LUND UNIVERSITY

Academic Training Lectures
CERN
4, **5**, 6, 7 April 2005

Monte Carlo Generators for the LHC

Torbjörn Sjöstrand

CERN and Lund University

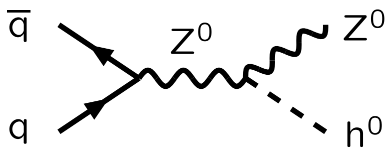
1. (Monday) Introduction and Overview; Matrix Elements
2. **(today) Parton Showers; Matching Issues**
3. (Wednesday) Multiple Interactions and Beam Remnants
4. (Thursday) Hadronization and Decays; Summary and Outlook

Event Physics Overview

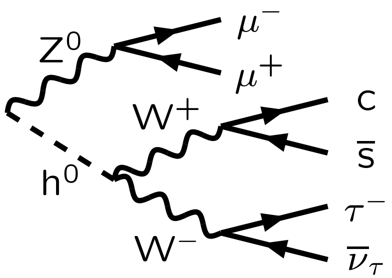
Repetition: from the “simple” to the “complex”,
or from “calculable” at large virtualities to “modelled” at small

Matrix elements (ME):

1) Hard subprocess:
$|\mathcal{M}|^2$, Breit-Wigners,
parton densities.

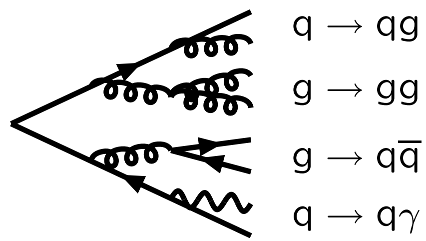


2) Resonance decays:
includes correlations.

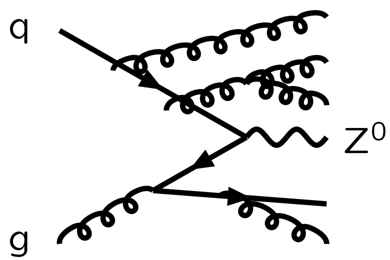


Parton Showers (PS):

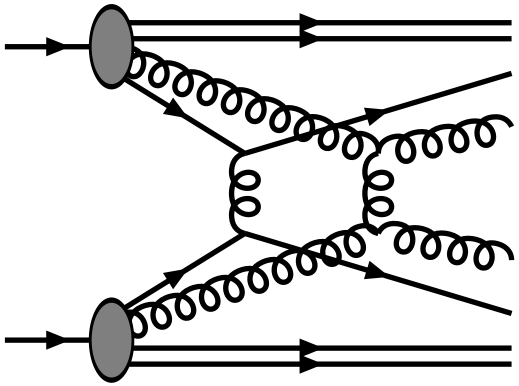
3) Final-state parton showers.



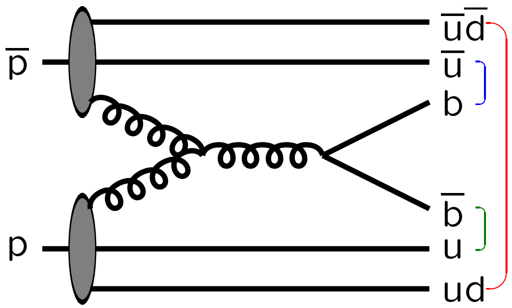
4) Initial-state parton showers.



5) Multiple parton–parton interactions.

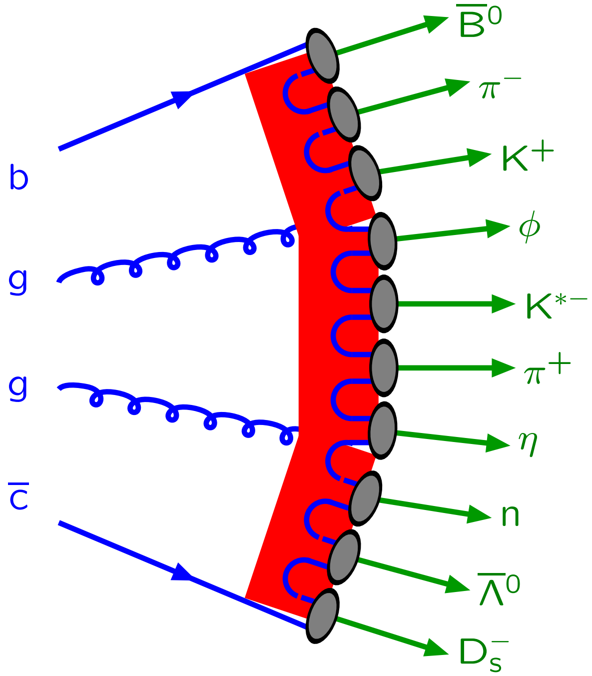


6) Beam remnants, with colour connections.

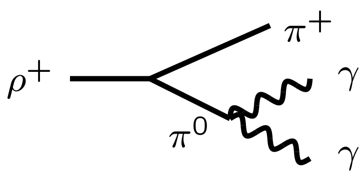


5) + 6) = Underlying Event

7) Hadronization



8) Ordinary decays: hadronic, τ, charm, . . .



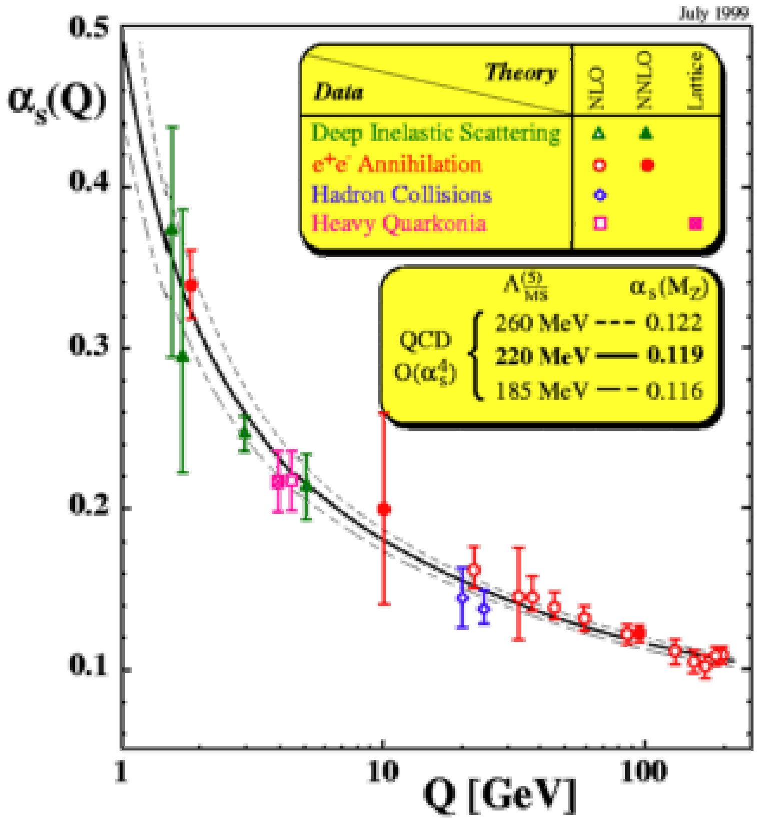
Divergences

Emission rate q $\longrightarrow$ qg diverges when

- collinear: opening angle $\theta_{\mathrm{qg}} \longrightarrow 0$
- soft: gluon energy $E_{\mathrm{g}} \longrightarrow 0$

Almost identical to e $\longrightarrow$ eγ (“bremsstrahlung”),
but QCD is non-Abelian so additionally

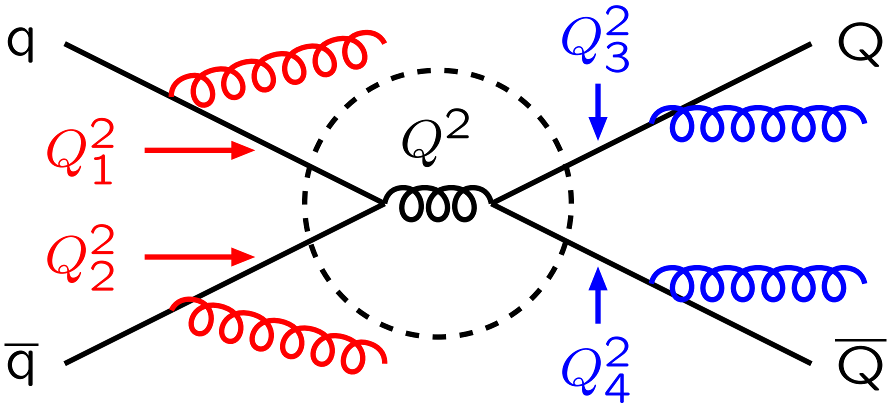
- g $\longrightarrow$ gg similarly divergent
- $\alpha_{\mathrm{s}}(Q^2)$ diverges for $Q^2 \longrightarrow 0$
  (actually for $Q^2 \longrightarrow \Lambda^2_{\mathrm{QCD}}$)



Big probability for one emission $\Longrightarrow$ also big for several
$\Longrightarrow$ with ME’s need to calculate to high order **and** with many loops
$\Longrightarrow$ extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.
Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \to n \;=\; (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$



ISR $2 \to 2$ FSR

FSR = Final-State Rad.;
timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing

ISR = Initial-State Rad.;
spacelike shower
$Q_i^2 \sim -m^2 > 0$ increasing

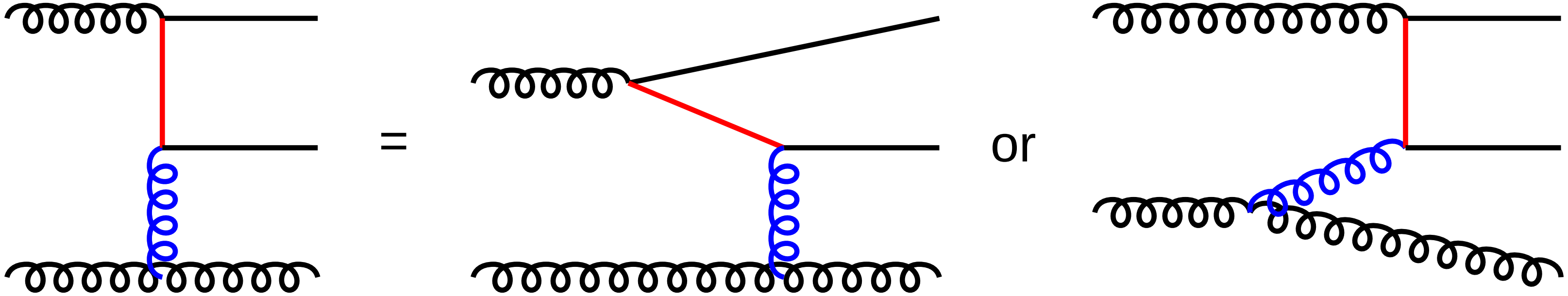
$2 \to 2$ = hard scattering (on-shell):

$$\sigma = \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \; f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Shower evolution is viewed as a probabilistic process,
which occurs with unit total probability:
the cross section is not directly affected,
but indirectly it is, via the changed event shape

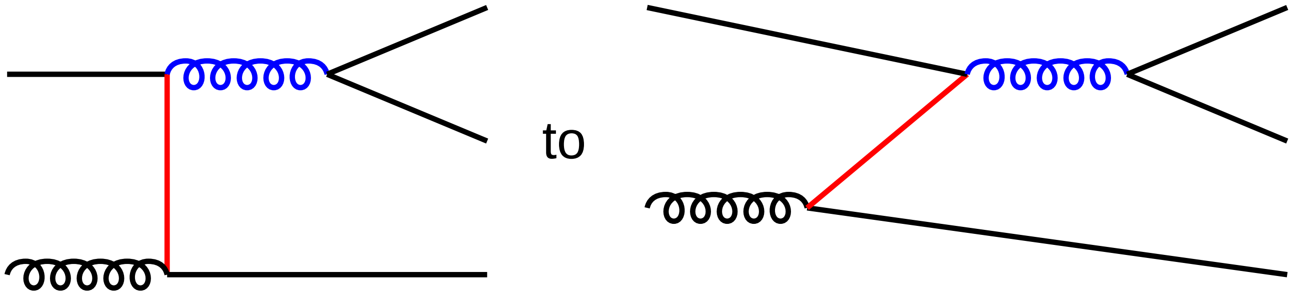
Doublecounting

A $2 \rightarrow n$ graph can be “simplified” to $2 \rightarrow 2$ in different ways:



$g \rightarrow q\bar{q} \oplus qg \rightarrow qg$ $\qquad$ $g \rightarrow gg \oplus gg \rightarrow q\bar{q}$

or deform

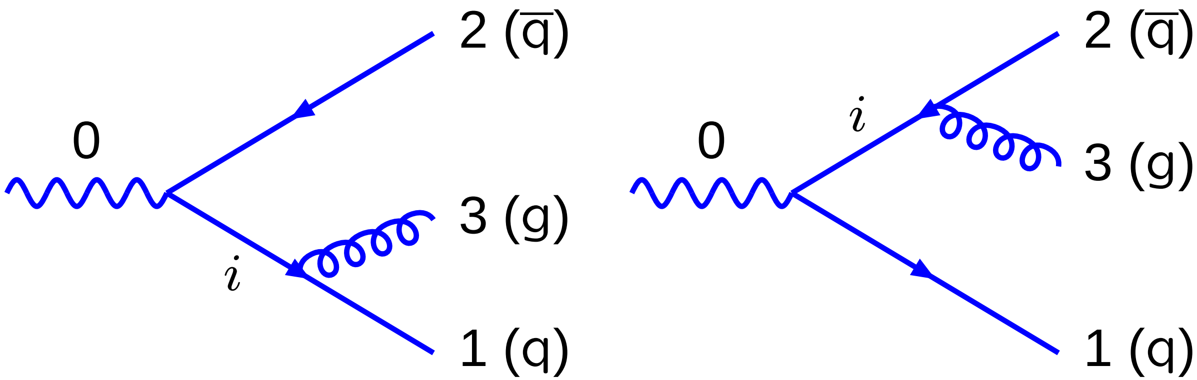


FSR $\qquad$ ISR

Do not doublecount: $2 \rightarrow 2$ = most virtual = shortest distance

Conflict: theory derivations often assume virtualities strongly ordered; interesting physics often in regions where this is not true!

