Critical phenomena in hadron- and lepton-induced reactions

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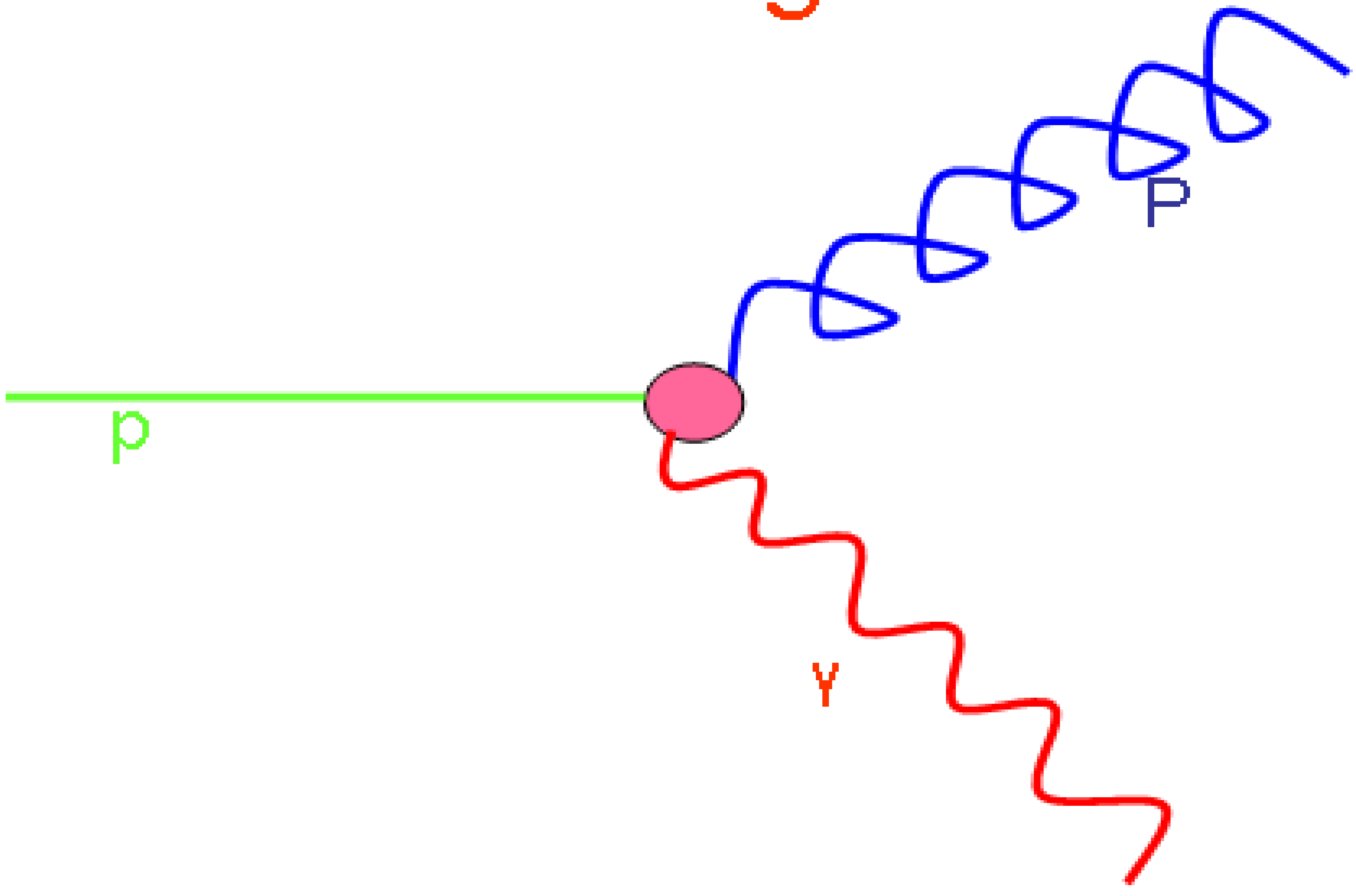
BITP, Kiev

Who is testing whom?



$$A(s,t,Q^2)$$

$$A(s,t,Q^2=m^2)$$ (on mass shell)

