

# Rare decays and flavour physics: a theoretical overview



*Gino Isidori*  
Laboratori Nazionali di Frascati

SPSC meeting Villar, 26<sup>th</sup> April 2004

- Introduction
  - The flavour sector of the SM & the flavour problem
- Rare FCNC Kaon decays within the SM
  - Generalities
  - The magnificent four:  
 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 e^+ e^-$  &  $K_L \rightarrow \pi^0 \mu^+ \mu^-$
- Rare FCNC decays beyond the SM
  - Minimal Flavour Violation
  - Beyond the MFV ansatz
- A few comments on rare B, D decays [ $B, D \rightarrow \mu^+ \mu^-$ ]
- Conclusions

## • Introduction

According to:

- solid cosmological evidences [cosmological constant, dark matter, matter-anti-matter asymmetry, inflation, ... ]
  - convincing theoretical arguments [inclusion of gravity, instability of the Higgs potential, neutrino masses, origin of flavour, ...]
- ( + some *religious beliefs* [string theory...] )

there is no doubt that the Standard Model

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_i, \Psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \Psi_i, \phi)$$

cannot be the end of the story...

...but (unfortunately?) we must admit that the SM works pretty well up to the e.w. scale

➔ Natural to consider the SM as an **effective theory**, or the low-energy limit of a more fundamental theory, with new degrees of freedom appearing above some energy threshold  $\Lambda_{\text{NP}}$

Given the great success of the SM up to LEP energies:

- $\Lambda_{\text{NP}} \geq \langle \phi \rangle \sim 250 \text{ GeV}$
- the new d.o.f. must respect the  $SU(3) \times SU(2) \times U(1)$  gauge symmetry



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_i, \psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \psi_i, \phi) + \sum_i \frac{c_i}{\Lambda_{\text{NP}}} \mathcal{O}_i^{(d \geq 5)} + \dots$$

general parameterization of the possible new heavy d.o.f. valid as long as we perform low-energy experiments

Key questions:

- How large can  $\Lambda_{\text{NP}}$  be?
- Which is the nature ( $\Leftrightarrow$  **symmetries**) of the new degrees of freedom?

➔ Natural to consider the SM as an **effective theory**, or the low-energy limit of a more fundamental theory, with new degrees of freedom appearing above some energy threshold  $\Lambda_{\text{NP}}$

Given the great success of the SM up to LEP energies:

- $\Lambda_{\text{NP}} \geq \langle \phi \rangle \sim 250 \text{ GeV}$
- the new d.o.f. must respect the  $SU(3) \times SU(2) \times U(1)$  gauge symmetry



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_i, \psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \psi_i, \phi) + \sum_i \frac{c_i}{\Lambda_{\text{NP}}} \mathcal{O}_i^{(d \geq 5)} + \dots$$

Key questions:

- How large can  $\Lambda_{\text{NP}}$  be?
- Which is the nature ( $\Leftrightarrow$  **symmetries**) of the new degrees of freedom?

general parameterization of the possible new heavy d.o.f. valid as long as we perform low-energy experiments

Flavour physics – and particularly precision studies of rare decays – provides a key ingredient to answer these questions

The flavour sector of the SM:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_i, \Psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \Psi_i, \phi)$$

3 identical replica of the basic fermion family

flavour degeneracy broken by the **Yukawa** interaction

$$\begin{aligned} Q_i Y_d^{ij} d_j \phi &\rightarrow Q_i M_d^{ij} d_j \\ Q_i Y_u^{ij} u_j \phi_c &\rightarrow Q_i M_u^{ij} d_j \end{aligned}$$

$$M_d = \text{diag}(m_d, m_s, m_b)$$

$$M_u^+ = \text{diag}(m_u, m_c, m_t) \times V_{\text{CKM}}$$

The flavour sector of the SM:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_i, \Psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \Psi_i, \phi)$$

3 identical replica of the basic fermion family

flavour degeneracy broken by the **Yukawa** interaction

Nowadays we have a good knowledge of all the 10 observables entries [6 masses + 4 CKM angles] of the quark mass matrices:

$$\begin{aligned} Q_i Y_d^{ij} d_j \phi &\rightarrow Q_i M_d^{ij} d_j \\ Q_i Y_u^{ij} u_j \phi_c &\rightarrow Q_i M_u^{ij} d_j \end{aligned}$$

$$M_d = \text{diag}(m_d, m_s, m_b)$$

$$M_u^+ = \text{diag}(m_u, m_c, m_t) \times V_{\text{CKM}}$$

- strong hierarchical structure
- no clear symmetric pattern

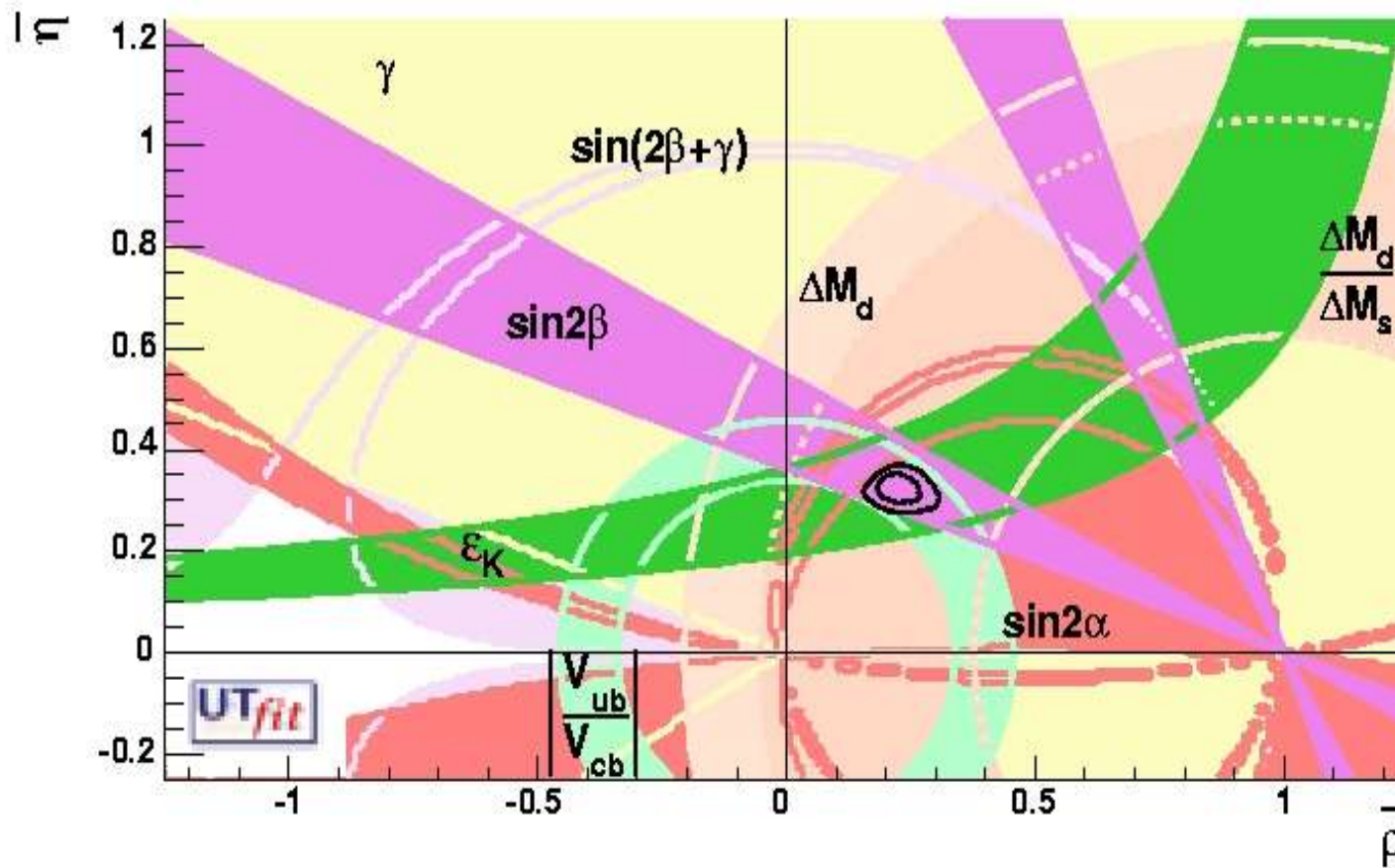
$$V_{\text{CKM}} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4) \quad \lambda = \sin\theta_c \approx 0.22$$

Wolfenstein, '83

$V_{ub}$  (points to  $A\lambda^3(\rho-i\eta)$ )

$V_{td}$  (points to  $A\lambda^3(1-\rho-i\eta)$ )

