Parton Distributions Functions at Hadron Colliders

Robert Thorne

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University College London

Thanks to Alan Martin, James Stirling and Graeme Watt

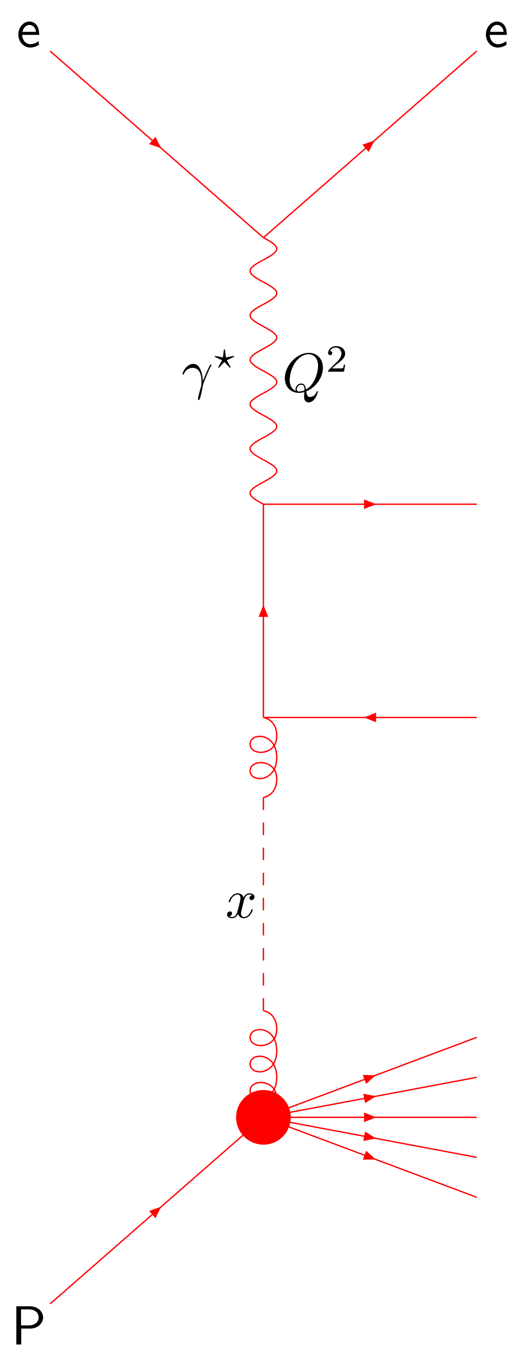
Strong force makes it difficult to perform analytic calculations of scattering processes involving hadronic particles.

The weakening of $\alpha_S(\mu^2)$ at higher scales $\longrightarrow$ the **Factorization Theorem**.

Hadron scattering with an electron factorizes.

Q^2 – Scale of scattering

$x = \frac{Q^2}{2m\nu}$ – Momentum fraction of Parton (ν=energy transfer)



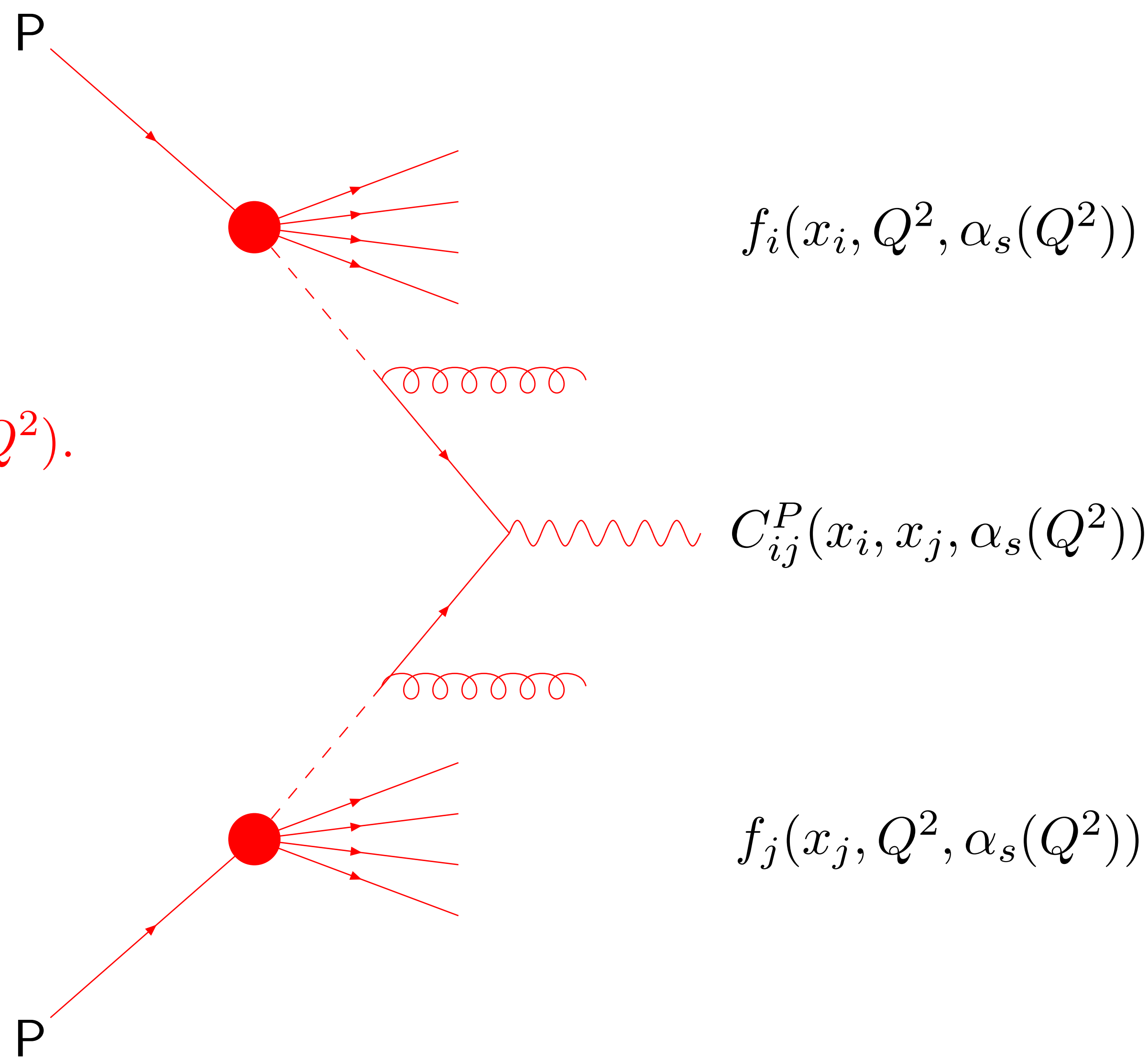
perturbative
calculable
coefficient function
$C_i^P(x, \alpha_s(Q^2))$

nonperturbative
incalculable
parton distribution
$f_i(x, Q^2, \alpha_s(Q^2))$

The coefficient functions $C_i^P(x, \alpha_s(Q^2))$ are process dependent (new physics) but are calculable as a power-series in $\alpha_s(Q^2)$.

$$C_i^P(x, \alpha_s(Q^2)) = \sum_k C_i^{P,k}(x)\alpha_s^k(Q^2).$$

Since the parton distributions $f_i(x, Q^2, \alpha_s(Q^2))$ are process-independent, i.e. **universal**, and evolution with scale is calculable, once they have been measured at one experiment, one can predict many other scattering processes.



General procedure.

Start parton evolution at low scale $Q_0^2 \sim 1\mathrm{GeV}^2$. In principle 11 different partons to consider.

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\mathrm{QCD}}$ so heavy parton distributions determined perturbatively. Leaves 7 independent combinations, or 6 if assume $s = \bar{s}$.

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \mathrm{sea} = 2 * (\bar{u} + \bar{d} + \bar{s}), \quad s + \bar{s} \quad \bar{d} - \bar{u}, \quad g.$$

Input partons parameterised as, e.g. (MRST/MSTW)

$$xf(x, Q_0^2) = (1-x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta.$$

For non-singlet combinations, valence quarks, $\bar{d} - \bar{u}$, δ expected to be ~ 0.5. For singlet combinations, sea and gluon, δ expected to be ~ 0.

Evolve partons upwards using LO, NLO (or NNLO) DGLAP equations.

$$\frac{df_i(x, Q^2, \alpha_s(Q^2))}{d \ln Q^2} = \sum_j P_{ij}(x, \alpha_s(Q^2)) \otimes f_j(x, Q^2, \alpha_s(Q^2))$$

Fit data for scales above $2 - 10\mathrm{GeV}^2$. Need many different types of experiment for full determination.

- Lepton-proton collider HERA – (DIS) $\longrightarrow$ small-x quarks. Also gluons from evolution, and $F_L(x, Q^2)$. Also, jets $\longrightarrow$ moderate-x gluon.

- Fixed target DIS – higher x – leptons (BCDMS, NMC, $\ldots$) $\longrightarrow$ up quark (proton) or down quark (deuterium) and neutrinos (CHORUS, NuTeV, CCFR) $\longrightarrow$ valence or singlet combinations.