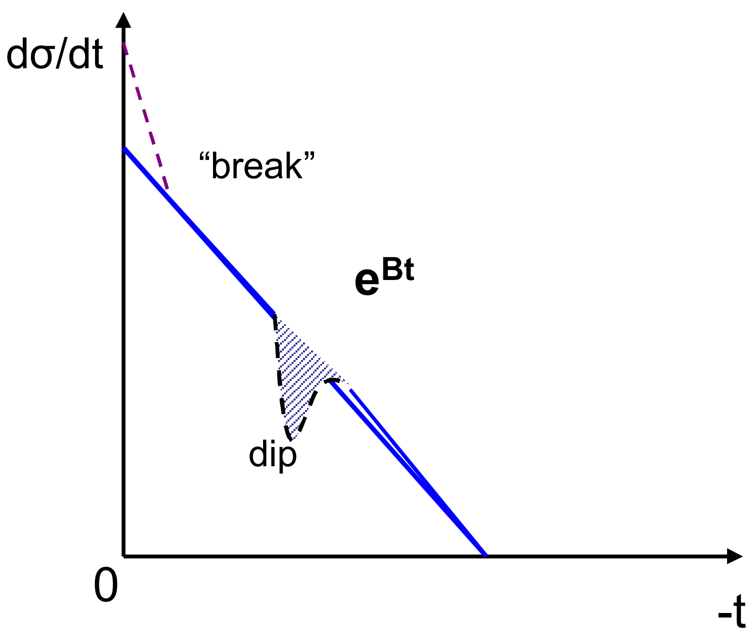
$$\Im m A(s,t=0,Q^2) \sim F_2$$ DIS

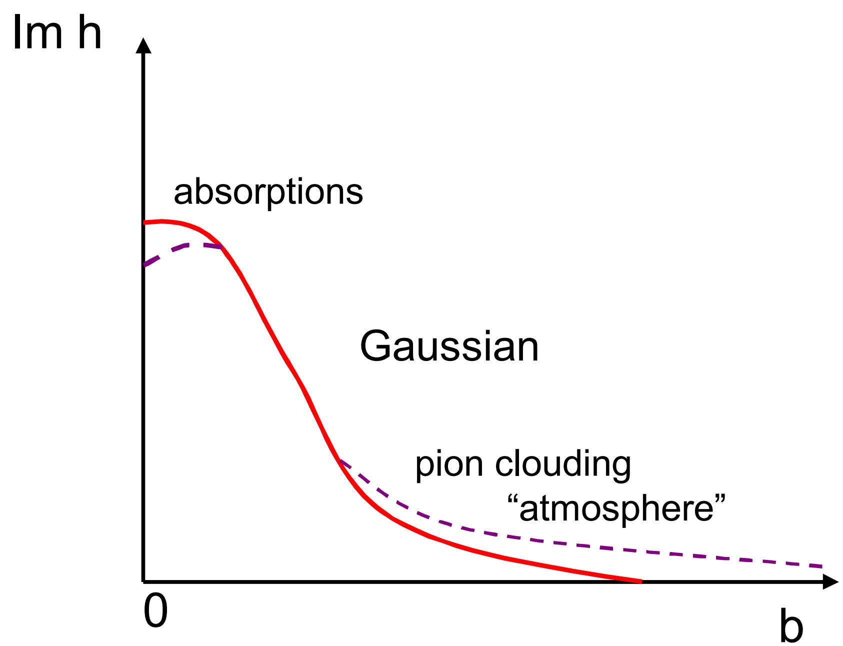
Geometrical scaling (GS), saturation and unitarity

1. On-shell (hadronic) reactions (s,t, Q^2=m^2);

$t \leftarrow\rightarrow b$ **transformation:** $h(s,b) = \int_0^\infty d\sqrt{-t}\sqrt{-t}A(s,t)$

and dictionary:



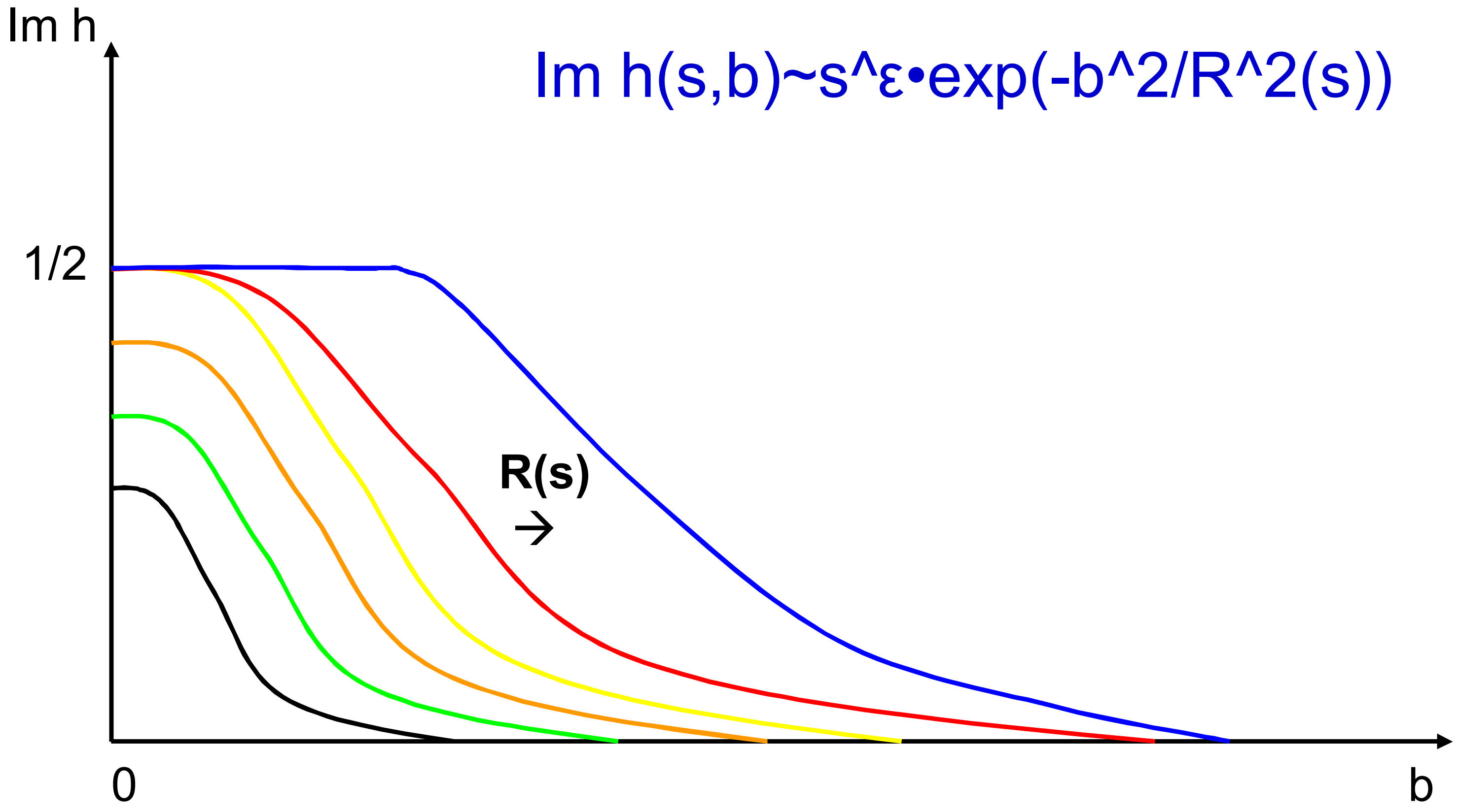


GS: A(s,b)=f(b/R(s)), BDL-saturation and s-channel unitarity

A GS example: A(s,b)~exp(-b^2/4R^2(s)).
A simple Regge (Pomeron) pole does not scale becase of an extra 1/ln s factor in the amplitude. A double Regge pole has the unique property of possessing GS and being compatible with unitarity.