$\rho$ - $\eta$  plane: ICHEP '04 status

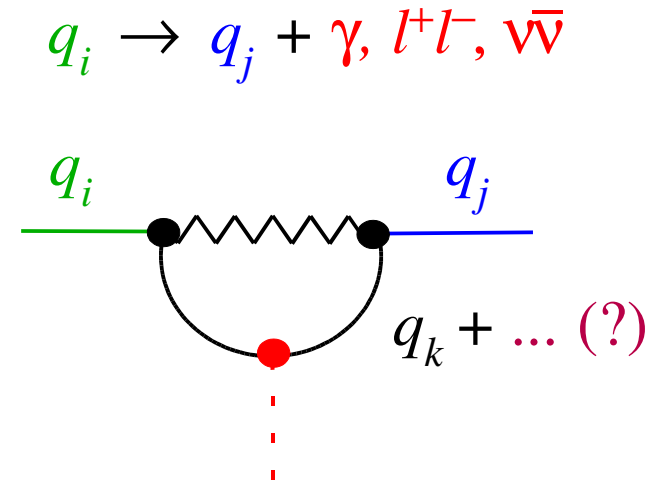


courtesy of  $UT_{fit}$   
M. Bona *et al.*

➔ Precision studies of rare decays can (slightly) improve our knowledge of the CKM matrix but their main interest is in probing the flavour structure of new physics:



Rare processes mediated by Flavor Changing Neutral Currents are the ideal candidates



- no SM tree-level contribution
- strong suppression – within the SM – by CKM hierarchy
- calculable with high precision within the SM if dominated by short-distance dynamics [*key point*]



precise determination of flavor mixing within the SM

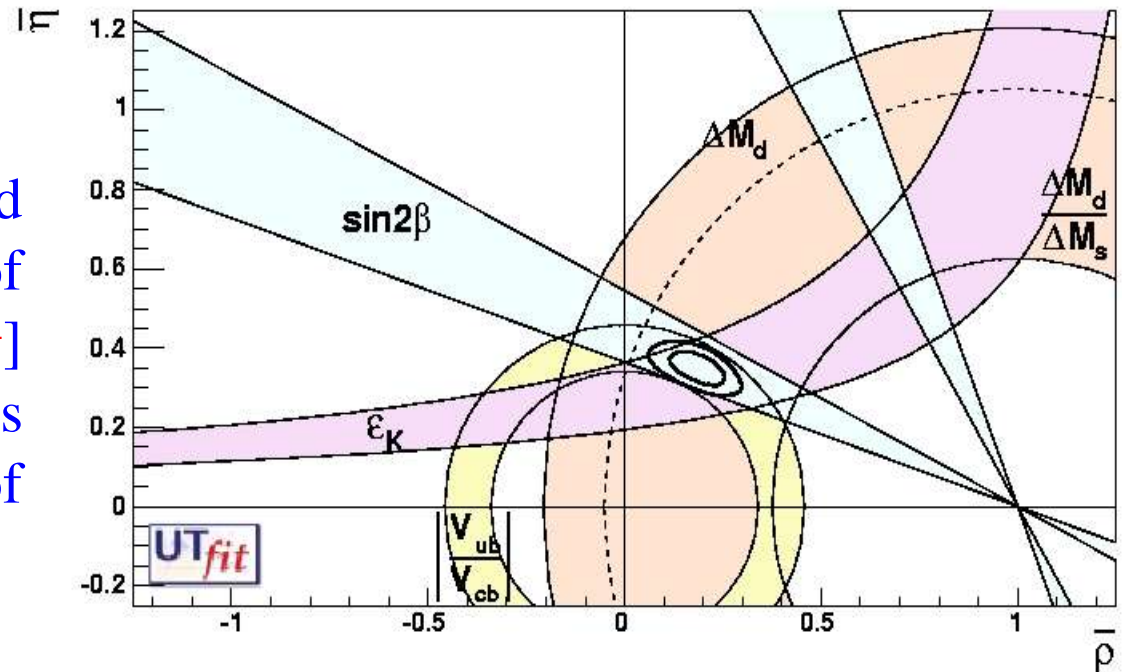


enhanced sensitivity to  
[ *the flavour structure of* ]  
physics beyond the SM



## The flavour problem:

Precise data on loop-induced flavour-changing processes of  $\Delta F=2$  type [*K-K & B-B mixing*] already provide stringent bounds on possible new degrees of freedom beyond the SM



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_i, \Psi_i) + \mathcal{L}_{\text{Higgs}}(A_i, \Psi_i, \phi) + \sum_i \frac{c_i}{\Lambda_{\text{NP}}} O_i^{(d \geq 5)} + \dots$$

E.g.:  $K^0-\bar{K}^0$  mixing  $\Rightarrow \Lambda_{\text{NP}} > 10^3 \text{ TeV}$  for  $O^{(6)} \sim (\bar{s}d)^2$

...while a natural stabilization of the Higgs sector  $\Rightarrow \Lambda_{\text{NP}} \sim 1 \text{ TeV}$

Two possible solutions:

- *pessimistic* [very unnatural]:  $\Lambda > 100 \text{ TeV}$ 
  - $\Rightarrow$  almost nothing to learn from rare decays  
(but also very difficult to find evidences of NP at LHC...)
- *natural*:  $\Lambda \sim 1 \text{ TeV}$  + flavor-mixing protected by additional symmetries
  - $\Rightarrow$  still a lot to learn from rare decays:
    - present fits of the CKM unitarity triangle involve only  $\Delta F=2$  loops + tree-level amplitudes  $\Rightarrow$  we know very little yet about rare  $\Delta F=1$  FCNC transitions
    - CKM fits provide mainly a consistency check of the SM hypothesis but do not provide a bound on the NP parameter space  $\Rightarrow$  only with the help of rare decays we can study the underlying flavor symmetry in a model-independent way

Towards a model independent approach to the flavour problem:

FLAVOUR STRUCTURE

ELECTROWEAK  
STRUCTURE

	$b \rightarrow s$	$b \rightarrow d$	$s \rightarrow d$
$\Delta F=2$ box	$\bar{Q}_b \gamma^\mu Q_s \bar{Q}_b \gamma_\mu Q_s$	...	
$\Delta F=1$ 4-quark box	$\vdots$		
gluon penguin			
$\gamma$ penguin			
$Z^0$ penguin			
$H^0$ penguin			

The FCNC matrix:  
  
each box correspond to an  
indep. combination of dim.-6  
 $SU(3) \times SU(2) \times U(1)$ -invariant  
operators

Towards a model independent approach to the flavour problem:

FLAVOUR STRUCTURE

ELECTROWEAK  
STRUCTURE

	$b \rightarrow s$	$b \rightarrow d$	$s \rightarrow d$
$\Delta F=2$ box	$\Delta M_d \quad A_{CP}(B_s \rightarrow \psi K)$	$\Delta M_s \quad A_{CP}(B_s \rightarrow \psi \phi)$	$\Delta M_K \quad \epsilon_K$
$\Delta F=1$ 4-quark box	$B_d \rightarrow \pi K, B_d \rightarrow \eta K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow \pi \pi, B_d \rightarrow \rho \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$\epsilon' / \epsilon,$ $A_{CP}(K \rightarrow 3\pi), \dots$
gluon penguin	$B_d \rightarrow X_s \gamma, B_d \rightarrow \pi K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow X_d \gamma, B_d \rightarrow \pi \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
$\gamma$ penguin	$B_d \rightarrow X_s l^+ l^-, B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi, B_s \rightarrow \pi K, \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
$Z^0$ penguin	$B_d \rightarrow X_s l^+ l^-, B_s \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$K_L \rightarrow \pi^0 l^+ l^-, K_L \rightarrow \pi^0 \nu \nu,$ $K^+ \rightarrow \pi^+ \nu \nu, \epsilon' / \epsilon, \dots$
$H^0$ penguin	$B_s \rightarrow \mu^+ \mu^-$	$B_d \rightarrow \mu^+ \mu^-$	$K_{L,S} \rightarrow \mu^+ \mu^-$

Towards a model independent approach to the flavour problem:

Th. error  $\lesssim 10\%$

decreasing SM contrib.



	$b \rightarrow s$ ( $\sim \lambda^2$ )	$b \rightarrow d$ ( $\sim \lambda^3$ )	$s \rightarrow d$ ( $\sim \lambda^5$ )
$\Delta F=2$ box	$\Delta M_d$ $A_{CP}(B_s \rightarrow \psi K)$	$\Delta M_s$ $A_{CP}(B_s \rightarrow \psi \phi)$	$\Delta M_K$ $\epsilon_K$
$\Delta F=1$ 4-quark box	$B_d \rightarrow \pi K, B_d \rightarrow \eta K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow \pi \pi, B_d \rightarrow \rho \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$\epsilon' / \epsilon,$ $A_{CP}(K \rightarrow 3\pi), \dots$
decreasing gluon penguin	$B_d \rightarrow X_s \gamma, B_d \rightarrow \pi K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow X_d \gamma, B_d \rightarrow \pi \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
SM $\gamma$ penguin	$B_d \rightarrow X_s l^+ l^-, B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi, B_s \rightarrow \pi K, \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
contrib. $Z^0$ penguin	$B_d \rightarrow X_s l^+ l^-, B_s \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$K_L \rightarrow \pi^0 l^+ l^-, K_L \rightarrow \pi^0 \nu \nu$ $K^+ \rightarrow \pi^+ \nu \nu, \epsilon' / \epsilon, \dots$
$H^0$ penguin	$B_s \rightarrow \mu^+ \mu^-$	$B_d \rightarrow \mu^+ \mu^-$	$K_{L,S} \rightarrow \mu^+ \mu^-$

# Towards a model independent approach to the flavour problem:

Th. error  $\lesssim 10\%$

decreasing SM contrib.



