Amplitudes of Elastic pp and $p\bar{p}$ Scattering

Erasmo Ferreira , A.K. Kohara and T. Kodama

Universidade Federal do Rio de Janeiro, Brazil

June 24, 2009

Topics

Near Forward Scattering Amplitude

Coulomb Phase

Analysis of Data

- 19 - 63 GeV
- 541-546 GeV
- 1800-1960 GeV
- Amplitudes: slopes and zeros

Derivative Dispersion Relations for Amplitudes

- DDR for $t = 0$

Derivative Dispersion Relations for Slopes

A reference: A.K. Kohara, T. Kodama, E.F. : hep-ph 0905.1955

Complete (simplified) amplitude

$$F^{C+N}(s,t) = F^{C}(s,t)e^{i\alpha\Phi(s,t)} + F^{N}(s,t)$$

F^C is the Coulomb part

$$F^{C} = (-/+)\ \frac{2\alpha}{|t|}\ F^2_{\mathrm{proton}}$$

with the proton electromagnetic form factor

$$F_{\mathrm{proton}} = (0.71/(0.71+|t|))^2$$

Parametrization of Near Forward Scattering Amplitude
For small angles

$$F^N(s,t) \approx F_R^N(s,0)e^{B_R t/2} + iF_I^N(s,0)e^{B_I t/2}$$

Usually B_R and B_I are treated as having equal values. We allow

$$B_R \neq B_I$$

For low $|t|$, the strong differential cross section has approximate form with single exponential slope

$$\frac{d\sigma}{dt} = \left|\frac{d\sigma}{dt}\right|_{t=0} e^{Bt}$$

with

$$B = \frac{\rho^2 B_R + B_I}{1+\rho^2}$$

WHAT WE DO HERE

$$B_R \neq B_I$$

Look at the Data

Derivative Dispersion Relations for Slopes

Four parameters describe $d\sigma/dt$ for small $|t|$

- Ratio at t=0

$$\rho = \frac{F_R^N(s,0)}{F_I^N(s,0)}$$

- σ from Optical Theorem

$$\sigma = 4\pi\ (0.389)\ \mathrm{Im}\ F_I^N(s,0)$$

- Slope of Real Amplitude

$$B_R$$

- Slope of Imaginary Amplitude

$$B_I$$

σ in milibarns , and amplitudes F_R, F_I in GeV^{-2} .

Normalization

With σ in mb and t in GeV^2 the practical expression for $d\sigma/dt$ is

$$\frac{d\sigma}{dt}$$

$$= 0.389\,\pi \left[\left[\frac{\rho\ \sigma\ e^{B_R t/2}}{0.389 \times 4\pi} + F^C \cos(\Phi) \right]^2 + \left[\frac{\sigma\ e^{B_I t/2}}{0.389 \times 4\pi} + F^C \sin(\Phi) \right]^2 \right]$$

Ref.: P. Gauron, B. Nicolescu, O.V. Selyugin - PLB 629 (2005) 83

The phase Φ was initially studied by West and Yennie , and different evaluations were made (Selyugin, Petrov, Predazzi, Prokudin, Kundrát-Lokajicek). We extend these, considering $B_R \neq B_I$. Start from West and Yennie

$$\Phi(s,t) = (-/+)\left[\ln\left(-\frac{t}{s}\right) + \int_{-4p^2}^{0} \frac{dt'}{|t'-t|}\left[1 - \frac{F^N(s,t')}{F^N(s,t)}\right] \right]$$

Sign $(-/+)$ for pp/p$\bar{\mathrm{p}}$ respectively. p is proton momentum in cm system, and at high energies $4p^2 \approx s$. For small $|t|$ we have

$$\frac{F^N(s,t')}{F^N(s,t)} = \frac{F_R^N(s,0)e^{B_R t'/2} + i\ F_I^N(s,0)e^{B_I t'/2}}{F_R^N(s,0)e^{B_R t/2} + i\ F_I^N(s,0)e^{B_I t/2}}$$

$$= \frac{c}{c+i}\ e^{B_R(t'-t)/2} + \frac{i}{c+i}\ e^{B_I(t'-t)/2}$$

where

$$c \equiv \rho e^{(B_R - B_I)t/2}$$

The integrals that appear are reduced to the form (KL)

$$I(B) = \int_{-4p^2}^{0} \frac{dt'}{|t'-t|}\left[1 - e^{B(t'-t)/2}\right]$$

solved in terms of exponential integrals (Abramowitz) as

$$I(B) = E_1\left[\frac{B}{2}\left(4p^2+t\right)\right] - E_i\left[-\frac{Bt}{2}\right] + \ln\left[\frac{B}{2}\left(4p^2+t\right)\right] - \ln\left[-\frac{Bt}{2}\right] + 2\gamma$$