GS was popular when observed (σ(el)~ σ(inel)~ σ(t)~B~lns, σ(el)/σ(t)~const) in the whole range of the CERN ISR, but was abandoned after, when the ratio σ(el)/σ(t) was found to rise definitely at the SPS and beyond. Interestingly, the departure from GS started well below the BDS saturation. We still do not know if GS at the ISR was merely an accident and whether there is some dynamical reason for its violation beyond the ISR energy region.

GS at the ISR



Does GS imply saturation? Not necessarily!

$ImH(s,b) = |h(s,b)|^2 + G_{in}(s,b)$, (h is associaed with the "opacity), Here from: $0 \leq |h(s,b)|^2 \leq \Im h(s,b)) \leq 1$. The Black Disc Limit (BDL) corresponds to $\Im h(s,b) = 1/2$, provided $h(s,b) = i(1 - \exp[i\omega(s,b)]/2$, with an imaginary eikonal $\omega(s,b) = i\Omega(s,b)$.

There is an alternative solution, that with the "minus" sign in $h(s,b) = [1 \pm \sqrt{1 - 4G_{in}(s,b)}]/2$, giving (S.Troshin and N.Tyurin (Protvino)): $h(s,b) = \Im u(s,b)/[1 - iu(s,b))]$, Solutions of the unitarity equation:

$$h(s,b) = \frac{1}{2}[1 \pm \sqrt{1 - 4G_{in}(s,b)}];$$

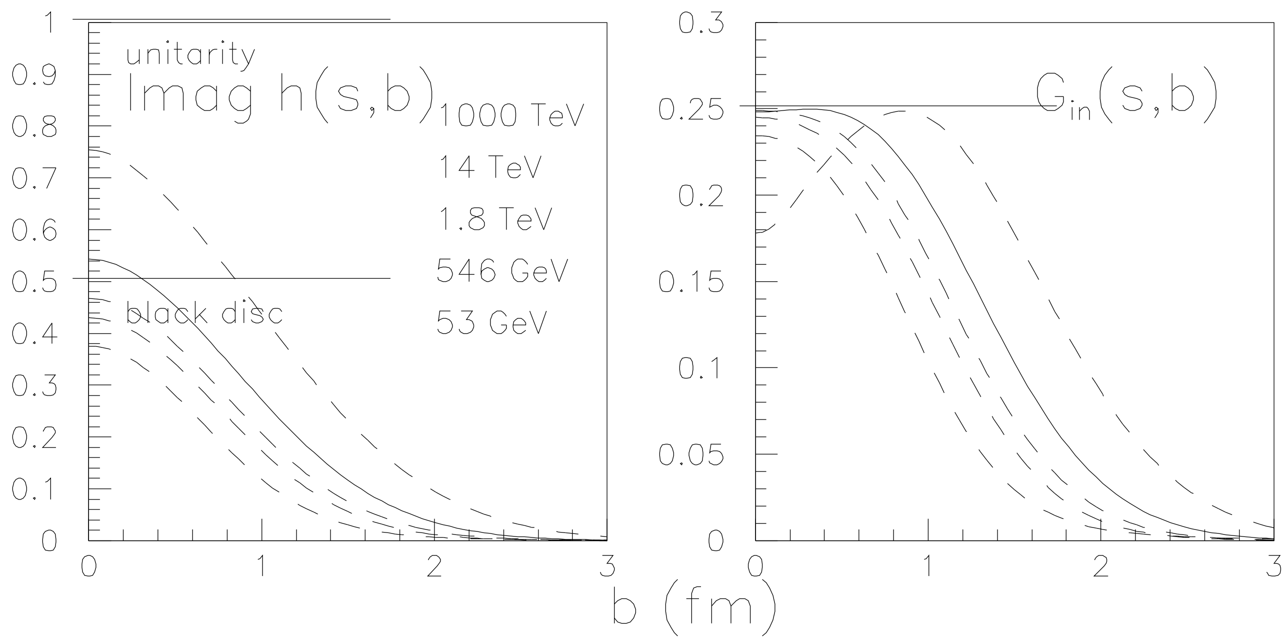
a) "Minus" (eikonal, exponential):

$$h(s,b) = \frac{i}{2}(1 - exp[i\omega(s,b)]);$$

b) "Plus" (rational):

$$h(s,b) = \frac{u(s,b)}{1 - iu(s.b)};$$

"Protvino-Serpukhov" (Logunov, Savrin, Troshin, Tyurin) P. Desgrolard, L. Jenkovszky, B. Struminsky, Eur. Phys. J. **C11** (1999) 143.



