# Saturation in nuclei

T. Lappi[1,2]

[1]IPhT, CEA/Saclay

[2]University of Jyväskylä

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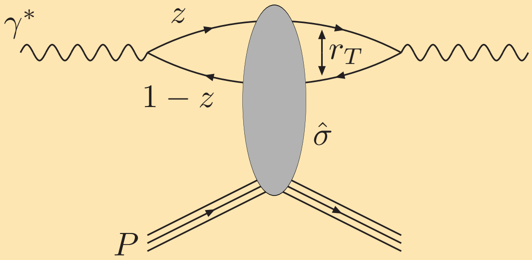
# Outline

- Connection between DIS and HIC: Wilson line
- Value of $Q_s$
- Applications
  - Gluon multiplicity at LHC AA collisions
  - Multiplicity distributions
  - Inclusive nuclear diffraction at eRHIC and LHeC

See **`www.eic.bnl.gov`** **`www.lhec.org.uk`**

# DIS in dipole frame

Small $x$: factorize $\gamma^*$ wavefunction — dipole cross section




Use:

- $S$-matrix real
- optical theorem

- $\Psi^{\gamma}_{L,T} \sim K_{0,1}\left(\sqrt{z(1-z)}Q|\mathbf{r}_T|\right)$
  - momentum scale $Q^2 \sim 1/\mathbf{r}_T{}^2$
- Diffractive: $t$ is FT of $\mathbf{b}_T$.

# DIS Observables from dipole cross section

$$\sigma_{\rm dip}(x, \mathbf{r}_T, \boldsymbol{\Delta}) = \int \mathrm{d}^2\mathbf{b}_T \frac{\mathrm{d}^2\sigma_{\rm dip}(x, \mathbf{r}_T, \mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} e^{i\mathbf{b}_T\cdot\boldsymbol{\Delta}}, \qquad \boldsymbol{\Delta}^2 = -t$$

▼

**Inclusive**

$$\sigma_{L,T}^{\gamma^* p} = \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma_{\rm dip}(x, \mathbf{r}_T)$$

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Inclusive **diffractive** (elastic dipole–target)

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$$\frac{\sigma_{L,T}^{D,tot}}{\mathrm{d}t} = \frac{1}{16\pi} \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma_{\mathrm{dip}}^2(x, \mathbf{r}_T, \boldsymbol{\Delta})$$

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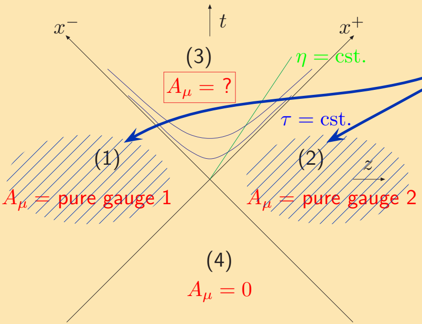
$$\frac{\sigma_{L,T}^{D,V}}{\mathrm{d}t} = \frac{1}{16\pi} \left| \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left(\Psi^{\gamma}\Psi^{*V}\right)_{L,T} \sigma_{\rm dip}(x, \mathbf{r}_T, \mathbf{\Delta}) \right|^2$$

# Calculating gluon production in AA from CGC

**C**lassical **Y**ang-**M**ills




2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}_T)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

$$U_{(1,2)}(\mathbf{x}_T) = Pe^{ig\int dx^- \frac{\rho(\mathbf{x}_T, x^-)}{\nabla_T{}^2}}$$

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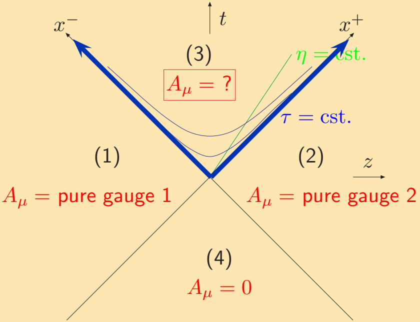