At high energies and small $|t|$ simplify $4p^2 + t \rightarrow s$ and $I(B)$ becomes

$$I(B) = E_1\left(\frac{Bs}{2}\right) - E_i\left(-\frac{Bt}{2}\right) + \ln\left(\frac{Bs}{2}\right) - \ln\left(-\frac{Bt}{2}\right) + 2\gamma$$

Then the real part of the phase is

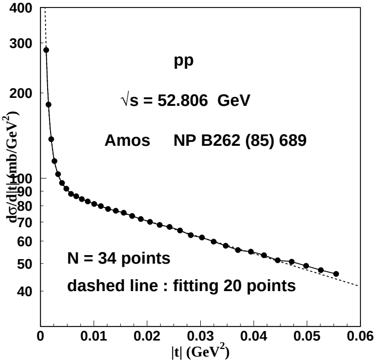
$$\Phi(s,t) = (-/+)\left[\ln\left(-\frac{t}{s}\right) + \frac{1}{c^2+1}\left[c^2 I(B_R) + I(B_I)\right]\right]$$

Scattering parameters in the literature

Table: Forward scattering parameters found in the literature.

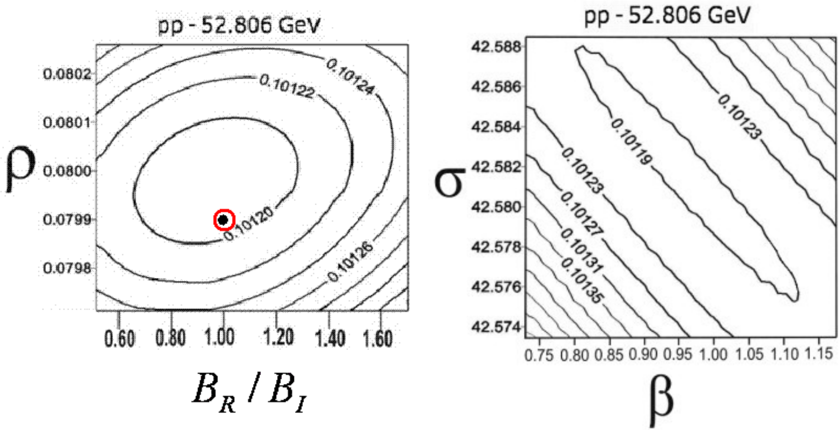
$\sqrt{s}$ (GeV)	σ (mb)	ρ	$B(\mathrm{GeV}^{-2})$
19.4	38.98 ± 0.04	0.019 ± 0.016	11.74 ± 0.04
23.5	38.94 ± 0.17	0.02 ± 0.05	11.80 ± 0.30
30.7	40.14 ± 0.17	0.042 ± 0.011	12.20 ± 0.30
44.7	41.79 ± 0.16	0.0620 ± 0.011	12.80 ± 0.20
52.8	42.67 ± 0.19	0.078 ± 0.010	12.87 ± 0.14
62.5	43.32 ± 0.23	0.095 ± 0.011	13.02 ± 0.27
541	62.20 ± 1.5	0.135 ± 0.015	15.52 ± 0.07
$1800^{(a)}$	72.20 ± 2.7	0.140 ± 0.069	16.72 ± 0.44
$1800^{(b)}$	80.03 ± 2.24	0.15	16.98 ± 0.25

52.8 GeV

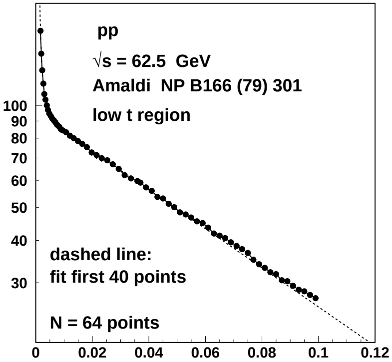


pp
√s = 52.806 GeV
Amos NP B262 (85) 689
N = 34 points
dashed line : fitting 20 points
dσ/d|t| (mb/GeV²)
|t| (GeV²)
400
300
200
100
90
80
70
60
50
40
0
0.01
0.02
0.03
0.04
0.05
0.06