P. Desgrolard, L.L. Jenkovszky and B.V. Struminsky

GS in DIS (K.Golec-Biernat and M.Wüsthof, 1999):

$$F_2(x,Q^2) = Q^2/4\pi^2\alpha_{em}(\sigma_T + \sigma_L),$$

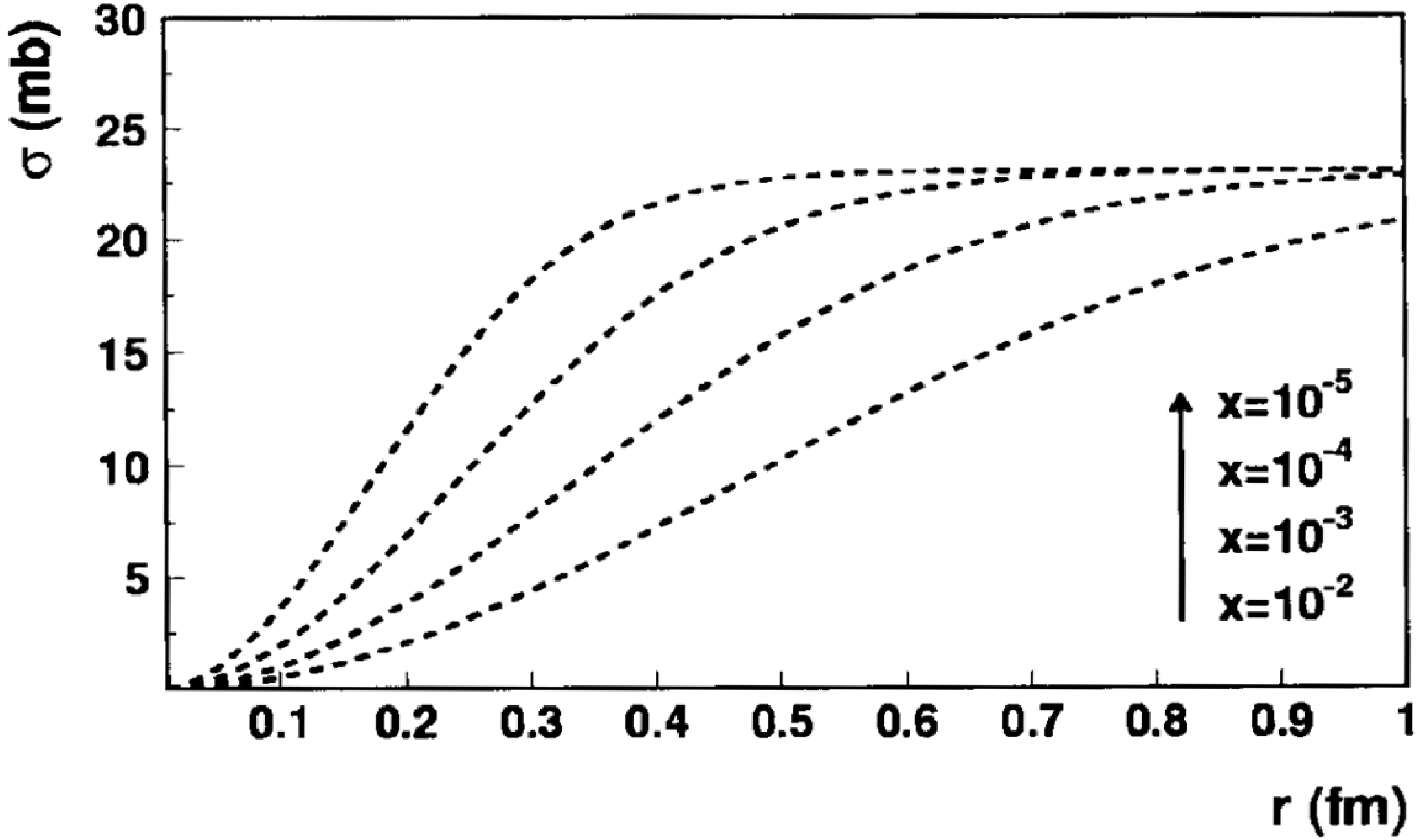
$$\sigma_{T,L} = \int d^2\mathbf{r}dz|\Psi_{T,L}(\mathbf{r},z,Q^2)|\hat{\sigma}(x,r),$$

$$\hat{\sigma}(x,r) = \sigma_0[1 - \exp[-r^2/(4R_0^2(x))]],$$

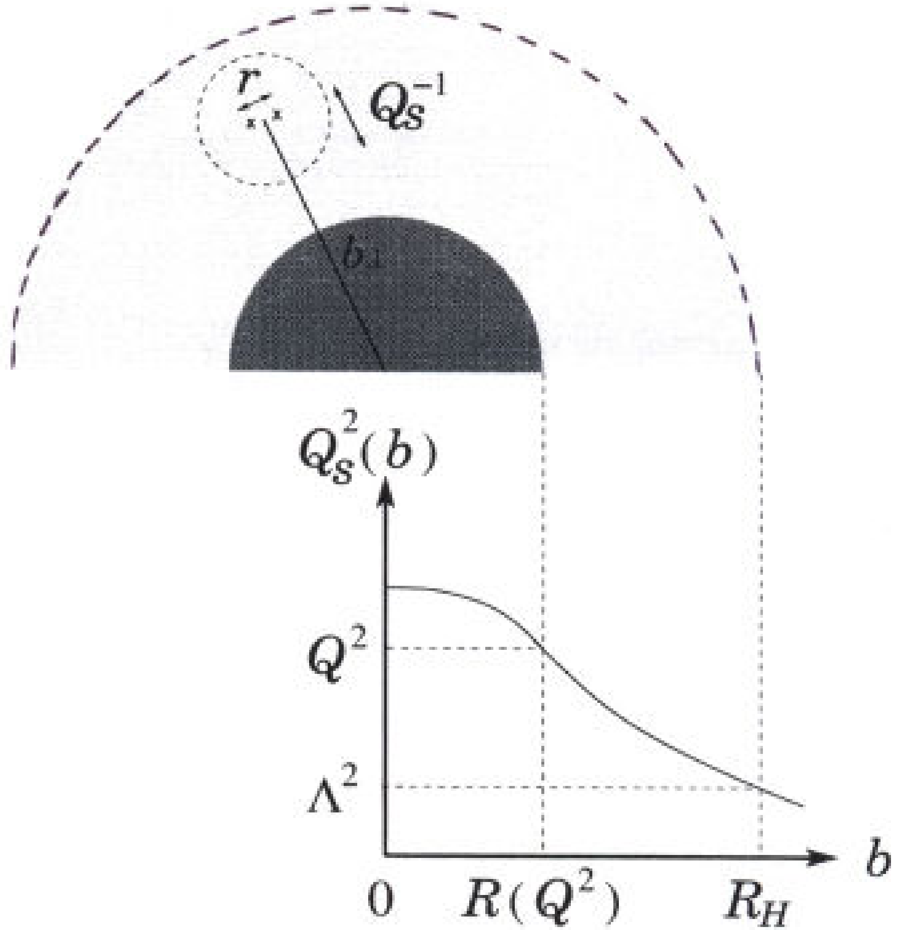
$$R_0^2(x) = \frac{1}{Q_0^2}(x/x_0)^\lambda,$$