From Matrix Elements to Parton Showers



$\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\overline{\mathrm{q}}\mathrm{g}$

$x_j = 2E_j/E_{\mathrm{cm}} \Rightarrow$

$x_1 + x_2 + x_3 = 2$

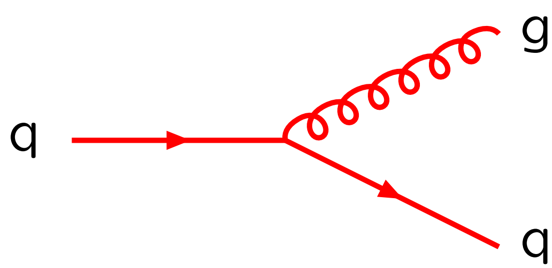
$$m_{\mathrm{q}} = 0: \quad \frac{\mathrm{d}\sigma_{\mathrm{ME}}}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{4}{3}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2$$

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\mathrm{cm}}^2} = \frac{Q^2}{E_{\mathrm{cm}}^2} \quad \Rightarrow \quad \mathrm{d}x_2 = \frac{\mathrm{d}Q^2}{E_{\mathrm{cm}}^2}$$

$$x_1 \approx z \quad \Rightarrow \quad \mathrm{d}x_1 \approx \mathrm{d}z$$

$$x_3 \approx 1 - z$$



$$\Rightarrow \mathrm{d}\mathcal{P} = \frac{\mathrm{d}\sigma}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{\mathrm{d}x_2}{(1-x_2)}\,\frac{4}{3}\,\frac{x_2^2 + x_1^2}{(1-x_1)}\,\mathrm{d}x_1 \approx \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,\frac{4}{3}\,\frac{1+z^2}{1-z}\,\mathrm{d}z$$

Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

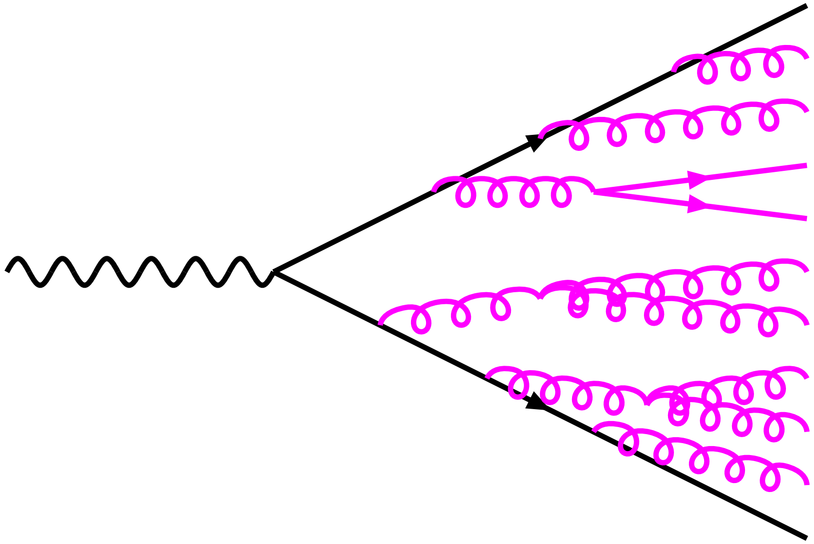
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z$$

$$P_{\mathrm{q}\to\mathrm{qg}} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{\mathrm{g}\to\mathrm{gg}} = 3\,\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{\mathrm{g}\to\mathrm{q\overline{q}}} = \frac{n_f}{2}(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs to stay away from nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \min(m_{ij}) \approx 1$ GeV,

$z_{\mathrm{min}}(E,Q) < z < z_{\mathrm{max}}(E,Q)$

or $p_\perp > p_{\perp\mathrm{min}} \approx 0.5$ GeV

The Sudakov Form Factor

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

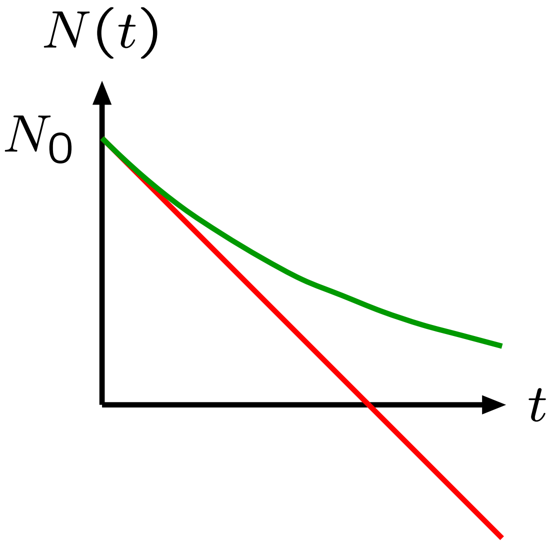
$\mathcal{P}_{\text{nothing}}(0 < t \le T) = \mathcal{P}_{\text{nothing}}(0 < t \le T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \le T)$

Subdivide further, with $T_i = (i/n)T$, $0 \le i \le n$:

$$\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \le T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \le T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(- \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
\end{aligned}$$

$$\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T) \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

Example: radioactive decay of nucleus



naively: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN_0 \Rightarrow N(t) = N_0\,(1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN(t) \Rightarrow N(t) = N_0\ \exp(-ct)$

generalizes to: $N(t) = N_0\ \exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

or: $\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -c(t)\ N_0\ \exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

sequence allowed: nucleus$_1 \to$ nucleus$_2 \to$ nucleus$_3 \to \ldots$

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z\ \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_\mathrm{max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_\mathrm{s}}{2\pi}P_{a\to bc}(z')\,\mathrm{d}z'\right)$$

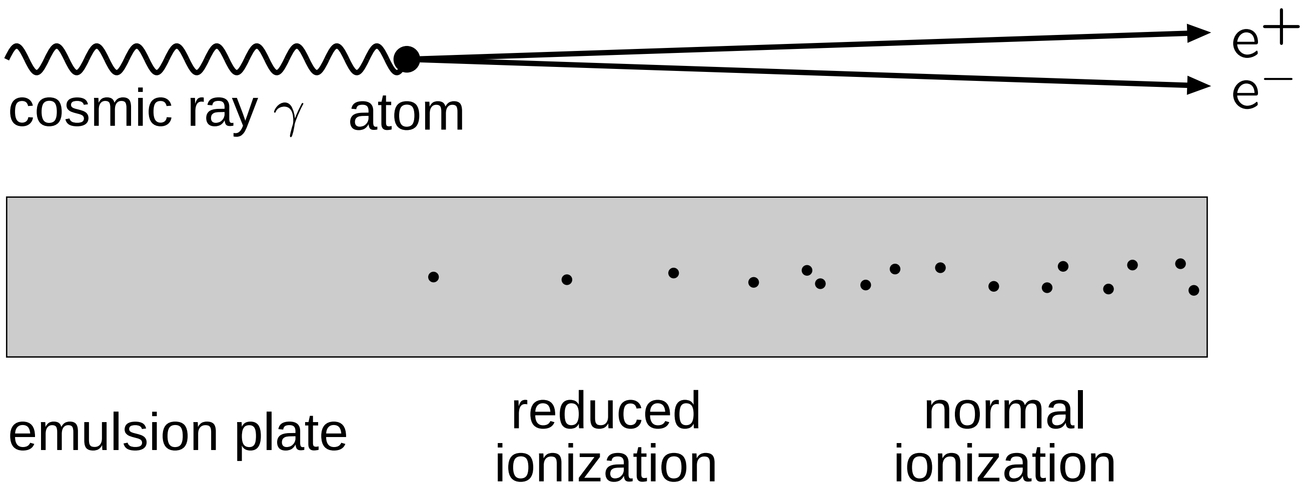
where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

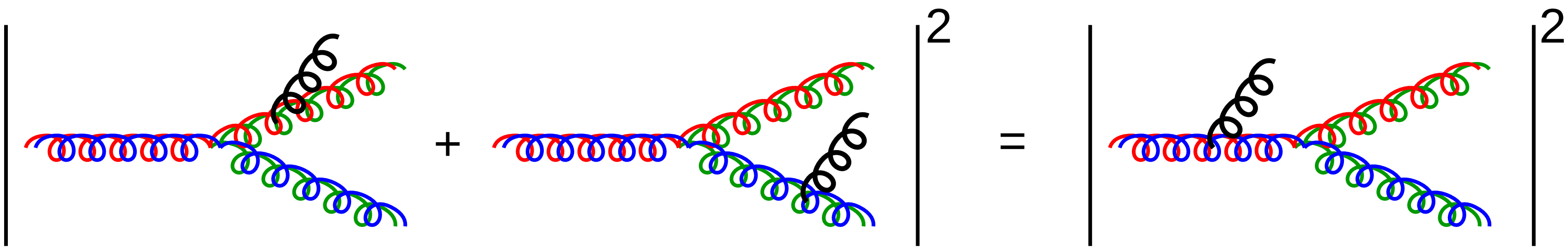
Note that $\sum_{b,c}\int \mathrm{d}Q^2 \int dz\ \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*
($\equiv 1$ if extended over whole phase space, else possibly nothing happens)

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



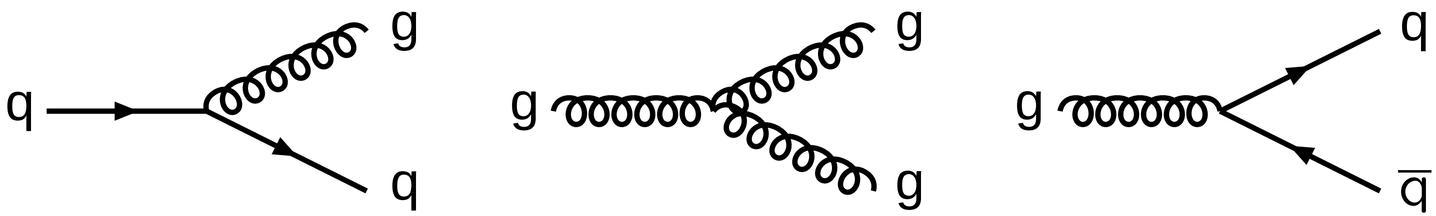
solved by • requiring emission angles to be decreasing

or • requiring transverse momenta to be decreasing

The Common Showering Algorithms

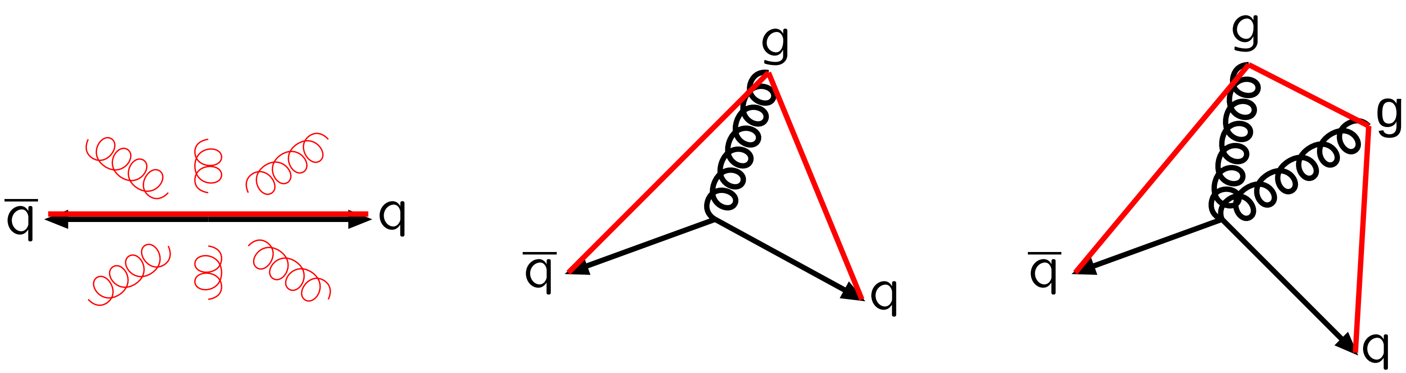
Three main approaches to showering in common use:

Two are based on the standard shower language
of $a \to bc$ successive branchings:



HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

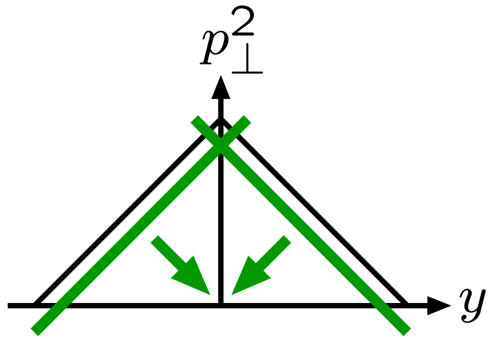
One is based on a picture of dipole emission $ab \to cde$:



ARIADNE: $Q^2 = p_\perp^2$; FSR mainly, ISR is primitive;
there instead LDCMC: sophisticated but complicated

Ordering variables in final-state radiation

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ "hardness" ordered

coherence brute force

covers phase space

ME merging simple

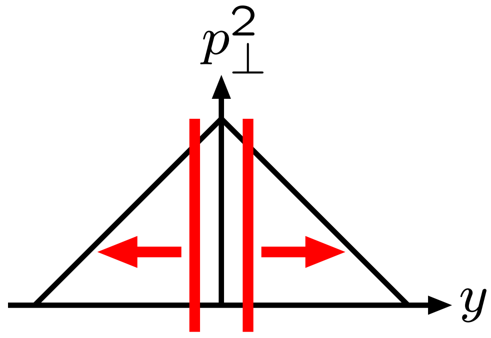
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \to -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

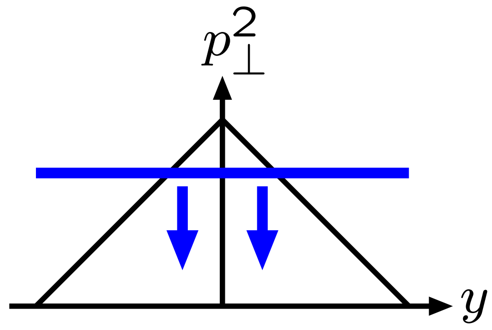
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \to \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ "hardness" ordered

coherence inherent

covers phase space

ME merging simple

g $\to$ q$\overline{\text{q}}$ messy

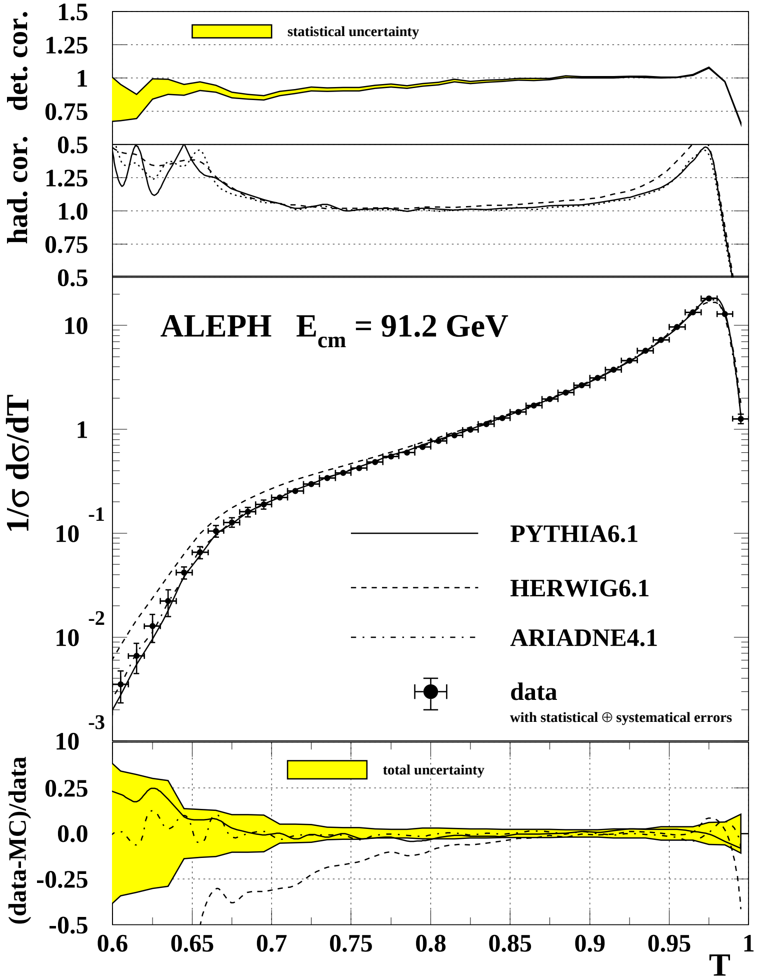
Lorentz invariant

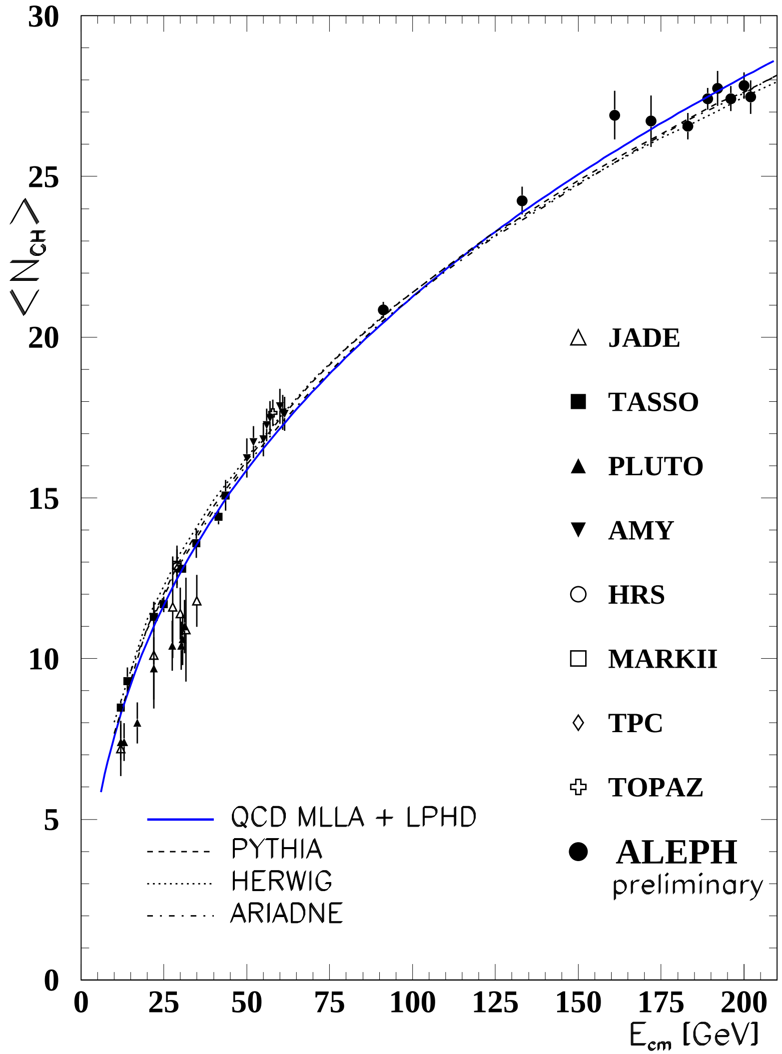
can stop/restart

ISR: more messy

Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically ARIADNE ($p_\perp^2$) $>$ PYTHIA (m^2) $>$ HERWIG (θ)





. . . and programs evolve to do even better . . .

Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{qg}} \approx \int \frac{\mathrm{d}Q^2}{Q^2} \int \mathrm{d}z \, \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z}$$
$$\approx \alpha_{\mathrm{s}} \ln\left(\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}}\right) \frac{8}{3} \ln\left(\frac{1-z_{\mathrm{min}}}{1-z_{\mathrm{max}}}\right) \sim \alpha_{\mathrm{s}} \ln^2$$

Rate for n emissions is of form:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{q}n\mathrm{g}} \sim (\mathcal{P}_{\mathrm{q}\to\mathrm{qg}})^n \sim \alpha_{\mathrm{s}}^n \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type $\alpha_{\mathrm{s}}^n \ln^{2n-1}$

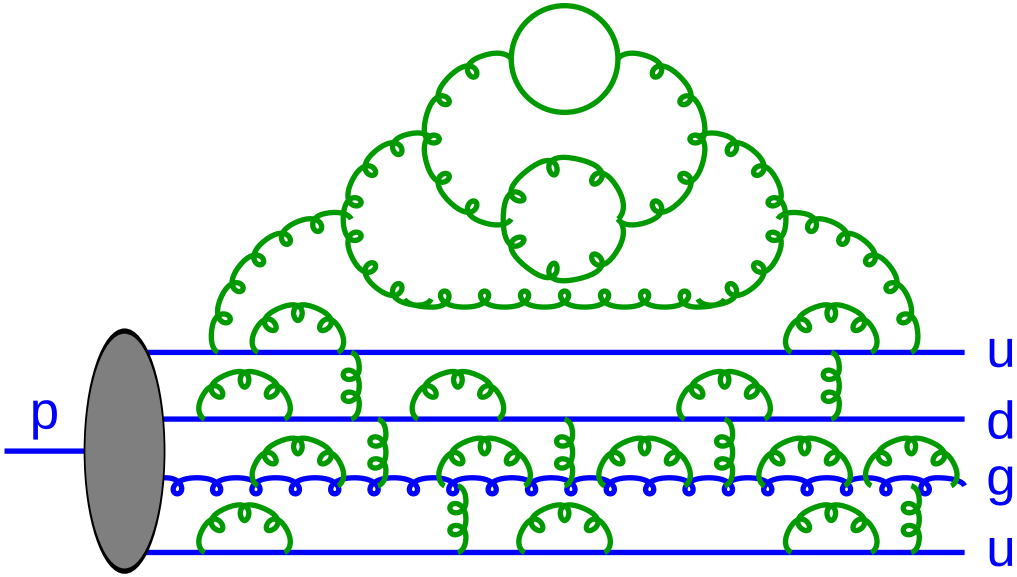
No existing generator completely NLL (NLLJET?), but

- energy-momentum conservation (and "recoil" effects)
- coherence
- $2/(1-z) \to (1+z^2)/(1-z)$
- scale choice $\alpha_{\mathrm{s}}(p_\perp^2)$ absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_{\mathrm{s}}^2)$ splitting kernels $P_{\mathrm{q}\to\mathrm{qg}}$ and $P_{\mathrm{g}\to\mathrm{gg}}$
- ...

$\Rightarrow$ far better than naive, analytical LL

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2) =$ number density of partons i
at momentum fraction x and probing scale Q^2.

Linguistics (example):

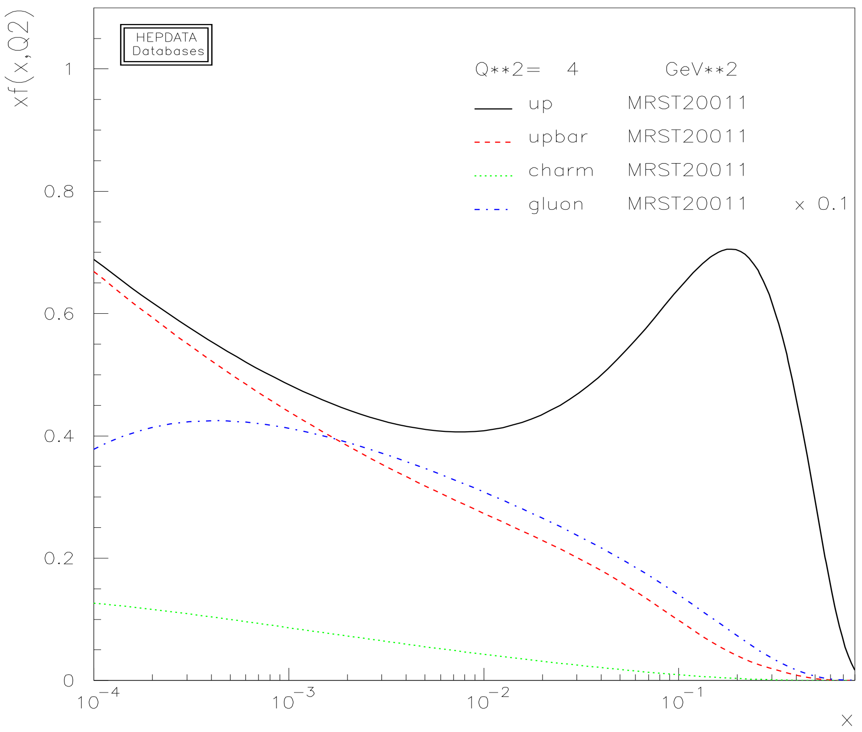
$$F_2(x, Q^2) \;=\; \sum_i e_i^2 \, x f_i(x, Q^2)$$

structure function parton distributions

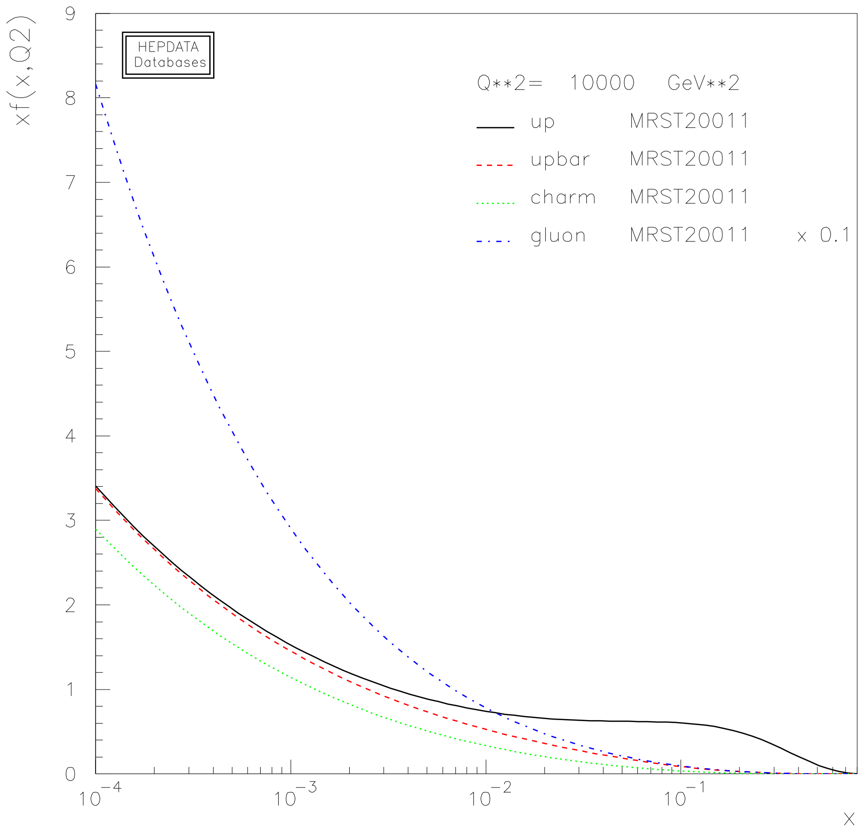
Absolute normalization at small Q_0^2 unknown.
Resolution dependence by DGLAP:

$$\frac{\mathrm{d} f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}\left(z = \frac{x}{x'}\right)$$

$Q^2 = 4$ GeV^2

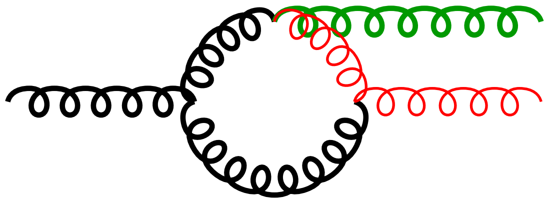


$Q^2 = 10000$ GeV^2

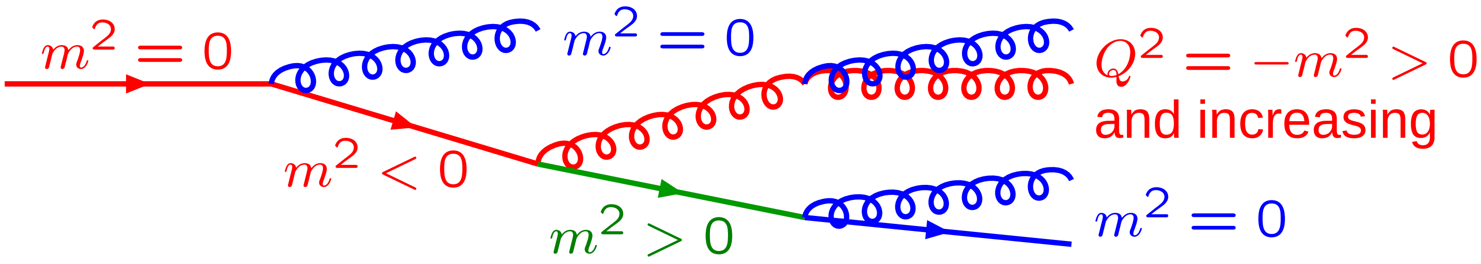


Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



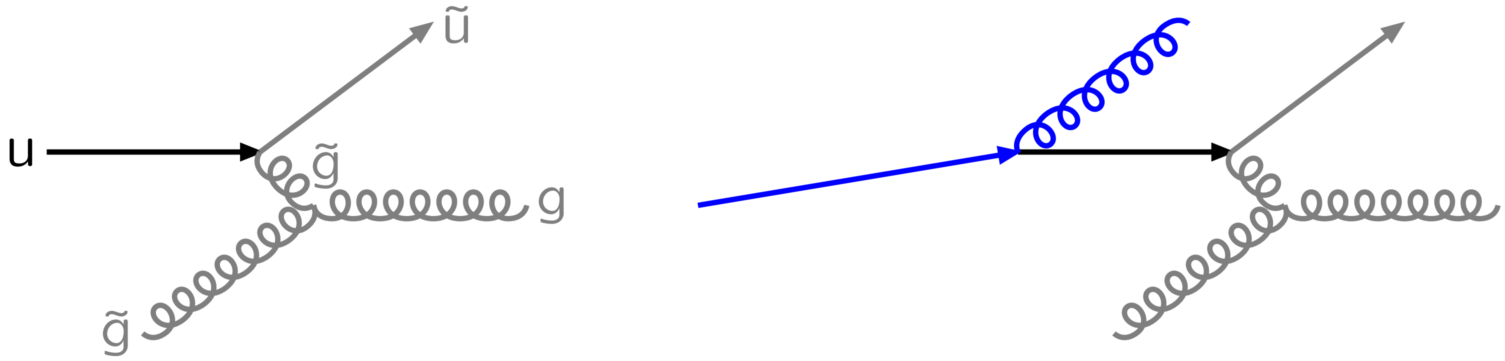
- Convenient reinterpretation:



Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low Q_0 and evolve, see what happens.
Inefficient:
1) have to evolve and check for *all* potential collisions, but 99.9...% inert
2) impossible to steer the production e.g. of a narrow resonance (Higgs)

Backwards evolution

Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"



Monte Carlo approach, based on *conditional probability*: recast

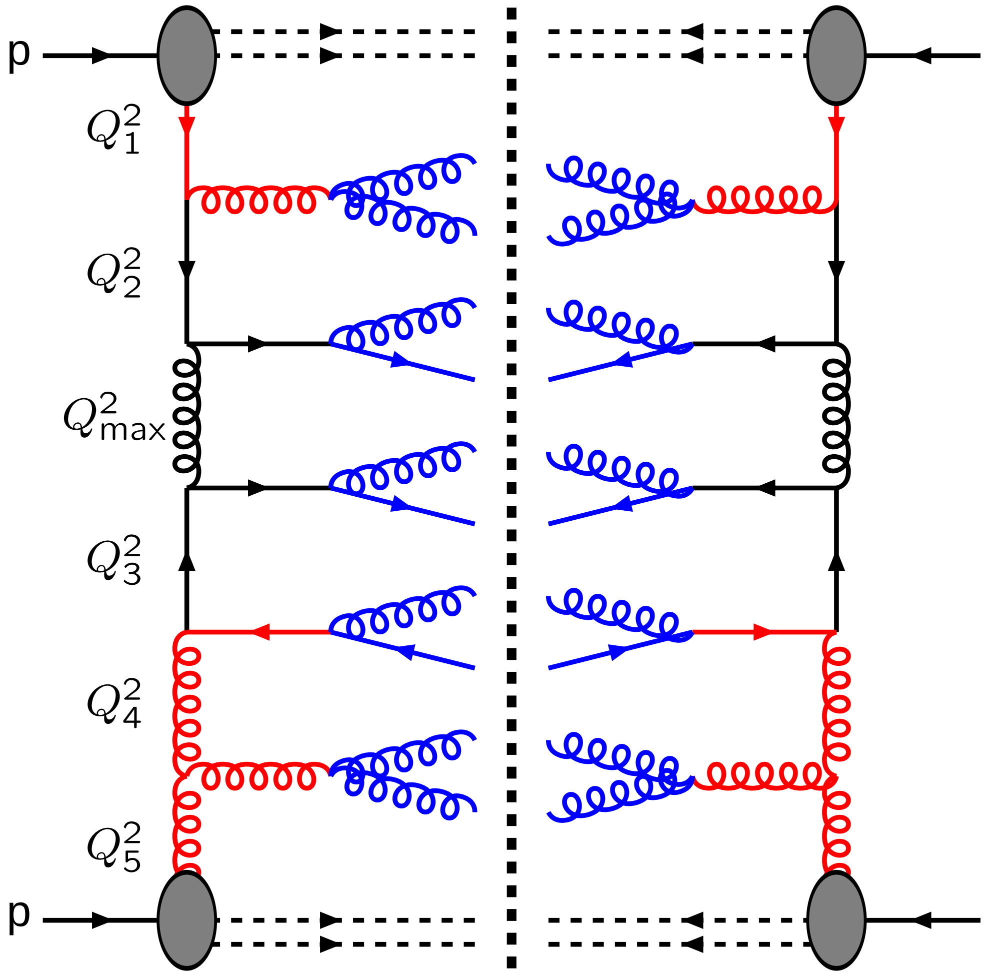
$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}(z)$$

with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

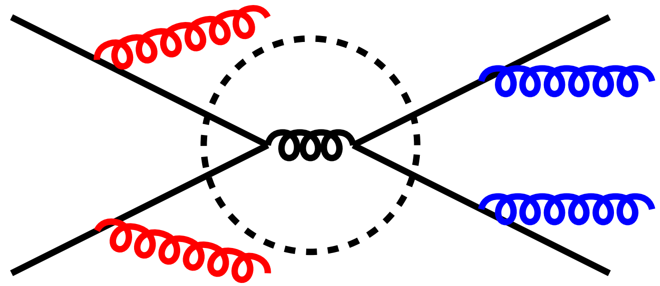
$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \frac{x' f_a(x',t)}{x f_b(x,t)} \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Ladder representation combines whole event:



cf. previously:



One possible
Monte Carlo order:

1) Hard scattering

2) Initial-state shower
from center outwards

3) Final-state showers

DGLAP: $Q^2_{\mathrm{max}} > Q_1^2 > Q_2^2 \sim Q_0^2$
$Q^2_{\mathrm{max}} > Q_3^2 > Q_4^2 > Q_5^2 \sim Q_0^2$

BFKL/CCFM: go beyond Q^2 ordering;
important at small x and Q^2

Initial-State Shower Comparison

Two(?) CCFM Generators:

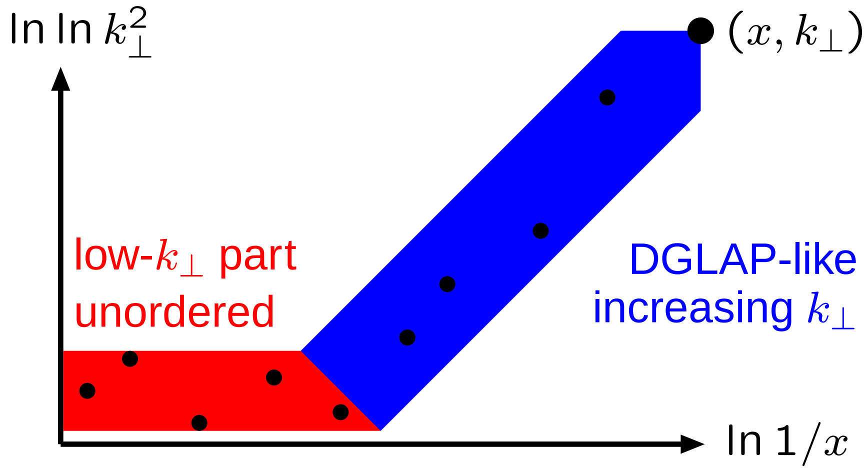
(SMALLX (Marchesini, Webber))

CASCADE (Jung, Salam)

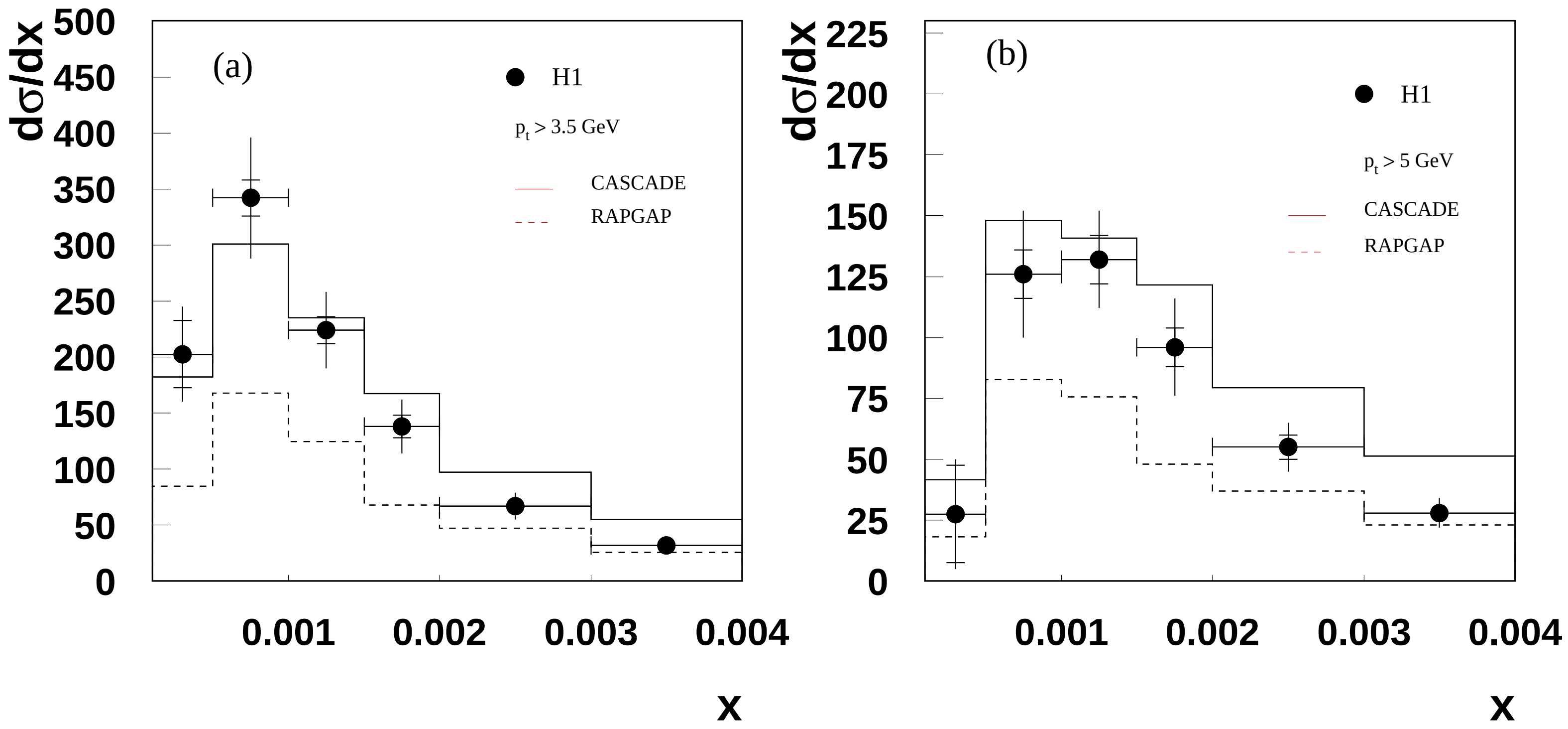
LDC (Gustafson, Lönnblad):

reformulated initial/final rad.

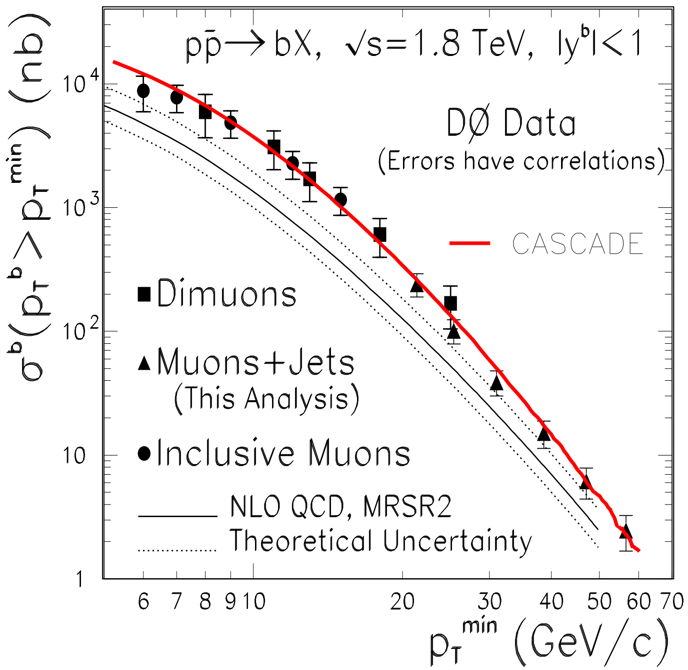
$\Longrightarrow$ eliminate non-Sudakov

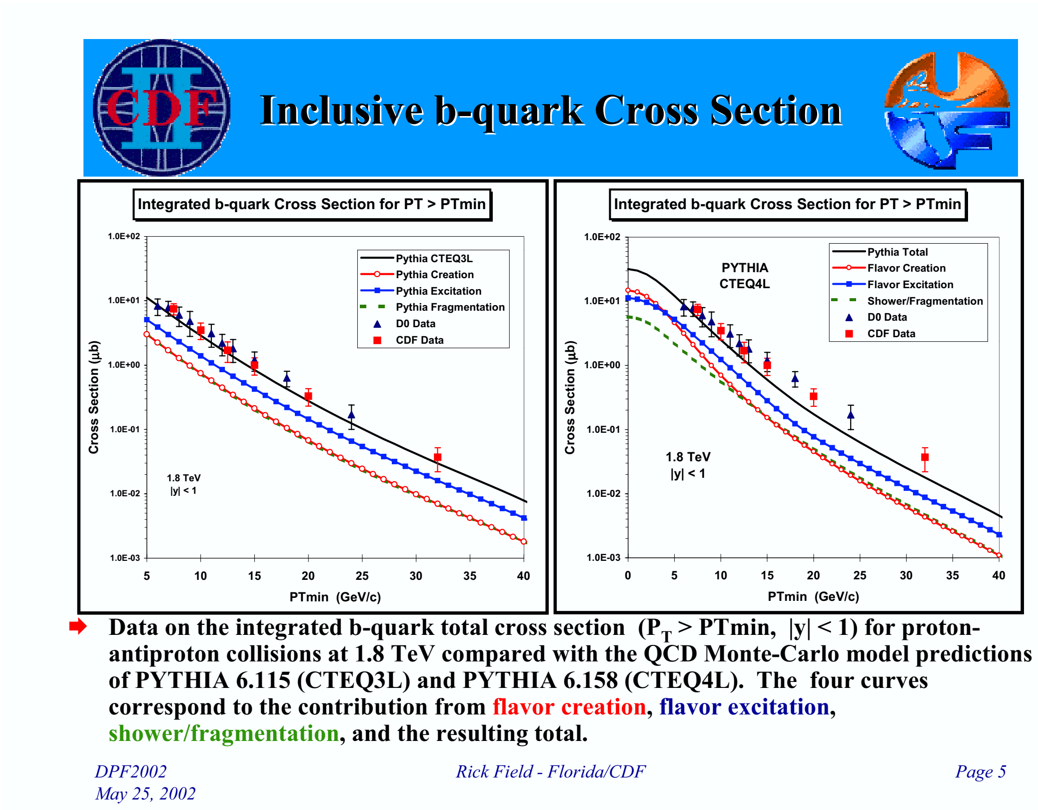


Test 1) forward (= p direction) jet activity at HERA



2) Heavy flavour production





but also explained by DGLAP with leading order pair creation
+ flavour excitation (≈ unordered chains)
+ gluon splitting (final-state radiation)

CCFM requires off-shell ME's + unintegrated parton densities

$$F(x,Q^2) = \int^{Q^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathcal{F}(x,k_\perp^2) + (\text{suppressed with } k_\perp^2 > Q^2)$$

so not ready for prime time in pp

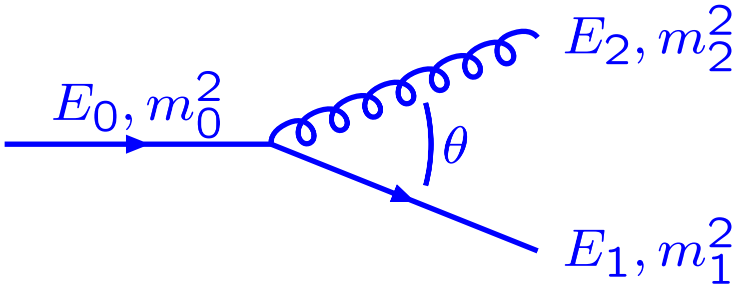
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z \cdot (\text{Sudakov})$$

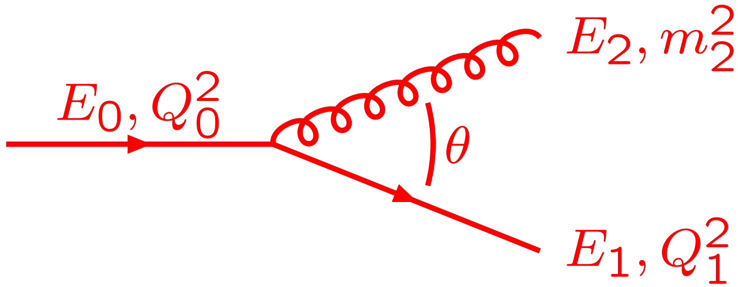
but

Final-state showers:
Q^2 timelike ($\sim m^2$)



decreasing E, m^2, θ
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ “minor” variations:
Q^2, shower vs. dipole, . . .

Initial-state showers:
Q^2 spacelike ($\approx -m^2$)

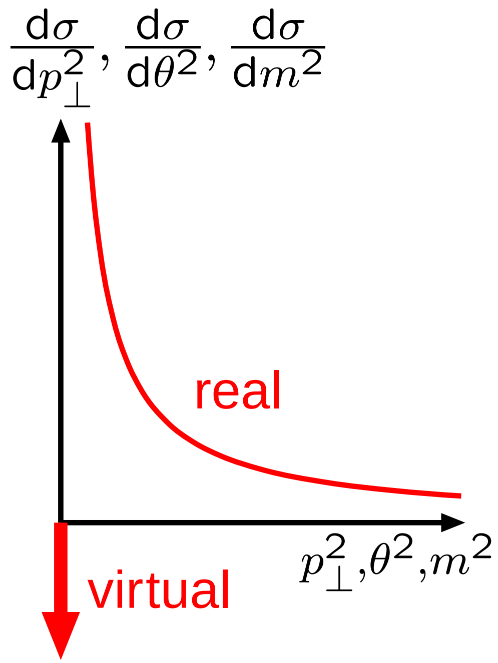


decreasing E, increasing Q^2, θ
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, . . .