	$b \rightarrow s$ ( $\sim \lambda^2$ )	$b \rightarrow d$ ( $\sim \lambda^3$ )	$s \rightarrow d$ ( $\sim \lambda^5$ )
$\Delta F=2$ box	$\Delta M_d$ $A_{CP}(B_s \rightarrow \psi K)$	$\Delta M_s$ $A_{CP}(B_s \rightarrow \psi \phi)$	$\Delta M_K$ $\epsilon_K$
$\Delta F=1$ 4-quark box	$B_d \rightarrow \pi K, B_d \rightarrow \eta K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow \pi \pi, B_d \rightarrow \rho \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$\epsilon' / \epsilon,$ $A_{CP}(K \rightarrow 3\pi), \dots$
decreasing gluon penguin	$B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
SM $\gamma$ penguin	$B_d \rightarrow X_s l^+ l^-$ $B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-$ $B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi, B_s \rightarrow \pi K, \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
contrib. $Z^0$ penguin	$B_d \rightarrow X_s l^+ l^-$ $B_s \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-$ $B_d \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$K_L \rightarrow \pi^0 l^+ l^-$ $K_L \rightarrow \pi^0 \nu \nu$ $K^+ \rightarrow \pi^+ \nu \nu, \epsilon' / \epsilon, \dots$
$H^0$ penguin	$B_s \rightarrow \mu^+ \mu^-$	$B_d \rightarrow \mu^+ \mu^-$	$K_{L,S} \rightarrow \mu^+ \mu^-$



= exp. error  $\lesssim 10\%$



= exp. error  $\sim 30-50\%$

- Rare FCNC Kaon decays within the SM

### General decomposition

of the decay amplitude for  $K \rightarrow \pi + l^+l^-/\nu\nu$  modes:

#### I. Clean electroweak short-distance component

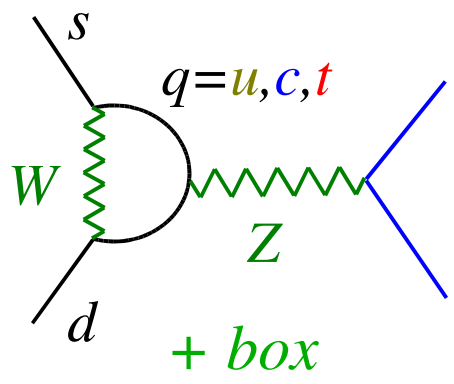
- similar -within the SM- for all the channels
- modified by NP in (very) different ways in the various channels

#### II. Long-distance component of e.m. origin

- relevant to  $K \rightarrow \pi + l^+l^-$  modes only
- insensitive to NP

# I. The clean electroweak short-distance amplitude

Z-penguin + W-box diagrams  $\Rightarrow$  scale-independent amplitude dominated by *short-distance* [top-quark] thanks to the *'hard'* GIM mechanism:



$$\Rightarrow A^q_{\text{loop}} \sim m_q^2 V_{qs}^* V_{qd} \sim \begin{cases} \Lambda_{\text{QCD}}^2 \lambda & \text{(u)} \\ m_c^2 \lambda + i m_c^2 \lambda^5 & \text{(c)} \\ m_t^2 \lambda^5 + i m_t^2 \lambda^5 & \text{(t)} \end{cases}$$

- Genuine  $O(G_F^2)$  effect
- QCD corrections small and known beyond LO
- large CPV-phase contributing to  $K_L \rightarrow \pi^0 + \nu\bar{\nu}, l^+l^-$

$$\mathcal{H}_{\text{eff}} = \sum_i C_i(M_W) Q_i \quad \begin{cases} Q_\nu = (\bar{s} d)_{V-A} (\bar{\nu} \nu)_{V-A} \\ Q_{9V} = (\bar{s} d)_{V-A} (\bar{l} l)_V \\ Q_{10A} = (\bar{s} d)_{V-A} (\bar{l} l)_A \end{cases}$$



## Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators ]

## Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators]

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

Buchalla & Buras '97

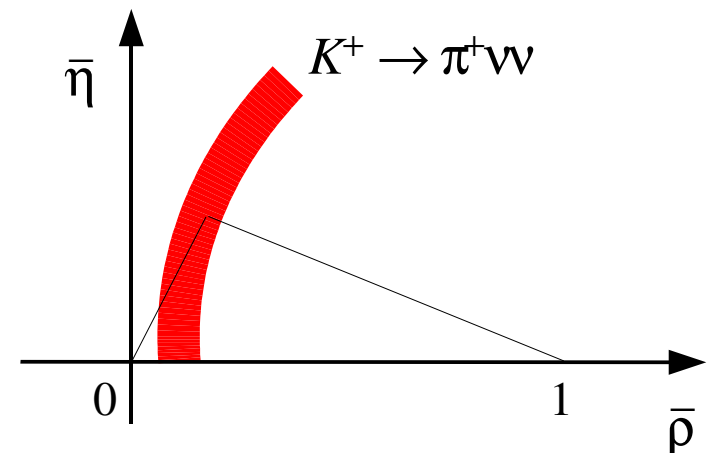
Lu & Wise '94    Falk *et al.* '00

large fraction of the present error still due to parametric CKM uncertainties

$$\text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \cdot 10^{-11}$$

$$\rho_c = 1.40 \pm 0.06 \quad \Rightarrow \quad \delta \text{BR}_{\text{th}} \approx 8\%$$

On-going theoretical activity to reduce  $\delta \text{BR}_{\text{th}}$  below the 5% level: [Munich](#), [Frascati](#)



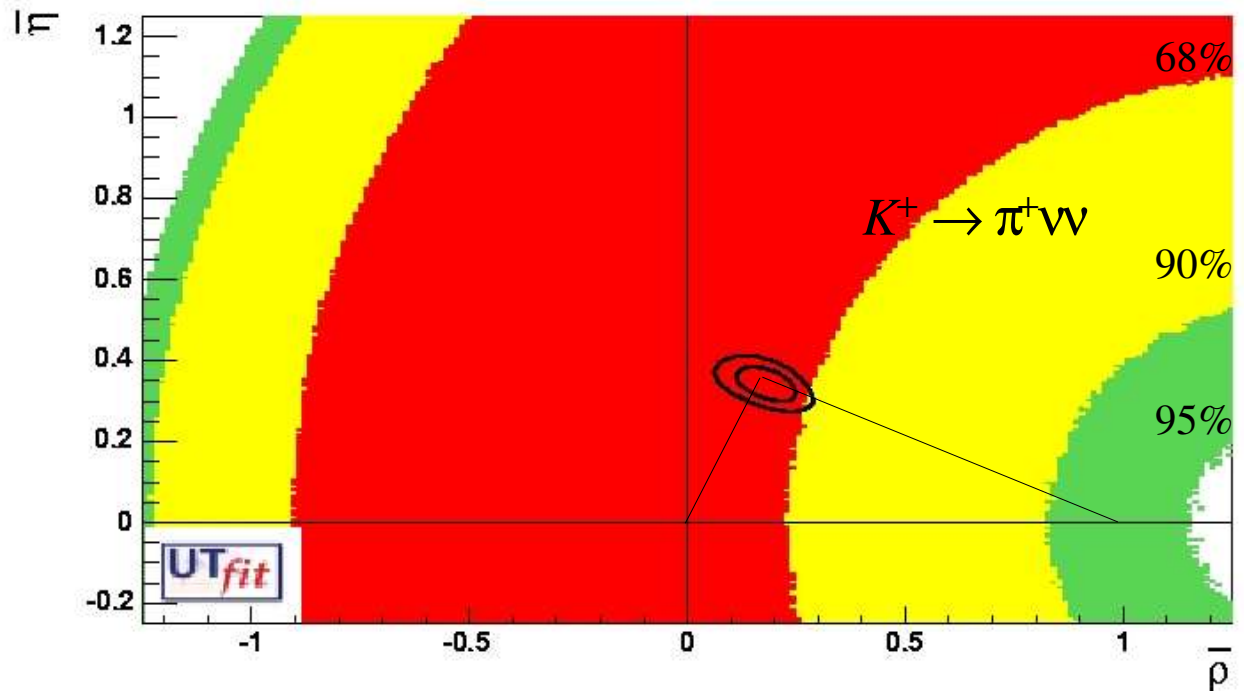
## Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators]

$$\boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}} \quad \text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \cdot 10^{-11}$$

$$\text{BR}(K^+)^{\text{exp}} = (1.47^{+1.9}_{-0.9}) \cdot 10^{-10}$$

E787+E949 [BNL] '  $\emptyset$



## Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators ]