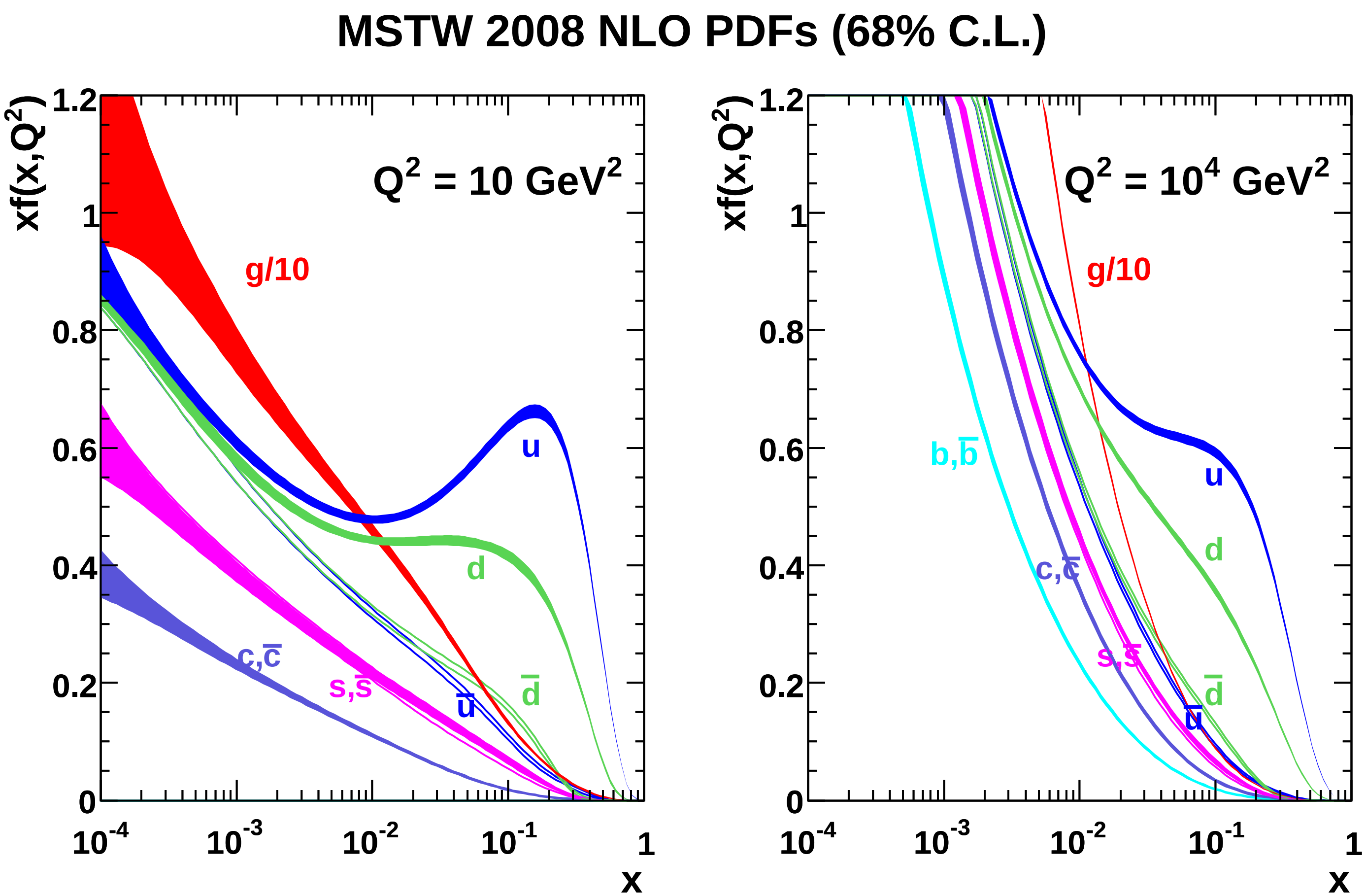
- Di-muon production in neutrino DIS – strange quarks and neutrino-antineutrino comparison $\longrightarrow$ asymmetry .

- Drell-Yan production of dileptons – quark-antiquark annihilation (E605, E866) – high-x sea quarks. Deuterium target – $\bar{u}/\bar{d}$ asymmetry.

- High-p_T jets at colliders (Tevatron) – high-x gluon distribution.

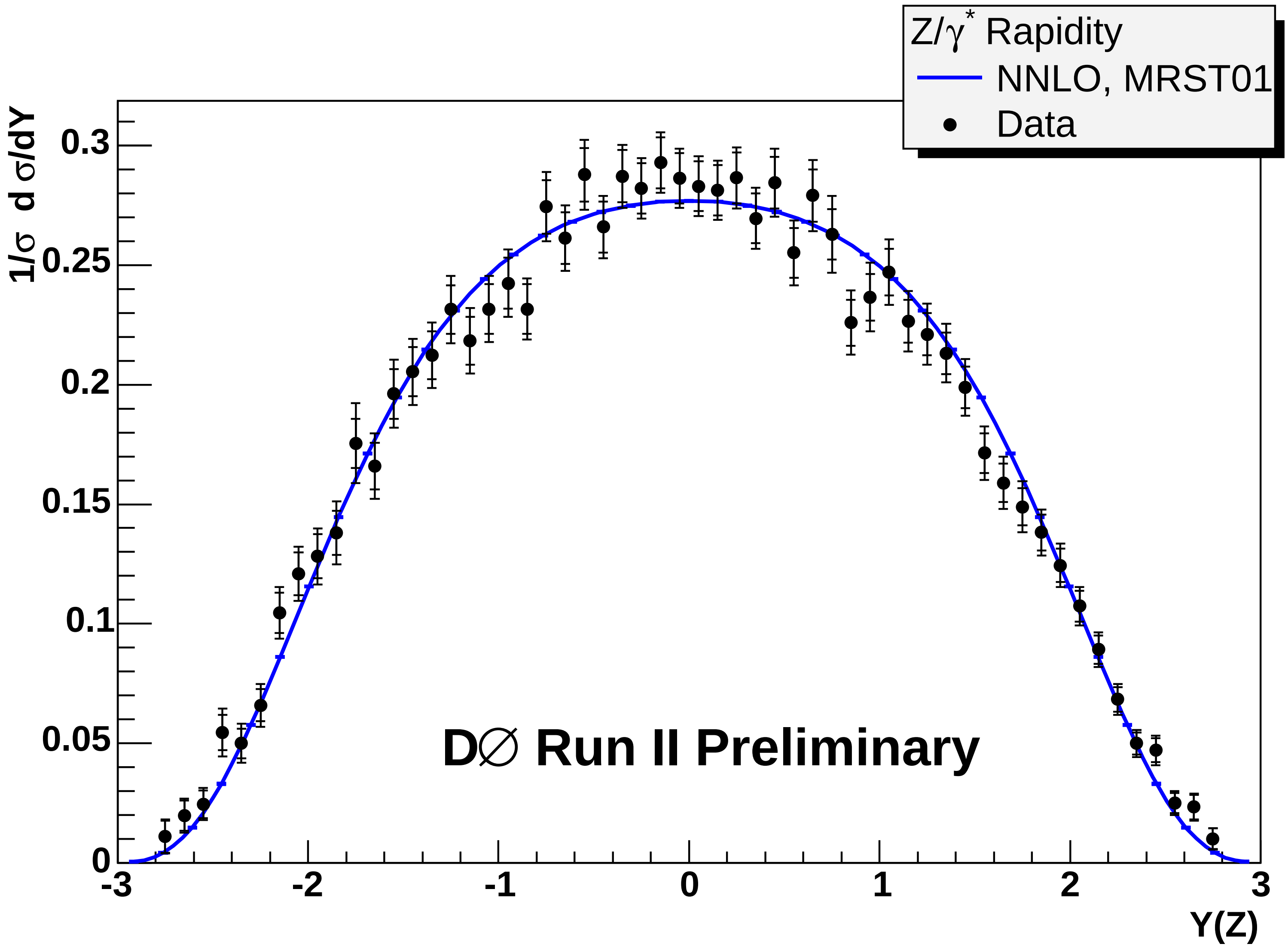
- W and Z production at colliders (Tevatron) – different quark contributions to DIS.

This procedure is generally successful and is part of a large-scale, ongoing project. Results in partons of the form shown.



Various choices of PDF – MSTW, CTEQ, NNPDF, Alekhin, ZEUS, H1, Jimenez-Delgado *et al* *etc*.. All LHC cross-sections rely on our understanding of these partons.

Excellent predictive power – comparison of MRST prediction for Z rapidity distribution with preliminary data.



Interplay of LHC and pdfs/QCD

Make predictions for all processes, both SM and BSM, as accurately as possible given current experimental input and theoretical accuracy.

Check against well-understood processes, e.g. central rapidity W, Z production (luminosity monitor), lowish-E_T jets, ...

Compare with predictions with more uncertainty and lower confidence, e.g. high-E_T jets, high rapidity bosons or heavy quarks .....

Improve uncertainty on parton distributions by improved constraints, and check understanding of theoretical uncertainties, and determine where NNLO, electroweak corrections, resummations $etc.$ needed.

Make improved predictions for both background and signals with improved partons and surrounding theory.

Spot new physics from deviations in these predictions. As a nice by-product improve our understanding of the strong sector of the Standard Model considerably.