Matrix Elements vs. Parton Showers

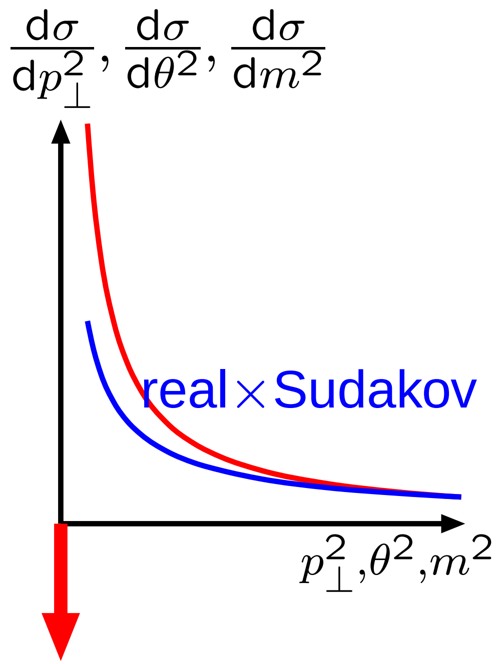
ME : Matrix Elements

- $+$ systematic expansion in α_s ('*exact*')
- $+$ powerful for multiparton Born level
- $+$ flexible phase space cuts
- $-$ loop calculations very tough
- $-$ negative cross section in collinear regions
  $\Rightarrow$ unpredictive jet/event structure
- $-$ *no easy match to hadronization*



PS : Parton Showers

- $-$ approximate, to LL (or NLL)
- $-$ main topology not predetermined
  $\Rightarrow$ inefficient for exclusive states
- $+$ process-generic $\Rightarrow$ simple multiparton
- $+$ Sudakov form factors/resummation
  $\Rightarrow$ sensible jet/event structure
- $+$ *easy to match to hadronization*



Matrix Elements and Parton Showers

Recall complementary strengths:

- ME's good for well separated jets
- PS's good for structure inside jets

Marriage desirable! But how?

Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing $\Longrightarrow$ no established orthodoxy

Three main areas, in ascending order of complication:

1) Match to lowest-order nontrivial process — merging

2) Combine leading-order multiparton process — vetoed parton showers

3) Match to next-to-leading order process — MC@NLO

Merging

= cover full phase space with smooth transition ME/PS

Want to reproduce $$W^{\mathrm{ME}} = \frac{1}{\sigma(\mathrm{LO})} \frac{\mathrm{d}\sigma(\mathrm{LO}+\mathrm{g})}{\mathrm{d(phasespace)}}$$

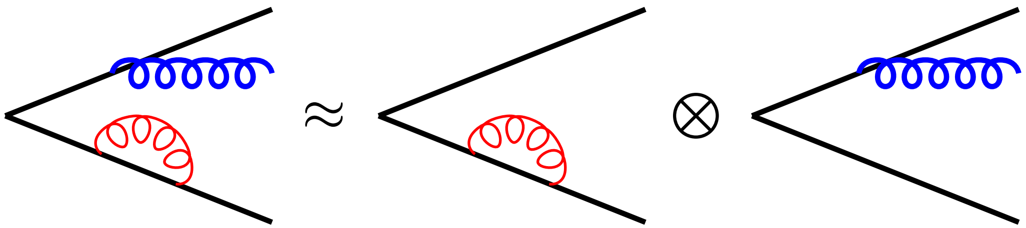
by shower generation + correction procedure

$$\overbrace{W^{\mathrm{ME}}}^{\text{wanted}} = \overbrace{W^{\mathrm{PS}}}^{\text{generated}} \; \overbrace{\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}}^{\text{correction}}$$

- Exponentiate ME correction by shower Sudakov form factor:

$$W^{\mathrm{PS}}_{\mathrm{actual}}(Q^2) = W^{\mathrm{ME}}(Q^2) \exp\left(- \int_{Q^2}^{Q^2_{\mathrm{max}}} W^{\mathrm{ME}}(Q'^2)\, \mathrm{d}Q'^2 \right)$$

- Do not normalize W^{ME} to $\sigma(\mathrm{NLO})$ (error $\mathcal{O}(\alpha_{\mathrm{s}}^2)$ either way)



$1 + \mathcal{O}(\alpha_{\mathrm{s}})$ ↓ $\qquad \int = 1$ ↓

$$\mathrm{d}\sigma = K \; \sigma_0 \; \mathrm{d}W^{\mathrm{PS}}$$

- Normally several shower histories $\Rightarrow$ $\sim$equivalent approaches

Final-State Shower Merging

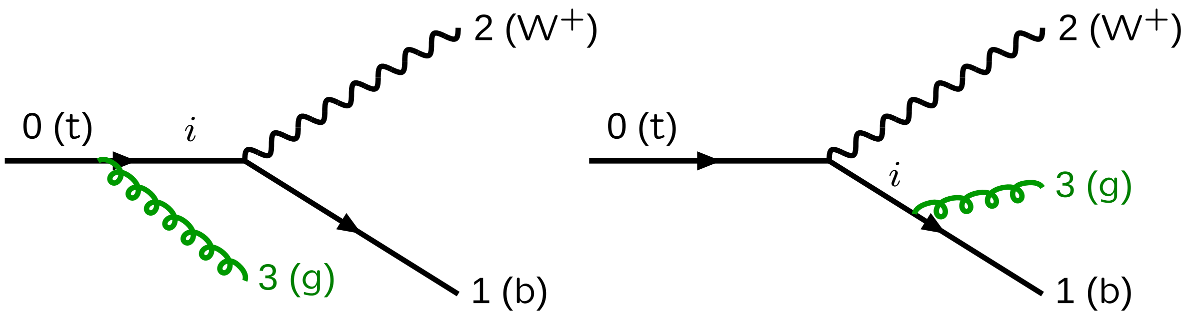
Merging with $\gamma^*/\mathrm{Z}^0 \to \mathrm{q\overline{q}g}$ for $m_\mathrm{q} = 0$ since long

(M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)

For $m_\mathrm{q} > 0$ pick $Q_i^2 = m_i^2 - m_{i,\mathrm{onshell}}^2$ as evolution variable since

$$W^{\mathrm{ME}} = \frac{(\ldots)}{Q_1^2 Q_2^2} - \frac{(\ldots)}{Q_1^4} - \frac{(\ldots)}{Q_2^4}$$

Coloured decaying particle also radiates:



ME $\frac{1}{Q_0^2 Q_1^2}$

matches

PS $\mathrm{b} \to \mathrm{bg}$

$\Rightarrow$ can merge PS with generic $a \to bc\mathrm{g}$ ME

(E. Norrbin & TS, NPB603 (2001) 297)

Subsequent branchings $\mathrm{q} \to \mathrm{qg}$: also matched to ME, with reduced energy of system

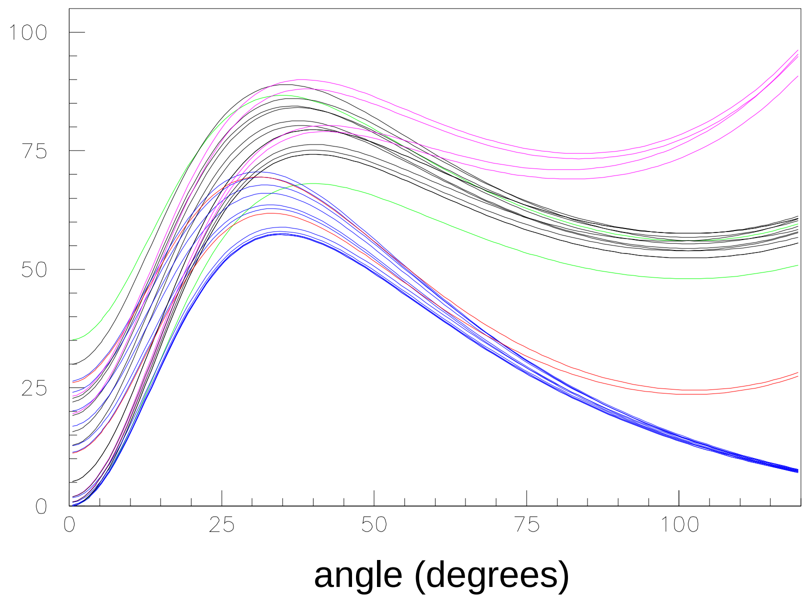
PYTHIA performs merging with generic FSR $a \to bc$g ME,

in SM: $\gamma^*/\mathrm{Z}^0/\mathrm{W}^\pm \to \mathrm{q}\overline{\mathrm{q}}$, $\mathrm{t} \to \mathrm{bW}^+$, $\mathrm{H}^0 \to \mathrm{q}\overline{\mathrm{q}}$,

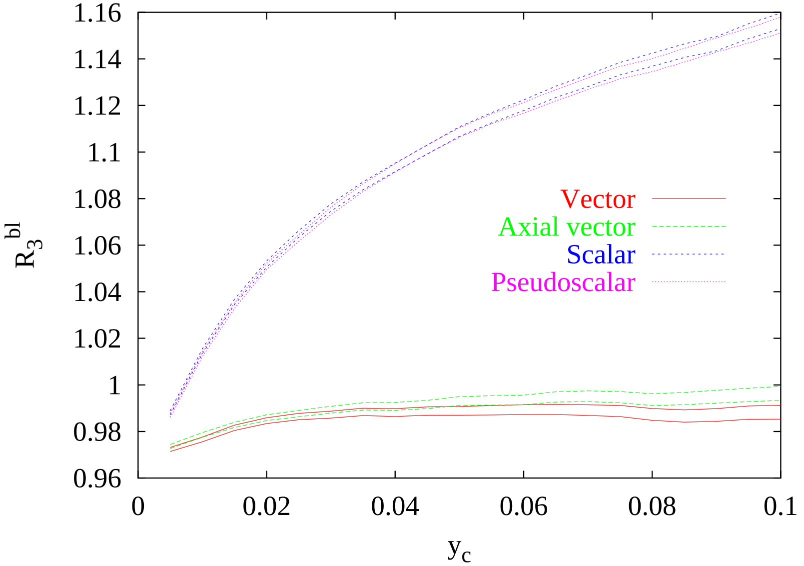
and MSSM: $\mathrm{t} \to \mathrm{bH}^+$, $\mathrm{Z}^0 \to \tilde{\mathrm{q}}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{W}^+$, $\mathrm{H}^0 \to \tilde{\mathrm{q}}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{H}^+$,

$\chi \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\chi \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\chi$, $\mathrm{t} \to \tilde{\mathrm{t}}\chi$, $\tilde{\mathrm{g}} \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\tilde{\mathrm{g}}$, $\mathrm{t} \to \tilde{\mathrm{t}}\tilde{\mathrm{g}}$

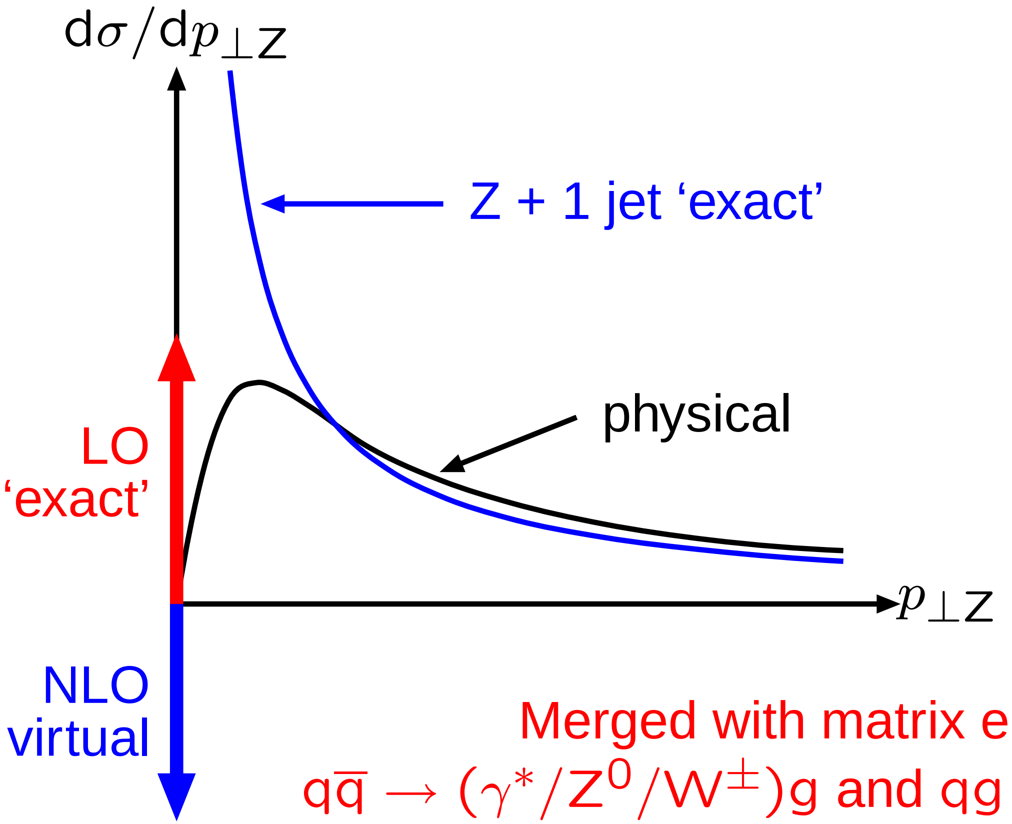
g emission for different colour, spin and parity:



$R_3^{\mathrm{bl}}(y_c)$: mass effects in Higgs decay:



Initial-State Shower Merging



resummation:
physical $p_{\perp Z}$ spectrum

shower: ditto
+ accompanying
jets (exclusive)

Merged with matrix elements for
$q\bar{q} \to (\gamma^*/Z^0/W^\pm)g$ and $qg \to (\gamma^*/Z^0/W^\pm)q'$:

(G. Miu & TS, PLB449 (1999) 313)

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{q\bar{q}'\to gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1$$

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{qg\to q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3$$

with $Q^2 = -m^2$
and $z = m_W^2/\hat{s}$

Merging in HERWIG

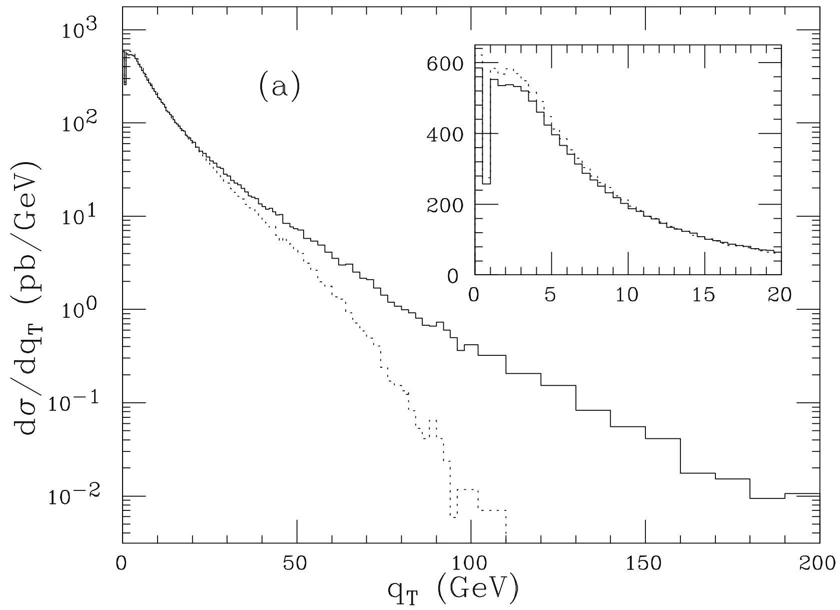
HERWIG also contains merging, for

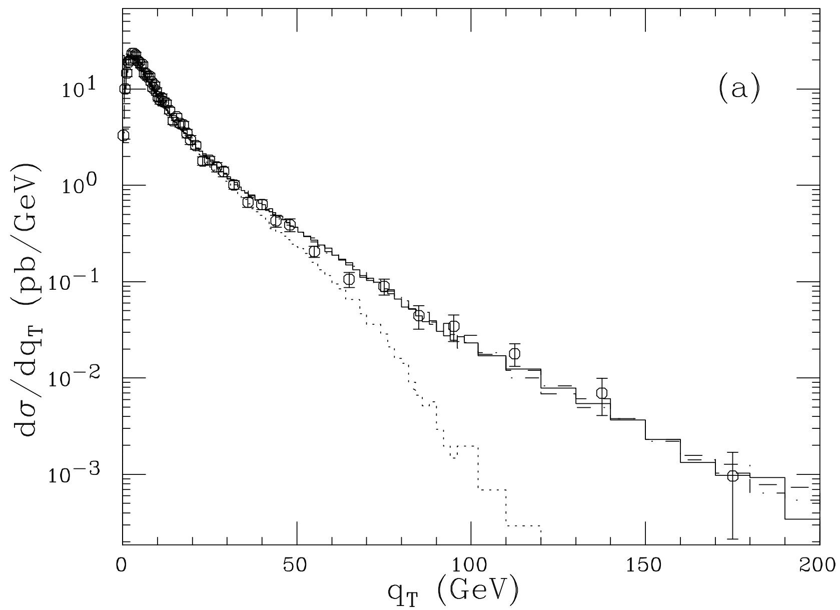
- $Z^0 \to q\bar{q}$
- $t \to bW^+$
- $q\bar{q} \to Z^0$

and some more

Special problem:
angular ordering does not cover full phase space; so
(1) fill in “dead zone” with ME
(2) apply ME correction
in allowed region