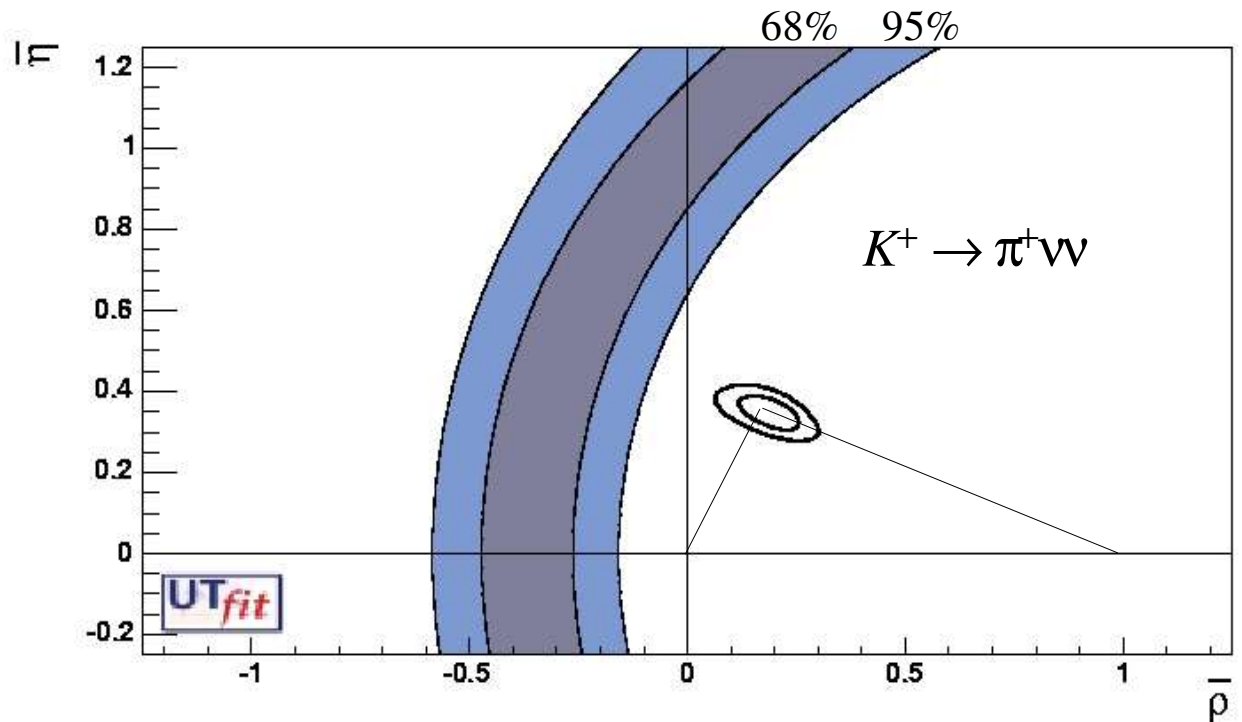
$$\boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}} \quad \text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \cdot 10^{-11}$$

if we could decrease the error at the 10% level...

$$\text{BR}(K^+)^{\text{exp}} = (1.47_{-0.9}^{+1.9}) \cdot 10^{-10}$$

$\downarrow$   
 $\pm 10\% \rightarrow$

The situation could become quite interesting...



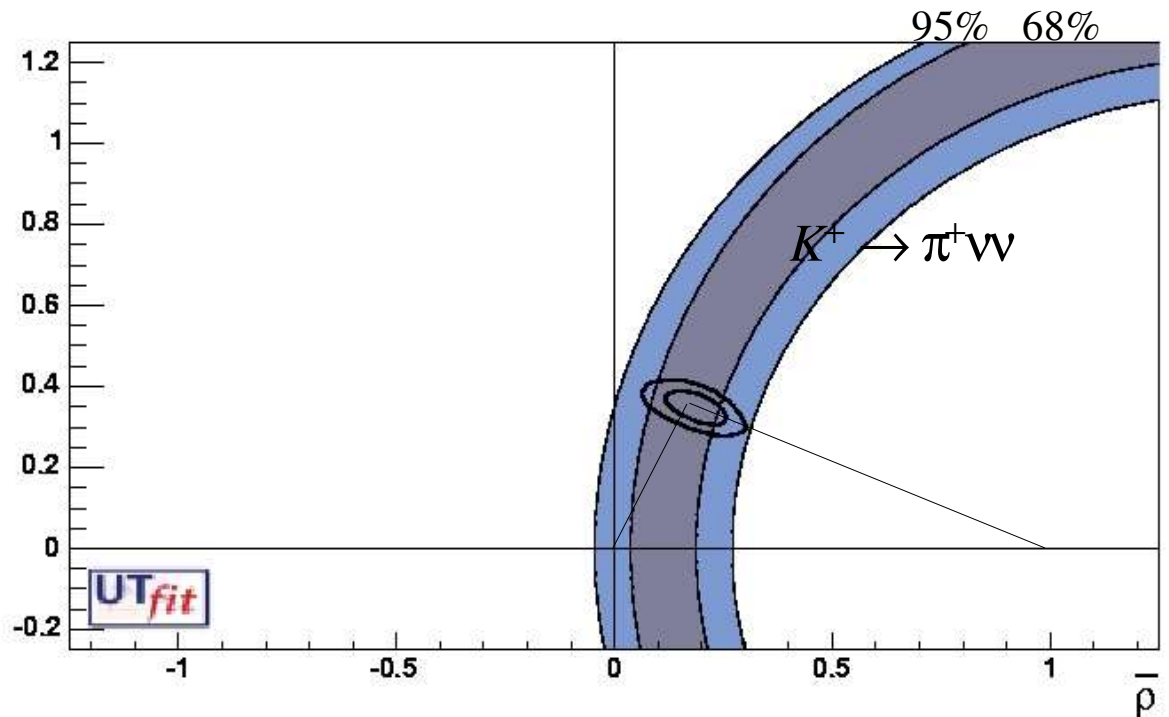
## Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators]

$$\boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}} \quad \text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \cdot 10^{-11}$$

**N.B.:** plotting the  $\text{BR}(K^+)^{\text{exp}}$  contour in the  $\rho$ - $\eta$  plane is only a fast way to compare it with the SM prediction:

the **main interest** of such measurement is not a more precise determination of  $V_{td}$  but the extraction of a key **information about NP**



# Neutrino modes:

- Hadronic matrix element of the leading operator directly extracted from  $K_{l3}$
- No sizable long distance corrections [only Z-penguin & W-box  $\Rightarrow$  hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms

$K_L \rightarrow \pi^0 \nu \bar{\nu} \Rightarrow$  peculiar CP structure  $\Rightarrow$  charm & l.d. effects further suppressed ( $\sim$  negligible)

$K_L \rightarrow \pi^0 \nu \bar{\nu}$

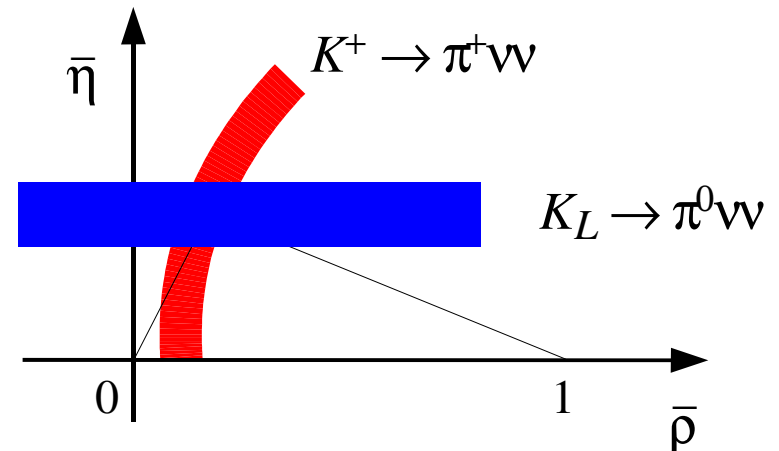
Littenberg, '89  
 Buchalla & Buras '97  
 Buchalla & G.I. '98

$$\text{BR}(K_L)^{[\text{SM}]} = 1.48 \cdot 10^{-11} \left[ \frac{m_t(m_t)}{166 \text{ GeV}} \right]^{2.3} \left[ \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}} \right]^2$$

$$= (3.0 \pm 0.6) \cdot 10^{-11}$$

th. error  $\sim 2\%$  !

control the amount of CPV within the SM

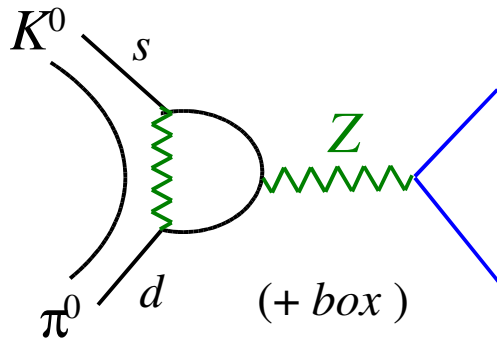


$$K_L \rightarrow \pi^0 l^+ l^-$$

The 3 components of the  $K_L \rightarrow \pi^0 l^+ l^-$  amplitude:

**A. direct CPV amplitude**

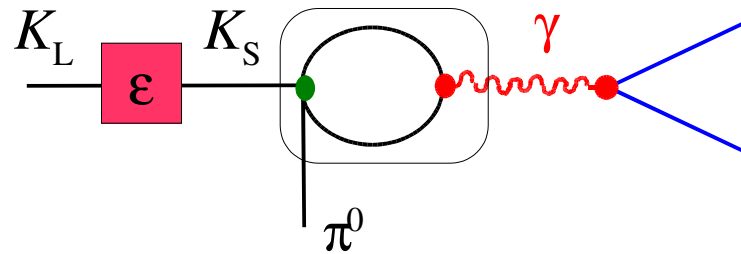
- short-distance dominated
- very similar to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$