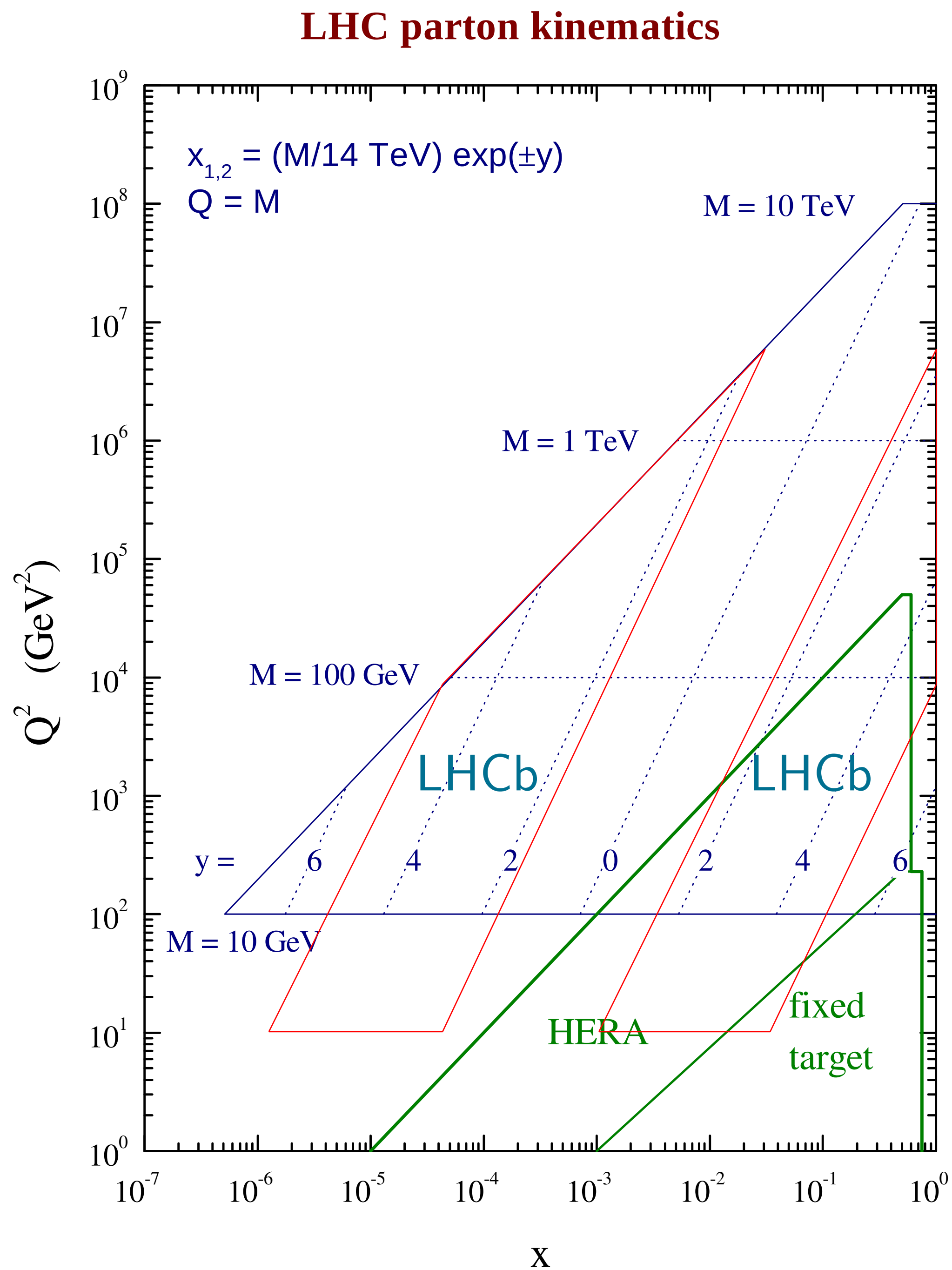
Remainder of talk describes this process in more detail.

Predictions at the LHC

New kinematic regime.

PDFs mainly extrapolated via evolution rather than measured directly.

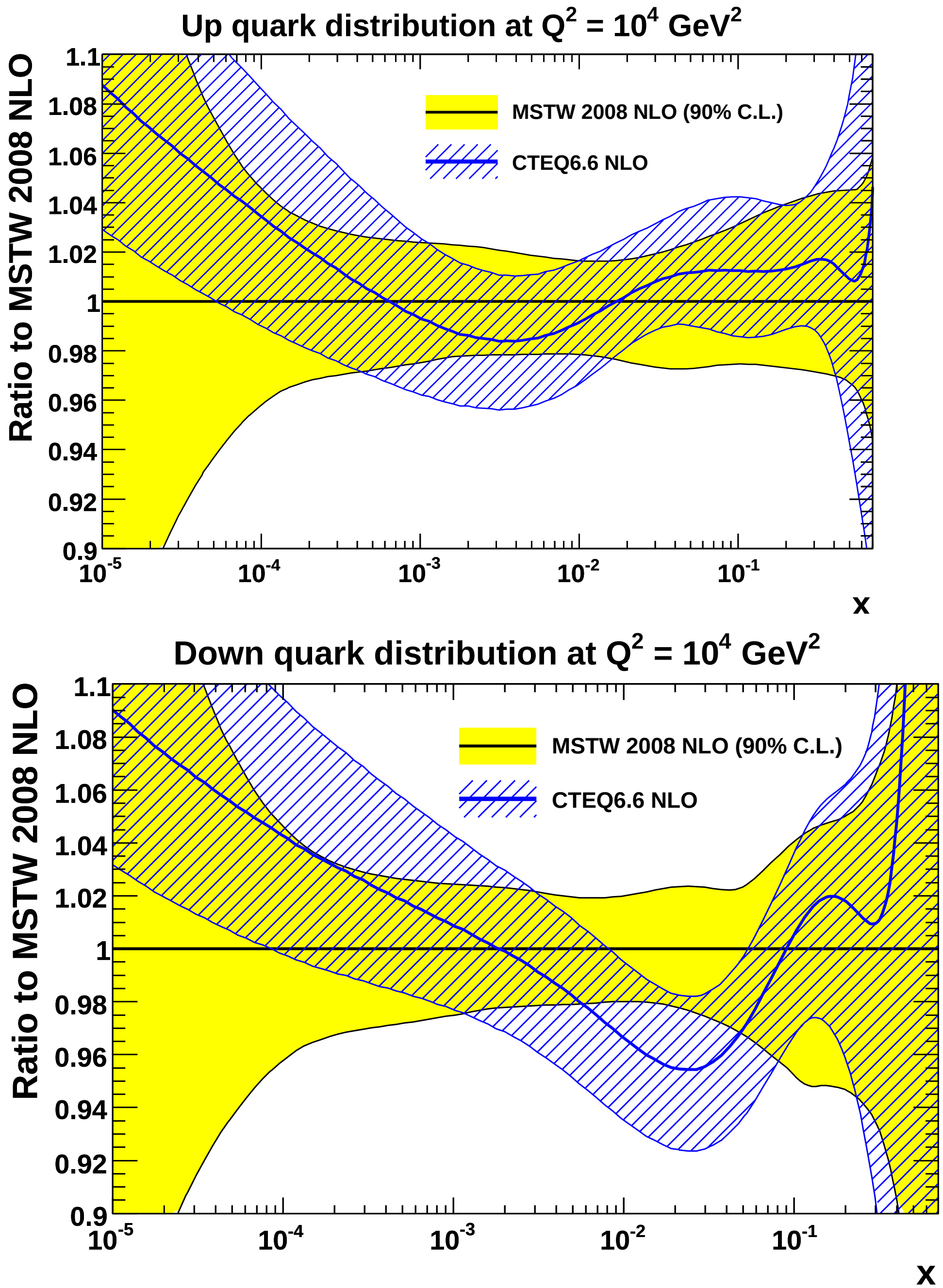
High scale and small-x parton distributions are vital for understanding processes at the LHC.



Uncertainty on MSTW u and d distributions, along with CTEQ6.

Reasonable agreement between groups.

Central rapidity $x = 0.006$ is ideal for uncertainty in W, Z (Higgs?) at the LHC.



Predictions for W and Z cross-sections for LHC with common fixed order QCD and vector boson width effects, and common branching ratios.

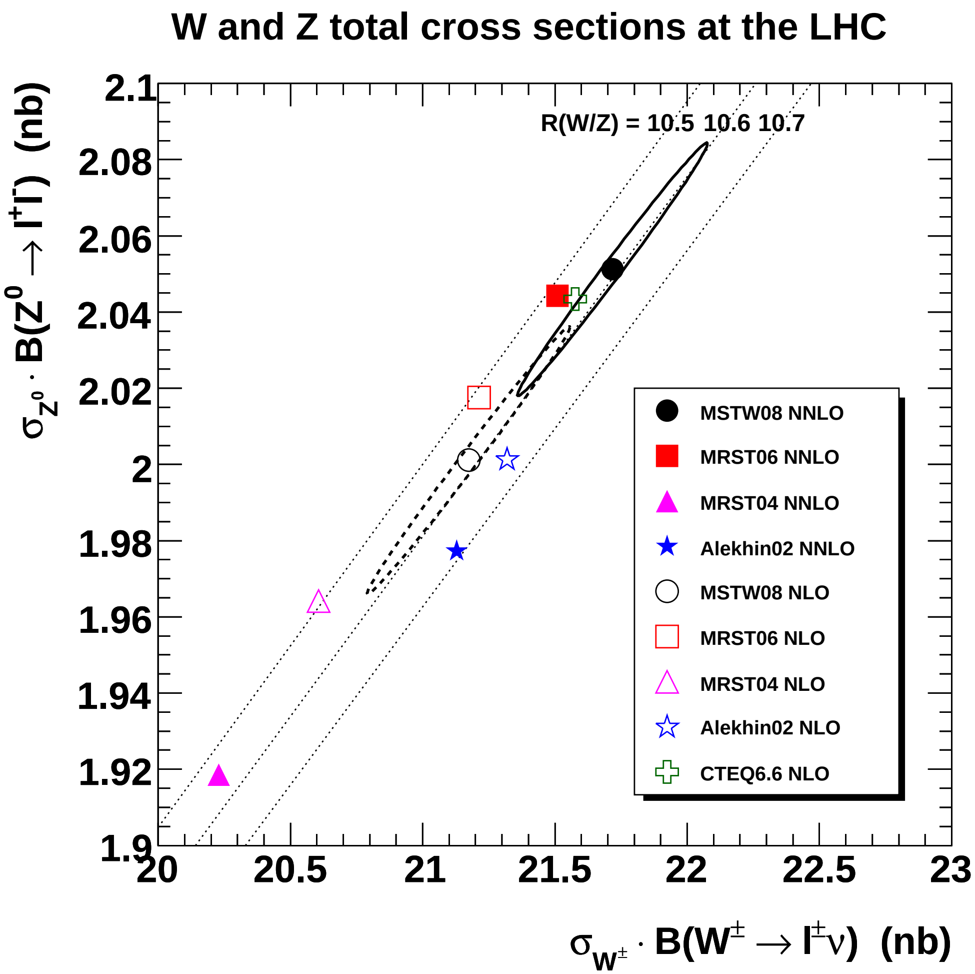
Good agreement at NLO for variety of PDFs.

Fairly significant change from NLO to NNLO mainly due to hard cross-section correction.

Some difference in W/Z ratio.

Generally all fine?

W, Z total cross-sections best-case scenario.



