Amplitudes in the Coulomb Interference Region of pp and pp̄ Scattering

Anderson Kendi Kohara, Erasmo Ferreira, Takeshi Kodama

Instituto de Física, Universidade Federal do Rio de Janeiro
C.P. 68528, Rio de Janeiro 21945-970, RJ, Brazil

The determination of the parameters of the pp and pp̄ amplitudes used for the description of scattering in the Coulomb interference region is discussed, with emphasis put on the possibility that the effective slope observed in the differential cross section is formed by different exponential slopes in the real and imaginary amplitudes (called B_R and B_I). For this purpose a more general treatment of the Coulomb phase is developed.

The differential cross section data in the range from 19 to 1800 GeV are analysed with four parameters (σ, ρ, B_I, B_R), and it is observed that unique determination of the parameters cannot be obtained from the available data. Correlations in pairs of the four quantities are investigated, showing ranges leading to the smaller χ^2 values.

1 Low |t| Region and Coulomb Phase

Along about 50 years since the beginning of the experimentation of pp and pp̄ scattering at high energies, theoretical models and interpretations are continuously developed, but the experimental data stopped increasing in quantity or quality, restricting the progress in several aspects. The interest in these systems is now renewed [1], due to the data that will come from RHIC and LHC .

The disentanglement of the squared moduli that represent the measured quantities in terms of the imaginary and real amplitudes is not at all an easy phenomenological task. In previous analyses of the pp and pp̄ data, the real and imaginary amplitudes were considered as having the same exponential dependence $\exp(Bt/2)$, where B is the slope of the log plot of $d\sigma/dt$. This simplifying assumption is not adequate, according to dispersion relations [2] and according to the theorem of A. Martin [3] that says that the position of the zero of the real amplitude is close and approaches $t = 0$ as the energy increases. Both results indicate that the slope of the real amplitude should be larger, and in the present work we investigate the description of the Coulomb interference region allowing for different real and imaginary slopes.

In elastic pp and pp̄ collisions, the combined nuclear and Coulomb amplitude is written

$$F^{C+N}(s,t) = F^C(s,t)e^{i\alpha\Phi(s,t)} + F_R^N(s,t) + i\,F_I^N(s,t) \ , \tag{1}$$

where the Coulomb part F^C and the proton form factor are

$$F^C = (-/+)\,\frac{2\alpha}{|t|}\,F_{\text{proton}}^2 \quad , \quad F_{\text{proton}} = (0.71/(0.71 + |t|))^2 \ .$$

Our treatment is simplified in the sense that we do not consider different form factors for spin flip amplitudes, assuming that this is fair enough for the purposes of this paper.

The phase Φ between Coulomb and nuclear amplitudes was first considered by H.A. Bethe and then by West and Yennie [4], followed by different evaluations worked out by several authors [5, 6, 7]. In the present work we extend these investigations in the very low $|t|$ range, considering the possibility of different slopes for the real and imaginary amplitudes.

For small angles we can approximate

$$F^N(s,t) \approx F_R^N(s,0)e^{B_R t/2} + iF_I^N(s,0)e^{B_I t/2} \; . \tag{2}$$

The parameter ρ and the Optical Theorem

$$\rho = \frac{F_R^N(s,0)}{F_I^N(s,0)} \;\; , \;\; \sigma = 4\pi \, (0.389) \, \text{Im} \, F_I^N(s,0)$$

and the slopes B_R, B_I are used to parametrize the differential cross section for small $|t|$. In these expressions, σ is in milibarns and the amplitudes F_R, F_I are in GeV^{-2} .

For low $|t|$ we have the usual slope B observed in $d\sigma/dt$ data

$$\frac{d\sigma}{dt} = \left| \frac{d\sigma}{dt} \right|_{t=0} e^{Bt} \; , \; \text{with} \;\; B = \frac{\rho^2 B_R + B_I}{1 + \rho^2} \; .$$

In our evaluation of $\Phi(s,t)$ we start from the expression obtained by West and Yennie [4]

$$\Phi(s,t) = (-/+) \left[\ln \left(-\frac{t}{s} \right) + \int_{-4p^2}^{0} \frac{dt'}{|t'-t|} \left[1 - \frac{F^N(s,t')}{F^N(s,t)} \right] \right] \; , \tag{3}$$

where the signs $(-/+)$ are applied to the choices pp/p$\bar{\text{p}}$ respectively. The quantity p is the proton momentum in center of mass system, and at high energies $4p^2 \approx s$.