$\longleftrightarrow$   
*interference*

**B. indirect CPV**

- determined by  $K_S \rightarrow \pi^0 l^+ l^-$

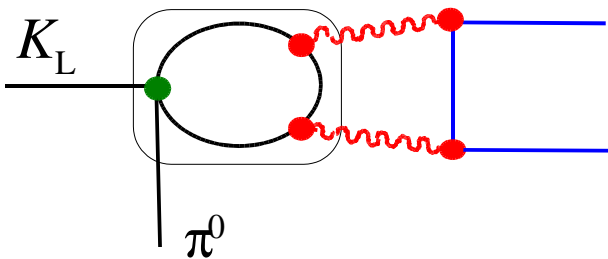


need exp. input

**C. CPC amplitude**

- no interference & different Dalitz plot
- predicted by theory with good accuracy

in terms of rate & spectrum of  $K_L \rightarrow \pi^0 \gamma \gamma$



need exp. input

$$K_L \rightarrow \pi^0 l^+ l^-$$

Thanks to some recent results by NA48-NA48/1:

$$\begin{aligned}
 B(K_S \rightarrow \pi^0 e^+ e^-)_{m_{ee} > 165 \text{ MeV}} &= (3.0_{-1.2}^{+1.5} \pm 0.2) \cdot 10^{-9} \\
 B(K_S \rightarrow \pi^0 \mu^+ \mu^-) &= (2.9_{-1.2}^{+1.4} \pm 0.2) \cdot 10^{-9} \\
 B(K_L \rightarrow \pi^0 \gamma \gamma)_{m_{\gamma\gamma} < 110 \text{ MeV}} &< 0.9 \cdot 10^{-8}
 \end{aligned}$$

+

Some related th. works:

Buchalla, D' Ambrósio, G.I. ' 03  
 G.I., Smith, Unterdorfer '04  
 Friot, Grenat, de Rafael ' 04

We finally have a clear picture of the various terms:

$$\begin{aligned}
 B(K_L \rightarrow \pi^0 l^+ l^-)^{[SM]} &= [ C_{\text{mix}} + C_{\text{int}} y_t + C_{\text{dir}} y_t^2 + C_{\text{CPC}} ] \cdot 10^{-12} & y_t &= \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}} \\
 & \begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \text{red} \quad \text{red} \quad \text{blue} \quad \text{green} \end{array} \\
 (e^+ e^-) &\approx 23 + (10 + 4) + 0 \Rightarrow (3.7 \pm 1.0) \cdot 10^{-11} \\
 (\mu^+ \mu^-) &\approx 5.4 + (2.5 + 1.8) + 5.2 \Rightarrow (1.5 \pm 0.3) \cdot 10^{-11}
 \end{aligned}$$



$$B(K_L \rightarrow \pi^0 e^+ e^-)^{[\text{SM}]} = (3.7 \pm 1.0) \cdot 10^{-11} \quad [ \approx 50\% \text{ due to short dist.} ]$$

$$B(K_L \rightarrow \pi^0 \mu^+ \mu^-)^{[\text{SM}]} = (1.5 \pm 0.3) \cdot 10^{-11} \quad [ \approx 30\% \text{ due to short dist.} ]$$

Errors on SM predictions dominated by the large (exp.) uncertainty on  $B(K_S \rightarrow \pi^0 l^+ l^-)$ , but irreducible theoretical error below 10%



$$B(K_L \rightarrow \pi^0 e^+ e^-)^{\text{exp}} < 2.8 \cdot 10^{-10} \quad [90\% \text{ CL}] \quad \text{KTeV ' 8}$$

$$B(K_L \rightarrow \pi^0 \mu^+ \mu^-)^{\text{exp}} < 3.8 \cdot 10^{-10} \quad [90\% \text{ CL}] \quad \text{KTeV ' 0}$$

not too far...



### Very interesting candidates for future dedicated experiments

- More observables to be studied [Dalitz plot, time-dependent distrib.]
- Different sensitivity to NP with respect to  $K_L \rightarrow \pi^0 \nu \nu$

the 3 decay modes  $K_L \rightarrow \pi^0 + e^+ e^-, \mu^+ \mu^-, \nu \nu$   
 are sensitive to different short-distance structures  
 $\Rightarrow$  3 independent info on NP

$$Q_\nu = (\bar{s} d)_{V-A} (\bar{\nu} \nu)_{V-A}$$

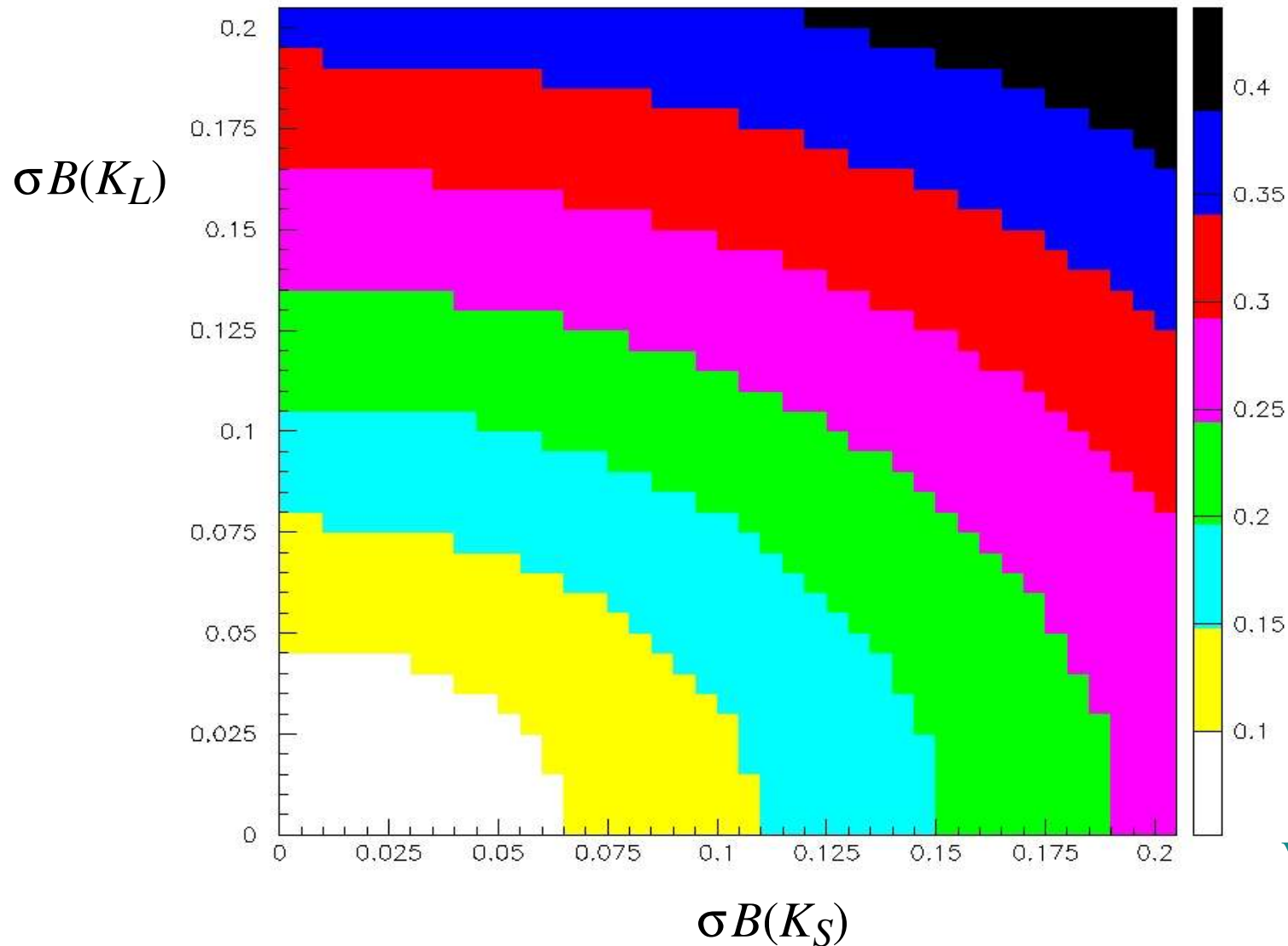
$$Q_{9V} = (\bar{s} d)_{V-A} (\bar{l} l)_V$$

$$Q_{10A} = (\bar{s} d)_{V-A} (\bar{l} l)_A$$

Relative error on  $\text{Im}(V_{ts}^* V_{td})$

vs.

relative exp. errors on  $B(K_S \rightarrow \pi^0 e^+ e^-)$  &  $B(K_L \rightarrow \pi^0 e^+ e^-)$



courtesy of  
V. Patera [KLOE]

- Rare FCNC decays beyond the SM

Natural solution of the flavour (+hierarchy) problem:

$\Lambda \sim 1 \text{ TeV}$  + flavor-mixing protected by additional symmetries

As long as we are interested only in low-energy rare processes, the most important feature of the NP model is the nature of this symmetry

