Saturation and Critical Phenomena in DIS

L.L. Jenkovszky[1], S.M. Troshin[2], and N.E. Tyurin[2]

[1]BITP, Kiev, 03680 Ukraine
[2]IHEP, Protvino, 142281 Russia

It is argued that the expected turn-down in $x - Q^2$ of the cross sections (structure functions $F_2(x, Q^2)$), assumed to result from the saturation of parton densities in the nucleon, is related to a phase transition from the (almost) ideal partonic gas, obeying Bjorken scaling, to a partonic "liquid". This can be quantified in the framework of statistical models, percolation and other approaches to collective phenomena of the strongly interacting matter. Similarities and differences between the case of lepton-hadron, hadron-hadron and nuclear collisions are discussed.

1 Introduction

Based on different observations, models and equations governing deep inelastic scattering (DIS) and related processes, a "saturation" regime is expected when certain values of low enough x and relevant Q^2 are reached. According to the dipole model of DIS, this regime already has been achieved and it is characterised by the "saturation radius" [1] $R_0^2 = (x/x_0)^\lambda / Q_0^2$, with $Q_0^2 = 1$ GeV^2, $x_0 = 3 \cdot 10^{-4}$ and $\lambda = 0.29$, found from a fit to the DIS data at $x < 0.01$. On more general grounds, saturation could be expected also from unitarity: the rapid (power-like) increase with $1/x$ of the structure functions/cross sections may suggest that unitarity corrections will tamper this rise, although formally the Froissart bound has never been proven for off-mass-shell particles, thus unitarity does not provide any rigorous limitation for such amplitudes [2, 3]. One more argument is physical: the rise of the structure function $F_2(x, Q^2)$ reflects the increase of the parton density (parton number in the nucleon). Since this number increases as a power, and the nucleon radius is known to increase as $\ln s$ (or, at most $\ln^2 s$) (shrinkage of the cone), the particle number density within a nucleon increases, inevitably, reaching a critical value where the partons start to coalesce (overlap, recombine etc). The qualitative picture of this phenomenon in the $Q^2 - 1/x$ plane is well known and cited in various contexts (see, e.g., [4]).

Quantitatively, the dynamics depend on many, poorly known, details, such as the properties of the constituents and their interaction within the nucleon.

Below we propose a novel approach to the saturation phenomenon in DIS and related processes based on the collective properties of the excited nucleon. Namely, we suggest that, below the saturation reggeon, the nucleon in DIS is seen as a gas of almost free partons. With their increasing density, the constituent gradually overlap and, starting from a certain value of x and Q^2, the gas of free partons coalesce condensing in a liquid of quarks and gluons. Saturation corresponds to the onset of the new phase.

The thermodynamic properties of such an excited nucleon are characterised by its temperature, pressure etc, and a relevant equation of state (EoS). The statistical treatment of partonic
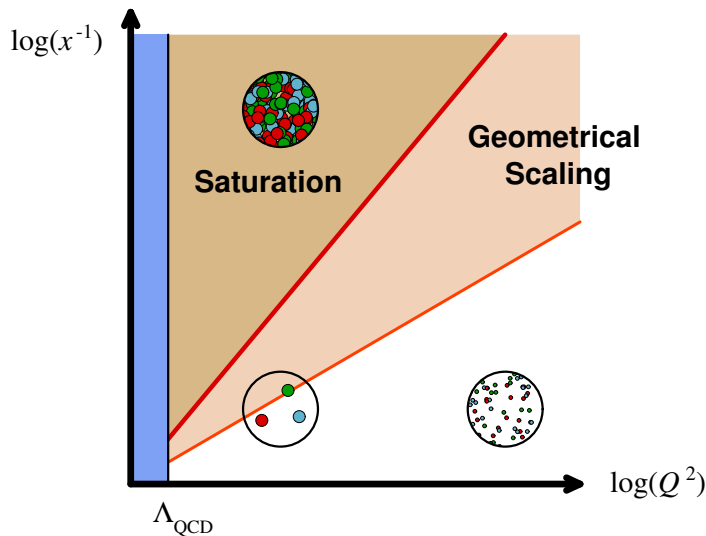
Figure 1: *Phase diagram of DIS, from Ref. [4].*

distributions is by far not new, see e.g. [5, 6, 7]. What is new in our approach, is the interpretation of the saturation in DIS as a manifestation of the transition from a dilute partonic gas to a liquid. The details (nature) of this (phase?) transition are not known. It can be of the first, of second order, or, moreover, be a smooth cross-over phenomenon. Our main argument is that the volume of the nucleon confining the partons (quarks and gluons) in the interior increases slower, at most as $\sim \ln^6 s$ (more likely, as $\sim \ln^3 s$), while the volume occupied by the interior, quarks and gluons, increases as a power, thus resulting in a limiting behaviour: a gas-to-liquid cross-over or a phase transition. The present contribution is a first step in understanding this complex process.