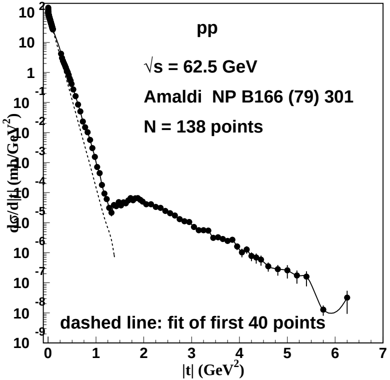
52.8 GeV - correlation of parameters



62.5 GeV

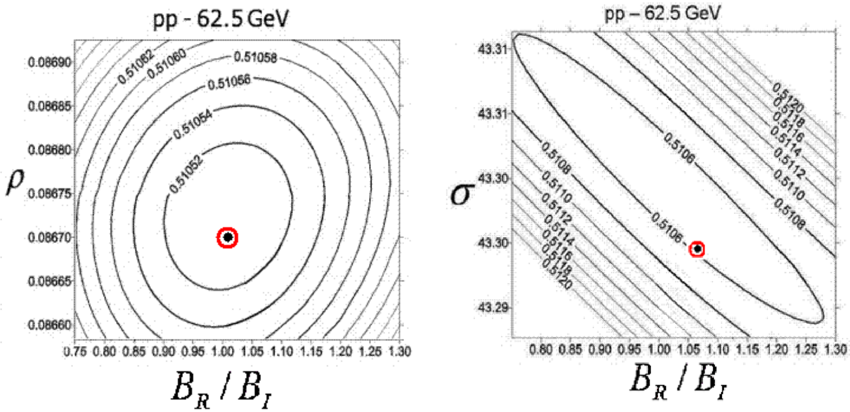


pp
√s = 62.5 GeV
Amaldi NP B166 (79) 301
low t region
dashed line:
fit first 40 points
N = 64 points
100
90
80
70
60
50
40
30
0
0.02
0.04
0.06
0.08
0.1
0.12

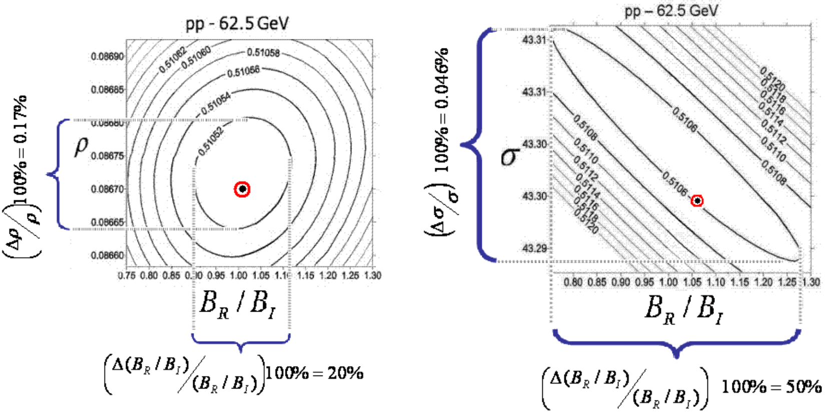


pp
√s = 62.5 GeV
Amaldi NP B166 (79) 301
N = 138 points
dashed line: fit of first 40 points
dσ/d|t| (mb/GeV²)
|t| (GeV²)
0
1
2
3
4
5
6
7

62.5 GeV - correlation of parameters



62.5 GeV - correlations of parameters

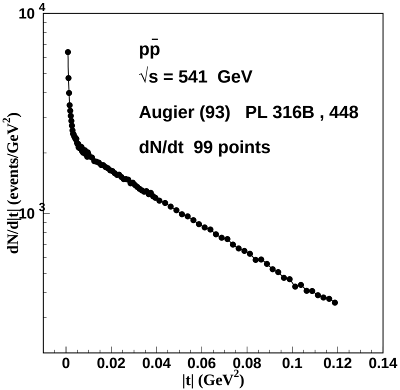


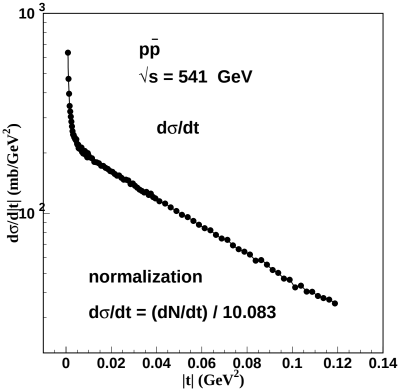
We fit selected data that have more quantity and quality. Results in general depend on the chosen set of low $|t|$ data. Thus : existing data do not allow safe determination of four parameters. Our results show remarkable low χ^2 values.