Important for agreement with data:





Vetoed Parton Showers

S. Catani, F. Krauss, R. Kuhn, B.R. Webber, JHEP 0111 (2001) 063; L. Lönnblad, JHEP0205 (2002) 046;
F. Krauss, JHEP 0208 (2002) 015; S. Mrenna, P. Richardson, JHEP0405 (2004) 040;
M.L. Mangano, in preparation

Generic method to combine ME's of several different orders to NLL accuracy; will be a 'standard tool' in the future

Basic idea:

- consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
- σ_i, $i \geq 1$, are divergent in soft/collinear limits
- absent virtual corrections would have ensured "detailed balance", i.e. an emission that adds to σ_{i+1} subtracts from σ_i
- such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
- so use shower routines to provide missing virtual corrections $\Rightarrow$ rejection of events (especially) in soft/collinear regions

Veto scheme:

1) Pick hard process, mixing according to $\sigma_0 : \sigma_1 : \sigma_2 : \ldots$,
above some ME cutoff, with large fixed α_{s0}

2) Reconstruct imagined shower history (in different ways)

3) Weight $W_\alpha = \prod_{\text{branchings}}(\alpha_s(k_{\perp i}^2)/\alpha_{s0}) \Rightarrow$ accept/reject

CKKW-L:

4) Sudakov factor for non-emission
on all lines above ME cutoff
$W_{\text{Sud}} = \prod_{\text{“propagators”}} \text{Sudakov}(k_{\perp \text{beg}}^2, k_{\perp \text{end}}^2)$

4a) CKKW : use NLL Sudakovs

4b) L: use trial showers

5) $W_{\text{Sud}} \Rightarrow$ accept/reject

6) do shower,
vetoing emissions above cutoff

MLM:

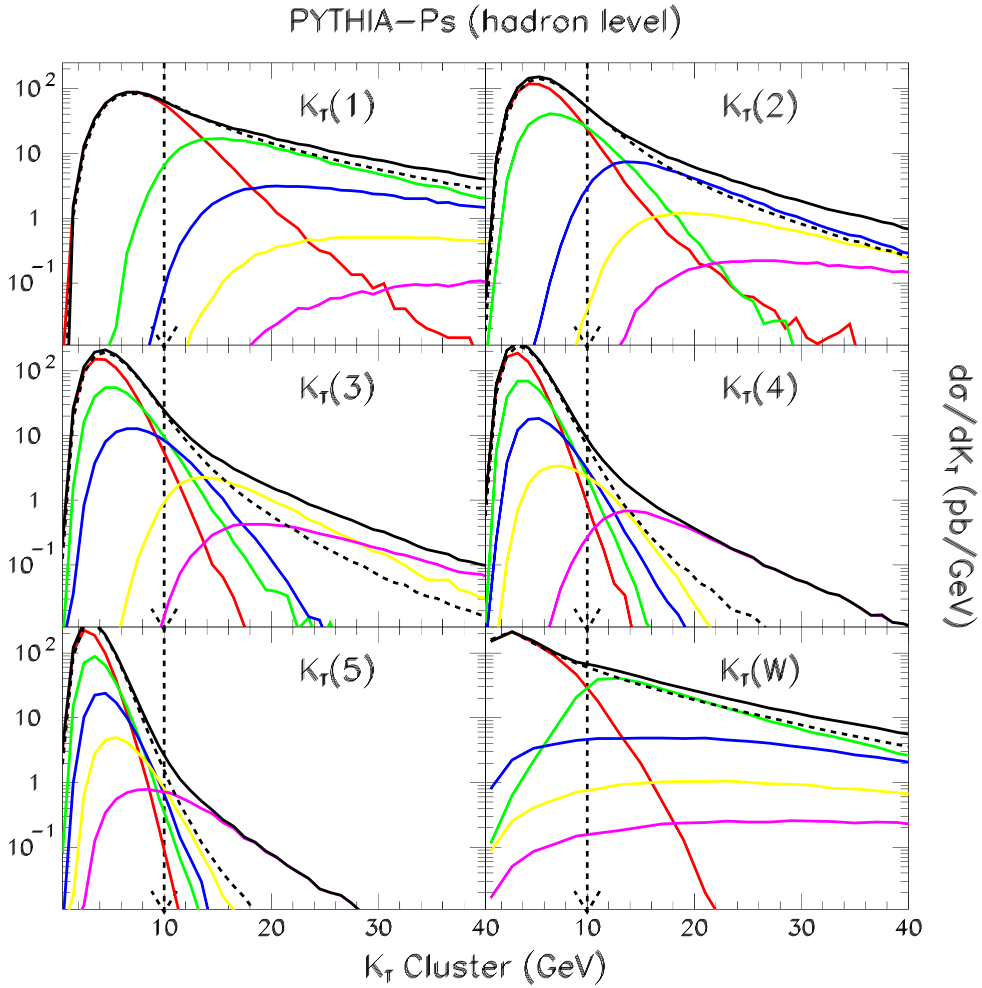
4) do parton showers

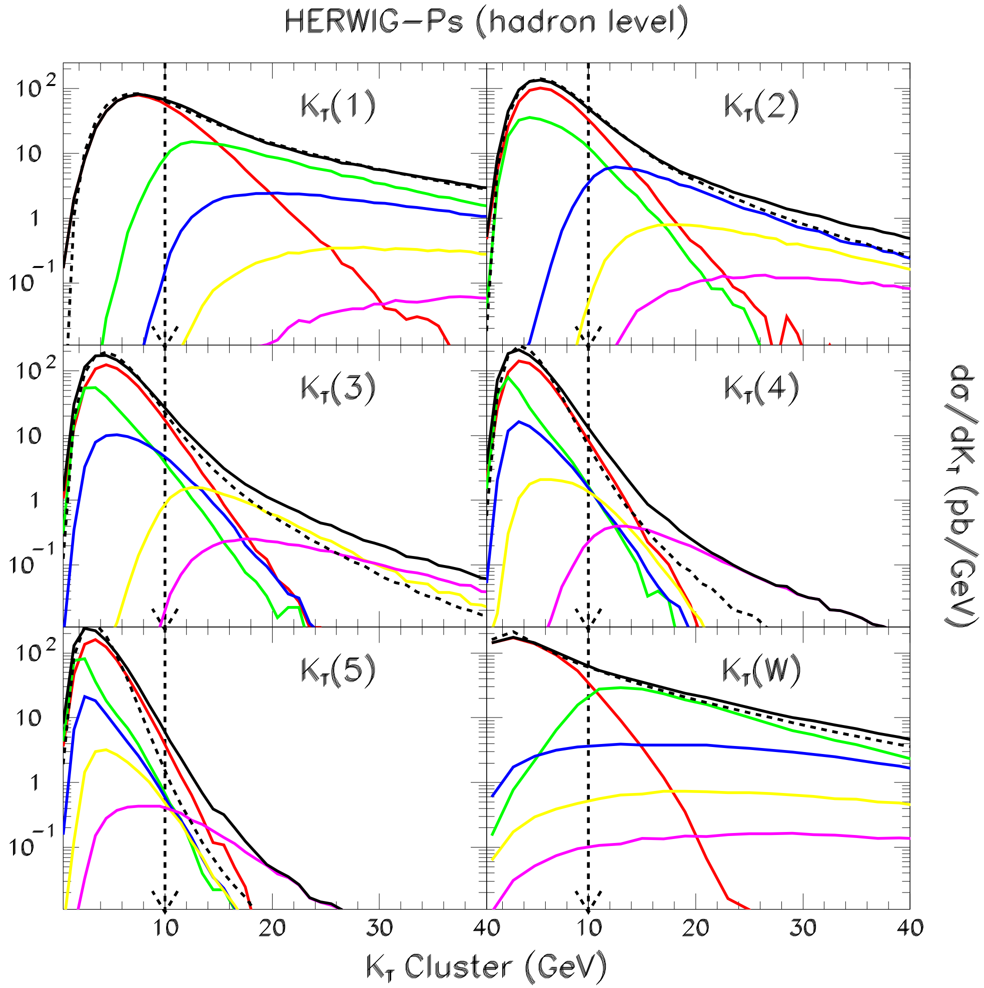
5) (cone-)cluster
showered event

6) match partons and jets

7) if all partons are matched,
and $n_{\text{jet}} = n_{\text{parton}}$,
keep the event,
else discard it

CKKW mix of W + (0, 1, 2, 3, 4) partons, hadronized and clustered to jets:





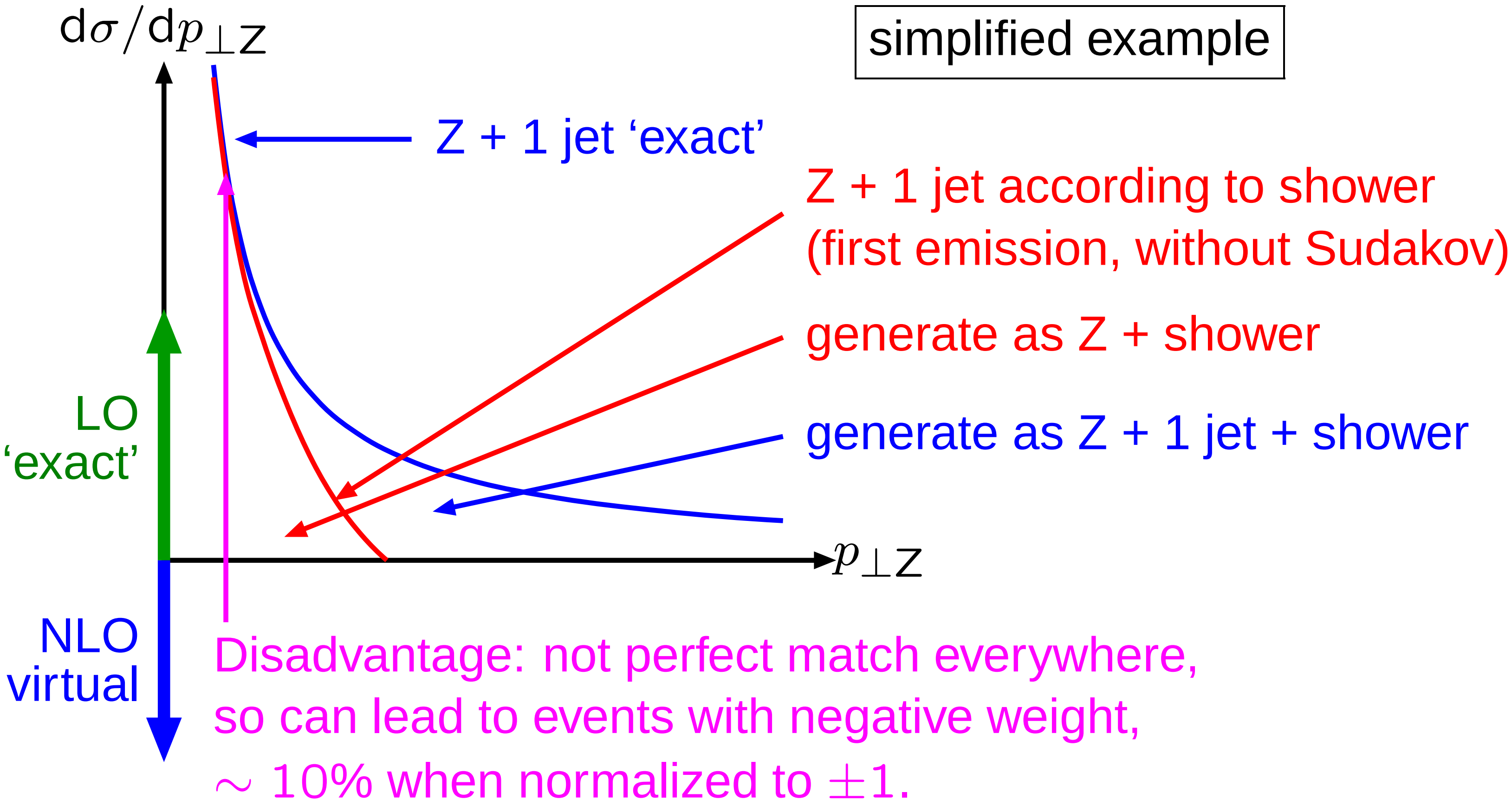
MC@NLO

Objectives:

- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of α_S.
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of “normal” events, that can be processed further.

Basic scheme (simplified!):

1) Calculate the NLO matrix element corrections to an n-body process (using the subtraction approach).
2) Calculate analytically (no Sudakov!) how the first shower emission off an n-body topology populates $(n+1)$-body phase space.
3) Subtract the shower expression from the $(n+1)$ ME to get the “true” $(n+1)$ events, and consider the rest of σ_{NLO} as n-body.
4) Add showers to both kinds of events.



MC@NLO in comparison:

- Superior with respect to “total” cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.

$\Rightarrow$ pick according to current task and availability.