$Q_0^2 = 1GeV^2$, $\sigma_0 = 23mb$, $x_0 = 3 * 10^{-4}$, $\lambda = 0.29$, $Q_s(x) \sim 1/R_0(x)$ being the saturation scale.

In spite of similarities, the saturation radia in hadron- and lepton-induced reactions have different physical meaning


σ (mb)
30
25
20
15
10
5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r (fm)
x=10^-5
x=10^-4
x=10^-3
x=10^-2

Hadron radius and saturation radius



(E.Ferreiro, E.Iancu. K.Itakara, and L.Mcerran, hep-ph:0206241)

Explicit model (P. Desgrolard, L. Jenkovszky, F. Paccanoni)

$$F_2^{(S,0)}(x,Q^2) = A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x,Q^2)},$$

with the "effective power"

$$\widetilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ell n\left(1 + \gamma_2 \ell n\left[1 + \frac{Q^2}{Q_0^2}\right]\right),$$

and

$$\Delta(x,Q^2) = \left(\widetilde{\Delta}(Q^2)\ell n\frac{x_0}{x}\right)^{f(Q^2)},$$

where

$$f(Q^2) = \frac{1}{2}\left(1 + e^{-Q^2/Q_1^2}\right).$$

SF for both low- and high x, S+NS:

$$F_2(x,Q^2) = F_2^{(S)}(x,Q^2) + F_2^{(NS)}(x,Q^2)$$

where

$$F_2^{(S)}(x,Q^2) = F_2^{(S,0)}(x,Q^2)\ (1-x)^{n(Q^2)},$$

with

$$n(Q^2) = \frac{3}{2}\left(1 + \frac{Q^2}{Q^2+c}\right),$$

$c = 3.5489$ GeV^2.

$$F_2^{(NS)}(x,Q^2) = B\ (1-x)^{n(Q^2)}\ x^{1-\alpha_r}\ \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_r}\ .$$

Various limits:

a) Large Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to \infty) \to A \ \exp\sqrt{\gamma_1 \ell n \ell n \frac{Q^2}{Q_0^2} \ \ell n \frac{x_0}{x}} \ ,$$

(asymptotic solution of the GLAP equation).

b) Low Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \, e^{\Delta(x,Q^2 \to 0)} \ \left(\frac{Q^2}{a}\right)^{1+\widetilde{\Delta}(Q^2 \to 0)}$$

with

$$\widetilde{\Delta}(Q^2 \to 0) \to \epsilon + \gamma_1 \gamma_2 \left(\frac{Q^2}{Q_0^2}\right) \ \to \ \epsilon,$$