In a related paper, Ref. [8], the phase structure of the hadronic matter in terms of its temperature T and its baryochemical potential μ, was studied in the framework of the percolation theory. The percolation mechanism was used in [9] to obtain a limiting energy dependence of the hadronic matter at $s \to \infty$.

2   Saturation

We define the saturation line (in the $x - Q^2$ plane) as the turning point (line) of the derivatives

$$B_Q(x,Q^2) = \frac{\partial F_2(x,Q^2)}{\partial(\ln Q^2)}, \quad B_x(x,Q^2) = \frac{\partial F_2(x,Q^2)}{\partial(\ln(1/x))}, \tag{1}$$

called B_Q or B_x slopes, where $F_2(x,Q^2)$ is a "reasonable" model for the structure function, i.e. one satisfying the basic theoretical requirements, yet fitting the data. For example, the model for $F_2(x,Q^2)$ of Ref. [10] interpolates between Regge behaviour at small Q^2 and the solution of the DGLAP evolution equation at asymptotically large values of Q^2, practically for
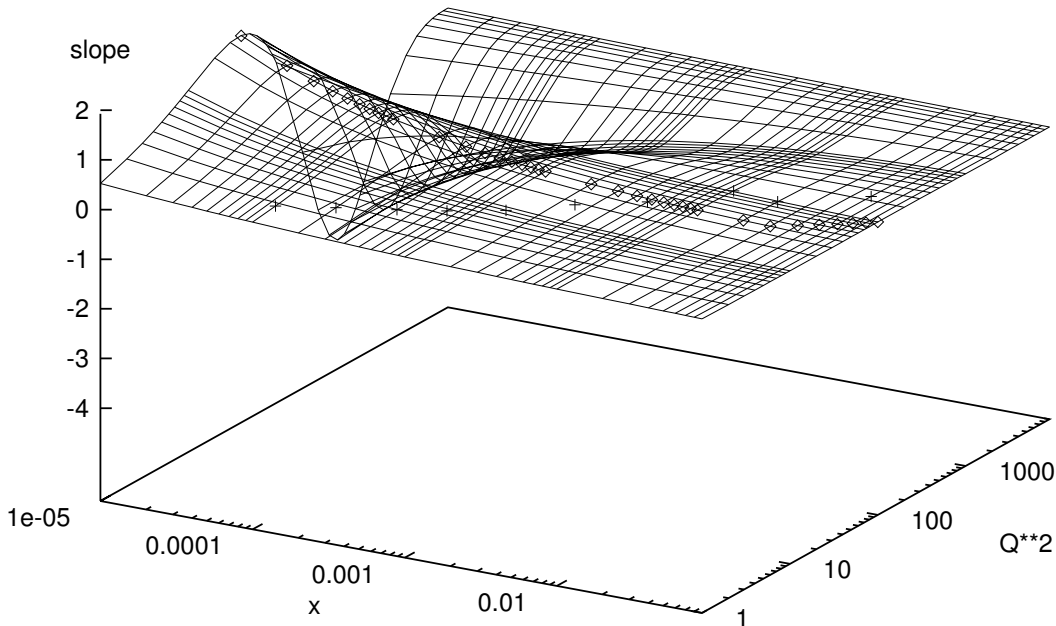
Figure 2: *The surface of $B_Q(x, Q^2)$ calculated in Ref. [10].*

all values of x. The resulting two-dimensional projection of the Q−slope is shown in Fig. 2. In our interpretation, the critical line (saturation = phase transition) occurs along the fold line on this figure (compare with a similar figure, Fig. 2 of Ref. [11], derived from a different model).

The main goal of this paper is the identification of this line (point) with the critical line (point) on the T, μ phase diagram of an excited nucleon viewed as a thermodynamical system. The thermodynamical approach to DIS may provide a new insight to this complex phenomenon. We are aware of the limited time scales in a deep inelastic scattering from the point of view of thermalisation, a familiar problem relevant to any thermodynamical description of hadronic systems. Let us only remind that the thermodynamic approach to high-energy scattering and multiple production, originated by Fermi and Landau's papers, were applied to hadrons, rather than heavy ions.