W, Z uncertainty – more details

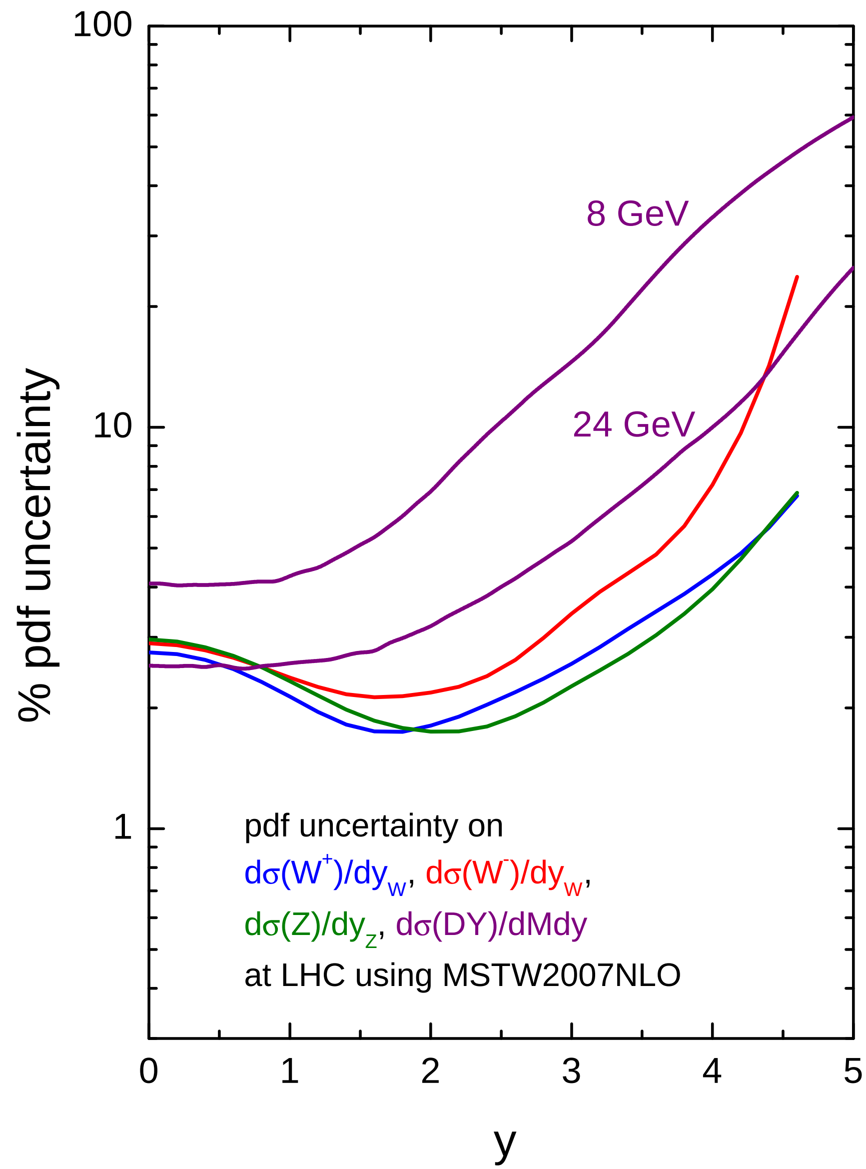
Uncertainty on $\sigma(Z)$ and $\sigma(W^+)$ grows at high rapidity.

Uncertainty on $\sigma(W^-)$ grows more quickly at very high y – depends on less well-known down quark.

Uncertainty on $\sigma(\gamma^\star)$ is greatest as y increases. Depends on partons at very small x.

Lots of interest in LHCb range.

Still only uncertainty from data with *perfect* framework.



Other Sources of Uncertainty

It is vital to consider theoretical/assumption-dependent uncertainties:

- Methods of determining "best fit" and uncertainties.
- Underlying assumptions in procedure, e.g. parameterisations and data used.
- Treatment of heavy flavours.
- PDF and α_S correlations.
- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements.
- Standard higher orders (NNLO)
- Resummations, e.g. small x ($\alpha_s^n \ln^{n-1}(1/x)$), or large x ($\alpha_s^n \ln^{2n-1}(1-x)$)
- low Q^2 (higher twist), saturation

Lead to differences in current partons, and to corrections in predicted cross-sections.

Parton Fits and Uncertainties. Two main approaches.

Parton parameterization and Hessian (Error Matrix) approach first used by H1 and ZEUS, and extended by CTEQ.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

The Hessian matrix H is related to the covariance matrix of the parameters by

$$C_{ij}(a) = \Delta\chi^2 (H^{-1})_{ij}.$$

We can then use the standard formula for linear error propagation.

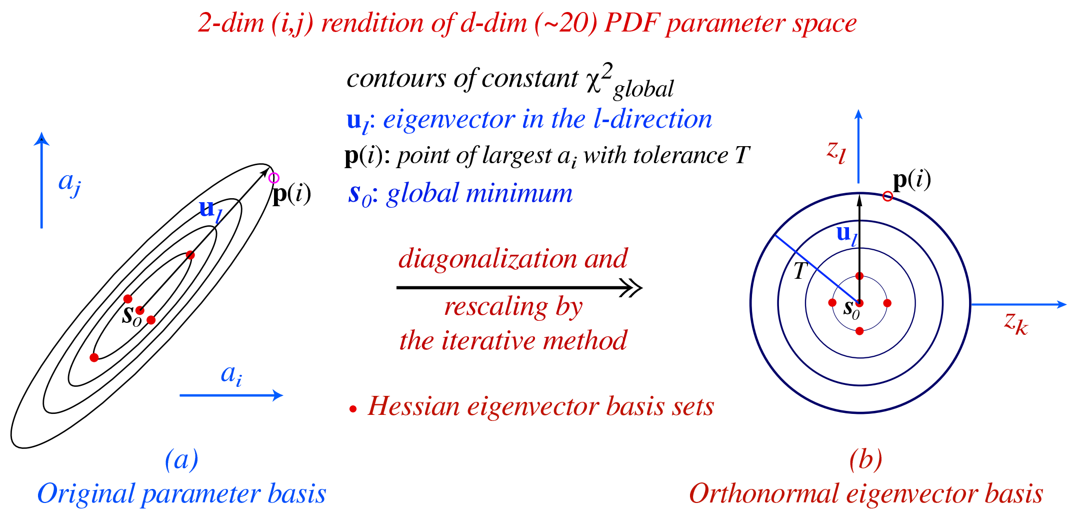
$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},$$

This is now the most common approach (sometimes *Offset method*).

Problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space.

Solved by finding and rescaling eigenvectors of H leading to diagonal form

$$\Delta\chi^2 = \sum_i z_i^2$$



Implemented by CTEQ, then others. Uncertainty on physical quantity then given by

$$(\Delta F)^2 = \sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2,$$

where $S_i^{(+)}$ and $S_i^{(-)}$ are PDF sets displaced along eigenvector direction.

Question of choosing "correct" $\Delta\chi^2$ given complication of errors in full fit and sometimes conflicting data sets.

CTEQ use $\Delta\chi^2 \sim 40$ and MRST/MSTW use more complicated approach – results in $\Delta\chi^2 \sim 15$, for one σ. Other fits less global, keep to $\Delta\chi^2 = 1$.

More recently **Neural Network** group (Ball *et al.*) limit parameterization dependence. Leads to alternative approach to "best fit" and uncertainties.

Statistical approach – construct a set of Monte Carlo replicas $\sigma^k(p_i)$ of the original data set $\sigma^{data}(p_i)$. Representation of $P[\sigma(p_i)]$ at points p_i.

Train a neural network for the parton distribution function on each replica, obtaining a representation of the pdfs $q_i^{(net)(k)}$. (Latter part follows Giele *et al.*)

The set of neural nets is a representation of the probability density – mean μ_O and deviation σ_O of observable O then given by

$$\mu_O = \frac{1}{N_{rep}} \sum_1^{N_{rep}} O[q_i^{(net)(k)}], \quad \sigma_O^2 = \frac{1}{N_{rep}} \sum_1^{N_{rep}} (O[q_i^{(net)(k)}] - \mu_O)^2.$$

Include information about measurements and errors in distribution of $\sigma^{data}(p_i)$.

This is statistically correct, and does not rely on the approximation of linear propagation of errors in calculating observables, but is more complicated and time intensive.

Currently uses DIS data sets, and very recently dimuon production in neutrino DIS.

Attractive but ambitious large-scale project being extended to more data types.

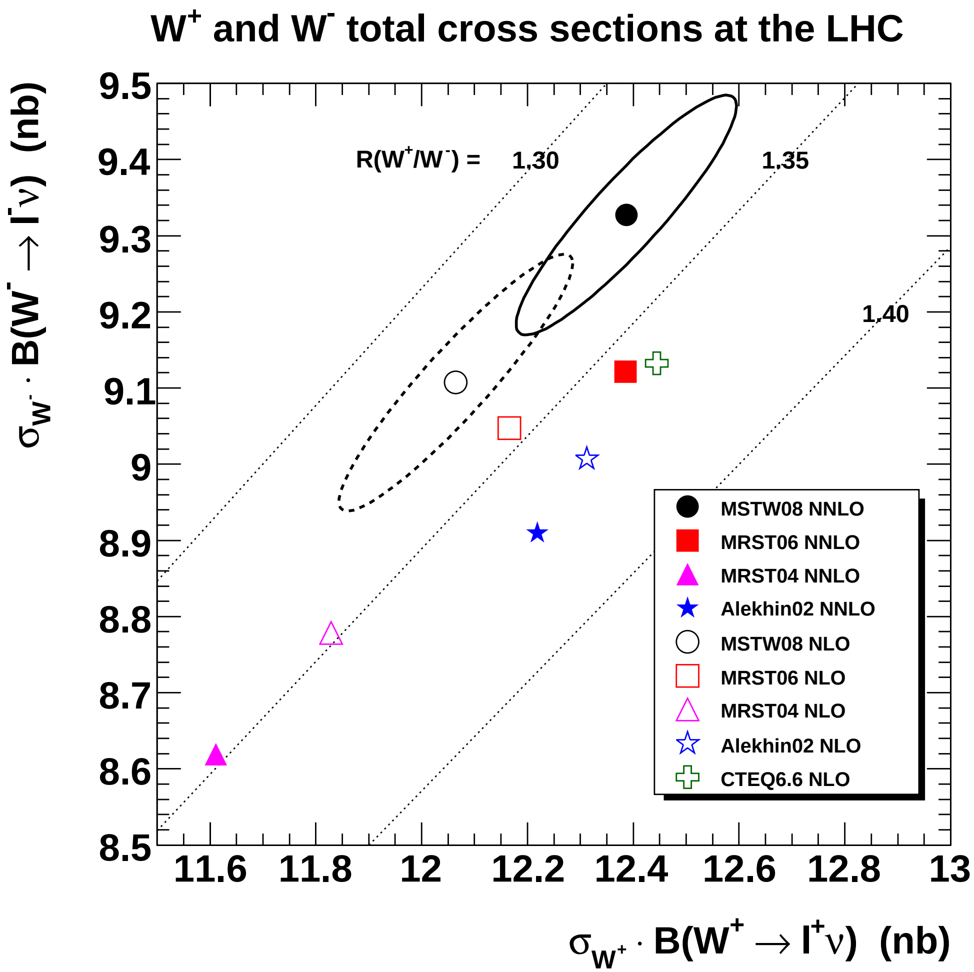
Parameterisations

MSTW predictions for W+ and W- cross-sections for LHC with common fixed order QCD and vector boson width effects, and common branching ratios.