$$f(Q^2 \to 0) \to 1,$$

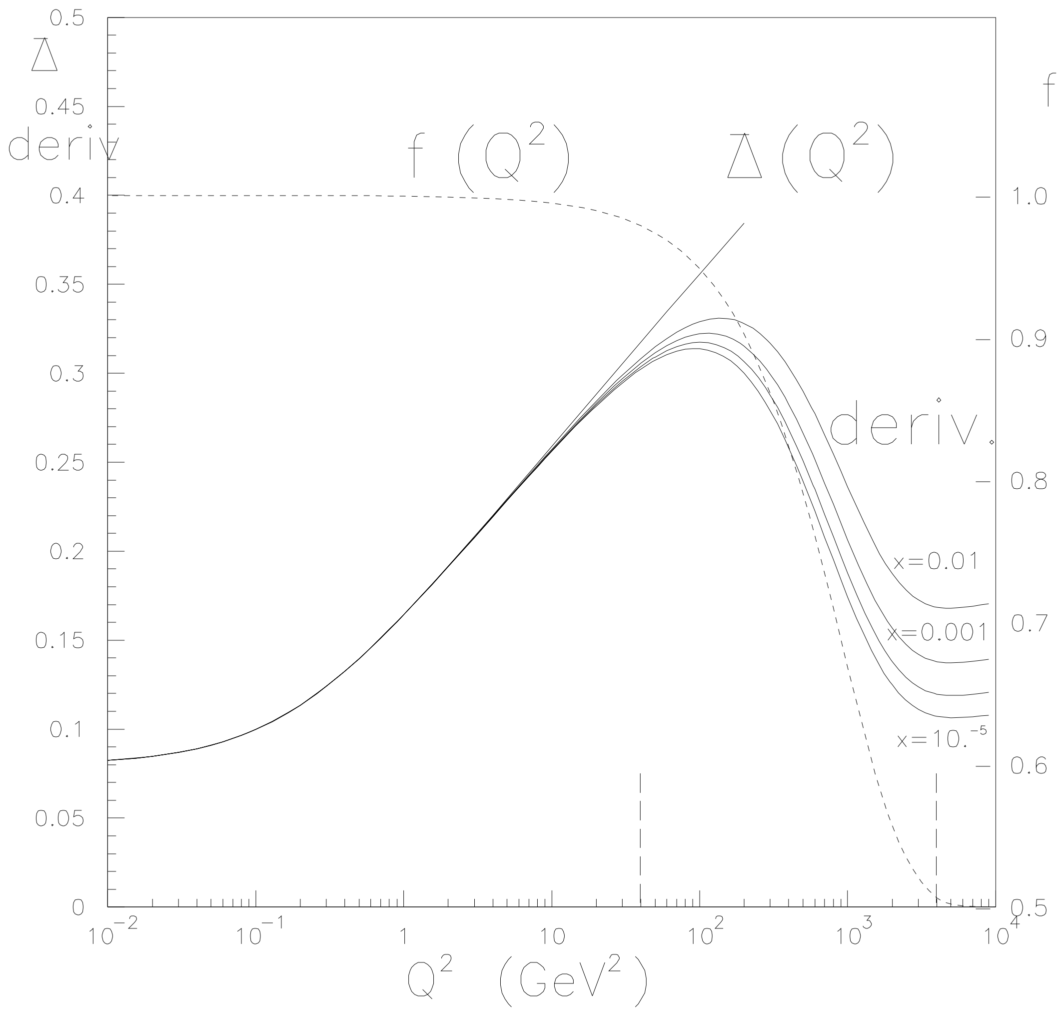
from which

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \ \left(\frac{x_0}{x}\right)^{\epsilon} \ \left(\frac{Q^2}{a}\right)^{1+\epsilon} \ \propto (Q^2)^{1+\epsilon} \ \to 0 \ ,$$

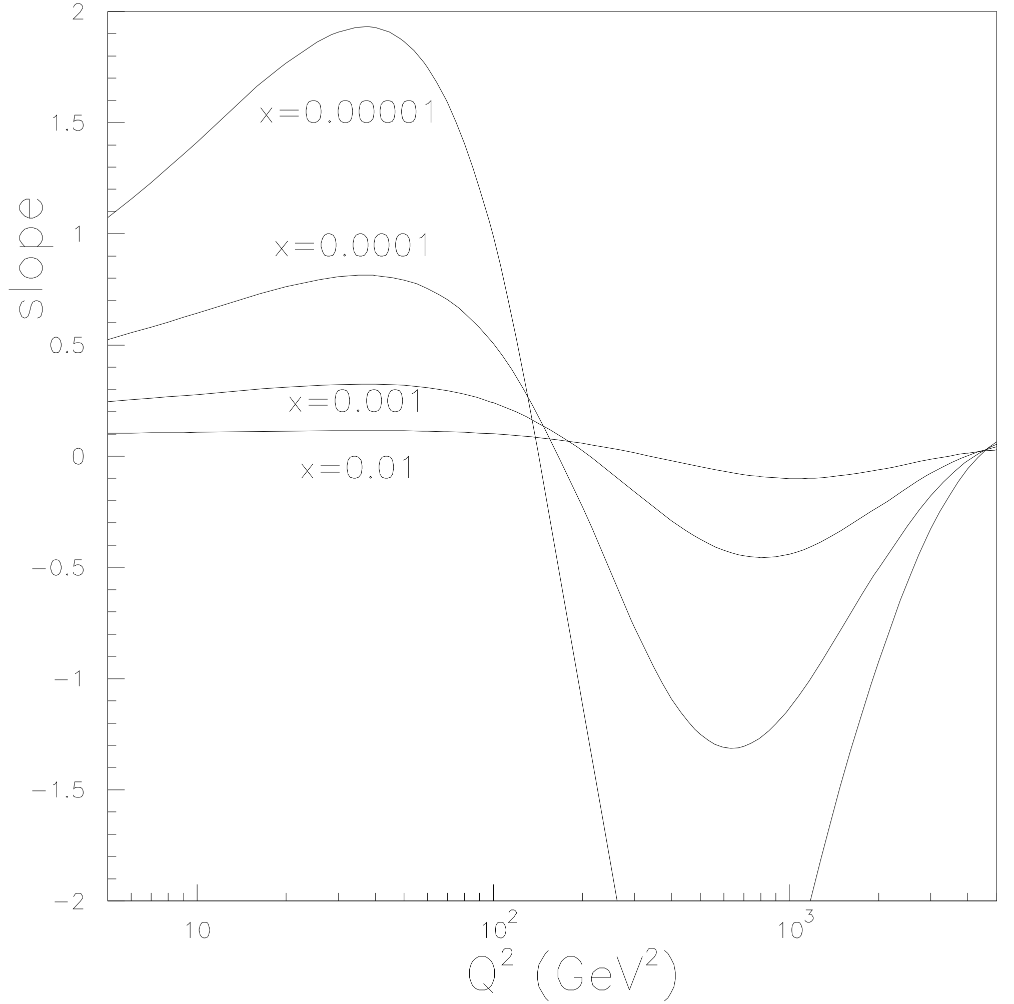
Q^2- and $x-$ SLOPES; SATURATION (and PHASE TRANSITION?)

1. $\frac{\partial F_2}{\partial(\ell n Q^2)}$ as a function of x and Q^2.

2. $\frac{\partial \ell n F_2}{\partial(\ell n(1/x))}$ as a function of Q^2 for some x values.