3 Statistical Models of Parton Distributions

The statistical model of parton distributions in DIS was considered and developed in quite a number of papers [5, 6, 7]. In its simplest version, one assumes [5] that inside the nucleon, the valence quarks, as well as the sea quarks and antiquarks and gluons form a noninteracting gas in equilibrium. This simple picture may be further developed in two directions: 1. Introduction of the effect of the finite size (and its energy dependence!) of the nucleon on statistical expression for the number of states for the unit energy interval; 2) Account for the Q^2 evolution, that can be calculated either from the DGLAP equation or phenomenologically, as e.g. in Ref. [10]. A likely scenario emerging [5, 6] for this high quark density $n_q = (n_q - n_{\bar{q}})/3$ system is that

quarks form Cooper pairs and new condensates develop.

The nucleon of mass M consists of a gas of massless particles (quarks, antiquarks and gluons) in equilibrium at temperature T in a spherical volume V with radius $R(s)$ increasing with squared c.m.s. energy s as $\ln s$ (or $\ln^2 s$). The invariant parton number density in phase space is given by [12]

$$\frac{dn^i}{d^3p^i\,d^3r^i} = \frac{dn}{d^3p\,d^3r} = \frac{gf(E)}{(2\pi)^3}, \tag{2}$$

where g is the degeneracy ($g = 16$ for gluons and $g = 6$ for q and $\bar{q}$ of a given flavour), $E, \mathbf{p}$ is the parton four-momentum and $f(E) = \Big(\exp[\beta(E-\mu)]\pm 1\Big)^{-1}$ is the Fermi or Bose distribution function with $\beta \equiv T^{-1}$. Quantities in the infinite momentum frame (IMF) are labelled by the subscript i.

The invariant parton density dn^i/dx in the IMF is related to dn/dE and $f(E)$ in the proton rest frame as follows [5]

$$\frac{dn^i}{dx} = \frac{gV(s)M^2x}{(2\pi)^2} \int_{xM/2}^{M/2} dE\,f(E), \tag{3}$$

and the structure function

$$F_2(x) = x \sum_q e_q^2 \Big[\Big(\frac{dn^i}{dx}\Big)_q + \Big(\frac{dn^i}{dx}\Big)_{\bar{q}}\Big]. \tag{4}$$

Without any account for the finite volume of the hadron, this SF disagrees with the data. Finite volume effects can be incorporated, following R.S. Bhalerao from Ref. [5], and the result is

$$dn/dE = gf(E)(VE^2/2\pi^2 + aR^2E + bR), \tag{5}$$

where V and R are energy dependent and a, b, in front of the surface and curvature terms, are unknown numerical coefficients. Their values are important for the final result, but they cannot be calculated from perturbative QCD. A rather general method to calculate these important parameters can be found in Ref. [13]. We intend to come back to this point in a subsequent publication.

4 Percolation

Percolation as a model of phase transition from colourless hadrons to a quark gluon plasma (QGP) was studied in a number of papers, recently in Ref. [8]. Below we apply the arguments of that paper to the saturation inside a nucleon, where extended (dressed) quark and gluons, rather then mesons and baryons considered in Ref. [8] percolate into a uniform new phase of matter. Both objects are coloured particle inside a colourless nucleon. We start with a short introduction to the subject.

Consider N spheres of radius R_0 and hence volume $V_0 = (4\pi/3)R_0^3$ in a "box" of size V, with $V \gg V_0$.

Percolation (*clustering*) of spheres, in three spacial directions, was studied for the case of:

a) arbitrary overlap [15] and b) for those with impenetrable hard core, allowing only partial overlap [6].

For the case a), the first percolation point (partial percolation), with 30% of occupation, occurs for the density $n = N/V$, $n_m \approx 0.35/V_0$, the largest cluster having the density of about $1.2/V_0$. A second percolation point occurs at $n_v = 1.22/V_0$, when 70% of space is covered by spheres, i.e. the vacuum disappears as a large-scale entity.

The existence of two percolation thresholds, one for the formation of the first spanning cluster of spheres and the second one for the disappearing of a spanning vacuum "cluster", is a general feature of the 3-dimensional percolation theory.

b) An impenetrable spherical core, with $R_c = R_0/2$, and the spheres can only partially overlap. Here again one has two percolation thresholds, at $\bar{n}_m \approx 0.34/V_0$ (close to the case a)), and vacuum percolation, at $\bar{n}_v = 2.0/V_0$, requiring a higher density compared to a).