Quoted uncertainty for ratio very small, i.e. $\approx 0.8\%$. Prediction sensitive to u and d quarks.

$$\frac{\sigma(W^+)}{\sigma(W^-)} \approx \frac{u(x)\bar{d}(x)}{d(x)\bar{u}(x)} \approx \frac{u(x)}{d(x)},$$

If $\bar{u}(x) \to \bar{d}(x), x \to 0$, which data implies, and most parameterisations assume.



Fit includes most recent neutrino DIS and Tevatron vector boson data. Uncertainties should account for this.

Significantly more difference than uncertainty from other PDFs, including MRST – (effect noted for W-asymmetry by Cooper-Sarkar). Very interesting for early data.

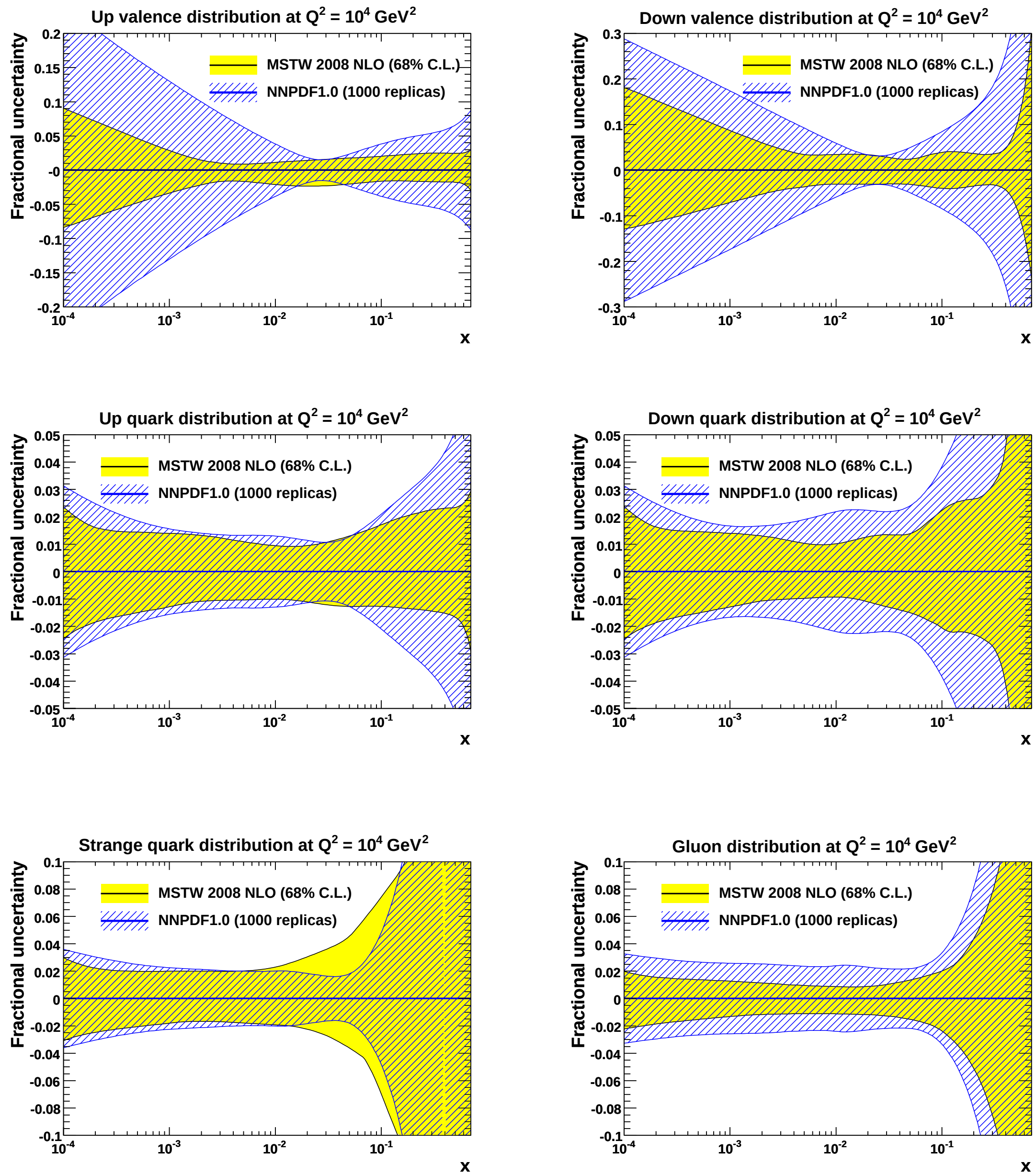
CTEQ/MRST uncertainties largely similar – exceptions to come.

Comparison of uncertainties to NLO NNPDF 1.0 set. Fixed strange fraction of light quarks (relaxed in 1.2).

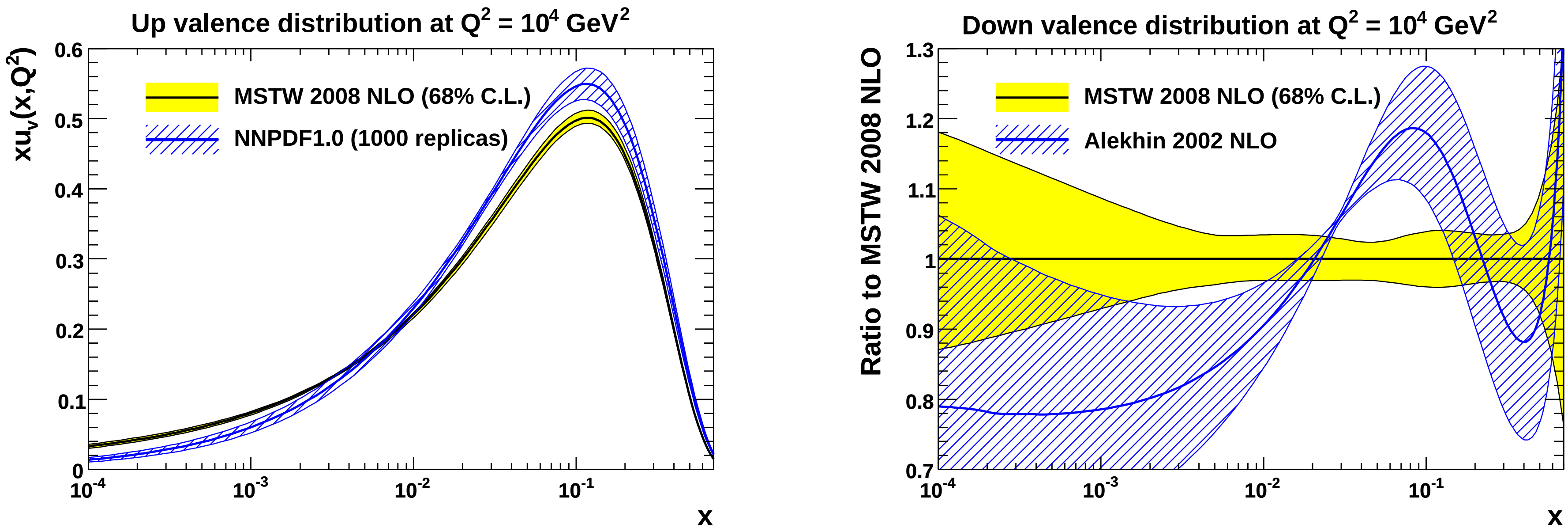
Often comparable despite input flexibility in NNPDF.

NNPDF 1.0 uses only DIS data – less constraint (central values of PDFs not always consistent).

Almost certainly real additional uncertainty on small-x valence and very high x. (Though extra data and sum rules play a role in reducing uncertainties even here.)