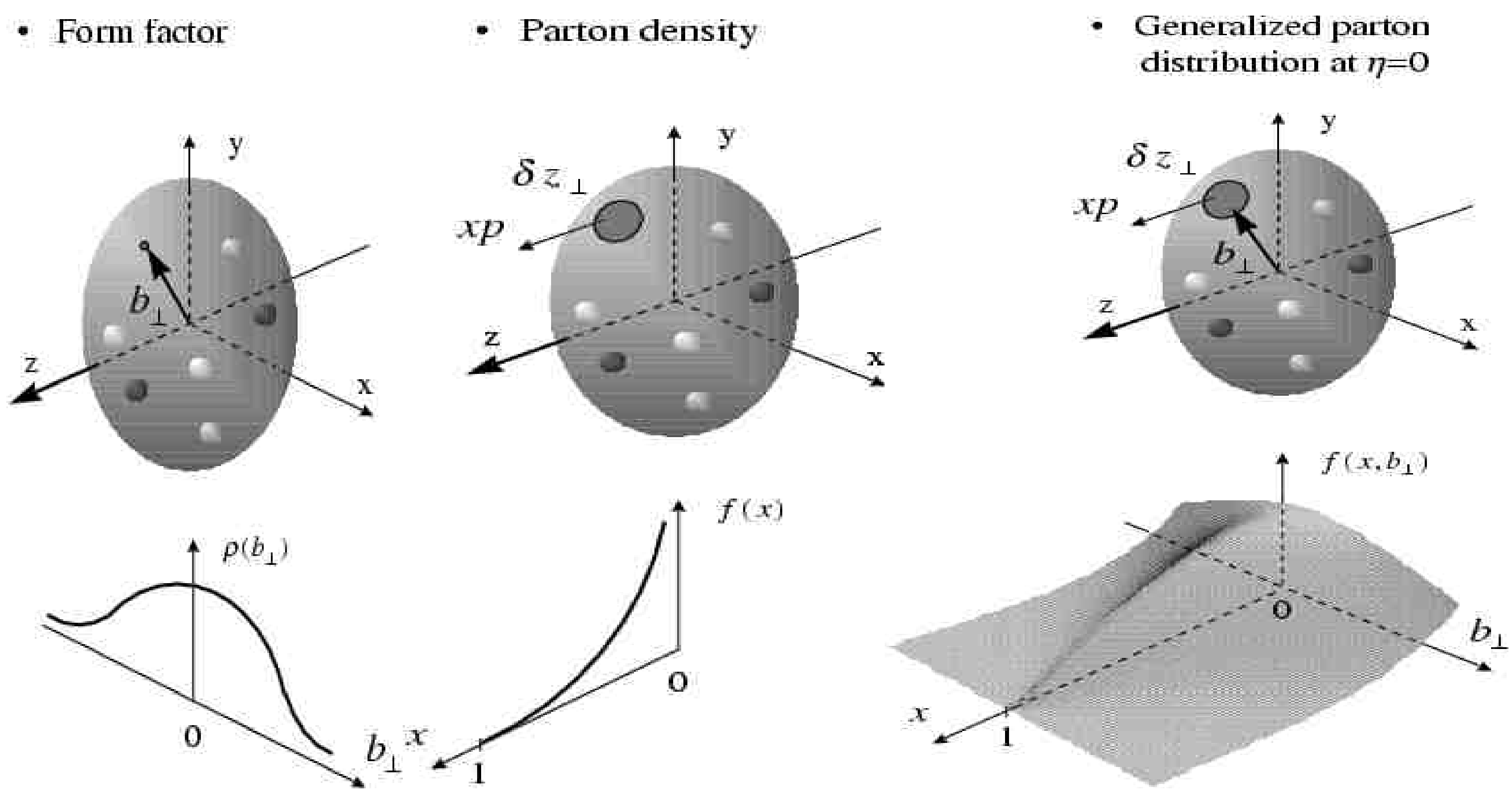


Δ
deriv
f (Q²)
Δ(Q²)
f
deriv.
x=0.01
x=0.001
x=10.⁻⁵
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1.0
0.9
0.8
0.7
0.6
0.5
10⁻²
10⁻¹
1
10
10²
10³
10⁴
Q² (GeV²)



slope
x=0.00001
x=0.0001
x=0.001
x=0.01
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
10
10^2
10^3
Q^2 (GeV^2)

Location of the saturation radius from a three-dimensional picture provided by hard exclusive reactions (DVCS or VMP)



To see the localization of the saturation radius one needs:

$$A(s,t,Q^2) \rightarrow A(s,t,Q^2=m^2) \text{ (on mass shell)}$$

$$A(s,t,Q^2) \rightarrow \Im m A(s,t=0,Q^2) \sim F_2 \quad \text{DIS}$$

Reconstruction of the DVCS amplitude from DIS

$$F_2 \sim \Im m A(\gamma^* p \rightarrow \gamma^* p)\Big|_{t=0} \rightarrow \Im m A(\gamma^* p \rightarrow \gamma p)\Big|_{t=0}$$

$$\rightarrow A(\gamma^* p \rightarrow \gamma p)\Big|_{t=0} \rightarrow A(\gamma^* p \rightarrow \gamma p)$$

or

$$\Im m A(\gamma^* p \rightarrow \gamma^* p)\Big|_{t=0} \sim F_2(x_B,Q^2) = x_B q(x_B,Q^2)$$

$$q(x_B,Q^2) \rightarrow q(\xi,\eta,t,x_B,Q^2) \rightarrow$$

?