4.1 GPD $\mathcal{H}(\mathbf{x}, \mathbf{t})$ and Impact Parameter Parton Distribution $\mathbf{q}(\mathbf{x}, \mathbf{b})$

The number of constituents in a nucleon can be found by integrating in b (impact parameter) the general parton distribution, e.g. that of Ref. [14].

$$q(x, b) = \frac{1}{2\pi} \int_0^\infty \sqrt{-t} \, d\sqrt{-t} \, \mathcal{H}(x, t) J_0(b\sqrt{-t}).$$

Here, contrary to $q(x)$, $q(x, b)$ is dimensional, with a dimension of squared mass m^2, interpreted as the transverse size of the extended parton in the hadron, $m = R^{-1}$, $q(x, b) = m^2 \tilde{q}(x, b)$, $\tilde{q}(x, b)$ being the partons number density [14]. In paper [14], m was a constant; here we choose it to depend on the photon virtuality: $m \to m_0 \ln(Q + m_\rho) = R^{-1}(Q)$, where m is the mass of the lightest vector meson, $m = m_\rho$. Note the inequality $q(x, b) \leq 1/S_q$, where $S_q(Q) = \ln^{-2}(Q + m_\rho)$.

Thus, the nucleon is composed of $N = 2\pi \int_0^1 dx \int_0^\infty b \, db \, \tilde{q}(x, b, Q)$ extended partons with the transverse area $S_q(Q)$.

4.2 A Toy EoS

One has for mesons

$$N_M(T) = 3\frac{\pi^2}{90} T^3.$$

On the other hand, mesons percolate whenever $n_v = 1.22/V_h$, where $V_h = (4\pi/3)R_h^3$, and we use for the probe meson radius $R_h = 1.1/\ln(Q + m_\rho)$, which in the limit of a real photon, $Q \to 0$, matches the relevant value $R_h = 0.8$ fm used in Ref. [8]. Solving $n_h(T) = n_f$ yields

$$T_M(T, Q) \simeq 171 \ln(Q + m_\rho) \, \text{MeV}$$

as the limiting temperature through meson fusion (cf. $T_\pi \simeq 240$ MeV of [8]). In a similar analysis of the phase diagram of hadronic matter [8], the (Q−independent) limiting temperature $T_\pi \approx 240$ MeV was obtained.

N.B. The parameters appearing below should be rescaled with account for the replacement $m \to \ln(Q + m_\rho)$!

The density of point-like nucleons of mass M is at $T = 0$

$$n_b(\mu, T = 0) = \frac{2}{3\pi^2}(\mu^2 - M^2)^{3/2},$$

or, by using the van der Waals approach of Ref. [6]

$$n_B(\mu, T = 0) = \frac{n_b(T, \mu)}{1 + n_b(T, \mu)V_e}.$$

With increasing nucleon density, the empty vacuum disappears for

$$\bar{n}_v \simeq 2/V_h(Q) \approx 0.93 \ln(Q + m_\rho)^3 \, \text{fm}^{-3},$$

which, for real photons ($Q = 0$) corresponds to about 5.5 times standard nuclear density. Solving $n_B(T = 0, \mu) = \bar{n}_v$ gives $\mu_v \simeq 1.12 \ln(Q + m_\rho) \, \text{GeV}$ for the limiting baryochemical potential at $T = 0$.

In the region of low or intermediate μ in the $T\mu$ diagram, one can approximate the density of point-like nucleons by the Boltzmann limit

$$n_b(\mu, T) \simeq \frac{2T^3}{\pi^2} \Big(\frac{M}{T}\Big)^2 K_2(M/T)e^{\mu/T} \simeq \frac{T^3}{2} \Big(\frac{2M}{\pi T}\Big)^{3/2} e^{(\mu - M)/T}.$$

As a result, one obtains a family of curves for the phase diagram $T(\mu)$ and for the EoS $p(T)$ for various values of Q and x (cf. Figs. 3 and 4 of Ref. [8]).

5   Conclusions

In this talk a new approach to the saturation phenomena in deeply virtual processes – DIS, DVCS, VMP – diffractive and non-diffractive – is suggested. The basic idea is a physical one: Bjorken scaling implies that the nucleon in DIS and related processes is a system of weakly interacting partonic gas, that can be described by means of Bose-Einstein or Fermi statistics. As the density increases (with decreasing x and relevant Q^2), seen as the violation of Bjorken scaling, the system reaches a coalescence point where the gas condenses, eventually to a liquid. The thermodynamic properties of this transition can be only conjectured, and further studies are needed to quantify this phenomenon.

Of two models presented in this note, more promising seems be the first one (Sec. 3). Further studies will show its viability.

An alternative measure of the onset of saturation and expected change of phase can be related to non-linear evolution equations. Saturation and a phase transition are expected when the non-linear contribution overshoots the linear term.

The phenomenon discussed in the present note may have much in common with the colour glass condensate proposed in the context of heavy ion collisions (see, e.g. [16] and earlier references therein). Apart from similarities (condensation of quarks and gluons as $x \to 0$) there are apparent differences: for example, hydrodynamical flow is not expected in DIS. In any case, the dynamics of the strong interaction is the same in lepton-hadron, hadron-hadron and heavy ion collisions.

Acknowledgements

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