For small $|t|$, assuming that $F^N(s,t')$ keeps the same form for large $|t'|$, we have

$$\frac{F^N(s,t')}{F^N(s,t)} = \frac{c}{c+i} \, e^{B_R(t'-t)/2} + \frac{i}{c+i} \, e^{B_I(t'-t)/2} \; , \; \text{with} \;\; c \equiv \rho e^{(B_R - B_I)t/2} \; .$$

The integrals that appear in the evaluation are reduced to the form [7]

$$I(B) = \int_{-4p^2}^{0} \frac{dt'}{|t'-t|} \left[1 - e^{B(t'-t)/2} \right] \; , \tag{4}$$

that is solved in terms of exponential integrals [8] as

$$I(B) = E_1 \left(\frac{Bs}{2} \right) - E_i \left(-\frac{Bt}{2} \right) + \ln \left(\frac{Bs}{2} \right) + \ln \left(-\frac{Bt}{2} \right) + 2\gamma \; . \tag{5}$$

where we have assumed the high energy simplification $4p^2 + t \to s$.

The real and imaginary parts of the phase are then written

$$\Phi_R(s,t) = (-/+) \left[\ln \left(-\frac{t}{s} \right) + \frac{1}{c^2+1} \left[c^2 I(B_R) + I(B_I) \right] \right] \; ,$$

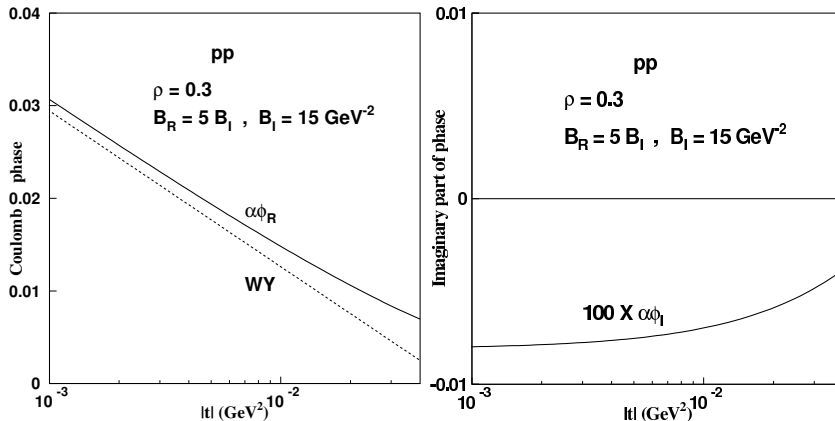$$\Phi_I(s,t) = (-/+) \frac{c}{c^2+1} \left[I(B_I) - I(B_R) \right] \; .$$

Figure 1: Comparison of the Coulomb phase $\alpha\Phi_R$ from our calculation with the West-Yennie (WY) expression in Eq. (7). The unrealistic large values of parameters $\rho = 0.3$ and $B_R/B_I = 5$ are chosen in order to enhance differences. The calculation is made at $\sqrt{s} = 50\,\text{GeV}$, but the values of $\alpha\Phi_R(s,t)$ do not show explicit dependence on s up to the LHC energy. The plot of $\alpha\Phi_I$ in the RHS shows extremely small values (notice the scale), about 100 times smaller than $\alpha\Phi_R$, so that we can safely put $\alpha\Phi_I$ equal to zero.

In the normalization that we use [9], with σ in mb and t in GeV^2, the practical expression for $d\sigma/dt$ in terms of the parameters σ, ρ, B_I and B_R is

$$\frac{d\sigma}{dt} = 0.389\,\pi\left[\left[\frac{\rho\,\sigma\,e^{B_R t/2}}{0.389 \times 4\pi} + F^C e^{\alpha\Phi_I}\cos(\alpha\Phi_R)\right]^2 + \left[\frac{\sigma\,e^{B_I t/2}}{0.389 \times 4\pi} + F^C e^{\alpha\Phi_I}\sin(\alpha\Phi_R)\right]^2\right]. \quad (6)$$

The expression for $\alpha\Phi_R$ can be compared with the expression from West and Yennie (WY)

$$\alpha\Phi_{WY} = (-/+)\alpha\left[\gamma + \ln\left(-\frac{Bt}{2}\right)\right]. \quad (7)$$

Figure 1 presents a numerical comparison, where we choose large unrealistic values $\rho = 0.3$, $B_R/B_I = 5$ to enhance possible differences. In the WY formula B is taken as the $d\sigma/dt$ average slope. The plot of $\alpha\Phi_I(s,t)$ in the RHS shows extremely small values so that the imaginary part can be put equal to zero.