- Rare FCNC decays beyond the SM

Natural solution of the flavour (+hierarchy) problem:

$\Lambda \sim 1 \text{ TeV}$  + flavor-mixing protected by additional symmetries

As long as we are interested only in low-energy rare processes, the most important feature of the NP model is the nature of this symmetry

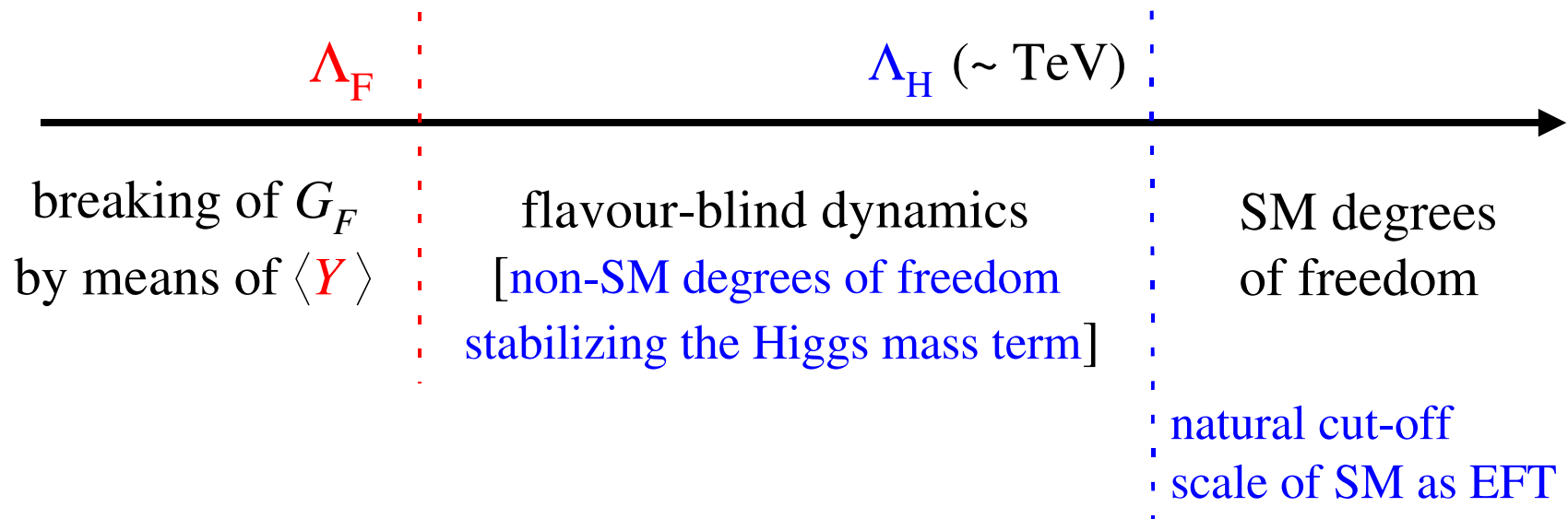
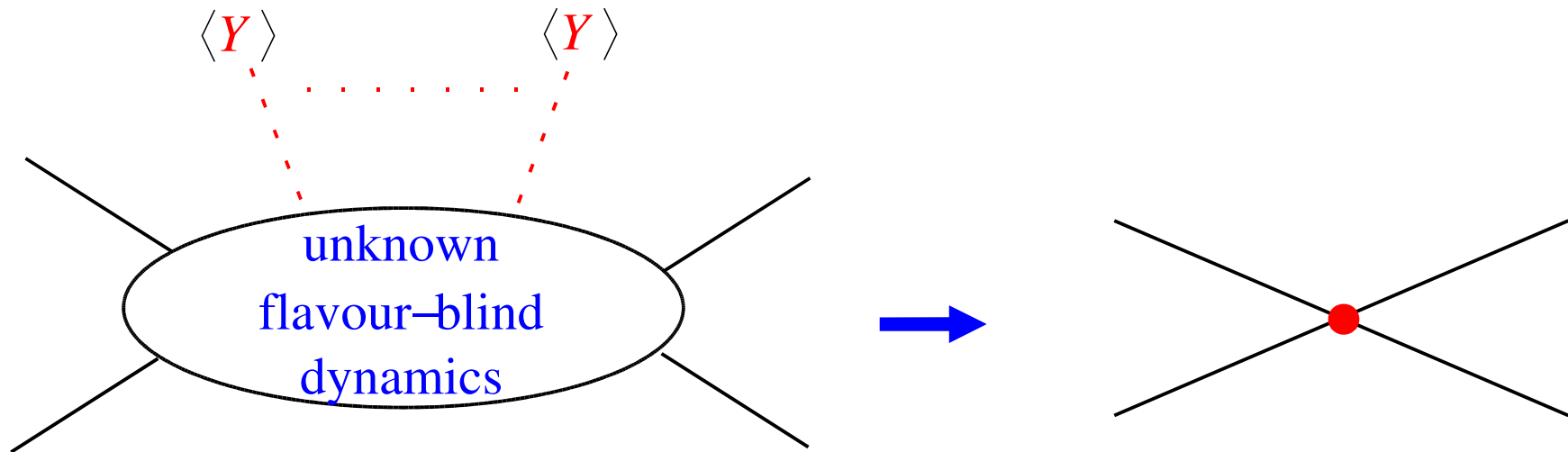


most restrictive possibility

Minimal Flavour Violation (MFV) hypothesis:

The breaking of the flavour symmetry occurs at very high scales and is mediated at low energies only by terms prop. to SM Yukawa couplings

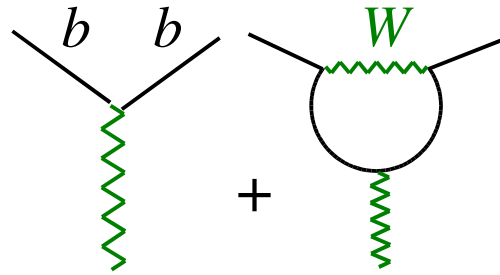
- natural implementation in many consistent scenarios [SUSY, technicolour, extra dimensions,...]
- possible to build a predictive low-energy EFT  
model-independent approach



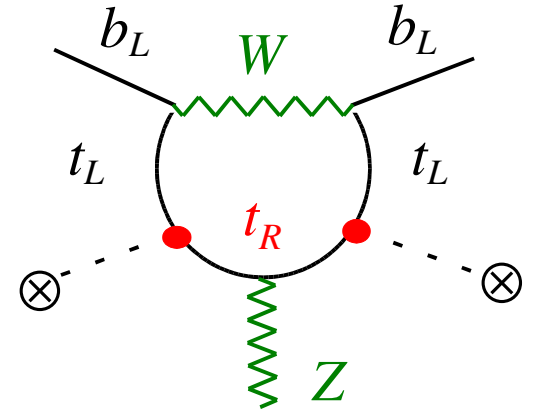
The MFV hypothesis can be considered as the most pessimistic scenario:  
 $\Rightarrow$  deviations from the SM in FCNCs bounded by flavour-conserving  
 e.w. precision observables

E.g.: Z-penguins &  $R_b$

$$R_b = \frac{\Gamma(Z \rightarrow bb)}{\Gamma(Z \rightarrow \text{had})}$$



breaking of  
universality  
→  
driven by



$$R_b = R_d \left[ 1 - \frac{G_F m_t^2}{2\pi^2 v^2} + \dots \right] \approx 0.2182 - 0.0024$$

$R_b^{\text{exp}} = 0.2163 \pm 0.0007$

↑  
tree + flavour univ. terms

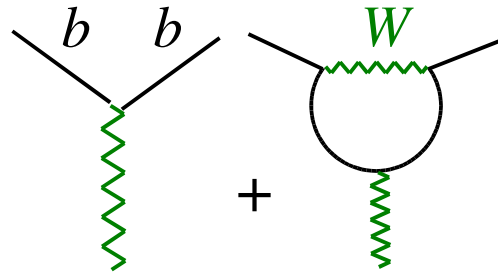
↓  
same e.w.-Yukawa  
structure of the leading  
 $K \rightarrow \pi \nu \nu$  amplitude

The  $O(10^{-3})$  accuracy on  $R_b$  of LEP let us to probe the genuine e.w.-Yukawa loop amplitude only at the 30% level

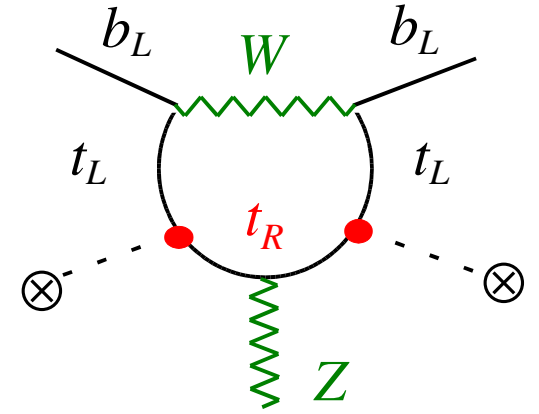
A 10% measurement of  $B(K^+ \rightarrow \pi^+ \nu \nu)$  [or  $B \rightarrow \pi \nu \nu$ ] would probe the same e.w.-Yukawa structure (assuming MFV) at the 6-8% level