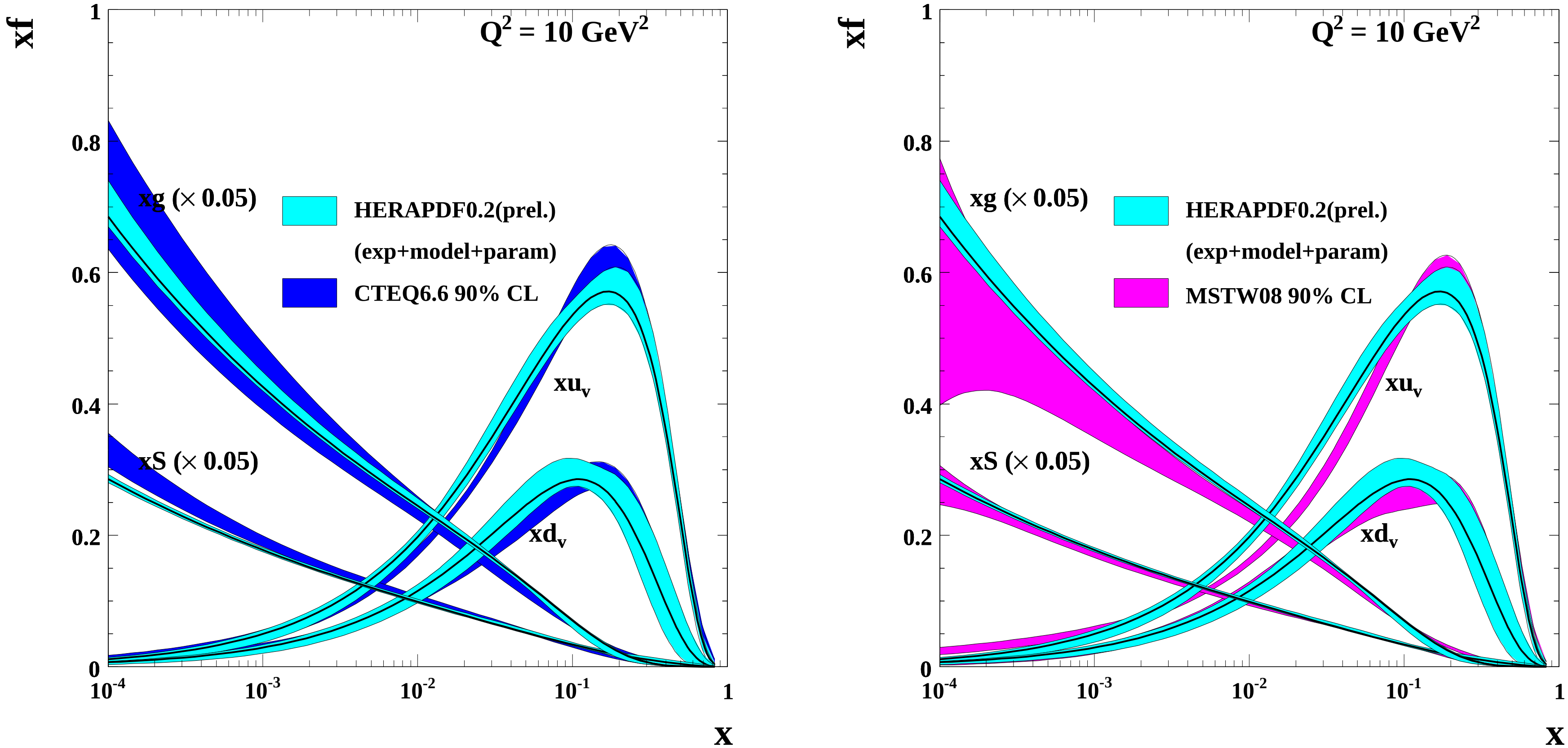
Note that PDF sets sometimes differ significantly in central values though. Largely due to data fit



Clearly seen comparing up valence for MSTW and NNPDF and and down valence for MSTW and Alekhin.

Other (in some cases far worse) examples.

HERA fits are only to HERA data, but use data averaged between H1 and ZEUS (reduced correlated errors) unavailable to others so far.

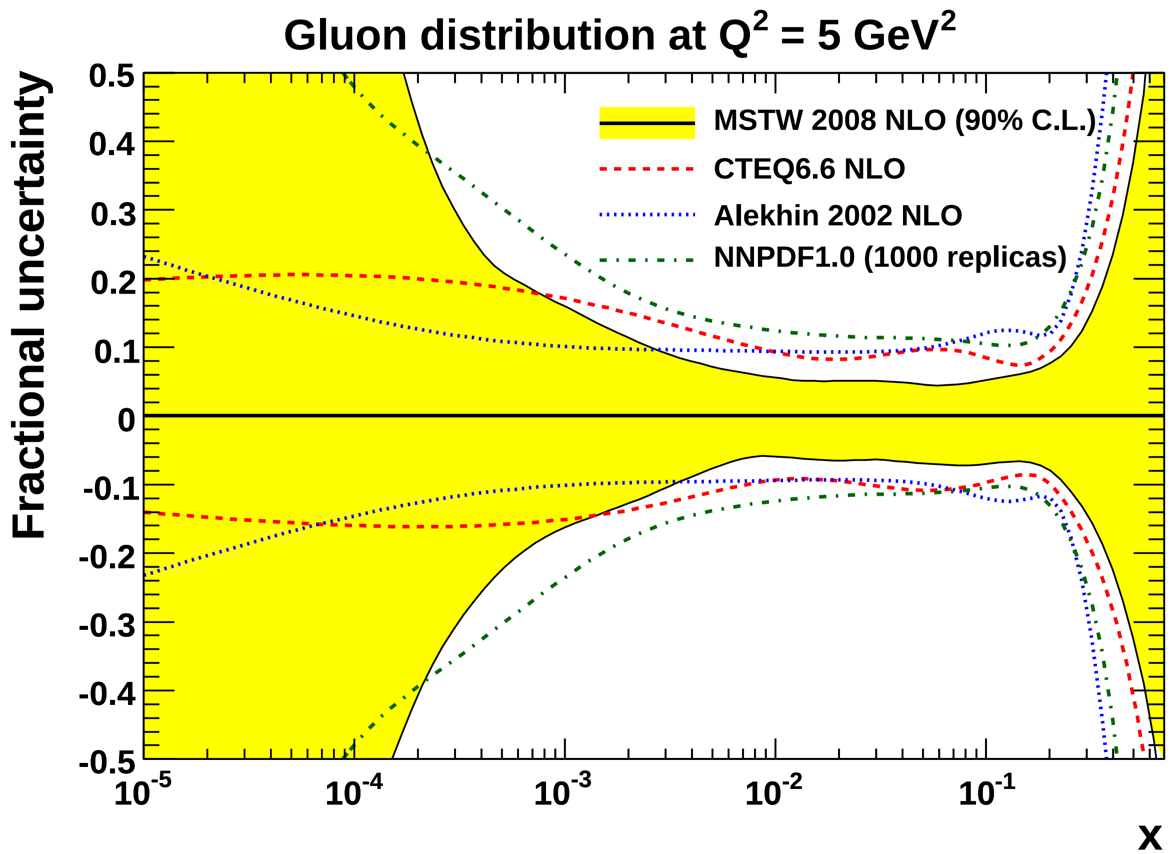


More consistent data sets $\rightarrow$ $\Delta\chi^2 = 1$ for uncertainties. Nevertheless similar to CTEQ/MSTW.

Significant differences in central values sometimes, and in shape of small-x gluon uncertainty.

Gluon Parameterisation - small x.

Different parameterisations lead to very different uncertainty for small x gluon.

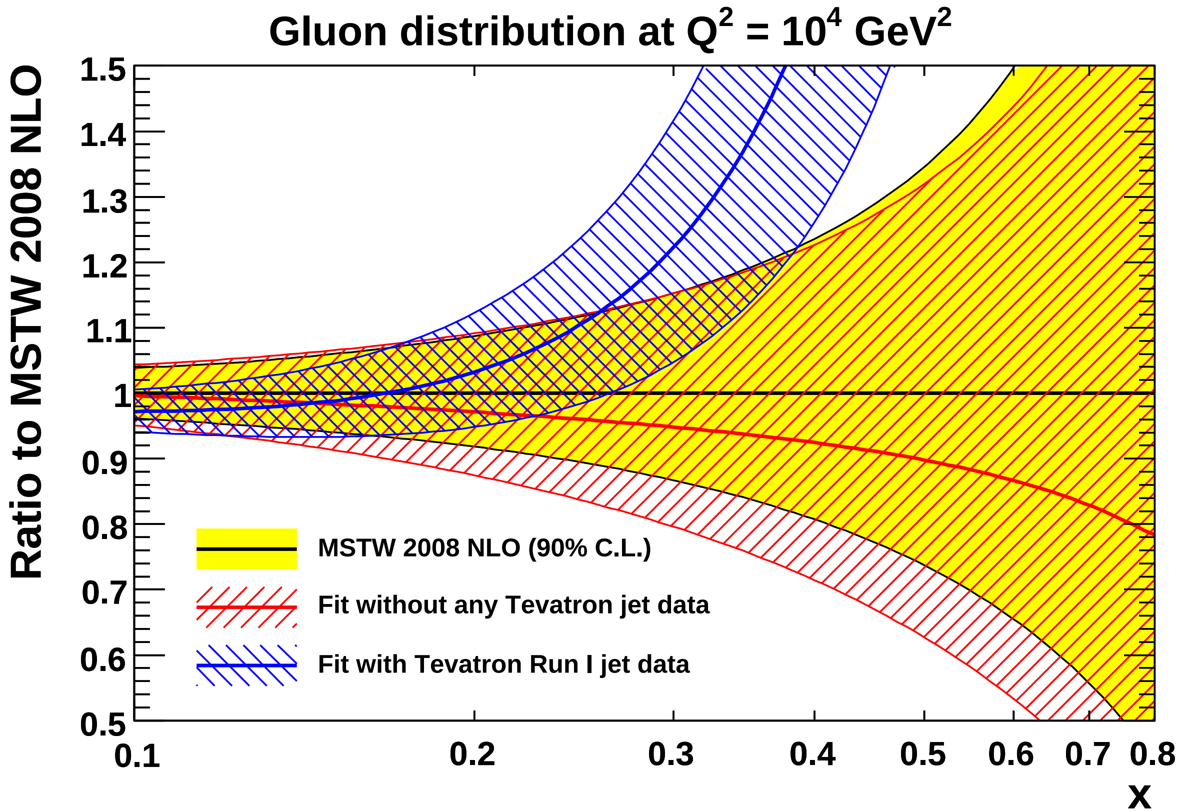


Most assume single power x^{λ} at input $\rightarrow$ limited uncertainty. If input at low Q^2 λ positive and small-x input gluon *fine-tuned* to ~ 0. Artificially small uncertainty.

MRST/MSTW and NNPDF more flexible (can be negative) $\rightarrow$ rapid expansion of uncertainty where data runs out.

Gluon Distribution - large x.