2   Analysis of Experimental Data

The data published with absolute values for $d\sigma/dt$ depend on evaluation of luminosity, using Coulomb interference forms different from ours, or sometimes take some parameter values (such as the total cross section) from other experiments. To have independent evaluation of the absolute normalization we consider the published data as event rates dN/dt, using Eq. (6) with a free normalization factor a_5, namely we put the published data as

$$\frac{dN}{dt} = a_5\,\frac{d\sigma}{dt}\,, \quad (8)$$

Table 1: Results of the analysis at 19.4 - 62.5 GeV. N = number of points fitted, in general with $|t| \leq 0.01\,\text{GeV}^2$; a_5 is a normalization factor (see text). We give results for fixed ratios B_R/B_I = 1 and 2. For 23.5 GeV the normalization factor found is 1. $\chi^2 = \sum \chi_i^2/(N-5)$.

$\sqrt{s}$	N	σ(mb)	ρ	$B_I(\text{GeV}^{-2})$	B_R	a_5	χ^2
19.4	45	38.92±0.09	0.0064±0.0044	11.25±0.84	$B_I, 2B_I$	1.040±0.004	1.015
19.4	45	38.92±0.09	0.01 (fixed)	11.25±0.84	$B_I, 2B_I$	1.042±0.004	1.015
30.6	14	40.35±0.12	0.021±0.003	12.00±1.2	B_I	0.9878±0.0028	0.7301
30.6	14	40.34±0.12	0.021±0.003	12.00±1.2	$2B_I$	0.9877±0.0028	0.7301
44.7	26	41.36±0.23	0.071±0.008	12.80±0.25	B_I	1.0296±0.0141	0.7307
44.7	26	41.33±0.23	0.072±0.008	12.80±0.25	$2B_I$	1.0304±0.0139	0.7269
52.8	12	42.89±0.08	0.063±0.005	13.00±0.2	B_I	0.9761±0.0030	0.0846
52.8	12	42.86±0.08	0.063±0.005	13.00±0.2	$2B_I$	0.9766±0.0030	0.0859
62.5	17	42.46±0.03	0.1043±0.0017	13.15±0.2	B_I	1.0435±0.0012	1.056
62.5	17	42.38±0.03	0.1064±0.0018	13.15±0.2	$2B_I$	1.0465±0.0012	1.041

with $d\sigma/dt$ written as in Eq. (6), and a_5 is determined for each experiment.

We use CERN-Minuit programs (PAW and ROOT) to obtain correlations for the parameters (σ, ρ, B_R, B_I and a_5), for values of energy where the data seem to have more quality and quantity. As shown in the tables, the values of the physical parameters are sensitive to the freedom given by a_5, although this factor is nearly 1. However, the visual quality of fittings are very good in either case (with or without a_5), producing superposed curves.

The results obtained depend strongly on the set of N low $|t|$ points selected for the analysis. In close inspection of the data in the range from 19.4 up to 62.5 GeV we observe a subtle knee in the data at about $|t| = 0.01\,\text{GeV}^2$. The parameter values and χ^2 become unstable when the number of points is larger than N. Examination of the contributions shows that at this position the real nuclear plus the Coulomb parts rapidly become very small compared to the imaginary part, with an exchange of dominating roles.

In the range 19.4 - 62.5 GeV, ρ is very small, the real nuclear amplitude has little influence and the values of ρ and B_R are undetermined, while σ and B_I are more stable. Actually we may consider that at these energies B_I is the most reliably determined parameter, as it is less dependent on normalization and on influence of the real amplitude.

2.1 Energy Range 19.4 to 62.5 GeV

Our attempts to determine parameters are described below. Some results are given in Table 1, where we stress the role of the normalization factor a_5 in the determination of the parameters at the lower energies. At 19.4 GeV the normalization a_5 is essential to lead to a positive value of ρ. Table 2 presents the results without free normalization factor. Notice the differences in parameter values in comparison with the treatment using a_5. To compare χ^2 values, we must take into account that they mean $\sum \chi_i^2/(N-5)$ and $\sum \chi_i^2/(N-4)$ respectively.