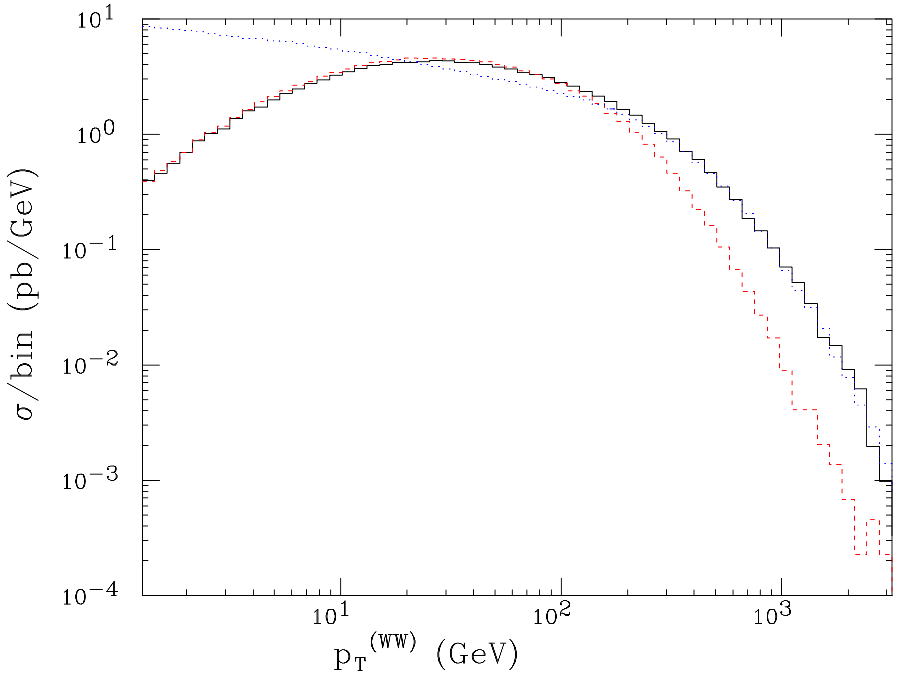
MC@NLO 2.31 [hep-ph/0402116]

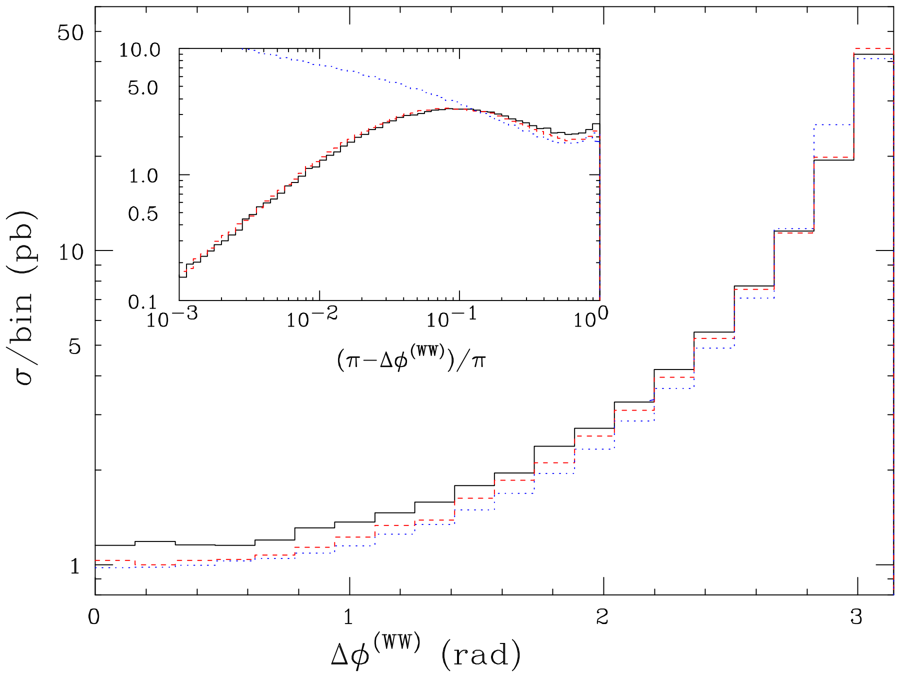
IPROC	Process
–1350–IL	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1360–IL	$H_1H_2 \to (Z \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1370–IL	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1460–IL	$H_1H_2 \to (W^+ \to) l^+_{\rm IL}\nu_{\rm IL} + X$
–1470–IL	$H_1H_2 \to (W^- \to) l^-_{\rm IL}\bar{\nu}_{\rm IL} + X$
–1396	$H_1H_2 \to \gamma^*(\to \sum_i f_i\bar{f}_i) + X$
–1397	$H_1H_2 \to Z^0 + X$
–1497	$H_1H_2 \to W^+ + X$
–1498	$H_1H_2 \to W^- + X$
–1600–ID	$H_1H_2 \to H^0 + X$
–1705	$H_1H_2 \to b\bar{b} + X$
–1706	$H_1H_2 \to t\bar{t} + X$
–2850	$H_1H_2 \to W^+W^- + X$
–2860	$H_1H_2 \to Z^0Z^0 + X$
–2870	$H_1H_2 \to W^+Z^0 + X$
–2880	$H_1H_2 \to W^-Z^0 + X$

(Frixione, Webber)

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented

W^+W^- Observables





These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

Solid: MC@NLO

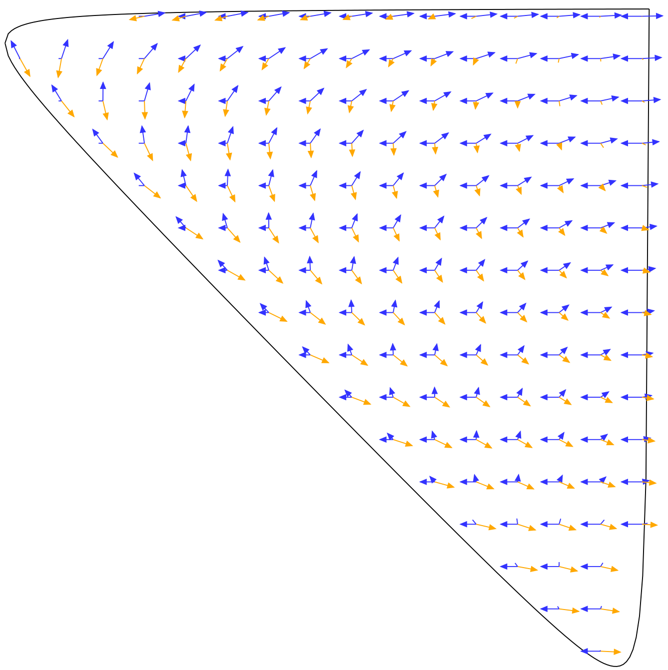
Dashed: HERWIG$\times\frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

HERWIG shower improvements

Quasi–Collinear Limit (Heavy Quarks)

$q\bar{q}g$–Phase space $(x, \bar{x})$



Sudakov-basis p, n with $p^2 = M^2$ ('forward'), $n^2 = 0$ ('backward'),

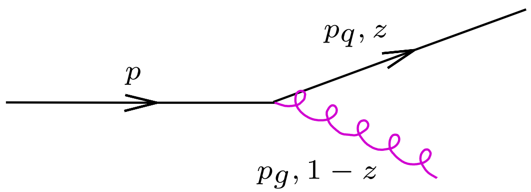
$$p_q = zp + \beta_q n - q_\perp$$
$$p_g = (1-z)p + \beta_g n + q_\perp$$

Collinear limit for radiation off heavy quark,

$$P_{gq}(z, \boldsymbol{q}^2, m^2) = C_F \left[\frac{1+z^2}{1-z} - \frac{2z(1-z)m^2}{\boldsymbol{q}^2 + (1-z)^2 m^2} \right] = \frac{C_F}{1-z} \left[1 + z^2 - \frac{2m^2}{z\tilde{q}^2} \right]$$

$\longrightarrow$ $\tilde{q}^2 \sim \boldsymbol{q}^2$ may be used as evolution variable.

Single emission:



New evolution variables

Kinematics to allow better treatment of heavy particles, avoiding overlapping regions in phase space, in particular for soft emissions

We choose $\tilde{q}^2$ as new evolution variable,

$$\tilde{q}^2 = \frac{\boldsymbol{q}^2}{z^2(1-z)^2} + \frac{m^2}{z^2} \quad \text{for} \quad q \to qg$$

and with the argument of running α_S chosen according to

$$\alpha_S(z^2(1-z)^2\tilde{q}^2)$$

angular ordering

$$\tilde{q}_{i+1} < z_i\tilde{q}_i \qquad \tilde{k}_{i+1} < (1-z_i)\tilde{q}_i$$

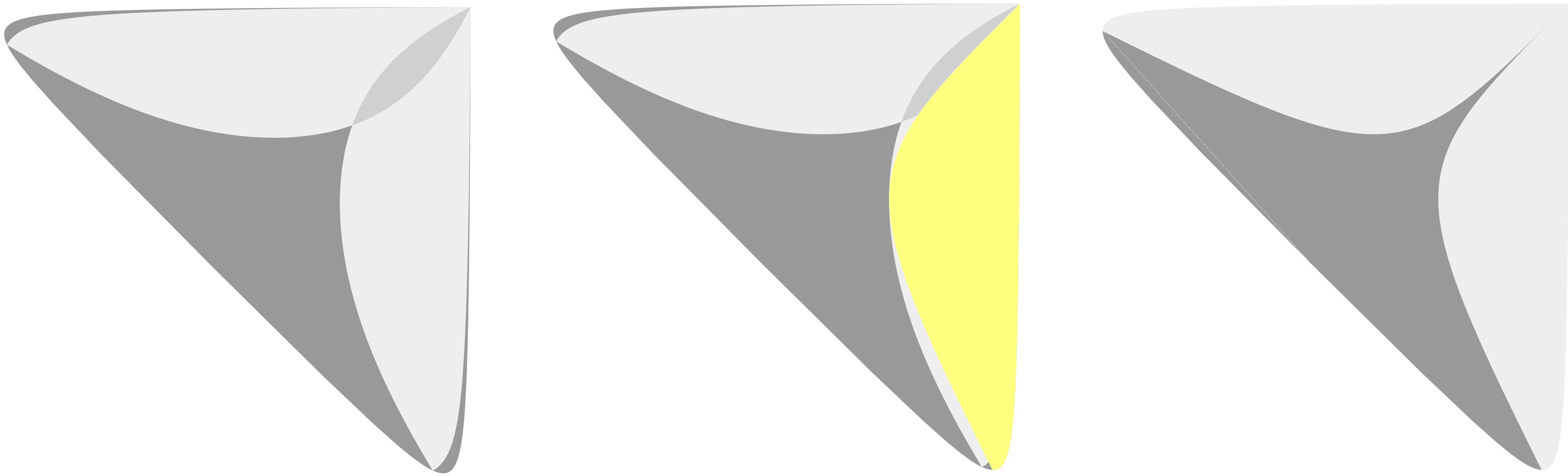
Technically: *reinterpretation* of known evolution variables, i.e. the branching probability for $a \to bc$ still is

$$dP(a \to bc) = \frac{d\tilde{q}^2}{\tilde{q}^2}\frac{C_i\alpha_S}{2\pi}P_{bc}(z,\tilde{q})\,dz$$

$\longrightarrow$ Sudakov's etc. technically remain the same!

$q\bar{q}g$ Phase Space old vs new variables

Consider $(x, \bar{x})$ phase space for $e^+e^- \longrightarrow q\bar{q}g$



HERWIG Comparison Herwig++

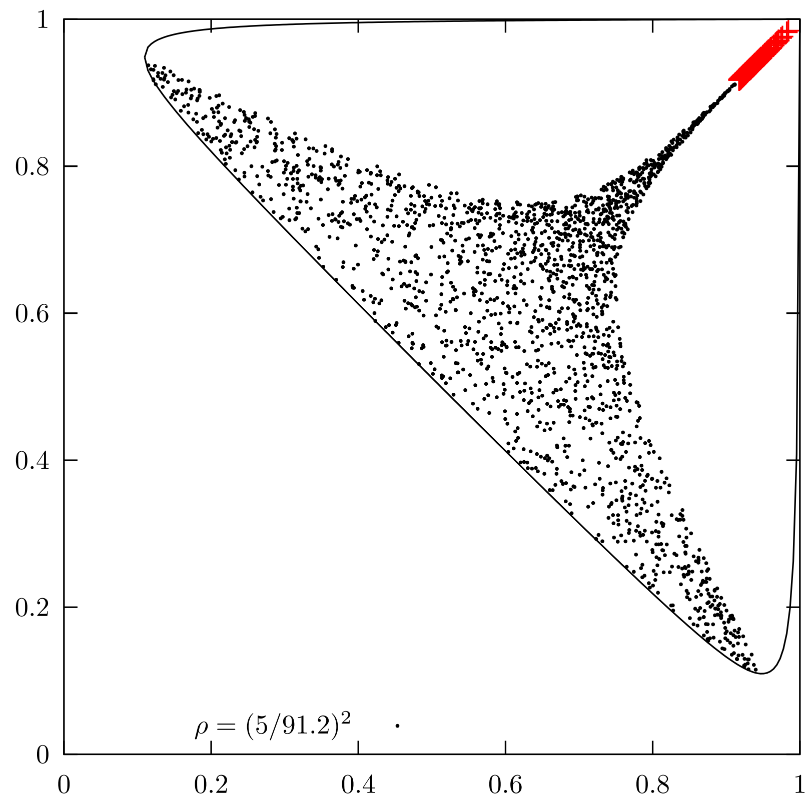
✗ Larger dead region with new variables.

✓ Smooth coverage of soft gluon region.

✓ No overlapping regions in phase space.

Hard Matrix Element Corrections

- Points $(x, \bar{x})$ in dead region chosen acc to LO $e^+e^- \to q\bar{q}g$ matrix element and accepted acc to ME weight.
- About 3% of all events are actually hard $q\bar{q}g$ events.
- Red points have weight > 1, practically no error by setting weight to one.
- Event oriented according to given $q\bar{q}$ geometry. Quark direction is kept with weight $x^2/(x^2 + \bar{x}^2)$.



PYTHIA shower improvements

Objective:
Incorporate several of the good points of the dipole formalism (like ARIADNE) within the shower approach ($\Rightarrow$ hybrid)

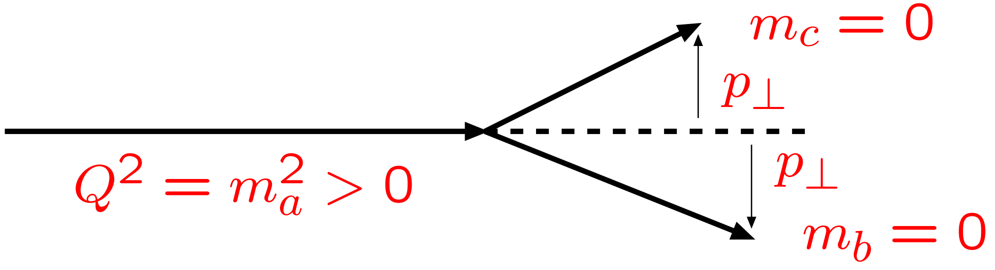
$\pm$ explore alternative $p_\perp$ definitions
$+$ $p_\perp$ ordering $\Rightarrow$ coherence inherent
$+$ ME merging works as before (unique $p_\perp^2 \leftrightarrow Q^2$ mapping; same z)
$+$ $g \to q\bar{q}$ natural
$+$ kinematics constructed after each branching
(partons explicitly on-shell until they branch)
$+$ showers can be stopped and restarted at given $p_\perp$ scale
(not yet worked-out for ISR+FSR)
$+$ $\Rightarrow$ well suited for ME/PS matching (L-CKKW, real+fictitious showers)
$+$ $\Rightarrow$ well suited for simple match with $2 \to 2$ hard processes
$++$ well suited for *interleaved multiple interactions*

Simple kinematics

Consider branching $a \to bc$ in lightcone coordinates $p^{\pm} = E \pm p_z$

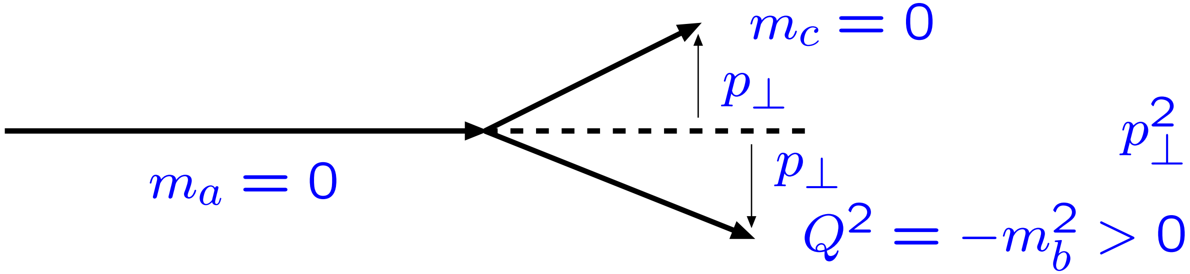
$$\left.\begin{array}{l} p_b^+ = z p_a^+ \\ p_c^+ = (1-z) p_a^+ \\ p^- \text{ conservation} \end{array}\right\} \Longrightarrow m_a^2 = \frac{m_b^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1-z}$$

Timelike branching:



$$p_\perp^2 = z(1-z)Q^2$$

Spacelike branching:



$$p_\perp^2 = (1-z)Q^2$$

Guideline, not final $p_\perp$!