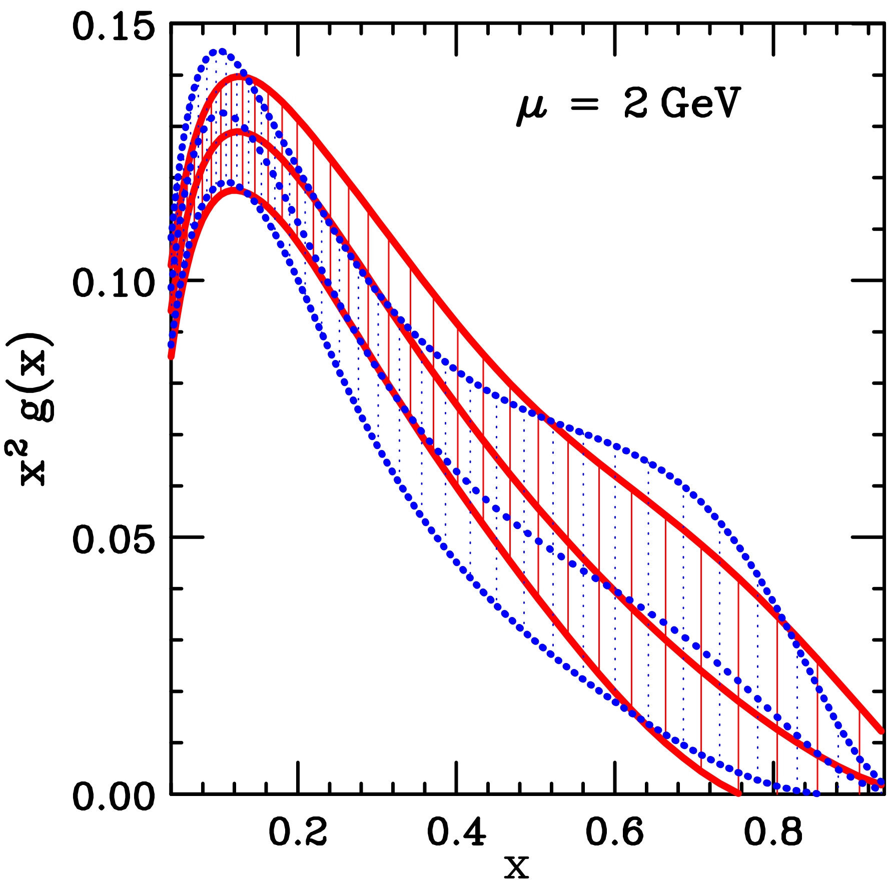
Constrained indirectly, but quite accurately, by DIS data, and directly by Tevatron high-p_T jets, now **Run I** and **Run II** available. *Slightly* confusing picture.

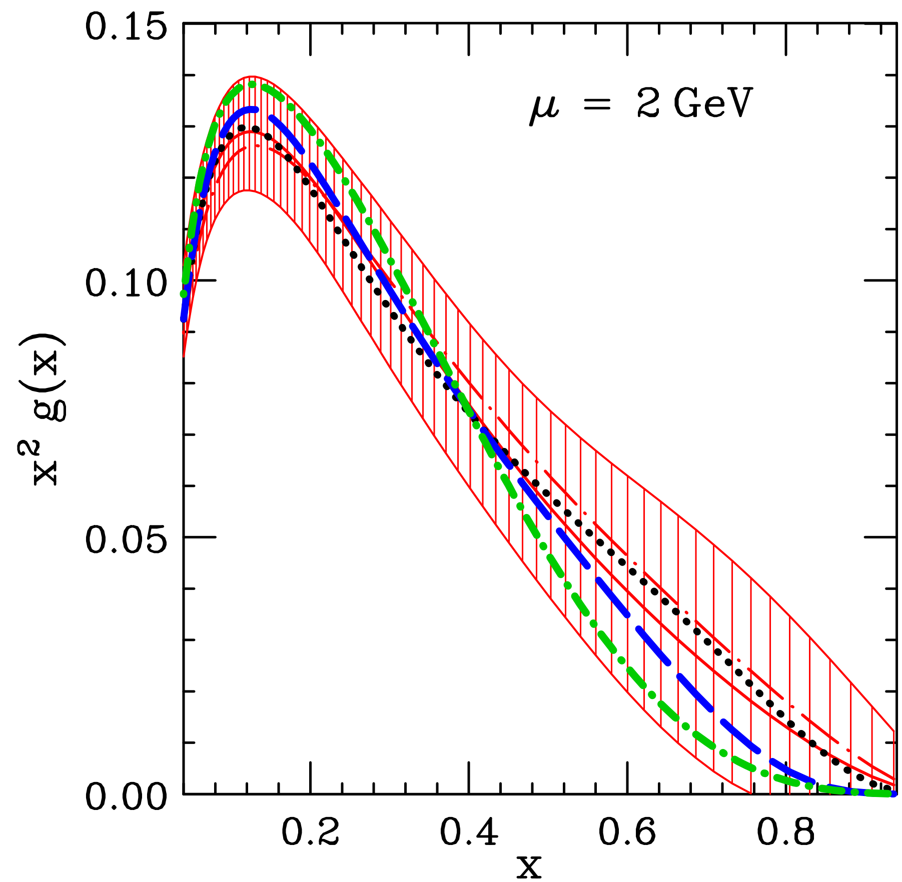


Only fit by MSTW and CTEQ. Former found gluon much softer for **Run II**. Fits not very consistent between runs.

CTEQ find more compatibility between **Run I** and **Run II** fits. Fit with both sets $\rightarrow$ little change – red CT09G, blue CTEQ6.6 (left).

Partially less strict with "consistency", partially difference in parameterisation, partially effectively *higher weight* to jet data in global fit.



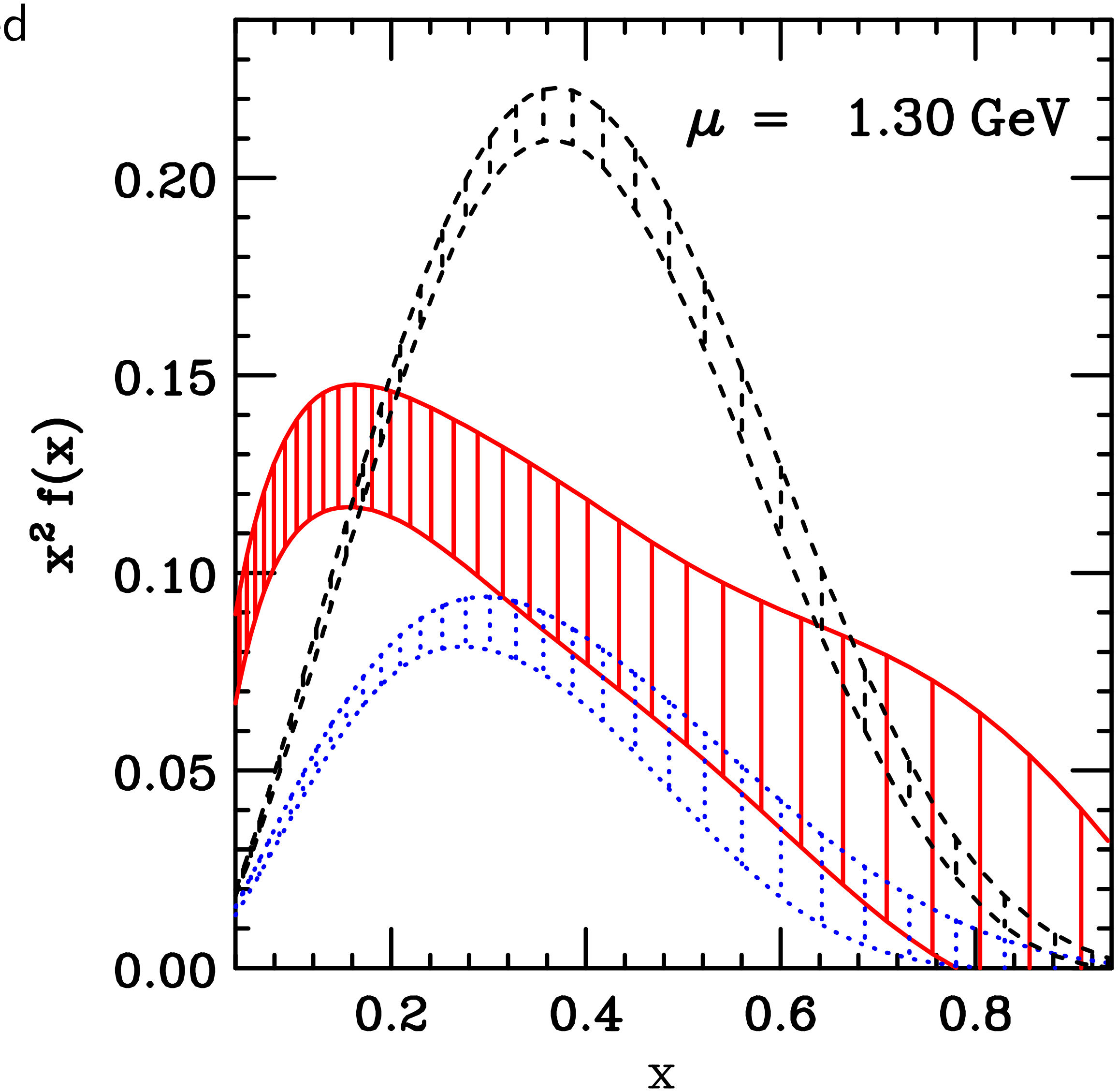


When fit to **Run II** data only and same procedure as MSTW blue (right) similar to MSTW green (right).

Generally high-x PDFs parameterised so will behave like $(1-x)^\eta$ as $x \to 1$. More flexibility in CTEQ.