E.g.:  $Z$ -penguins &  $R_b$

$$R_b = \frac{\Gamma(Z \rightarrow bb)}{\Gamma(Z \rightarrow \text{had})}$$



breaking of  
universality  
→  
driven by



$$R_b = R_d \left[ 1 - \frac{G_F m_t^2}{2\pi^2 \sqrt{2}} + \dots \right] \approx 0.2182 - 0.0024$$

$R_b^{\text{exp}} = 0.2163 \pm 0.0007$

↑  
tree + flavour univ. terms

↓  
same e.w.-Yukawa  
structure of the leading  
 $K \rightarrow \pi \nu \nu$  amplitude

- Even within the most pessimistic NP scenario  $O(30-50\%)$  deviations from SM possible in BR(**rare short-distance dominated FCNC decays**)
- **$O(10\%)$**  measurements of BR(**rare**) probe NP parameter space of NP not cover yet by LEP

## Beyond Minimal Flavour Violation

[new sources of flavour symmetry breaking at the TeV scale]

- A priori the most natural possibility naturally appearing in several specific scenarios [e.g. SUSY: huge literature]
- challenged -at present- by the good agreement with SM in  $\Delta F=2$  sector



# Beyond Minimal Flavour Violation

[new sources of flavour symmetry breaking at the TeV scale]

- A priori the most natural possibility naturally appearing in several specific scenarios [e.g. SUSY: [huge literature](#)]
- challenged -at present- by the good agreement with SM in  $\Delta F=2$  sector

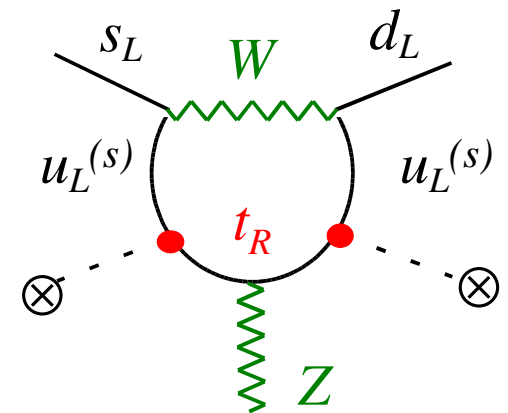
## General features:

- Some decoupling between  $\Delta F=2$  &  $\Delta F=1$  [i.e.:  $\delta_{\text{NP}}(\Delta F=1) \sim 100\%$  vs.  $\delta_{\text{NP}}(\Delta F=2) \sim 10\%$ ] possible thanks to the interplay between SU(2)·U(1) & flavour symm. breaking

Colangelo & G.I. '98,

Nir & Worah '98;

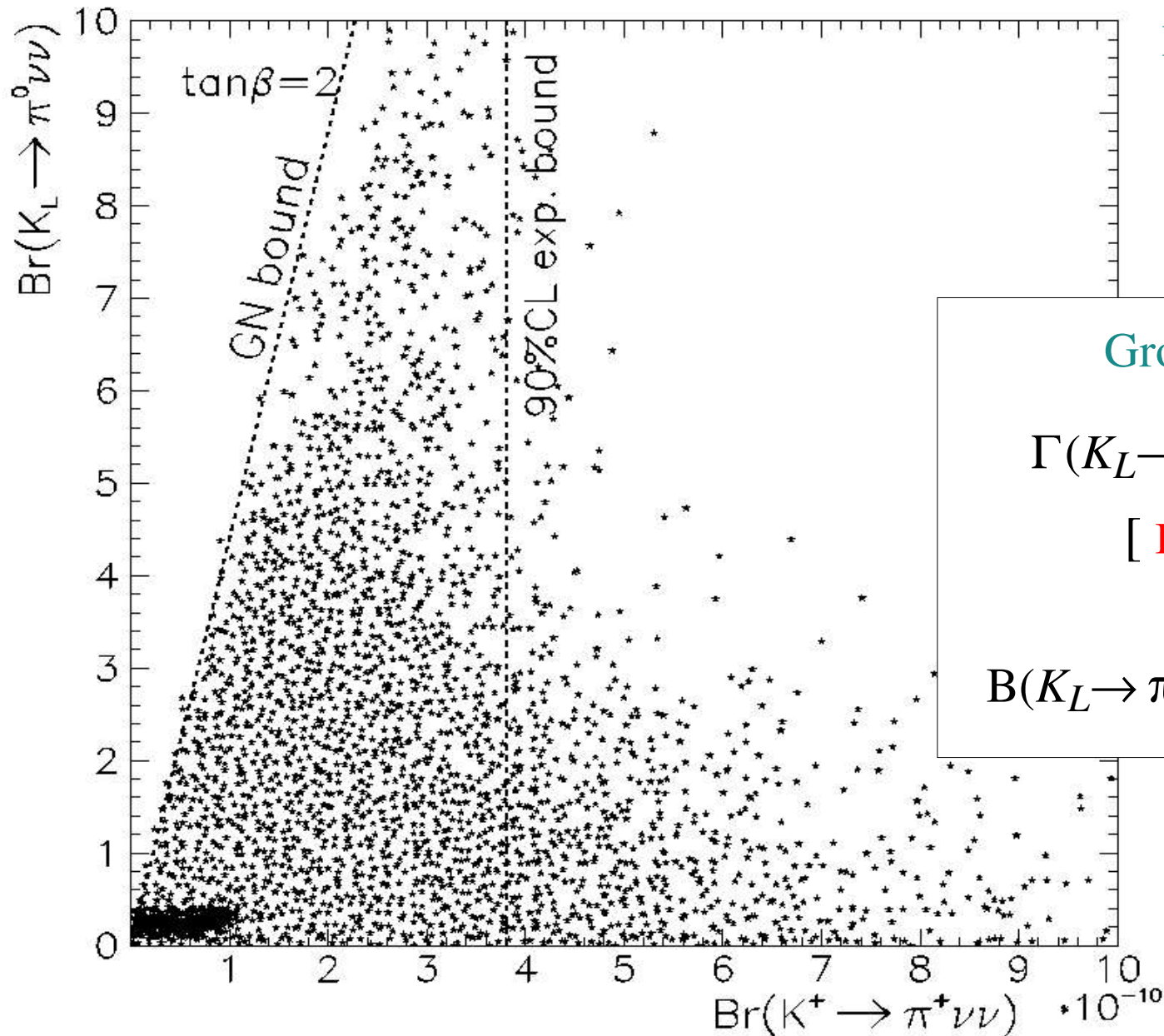
Buras, Romanino & Silvestrini, '98



- Rare kaon decays are particularly sensitive to new sources of flavour symm. breaking because of the severe CKM suppression [ $V_{ts}^* V_{td} \sim \lambda^5$ ]

E.g.:  $B(K \rightarrow \pi \nu \nu)$  within generic MSSM

[including all the phenomenological constraints from  $\epsilon_K$ ,  $\Delta M_K$ ,  $b \rightarrow s \gamma$ , ... ]



Buras *et al.* '04

Grossman-Nir bound:

$$\Gamma(K_L \rightarrow \pi^0 \nu \nu) < \Gamma(K^+ \rightarrow \pi^+ \nu \nu)$$

$$[ \text{Im}(A) < |A| ]$$



$$B(K_L \rightarrow \pi^0 \nu \nu) < 4.4 B(K^+ \rightarrow \pi^+ \nu \nu)$$

General features on non-MFV models:

- Possible some decoupling between  $\Delta F=2$  &  $\Delta F=1$  [i.e.:  $\delta_{NP}(\Delta F=1) \sim 100\%$  vs.  $\delta_{NP}(\Delta F=2) \sim 10\%$ ]  $\Leftrightarrow$  interplay between SU(2) · U(1) & flavour symm. breaking

- Rare K decays are particularly sensitive to new sources of flavour-symmetry breaking



If a 10% deviation from SM is clearly established in time-dependent CPV asymmetries in B decays  $\longrightarrow$