$\sqrt{s}$ GeV	σ (mb)	ρ	B_I (GeV^{-2})	B_R (GeV^{-2})	B_R/B_I	χ^2
19.4	40.38±0.07	0.019 (fixed)	14.54±0.26	B_I , $2B_I$	1 , 2	1.299
23.5	39.82±1.48	0.0186±0.0137	14.91±9.25	35.2±177.6	2.36	0.295
30.7	40.02±0.05	0.027 (fixed)	11.78±0.24	B_I , $2B_I$	1 , 2	0.536
44.7	41.84±0.29	0.0543±0.0037	12.98±0.63	16.13±15.51	1.243	0.611
52.8	42.58±0.82	0.0799±0.0086	13.41±1.85	14.11±33.68	1.052	0.114
62.5	43.30±0.16	0.0867±0.0034	13.30±0.36	13.90±10.01	1.045	0.539

Changes are varied. Examples: ρ at 44.699 GeV changes from 0.062 to 0.054 ; error bar in σ at 52.8 changes from $\pm$0.19 to $\pm$0.82 mb .

541-546 GeV - event rate dN/dt





Normalization using Coulomb interaction

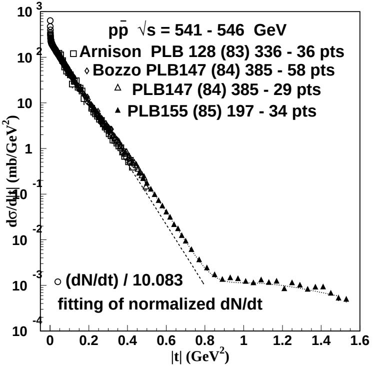
Fitting dN/dt with Coulomb interference expression leads to normalization factor 10.083 . We have also calculated with fixed normalization factor 10.6 (for more perfect matching with other data), with results shown in the table.

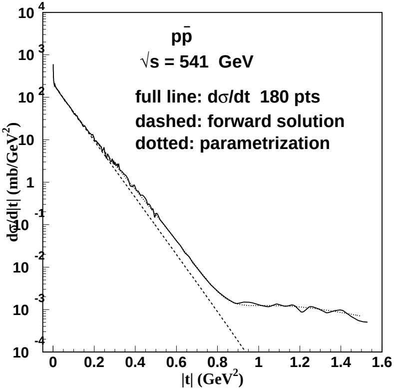
Table: Forward scattering parameters 541-546 GeV

σ (mb)	ρ	$B_I(\mathrm{GeV}^{-2})$	$B_R(\mathrm{GeV}^{-2})$	normalization	χ^2
63.90±0.38	0.172±0.009	15.36±0.15	15.45±4.58	10.083±0.135	1.097
63.65±0.91	0.160±0.017	15.16±0.12	2 B_I (fixed)	10.268±0.358	1.115
62.69±0.04	0.148±0.003	15.38±0.04	17.66±1.44	10.6 (fixed)	1.119
62.84±0.15	0.146±0.007	15.24±0.09	2 B_I(fixed)	10.6 (fixed)	1.126

The χ^2 values do not vary strongly as B_R/B_I changes, showing that the forward data can be described, within errors, by scattering parameters in different ranges.

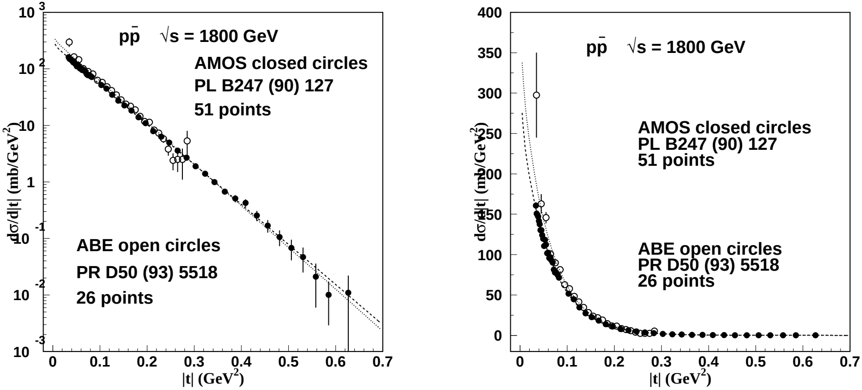
541-546 GeV - matching other data and parametrization for the full $|t|$ range





1800 GeV : two Fermilab experiments

Dashed line fits Amos (E-710) points ; dotted line fits Abe (CDF) data.



