-energy singlet (triplet) $\Lambda\overline{\Lambda}$ scattering, $\langle (...) \rangle \equiv \int W_{\mathbf{v}}(\mathbf{r}^*)\,(...)\,d^3\mathbf{r}^*$;

the averaging is performed over the distribution of distance between the sources of Λ and $\overline{\Lambda}$ particles.

- At sufficiently large values of k , one should expect that [8]:

$$B_s^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) = 0\,, \quad B_t^{(\Lambda\overline{\Lambda})}(\mathbf{k},\mathbf{v}) = 0 \quad .$$

In this case the angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\overline{\Lambda} \rightarrow p + \pi^+$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \qquad T = 0 \ .$$

7. Angular correlations in the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$ and the "mixed phase"

- Thus, at sufficiently large relative momenta (for example, $k \geq m_\pi$) one should expect that the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\overline{\Lambda} \to \overline{p} + \pi^+$, in the framework of the model of one-particle sources are absent. In this case two-particle (and multiparticle) sources may be, in principle, the cause of the spin correlations. Such a situation may arise, if at the considered energy the dynamical trajectory of the system passes through the so-called "mixed phase"; then the two-particle sources, consisting of the free quark and antiquark, start playing a noticeable role. For example, the process $s\bar{s} \to \Lambda\overline{\Lambda}$ may be discussed.

  The *CP* parity of the fermion-antifermion pair is $CP = (-1)^{S+1}$.

- In the case of one-gluon exchange, $CP = 1$, and then $S = 1$, i.e. the $\Lambda\overline{\Lambda}$ pair is generated in the triplet state; in doing so, the "trace" of the correlation tensor $T = 1$.

- Even if the frames of one-gluon exchange are overstepped, the quarks s and $\bar{s}$, being ultrarelativistic, interact in the triplet state ($S = 1$). In so doing, the primary CP parity $CP = 1$, and, due to the CP parity conservation, the $\Lambda\bar{\Lambda}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x. Then at large relative momenta $T = x > 0$.

- Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the annihilation process $\gamma\gamma \to \Lambda\bar{\Lambda}$, in this case the trace of the correlation tensor is described by the formula (the process $gg \to \Lambda\bar{\Lambda}$ is implied) :

$$T = 1 - \frac{4(1-\beta^2)}{1 + 2\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta} \quad ,$$

where β is the velocity of Λ (and $\overline{\Lambda}$) in the c.m. frame of the $\Lambda\overline{\Lambda}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame (see [14]) . At small β ($\beta << 1$) the $\Lambda\overline{\Lambda}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$) , whereas at $\beta \approx 1$ – in the triplet state ($S = 1$, $T = 1$) . Let us remark that at ultrarelativistic velocities β (i.e. at extremely large relative momenta of Λ and $\overline{\Lambda}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\Lambda\overline{\Lambda}$ pair ($T = 1$) .

- In the general case, the appearance of angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\overline{\Lambda} \rightarrow \overline{p} + \pi^+$ with the nonzero values of the "trace" of the correlation tensor T at large relative momenta of the Λ and $\overline{\Lambda}$ particles may testify to the passage of the system through the "mixed phase" .

8. Summary

- It is advisable to investigate the spin correlations of $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ pairs produced in relativistic heavy ion collisions.
- The spin correlations are studied by the method of angular correlations – method of moments.
- The spin correlations, as well as the momentum-energy ones, make it possible to determine the space-time characteristics of the generation region and, besides, the parameters of low-energy scattering of Λ on Λ and Λ on $\overline{\Lambda}$. They should be investigated jointly with the momentum-energy correlations.

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Thank you !