Transverse-momentum-ordered showers

1) *Define* $\mathrm{p}^2_{\perp\mathrm{evol}} = z(1-z)Q^2 = z(1-z)M^2$ for FSR
$\mathrm{p}^2_{\perp\mathrm{evol}} = (1-z)Q^2 = (1-z)(-M^2)$ for ISR

2) *Evolve* all partons *downwards* in $\mathrm{p}_{\perp\mathrm{evol}}$ from common $p_{\perp\mathrm{max}}$

$$\mathrm{d}\mathcal{P}_a = \frac{\mathrm{dp}^2_{\perp\mathrm{evol}}}{\mathrm{p}^2_{\perp\mathrm{evol}}} \frac{\alpha_\mathrm{s}(\mathrm{p}^2_{\perp\mathrm{evol}})}{2\pi} P_{a\to bc}(z)\,\mathrm{d}z\,\exp\left(-\int_{\mathrm{p}^2_{\perp\mathrm{evol}}}^{p^2_{\perp\mathrm{max}}}\cdots\right)$$

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{dp}^2_{\perp\mathrm{evol}}}{\mathrm{p}^2_{\perp\mathrm{evol}}} \frac{\alpha_\mathrm{s}(\mathrm{p}^2_{\perp\mathrm{evol}})}{2\pi} \frac{x' f_a(x', \mathrm{p}^2_{\perp\mathrm{evol}})}{x f_b(x, \mathrm{p}^2_{\perp\mathrm{evol}})} P_{a\to bc}(z)\,\mathrm{d}z\,\exp\left(-\cdots\right)$$

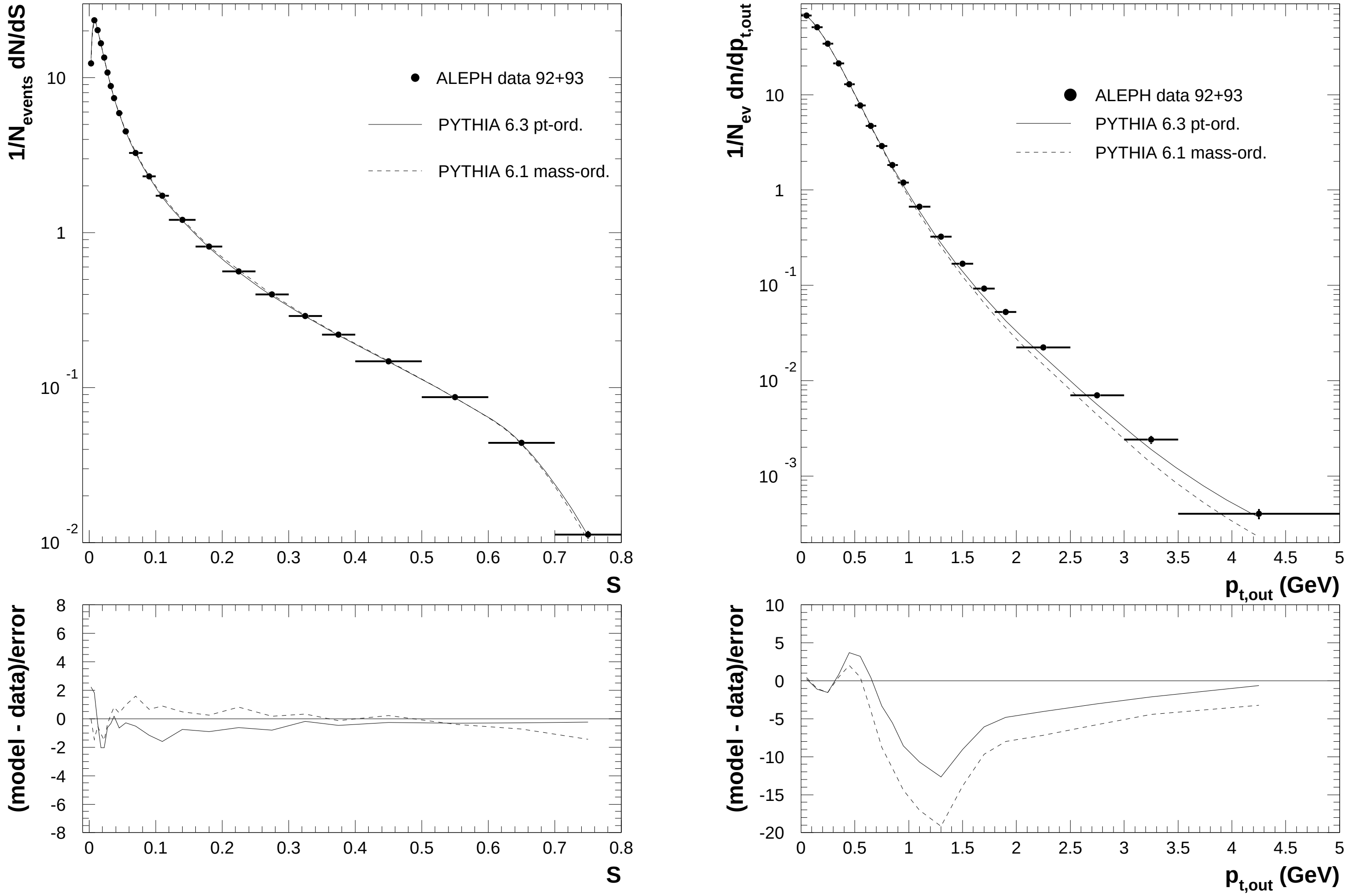
Pick the one with *largest* $\mathrm{p}_{\perp\mathrm{evol}}$ to undergo branching; also gives z.

3) Kinematics: *Derive* $Q^2 = \pm M^2$ by inversion of 1), but then interpret z as *energy fraction* (not lightcone) in “dipole” rest frame, so that *Lorentz invariant* and matched to matrix elements.
Assume yet unbranched partons on-shell and shuffle $(E, \mathbf{p})$ inside dipole.

4) *Iterate* $\Rightarrow$ combined sequence $p_{\perp\mathrm{max}} > p_{\perp 1} > p_{\perp 2} > \ldots > p_{\perp\mathrm{min}}$.

Testing the FSR algorithm

Tune performed by Gerald Rudolph (Innsbruck)
based on ALEPH 1992+93 data:



Quality of fit

Distribution of	nb.of interv.	$\sum \chi^2$ of model PY6.3 $p_\perp$-ord.	PY6.1 mass-ord.
Sphericity	23	25	16
Aplanarity	16	23	168
1$-$Thrust	21	60	8
$\text{Thrust}_{\text{minor}}$	18	26	139
jet res. y_3(D)	20	10	22
$x = 2p/E_{\text{cm}}$	46	207	151
$p_{\perp\text{in}}$	25	99	170
$p_{\perp\text{out}} < 0.7$ GeV	7	29	24
$p_{\perp\text{out}}$	(19)	(590)	(1560)
x(B)	19	20	68
sum $N_{\text{dof}} =$	190	497	765

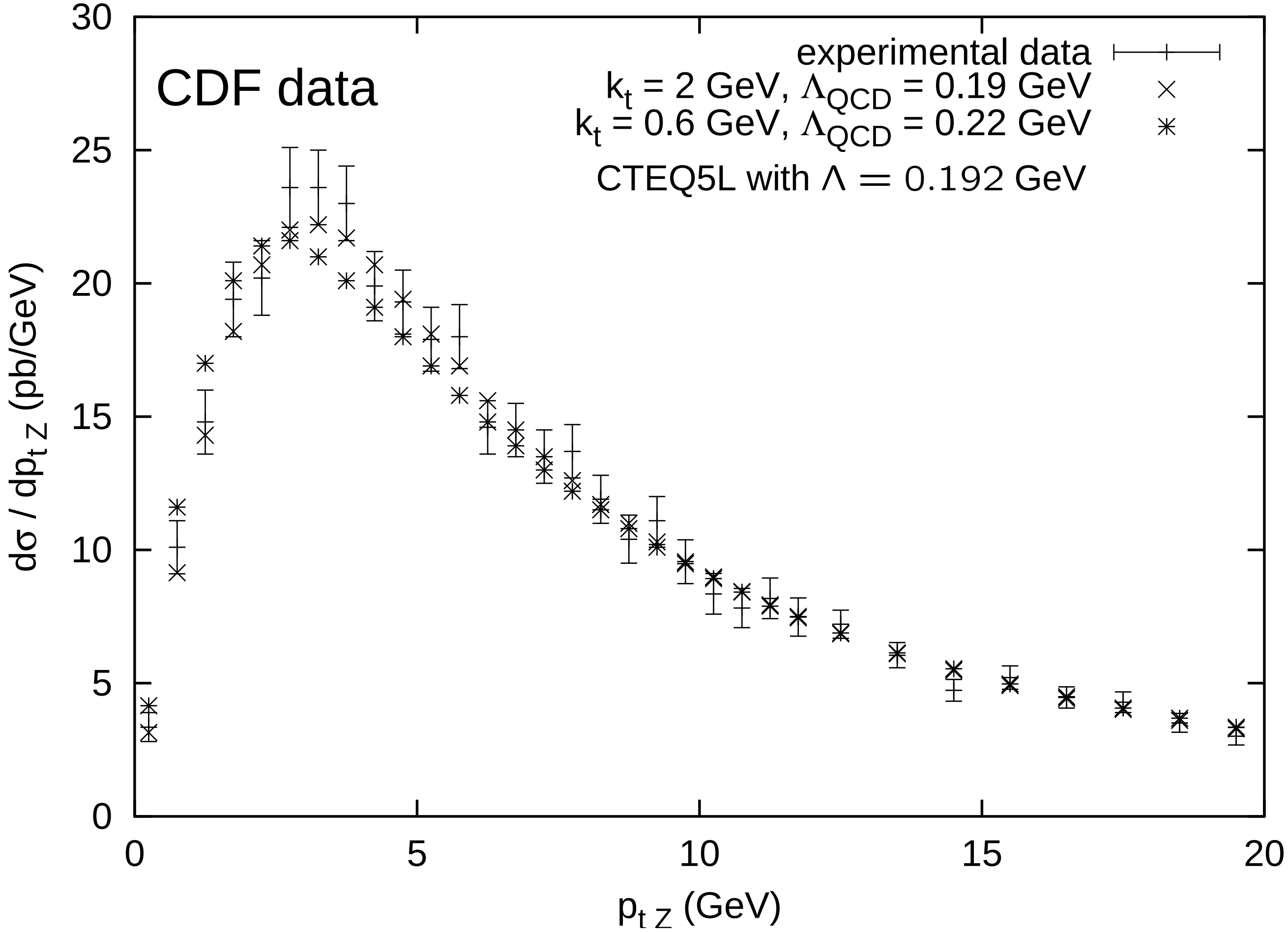
Generator is not assumed to be perfect, so
add fraction p of value in quadrature to the definition of the error:

p	0%	0.5%	1%
$\sum \chi^2$	523	364	234

for $N_{\text{dof}} = 196 \Rightarrow$ generator is 'correct' to $\sim$1%
except $p_{\perp\text{out}} > 0.7$ GeV (10%–20% error)

Testing the ISR algorithm

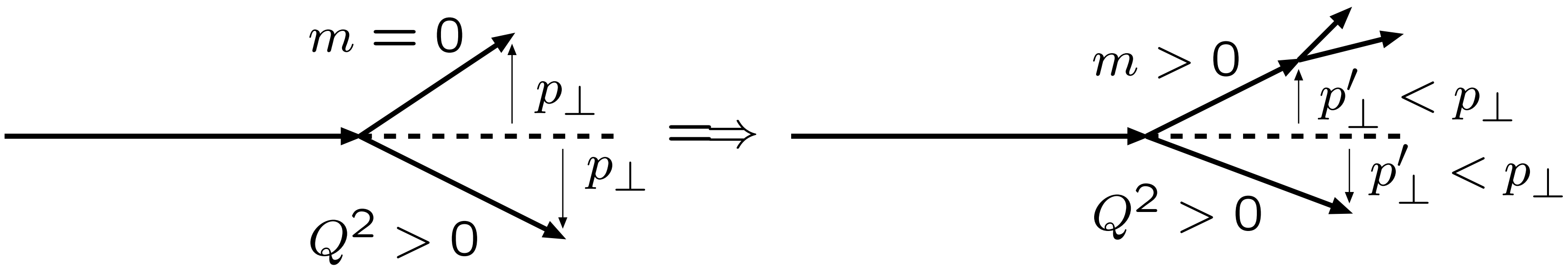
Still only begun. . .

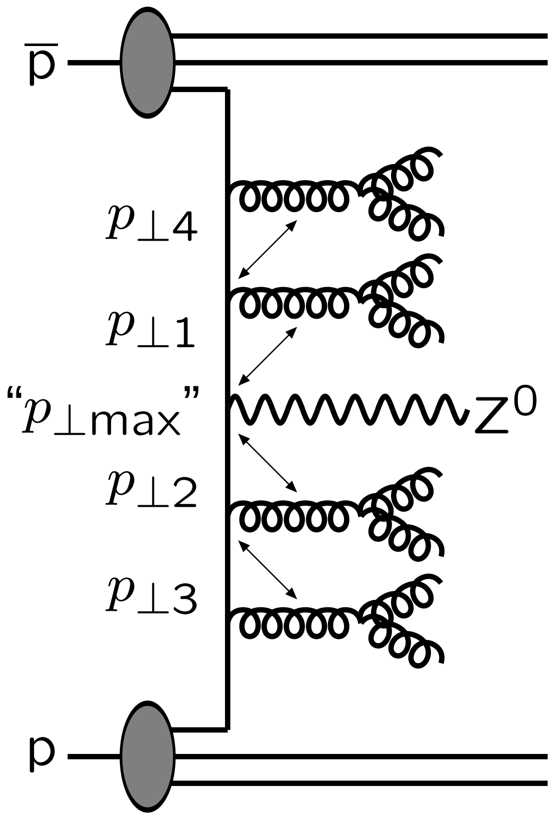


. . . but so far no showstoppers

Combining FSR with ISR

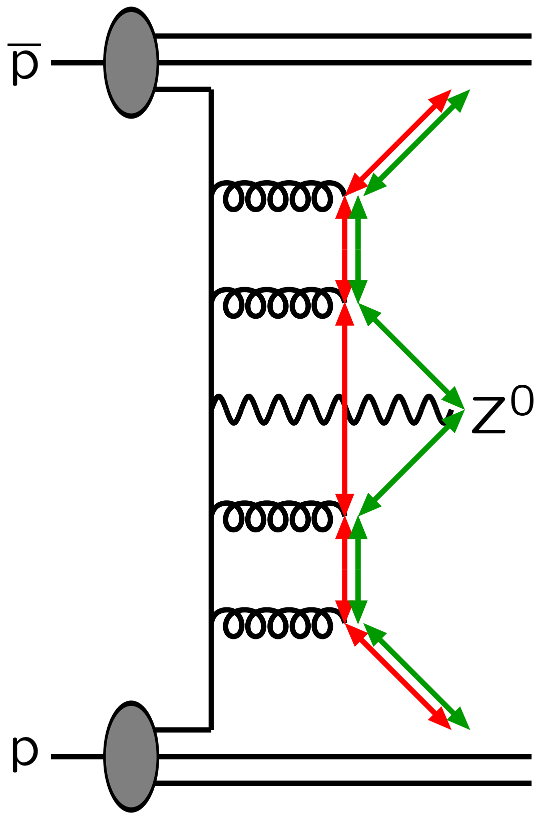
Evolution of timelike sidebranch cascades can reduce $p_\perp$:





Old:

Z^0 takes recoil



New:

Z^0 takes recoil

or

Z^0 unaffected by FSR (latter later)

Shower Summary

• Showers bring us *from* few-parton "pencil-jet" topologies
to multi-broad-jet states. •

• Necessary complement to matrix elements: •
⋆ Do not trust off-the-shelf ME for $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \lesssim 1$ ⋆
⋆ Do not trust unmatched PS for $R \gtrsim 1$ ⋆

• Two main lines of evolution: •

⋆ (1) Improve algorithm as such: evolution variables, kinematics,
NLL, small-x, $k_\perp$ factorization, BFKL/CCFM, ... ⋆

⋆ (2) Improve matching ME-PS: merging,
vetoed parton showers, MC@NLO ⋆

⋆ $\Rightarrow$ active area of development; high profile ⋆

• Tomorrow: Multiple parton–parton interactions; the other
perturbative mechanism of complicating a simple few-parton topology •