1800 GeV : parameter values and comments

Parameters (with fixed $\rho = 0.14$ and $\rho = 1.0$) are given in the table. Although there are large variation bars, notice that the lowest χ^2 are obtained with B_R larger than B_I, for both experiments.

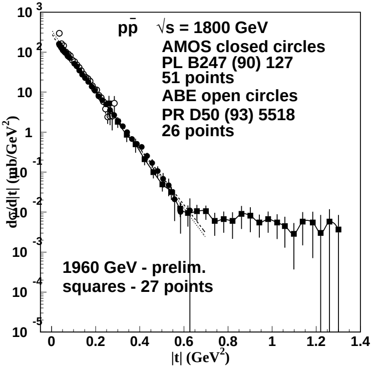
Table: Forward scattering parameters at 1800 GeV

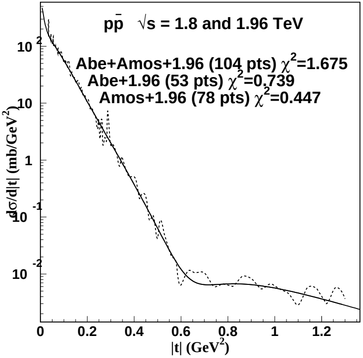
Exp.	σ(mb)	ρ	B_I(GeV^{-2})	B_R(GeV^{-2})	χ^2
E710	72.75±0.19	0.14 (fixed)	16.30±0.04	115.57± 164.20	0.6020
E710	71.82±0.18	0.14 (fixed)	16.28±0.04	B_I (fixed)	0.6060
E710	72.65±0.19	1.0 (fixed)	16.28±0.04	167.93± 48.56	0.5961
CDF	80.92±0.44	0.14 (fixed)	17.00±0.09	72.01± 116.15	1.771
CDF	9.98±0.43	0.14 (fixed)	16.98±0.09	B_I (fixed)	1.775
CDF	80.16±0.43	1.0 (fixed)	16.87±0.09	85.73± 16.94	1.705

Observing the large differences in χ^2, we learn that the E710 data are more compatible with the forward scattering basic expression for $d\sigma/dt$ than the CDF data.

Put together 1.8 and 1.96 TeV

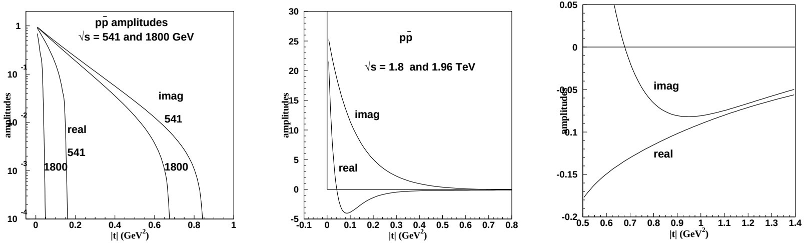
Parametrization for full $|t|$ range is smoother when E-710 is put together with new 1.96 TeV data , compared to matching with CDF.





Comments on 541/546 GeV and 1.8/1.96 TeV

$d\sigma/dt$ is obtained from real and imaginary amplitudes shown below, with their characteristic slopes and zeros, normalized to 1 at $|t| = 0$, obtained with parameterization used in previous work. Notice linear vertical scale in the plots for large $|t|$.



Characteristic features for 541/546 and 1800/1960 GeV

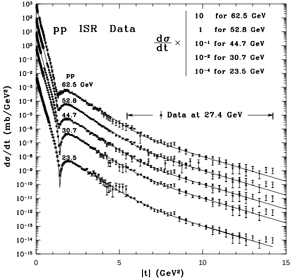
Some values obtained from the full t dependence, that can be read off from the plots of the 541/546 and 1800/1960 amplitudes, are given in the table. These are possible, but not necessarily correct, representations of the data. Notice the large values of ρ.

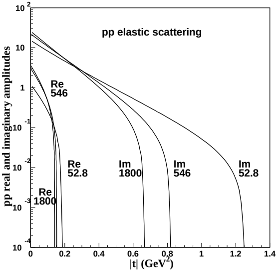
Table: Remarks: (1)Abe+Amos+1.96 ; (2) Amos+1.96 ; (3) Abe+1.96 .