Figure 2 shows examples of our fitting solutions, with data at 19.4, 44.7 and 62.5 GeV. At 44.7 and 62.5 GeV there are two lines, with and without factor a_5, and all with $B_R = 2B_I$, and following the parameters given in the same table. The lines are superposed, and cannot be distinguished by eye, although σ and ρ are different. We thus see the χ^2 criterion cannot determine parameters. It is remarkable that the fitting lines work very well much beyond the

Table 2: Results without free normalization factor. At 23.5 and 30.6 GeV the values of parameters for ratios 1 and 2 are the same in the digits exhibited. $\chi^2 = \sum \chi_i^2/(N-4)$.

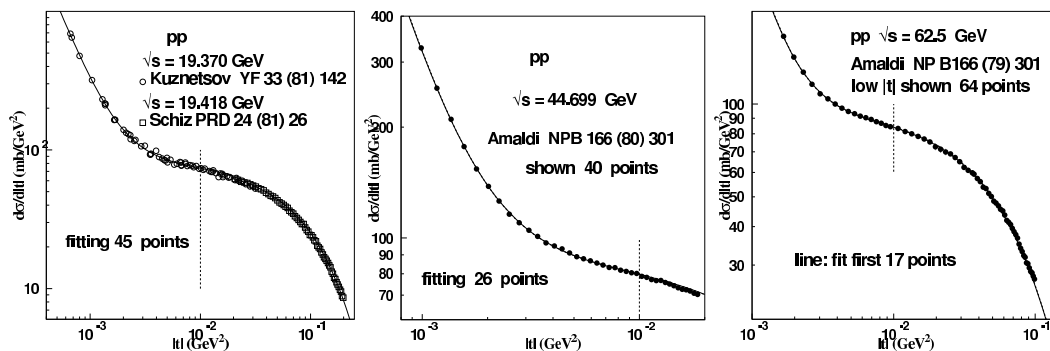| $\sqrt{s}$	N	σ(mb)	ρ	B_I(GeV^{-2})	B_R	a_5	χ^2
23.5	24	39.53±0.08	0.014±0.003	11.20±1.12	$B_I, 2B_I$	—	0.3028
30.6	14	40.30±0.10	0.034±0.003	12.00±1.0	$B_I, 2B_I$	—	0.7160
44.7	26	41.81±0.03	0.054±0.002	12.94±0.41	$2B_I$	—	0.9179
52.8	12	42.48±0.08	0.077±0.005	13.00±0.20	B_I	—	0.1422
52.8	12	42.46±0.08	0.078±0.005	13.00±0.20	$2B_I$	—	0.1395
62.5	17	43.27±0.03	0.086±0.002	13.15±0.12	B_I	—	1.141
62.5	17	43.25±0.03	0.086±0.002	13.15±0.12	$2B_I$	—	1.146

Figure 2: $d\sigma/dt$ and fitting curves at 19.4, 44.7 and 62.5 GeV. At 44.7 and 62.5 GeV there are two superposed curves, solid and dashed, obtained with and without normalization factor a_5. At 19.4 GeV the curve drawn up to 0.02 GeV^2 includes a factor $a_5 = 1.040$ to describe Kuznetsov's data. The vertical dashed lines show the limit of the N points used in the fitting. The curves follow well the data much beyond these limits.

fitting limit points (indicated by a vertical dashed lines) given by N; however fitting using a larger number instead of N introduces instability in the determination of the quantities. Thus, finding parameters and producing a line describing well the points are two different questions.

2.2 p$\bar{\text{p}}$ Scattering at 541, 1800 and 1960 GeV

Figure 3 shows the good agreement of $d\sigma/dt$ at low $|t|$, obtained from dN/dt by adjustment of the Coulomb interference, with the data of G. Arnison et al [17] in a $|t|$ range that partially covers the dN/dt event rate, and also the data of Bozzo et al. [18], including high $|t|$ values.