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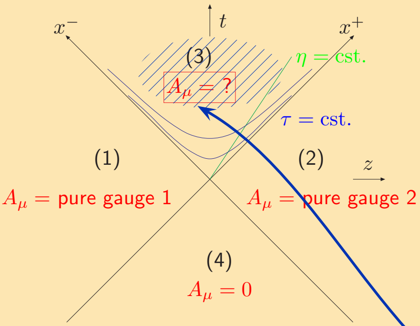
At $\tau = 0$:

$$\begin{aligned} A^i\Big|_{\tau=0} &= A^i_{(1)} + A^i_{(2)} \\ A^\eta\big|_{\tau=0} &= \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}] \end{aligned}$$

# Calculating gluon production in AA from CGC

## Classical Yang-Mills

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Solve numerically Yang-Mills equations for $\tau > 0$

This is the **glasma** field. ▶ Average over $\rho$.

# Relation between DIS and AA

The same Wilson line

$$U(\mathbf{x}_T) = P \exp\left\{ig \int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}_T, x^-)\right\}$$

Pure gauge fields (LC gauge) in the initial condition for AA:

$$A^i_{\text{(one nucleus)}} = \frac{i}{g} U(\mathbf{x}_T) \partial_i U^\dagger(\mathbf{x}_T)$$

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And dipole cross section for DIS

$$\hat{\sigma}(\mathbf{r}_T) = \int \mathrm{d}^2\mathbf{b}_T \frac{1}{N_\mathrm{c}}\left\langle 1 - U^\dagger\left(\mathbf{b}_T + \frac{\mathbf{r}_T}{2}\right) U\left(\mathbf{b}_T - \frac{\mathbf{r}_T}{2}\right)\right\rangle$$

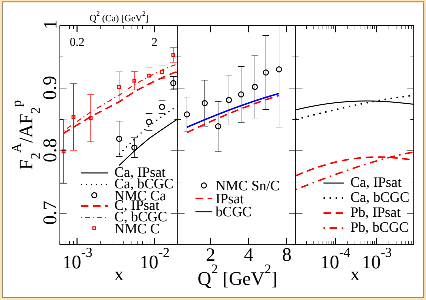
$U$ unitary ▶ automatically satisfy unitarity limit for $S$-matrix

$1/Q_\mathrm{s}$ is correlation length in $\mathbf{r}_T$

# Value of $Q_s$ at RHIC

## Value of $Q_s$ in nuclei, estimates based on DIS

Freund, Rummukainen, Weigert (-02)
Kowalski, Teaney (-03)
Armesto, Salgado, Wiedemann (-04)
Kowalski, T.L., Venugopalan (-07), plot




(Fit existing eA data)

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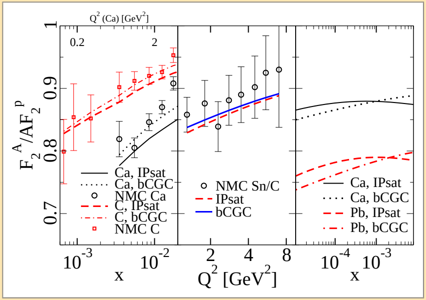


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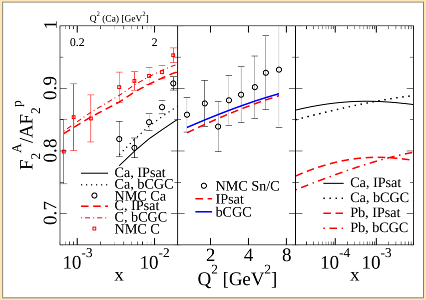


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CGC/glasma initial gluon multiplicity: $dN/dy = 1000$ for this $Q_s$

Kharzeev, Levin, Nardi, Krasnitz, Nara, Venugopalan, TL . . .