$\sqrt{s}$ (GeV)	σ (mb)	ρ	B_I (GeV^{-2})	B_R (GeV^{-2})	$\lvert t_0^R \rvert$ (GeV^2)	$\lvert t_0^I \rvert$ (GeV^2)	χ^2
541/546	63.05	0.12	13.88	25.79	0.16	0.85	1.32
1800/1960 (1)	73.98	1.17	15.50	85.43	0.05	0.69	1.68
1800/1960 (2)	73.95	0.75	15.47	84.12	0.05	0.69	0.45
1800/1960 (3)	88.49	1.21	17.94	61.43	0.05	0.69	0.74

Cross sections and amplitudes

Review of the description of ISR pp data and the behaviour of the amplitudes

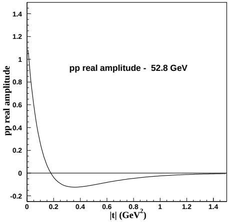


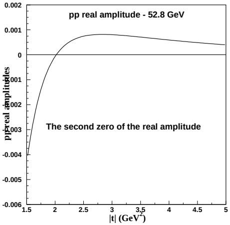


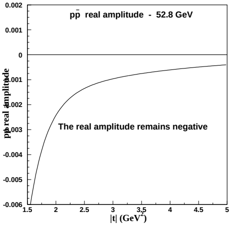
References : F. Pereira and E.F. - PRD59 (1998) 014008, PRD 61 (2000) 077507, Int.J. Mod. Phys. E16 (2007) 2893

Real amplitudes for pp and $\bar{p}p$ scattering

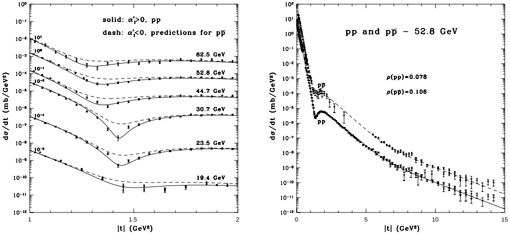
The real pp amplitude has a second zero (near the zero of the imaginary part). The real $\bar{p}p$ amplitude does not have the second zero , DUE TO SIGN OF 3-GLUON EXCHANGE . This causes the differences in the dips.



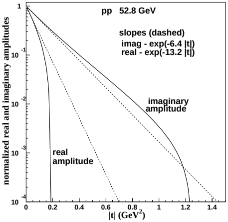


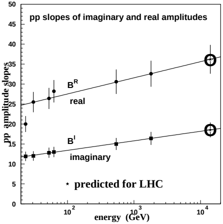


Comparison of dips in pp and ppbar scattering



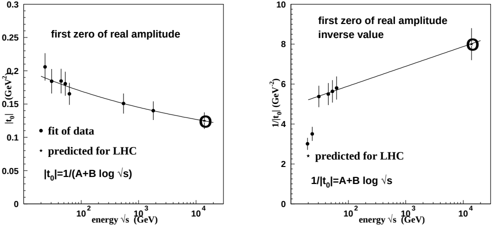
Slopes of real and imaginary amplitudes for pp scattering





Ref.: F. Pereira and E.F. - Int.J. Mod. Phys. E16 (2007) 2893

Displacement of the first zero of the real amplitude



Ref.: A. Martin - Phys. Lett. B404 (1997) 137

Derivative Dispersion Relations

Simplest even/odd Derivative Dispersion Relations (DDR)

$$\frac{\mathrm{Re}F_{+}(s,t)}{s} = \frac{K}{s} + \left[\tan\left(\frac{\pi}{2}\frac{d}{d\log s}\right)\right]\left[\frac{\mathrm{Im}F_{+}(s,t)}{s}\right]$$

$$\frac{\pi}{2}\frac{d}{d\log s}\left[\frac{\mathrm{Re}F_{-}(s,t)}{s}\right] = -\left[\left(\frac{\pi}{2}\frac{d}{d\log s}\right)\cot\left(\frac{\pi}{2}\frac{d}{d\log s}\right)\right]\left[\frac{\mathrm{Im}F_{-}(s,t)}{s}\right]$$

Amplitudes for pp and $\bar{p}p$ channels: $F_{\mathrm{pp}} = F_{+} + F_{-}$ and $F_{\bar{\mathrm{p}}\mathrm{p}} = F_{+} - F_{-}$. For each channel normalization by the optical theorem $\sigma(s) = \mathrm{Im}F(s,t=0)/s$, and exponential slopes for low $|t|$ $\mathrm{Re}F(s,t) = \mathrm{Re}F(s,0)\exp(-B^{R}|t|/2)$, $\mathrm{Im}F(s,t) = \mathrm{Im}F(s,0)\exp(-B^{I}|t|/2)$.