$$\rightarrow \xi q(\xi,\eta,t,x_B,Q^2) = GPD(\xi,\eta,t,x_B,Q^2)$$

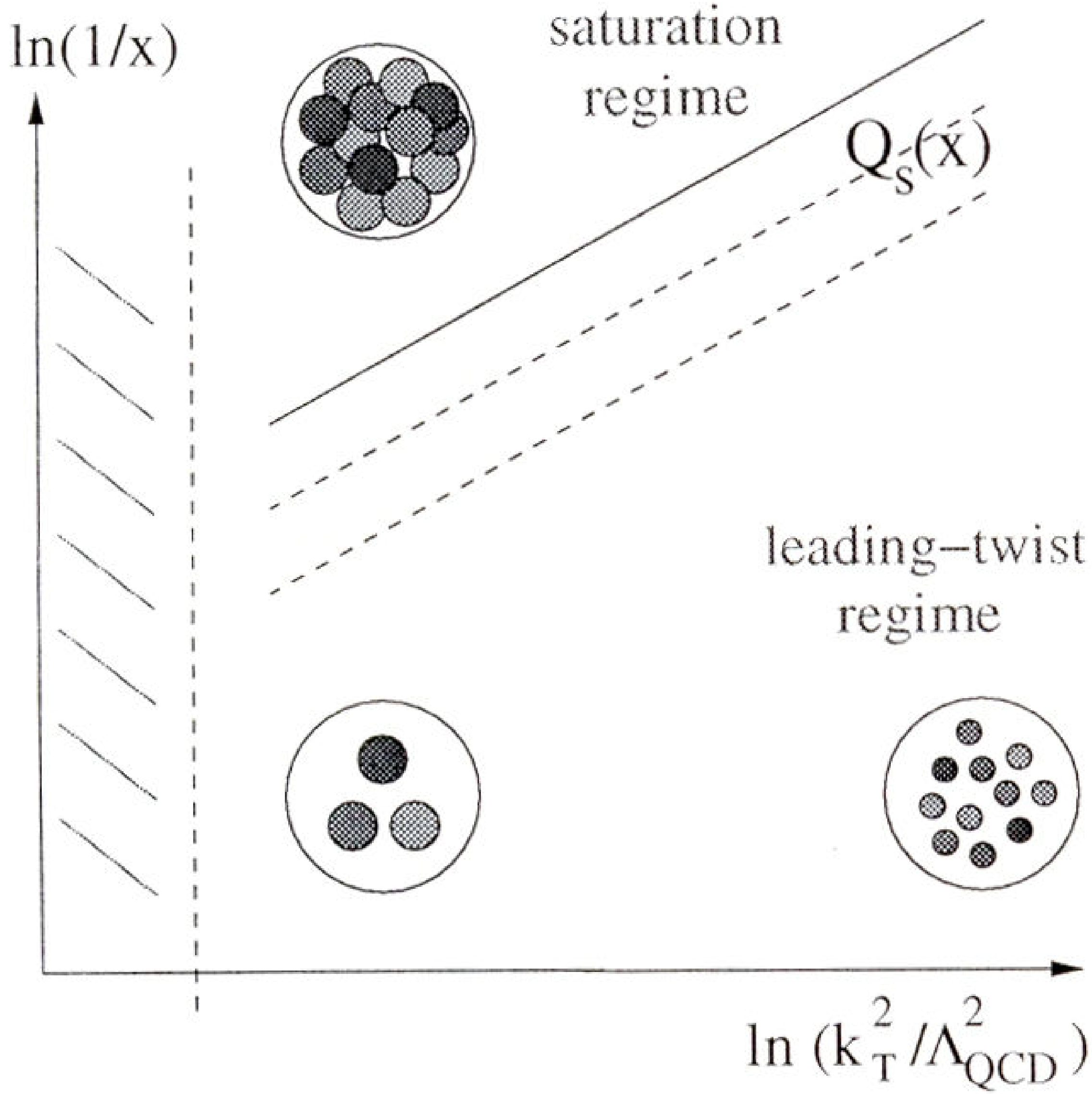
Scattering amplitude, with the correct analytic and asymptotic properties (M. Capua, S. Fazio, R. Fiore, L. Jenkovszky and F. Paccanoni, PLB, 2007)

$$A(s,t,Q^2)_{\gamma^* p\to\gamma p} = -A_0 e^{b\alpha(t)} e^{b\beta(z)} (-is/s_0)^{\alpha(t)} =$$

$$-A_0 e^{(b+L)\alpha(t)+b\beta(z)},$$

$$\beta(z) = \alpha(0) - \alpha_1 \ln(1-\alpha_2 z),$$

where $z = t - Q^2, \quad L \equiv \ln(-is/s_0).$



ln(1/x)
saturation
regime
$Q_s(x)$
leading–twist
regime
ln (k_T^2/Λ_{QCD}^2)

Summary:

(Collective properties and critical phenomena in DIS and in DVCS)

The nucleon/nucleus excited by an energetic photon may be treated below the saturation scale as a system of quasi-free partons (quarks and gluons), obeying the equation of state p(t)~T^4, similar to that used in the collision of nucleons and nuclei. With increasing density, the free gas on partons may undergo a phase transition similar to the vapor-liquid phase transition. The details of the thermodynamic properties (EoS) are not yet known. Tentatively, one can start from the ideal EoS p(t), p(T)~T^4, corresponding to the regime of the linear DGLAP evolution equation, to be replaced by a new, nonlinear phase of the saturated matter p(T)~T^4+f(T). The details of this phase transition are not know, but it occurs at a critical temperature T(R_s(x)) which depends on the saturation radius.

The end

Black disc limit?

$$h(s,b) = \frac{1}{2s}\int_0^\infty dqqJ_o(bq)A(s-q^2), \quad q = \sqrt{-t};$$

$$Imh(s,b) = |h(s,b)|^2 + G_{in}(s,b);$$

Unitarity limit:

$$0 \le |h(s,b)|^2 \le Imh(s,b) \le 1;$$

Black Disc Limit (BDL):

$$\sigma_{el} = \sigma_{in} = 1/2\sigma_{tot} \rightarrow Imh = 1/2.$$