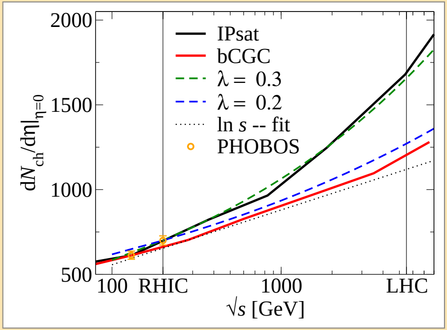
Numbers from DIS and AA work really well (even too well . . . )

# Value of $Q_s$ at LHC?

## What do we **not** know about $Q_s$

What is the CGC prediction for the LHC multiplicity?

$$\frac{dN}{d\eta} = c\frac{1}{\alpha_s}\pi R_A^2 Q_s^2$$



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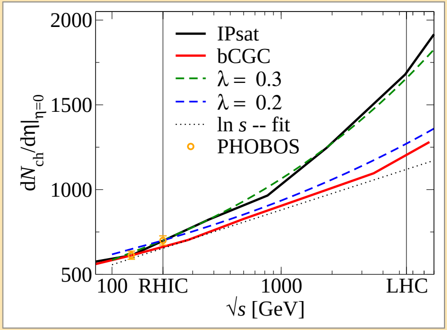


**Uncertainty dominated by energy dependence of $Q_s$**

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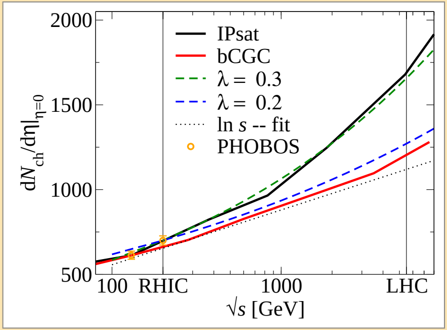


**Uncertainty dominated by energy dependence of $Q_s$**

- Fits to HERA give range $\lambda = 0.2 \ldots 0.3$ ($Q_s^2 \sim x^{-\lambda}$)
- AA at LHC will constrain interpretation of HERA

# Glittering Glasma and negative binomial

$$W[\rho] = \exp[-\int d^2\mathbf{x}_T \frac{\rho^a(\mathbf{x}_T)\rho^a(\mathbf{x}_T)}{g^4\mu^2}]$$

Correlations simple in MV model and dilute limit (small $\rho$)

- 2-particle correlation Dumitru, Gelis, McLerran, Venugopalan -08
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Moment $m_q \equiv \langle N^q \rangle$ − disc.

$$m_q = (q-1)!\,k\left(\frac{\bar{n}}{k}\right)^q$$

$$k \approx \frac{(N_c{}^2-1)Q_s^2 S_\perp}{2\pi}$$

$$\bar{n} = f_N \frac{1}{\alpha_s} Q_s^2 S_\perp$$

**This is a neg. bin.**

Old experimental observation
(approximative)
Natural consequence of glasma

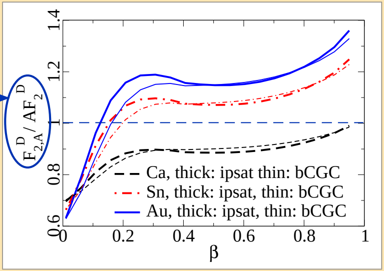
# Diffraction in nuclei is extreme QCD

**Compute diffractive structure functions $F_2^D$ in nuclei**

Diffraction enhanced in eA compared to ep

Frankfurt, Strikman, Guzey (-96 . . . ), Levin, Lublinsky (-99. . . ), Goncalves, Kugeratski, Navarra (-05 . . . ), Kowalski, T.L., Marquet, Venugopalan (-08, plot) , . . .

Talk by H. Kowalski in this workshop




(plot: coherent)

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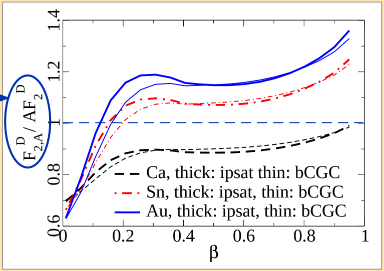


(plot: coherent)

Expect diffractive fraction to increase:

15% in ep ▶ 20%-25% for eA.

Combination of

- Enhanced diffraction
- Shadowing in inclusive

# Conclusions

- Common framework for AA, eA
- Dominant scale $Q_s$
- Examples
  - Initial gluon multiplicity in AA, uncertainty in $\lambda$
  - Multiplicity distribution
  - Diffraction enhanced in eA, how to measure this?