Ref.: E.F. - Int. J. Mod. Phys. E 16 (2007) 2893
E.F. and J. Sesma - J. Math. Phys. 49 (2008) 033504

In terms of measured quantities, for $t = 0$

$$\frac{1}{2}\left(\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} + \sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\right)$$

$$= \frac{K}{s} + \frac{1}{2}\left[\frac{\pi}{2}\frac{d}{d\log s} + \frac{1}{3}\left(\frac{\pi}{2}\frac{d}{d\log s}\right)^3 + \ldots\right]\left[\sigma_{\mathrm{pp}} + \sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

and

$$\frac{1}{2}\frac{\pi}{2}\frac{d}{d\log s}\left[\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} - \sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

$$= -\frac{1}{2}\left[1 - \frac{1}{3}\left(\frac{\pi}{2}\frac{d}{d\log s}\right)^2 - \ldots\right]\left[\sigma_{\mathrm{pp}} - \sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

To use DDR's , the energy dependence of total cross section is needed. Consider two forms :

$$\sigma = D + d_0 \log^2(s/s_0) + d_1 s^{-\mu_1} - \tau a_2 s^{-\eta_2}$$

$$\sigma = a_0 s^{\epsilon} + a_1 s^{-\eta_1} - \tau a_2 s^{-\eta_2}$$

where $\tau = +1$ and -1 for pp and $\bar{\mathrm{p}}$p respectively, s is in GeV^2, and $s_0 = 25\ \mathrm{GeV}^2$. Then closed forms can be written. Respective Even-DDR $\Longrightarrow$

$$\frac{1}{2}\left(\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} + \sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\right) = \frac{K}{s} + 2d_0\left(\frac{\pi}{2}\right)\log(s/s_0) - d_1 \tan\left(\frac{\pi}{2}\mu_1\right)s^{-\mu_1}$$

$$\frac{1}{2}\left(\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} + \sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\right) = \frac{K}{s} + a_0 \tan\left(\frac{\pi}{2}\epsilon\right)s^{\epsilon} - a_1 \tan\left(\frac{\pi}{2}\eta_1\right)s^{-\eta_1}$$

For the odd-DDR we arrive at a closed form

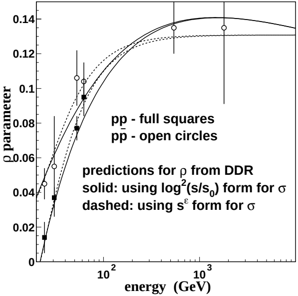
$$\frac{1}{2}\left[\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} - \sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\right] = -a_2 \cot\left(\frac{\pi}{2}\eta_2\right) s^{-\eta_2}$$

$$= \frac{1}{2}\cot\left(\frac{\pi}{2}\eta_2\right)\left[\sigma_{\mathrm{pp}} - \sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

These relations connect measurable quantities.

Predictions for ρ from DDR

Predictions for ρ from DDR using parametrizations $\sigma(s)$, compared to measured values.



Derivative Dispersion Relations for Slopes

With exponential amplitudes in the DDR's, expanding for small $|t|$, and taking that even DDR's at $|t| = 0$ are valid $\Longrightarrow$

$$B^R_{\mathrm{pp}}\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}} + B^R_{\bar{\mathrm{p}}\mathrm{p}}\sigma_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}} = \tan\left[\frac{\pi}{2}\frac{d}{d\log s}\right]\left[B^I_{\mathrm{pp}}\sigma_{\mathrm{pp}} + B^I_{\bar{\mathrm{p}}\mathrm{p}}\sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

Analogously from the odd DDR $\Longrightarrow$

$$\frac{d}{d\log s}\left[B^R_{\mathrm{pp}}\rho_{\mathrm{pp}}\sigma_{\mathrm{pp}} - B^R_{\bar{\mathrm{p}}\mathrm{p}}\rho_{\bar{\mathrm{p}}\mathrm{p}}\sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

$$= -\left[\left(\frac{d}{d\log s}\right)\cot\left(\frac{\pi}{2}\frac{d}{d\log s}\right)\right]\left[B^I_{\mathrm{pp}}\sigma_{\mathrm{pp}} - B^I_{\bar{\mathrm{p}}\mathrm{p}}\sigma_{\bar{\mathrm{p}}\mathrm{p}}\right]$$