The lowest $|t|$ values reached in measurements of p$\bar{\text{p}}$ elastic scattering at about 540 GeV are reported with event rates [16] only, with 99 points in the interval $0.000875 \leq |t| \leq 0.11875$ GeV^2. We use the Coulomb interference to find the normalization factor connecting event rate and differential cross-section. We fit the 37 points up to $|t| = 0.01$, finding the normalization factor $a_5 = 10.6$ with $\chi^2 = 1.531$ and the parameter values

$$\sigma = (62.93 \pm 0.09)\,\text{mb}\ ,\ \ \rho = 0.145 \pm 0.004\ ,\ \ B_I = (15.50 \pm 0.40)\ \text{GeV}^{-2},\ \ B_R = 2B_I\ .$$

The data at 1800 GeV from two Fermilab experiments are limited by $|t| \geq 0.03$ GeV^2 and

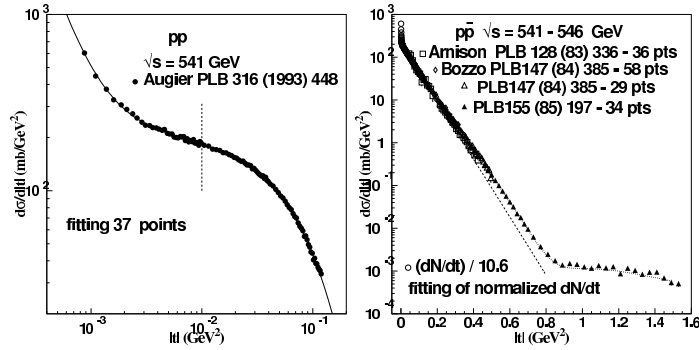Figure 3: The normalized event rate data at $541\,\mathrm{GeV}$ are in good agreement with the $d\sigma/dt$ data [17, 18] at the same energy and higher $|t|$ values.
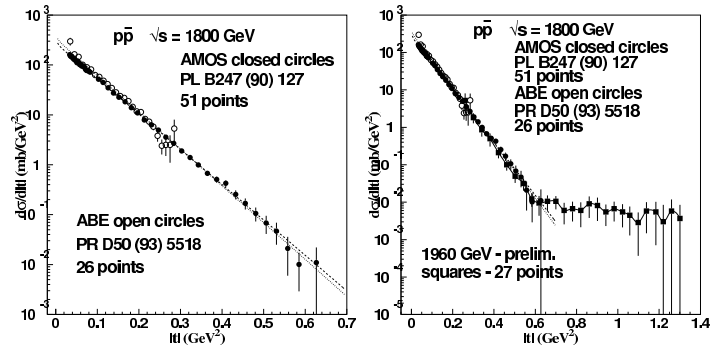

Figure 4: The $1800\,\mathrm{GeV}$ data from E710, with 51 points fitted by the dashed line, and from CDF, with 26 points fitted by the dotted line. The RHS plot shows also the preliminary 27 points at $1.96\,\mathrm{TeV}$.

do not reach the Coulomb interference region. Thus the determination of forward scattering parameters is doubtful. We obtain for E710 (Amos et al.) $\sigma = (72.748 \pm 0.186)\,\mathrm{mb}$, $\rho = 0.14$ (fixed), $B_I = (16.30 \pm 0.04)\ \mathrm{GeV}^{-2}$,$B_R = 7B_I$, with $\chi^2 = 0.60$. Figure 4 shows the fittings of the E-710 (Amos, with 51 points) and CDF (Abe, with 26 points).

3   Remarks and Conclusions

We have performed a detailed analysis of the $d\sigma/dt$ obtained in the ISR/SPS (CERN) and Tevatron (Fermilab) experiments. with freedom of different slopes for the real and imaginary amplitudes, namely $B_R \neq B_I$. The main conclusion is that different values of the slopes, in particular the possibility of $B_R > B_I$, in accordance with the expectations from Martin's theorem [3] and from dispersion relations [2], are perfectly consistent within the present values and errors of experimental data.

We investigate the four physical quantities relevant for the elastic forward processes, namely, σ, ρ, B_R and B_I. Studying the behaviour of χ^2 values and the statistically equivalent parameter ranges, we observe that the available data for small $|t|$ at the energies $20 - 2000\,\mathrm{GeV}$ are not sufficient for a good determination of these four parameters, which have ample freedom, with equivalent correlated ranges. The whole analysis shows that the description of the amplitudes requires efficient external inputs, such as dispersion relations.

We stress that this paper does not intend to give the best final values for the scattering parameters. Instead, we show that the analysis of the data leave rather ample possibilities that must be further investigated. We call attention to details that must be taken into account in the acquisition and treatment of future data from RHIC and LHC, expecting to have different situations, with much better statistics and accuracy in the measurements at the Coulomb interference region, together with a systematic energy scan program.

Acknowledgements

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