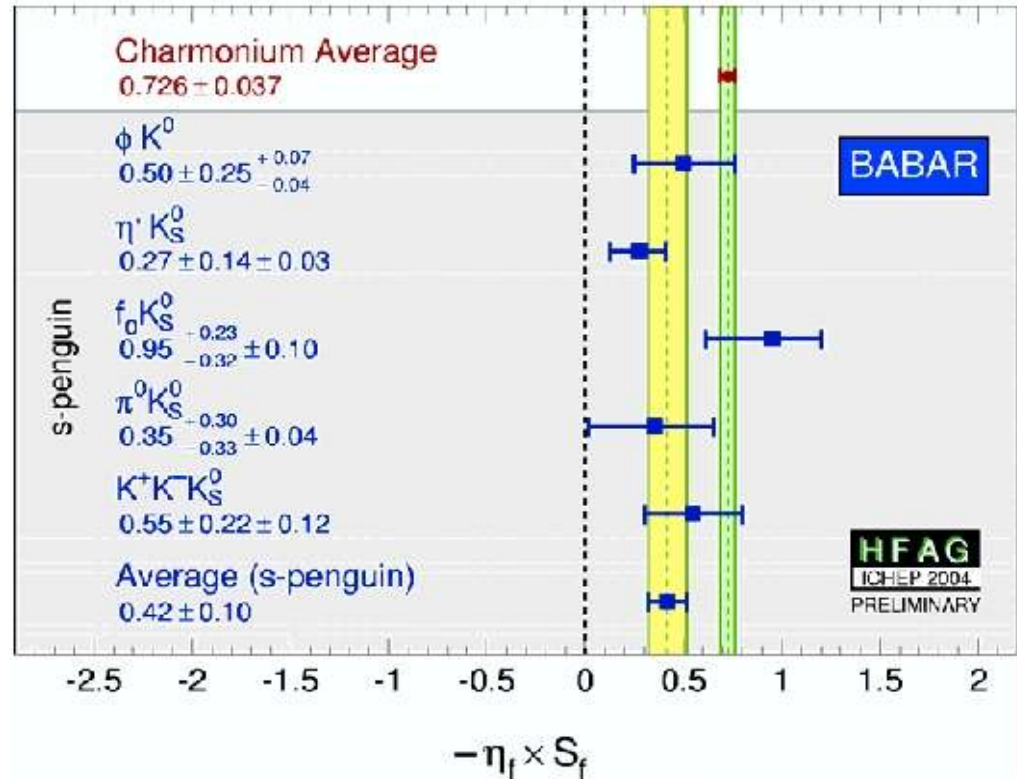


high chances to find O(1) non-standard effects in (at least some) rare K decays



All new!

Giorgi, ICHEP '04

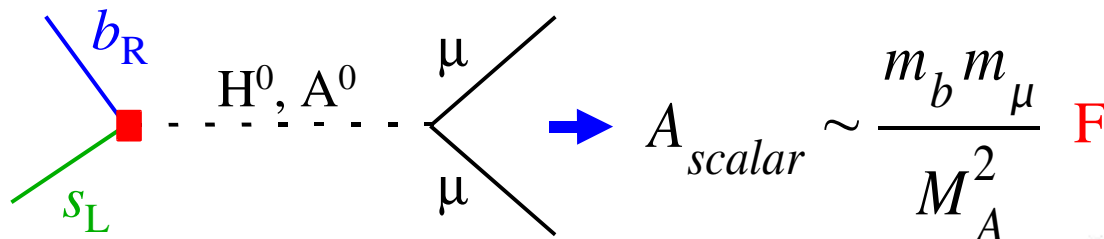


2.7 $\sigma$  from s-penguin to  $\sin 2\beta$  ( $c\bar{c}$ )

- A few comments on rare B, D decays

- The NP sensitivity of inclusive FCNC B decays [ $B \rightarrow X_{s,d} + \nu\nu, l^+l^-$ ] is quite similar to the one of  $K \rightarrow \pi + \nu\nu, l^+l^-$  [strict correlation in MFV models, different flavour structure in the general case]

- A unique opportunity in B decays is the possibility to probe non-standard scalar FCNCs via  $B_{s,d} \rightarrow \mu^+\mu^-$  decays:

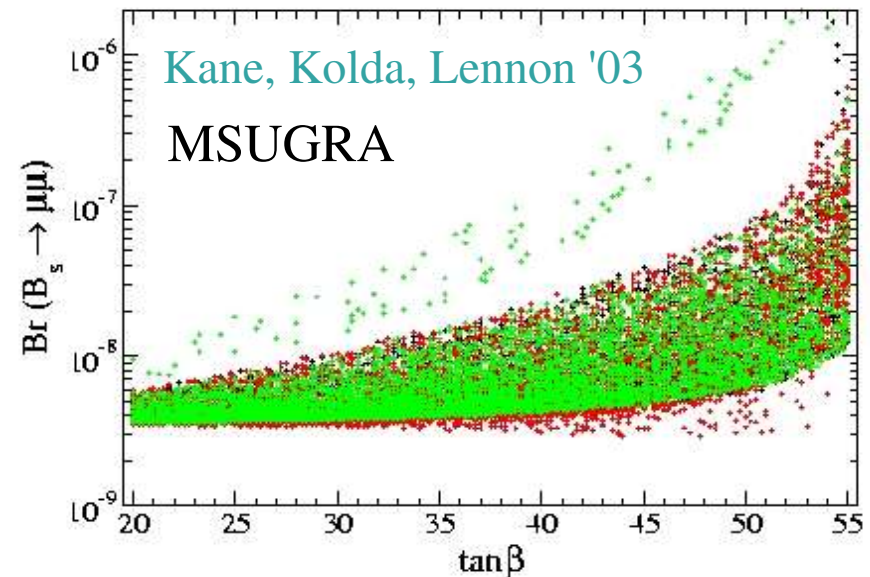


$$\text{BR}(B_s)^{\text{SM}} \approx 3 \cdot 10^{-9}$$

$$\text{BR}(B_d)^{\text{SM}} \approx 1 \cdot 10^{-10}$$

Scalar amplitude competitive with SM because of helicity suppression + large b-Yukawa coupling

sizeable (& unique) NP effects in models with more than one Higgs doublet (even within MFV)



- A few comments on rare B, D decays

- The NP sensitivity of inclusive FCNC B decays [ $B \rightarrow X_{s,d} + \nu\nu, l^+l^-$ ] is quite similar to the one of  $K \rightarrow \pi + \nu\nu, l^+l^-$  [strict correlation in MFV models, different flavour structure in the general case]
- A unique opportunity in B decays is the possibility to probe non-standard **scalar FCNCs** via  $B_{s,d} \rightarrow \mu^+\mu^-$  decays
- On general grounds, **rare D decays** are less interesting than B & K decays: dominated by long-distance dynamics which is difficult to control

Notable exceptions:

$$D \rightarrow \mu^+\mu^- \quad \text{BR}^{\text{SM}} < 10^{-12} \quad \text{BR}^{\text{exp}} < 3 \cdot 10^{-6}$$

$$D \rightarrow K, \pi + \nu\nu \quad \text{BR}^{\text{SM}} < 10^{-15}$$

+

LFV modes

Interesting windows for  
models beyond MFV

## • Conclusions

*Which is the scale of flavour symmetry breaking?*

*Are there new sources of flavour-symm. breaking beyond the Yukawa?*

Short-distance dominated rare decays are a key ingredient to answer these fundamental questions and, more in general, to explore the flavour structure of physics beyond the SM

LHC will *‘kill the penguins’*  
revealing the *‘anatomy of the internal constituents’*  
but this is not enough  
we also need to carefully study *‘penguin ethology’*  
via dedicated rare-decay experiments

Th. error  $\lesssim 10\%$

decreasing SM contrib. 

	$b \rightarrow s$ ( $\sim \lambda^2$ )	$b \rightarrow d$ ( $\sim \lambda^3$ )	$s \rightarrow d$ ( $\sim \lambda^5$ )
$\Delta F=2$ box	$\Delta M_d$ $A_{CP}(B_s \rightarrow \psi K)$	$\Delta M_s$ $A_{CP}(B_s \rightarrow \psi \phi)$	$\Delta M_K$ $\epsilon_K$
$\Delta F=1$ 4-quark box	$B_d \rightarrow \pi K, B_d \rightarrow \eta K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow \pi \pi, B_d \rightarrow \rho \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$\epsilon' / \epsilon,$ $A_{CP}(K \rightarrow 3\pi), \dots$
decreasing gluon penguin	$B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K,$ $A_{CP}(B_d \rightarrow \phi K), \dots$	$B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi,$ $A_{CP}(B_d \rightarrow \pi \pi), \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
SM $\gamma$ penguin	$B_d \rightarrow X_s l^+ l^-$ $B_d \rightarrow X_s \gamma$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-$ $B_d \rightarrow X_d \gamma$ $B_d \rightarrow \pi \pi, B_s \rightarrow \pi K, \dots$	$K_L \rightarrow \pi^0 l^+ l^-,$ $\epsilon' / \epsilon, \dots$
contrib. $Z^0$ penguin	$B_d \rightarrow X_s l^+ l^-$ $B_s \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$B_d \rightarrow X_d l^+ l^-$ $B_d \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \pi K, B_s \rightarrow KK, \dots$	$K_L \rightarrow \pi^0 l^+ l^-, K_L \rightarrow \pi^0 \nu \nu$ $K^+ \rightarrow \pi^+ \nu \nu, \epsilon' / \epsilon, \dots$
$H^0$ penguin	$B_s \rightarrow \mu^+ \mu^-$	$B_d \rightarrow \mu^+ \mu^-$	$K_{L,S} \rightarrow \mu^+ \mu^-$



= exp. error  $\lesssim 10\%$



= exp. error  $\sim 30-50\%$

In the kaon sector we can identify 4 outstanding modes:

$$K^+ \rightarrow \pi^+ \nu \nu \quad K_L \rightarrow \pi^0 \nu \nu \quad K_L \rightarrow \pi^0 e^+ e^- \quad K_L \rightarrow \pi^0 \mu^+ \mu^-$$

- Measurements of these modes at **10% level** (or below) is necessarily a major achievement: substantial improve in understanding flavour dynamics at the TeV scale (true for any realistic NP model)
- In planning new experiments, worth to think to the possibility of measuring more than one of these four channels in the same set-up (or with minor modifications)