Evaluation require the energy dependence of amplitude slopes. We assume that B_I is the same for both pp and $\bar{\mathrm{p}}\mathrm{p}$ channels, with

$$B^I_{\mathrm{pp}} = B^I_{\bar{\mathrm{p}}\mathrm{p}} = B^I(s) = c^I_1 + c^I_2\ \log s$$

Using the parametrized $\sigma(s)$, closed forms can be obtained.
For the even DDR for slopes we have two separate cases:
I- σ of $\log^2(s)$ kind $\Longrightarrow$

$$B^R_{\rm pp}\sigma_{\rm pp}\rho_{\rm pp} + B^R_{\rm \bar{p}p}\sigma_{\rm \bar{p}p}\rho_{\rm \bar{p}p} = B^I\left[2\pi d_0 \log(\frac{s}{s_0}) - 2d_1\tan(\frac{\pi}{2}\mu_1)s^{-\mu_1}\right]$$

$$+c_2^I\;\pi\;\left[d_0\log^2(\frac{s}{s_0}) + d_0\frac{\pi^2}{2} + D + d_1\sec^2(\frac{\pi}{2}\mu_1)s^{-\mu_1}\right]$$

II- σ of s^ϵ kind $\Longrightarrow$

$$B^R_{\rm pp}\sigma_{\rm pp}\rho_{\rm pp} + B^R_{\rm \bar{p}p}\sigma_{\rm \bar{p}p}\rho_{\rm \bar{p}p} = B^I\left[2a_0\tan(\frac{\pi}{2}\epsilon)\;s^\epsilon - 2a_1\tan(\frac{\pi}{2}\eta_1)\;s^{-\eta_1}\right]$$

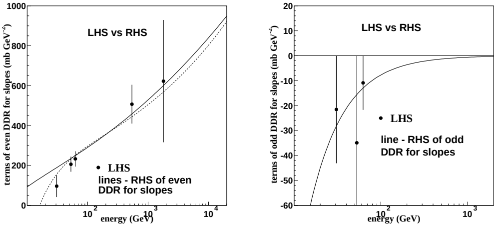
$$+c_2^I\;\pi\;\left[a_0\sec^2(\frac{\pi}{2}\epsilon)\;s^\epsilon + a_1\sec^2(\frac{\pi}{2}\eta_1)\;s^{-\eta_1}\right]$$

For the odd DDR for slopes $\Longrightarrow$

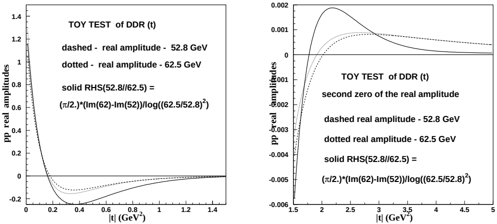
$$B^R_{\rm pp}\rho_{\rm pp}\sigma_{\rm pp} - B^R_{\rm \bar{p}p}\rho_{\rm \bar{p}p}\sigma_{\rm \bar{p}p}$$

$$= -2a_2\ s^{-\eta_2}\big[B^I \cot(\frac{\pi}{2}\eta_2) + c^I_2\ \frac{\pi}{2}\ {\rm cosec}^2(\frac{\pi}{2}\eta_2)\big]$$

Knowing $B_I(s)$ we may draw the RHS of the DDR for slopes. The LHS is given by the data, using B^R slopes of the pp and $\rm \bar{p}p$ amplitudes. The results are shown in the figure , where we observe compatibility, confirming both the values of B_R and B_I slopes and the sense of the new dispersion relations.



Experimental points (full circles): LHS of the even and odd DDR for slopes. Lines : RHS . For the even case full line for σ of form $\log^2(s/s_0)$ and dashed line for σ with s^{ϵ}.



Experimental points (full circles): LHS of the even and odd DDR for slopes. Lines : RHS . For the even case full line for σ of form $\log^2(s/s_0)$ and dashed line for σ with s^ϵ.

Final Remarks

- Hope that LHC data on elastic $d\sigma/dt$ will be of good quality and high statistics
- Treatment of the forward scattering range must include more tools of dispersion relations (eg. dispersion relations for slopes)
- Theoretical models expected to exhibit detailed behaviour of real and imaginary amplitudes, particularly in the forward direction