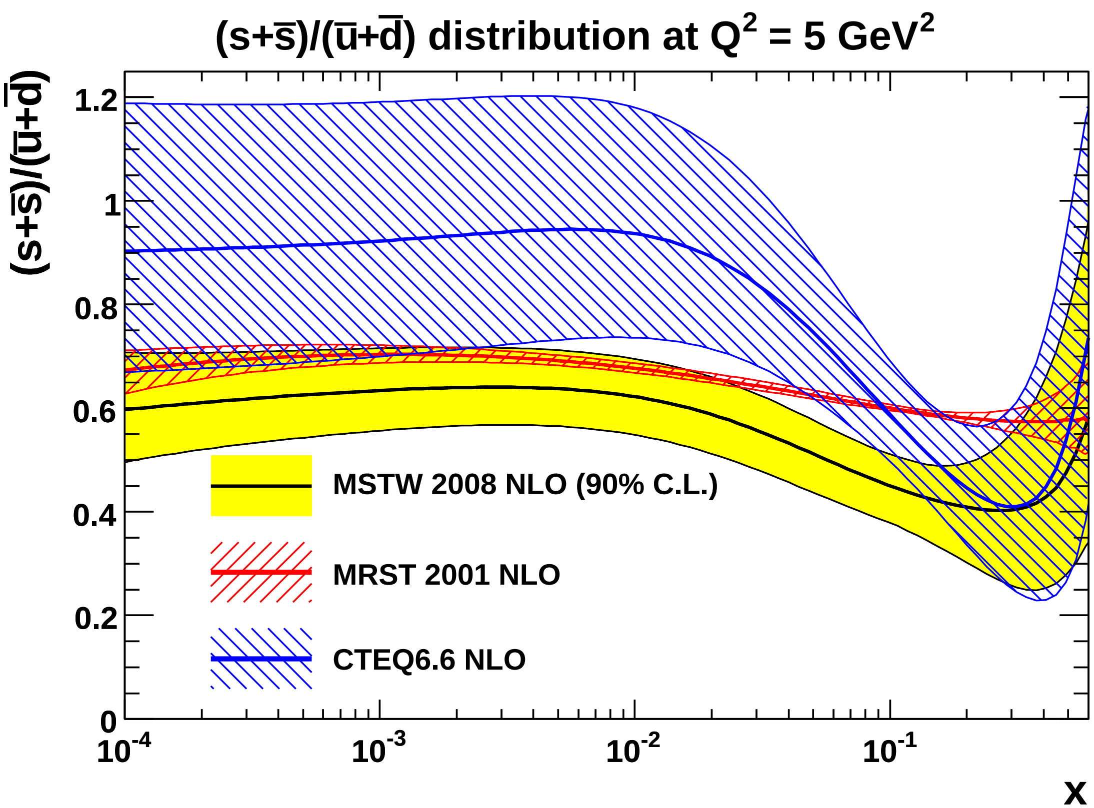
Very hard high-x gluon distribution (more-so even than NNPDF uncertainties).

However, is gluon, which is radiated from quarks, harder than the up valence distribution for $x \to 1$?



Strange Quarks

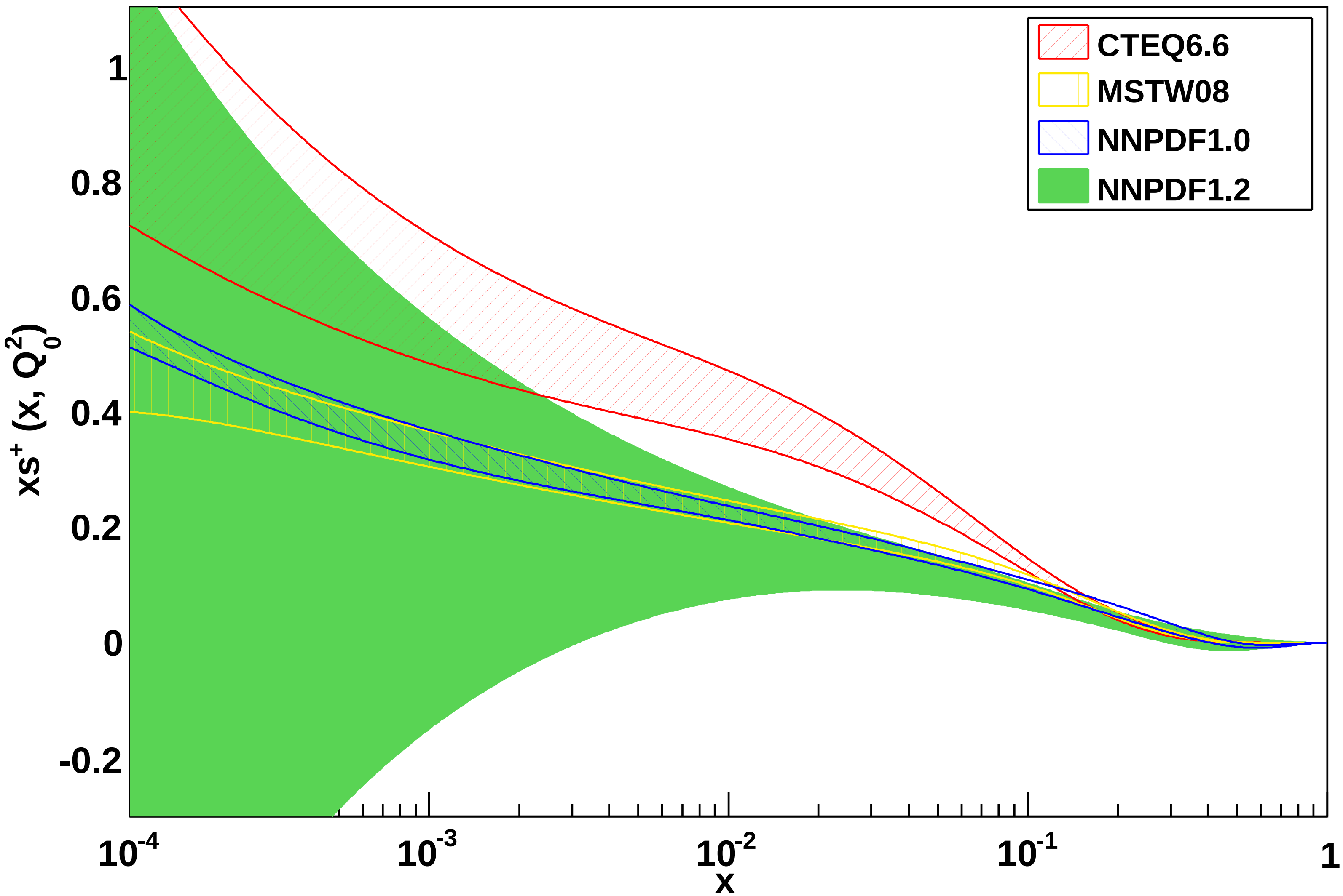
Direct fit to $s, \bar{s}$ from dimuon data leads to significant uncertainty increase compared to assumption of fixed fraction of sea used until recently. Constraint for $x \geq 0.01$.



MSTW assumes shape of strange given by theory assumption that suppression of form of massive quarks. Significantly different to CTEQ fitting to same data assuming only same small-x power for strange as light quarks.

Difference in region of data! Effect of nuclear corrections and/or heay quark treatment?

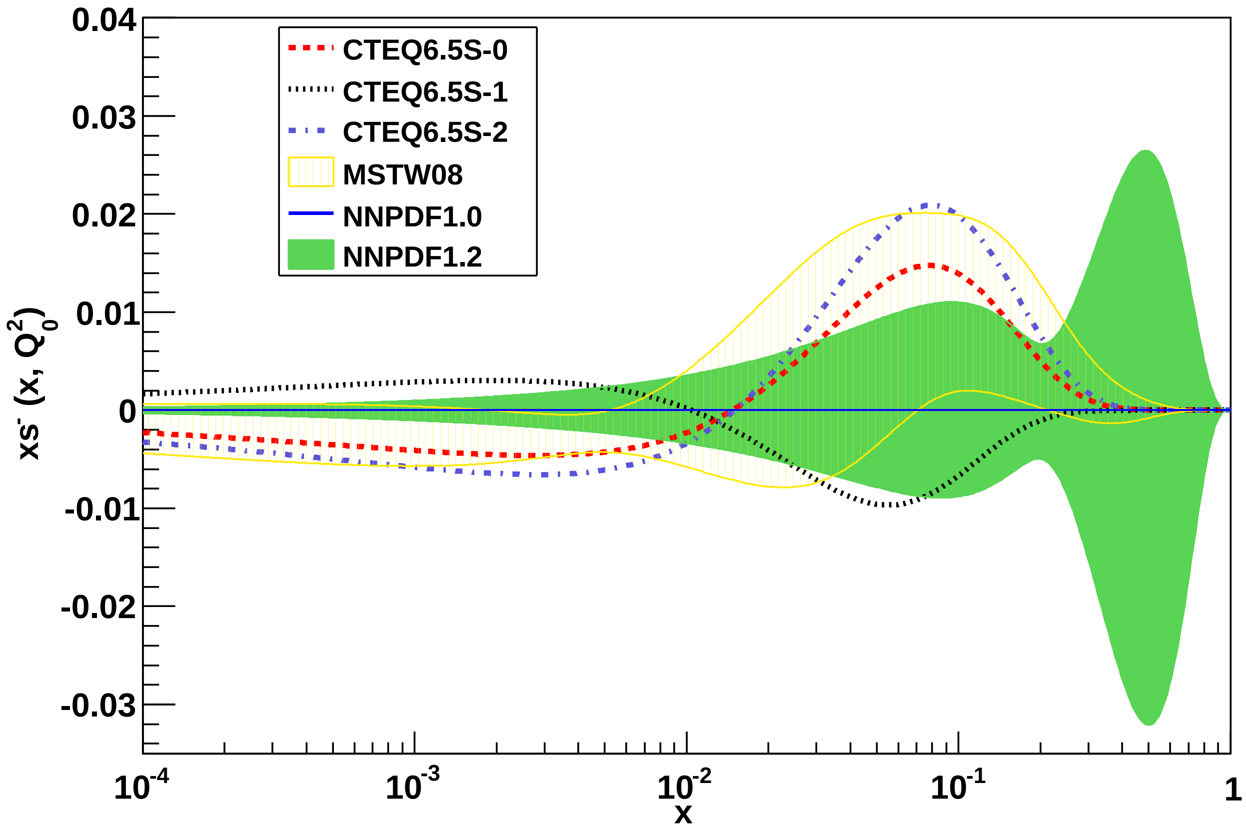
NNPDF1.2, which includes dimuon data, have no theoretical constraint on strange quark distribution at all at small x.



Overestimate of uncertainty? Impacts on small-x light quarks.

Strange asymmetry.

Most recent sets obtain $s - \bar{s}$ for first time from differences in $\nu, \bar{\nu}$ dimuon production.



All tend towards positive momentum asymmetry, but all fairly consistent with zero, or with enough to remove (or seriously) reduce NuTeV anomaly on $\sin^2\theta_W$.

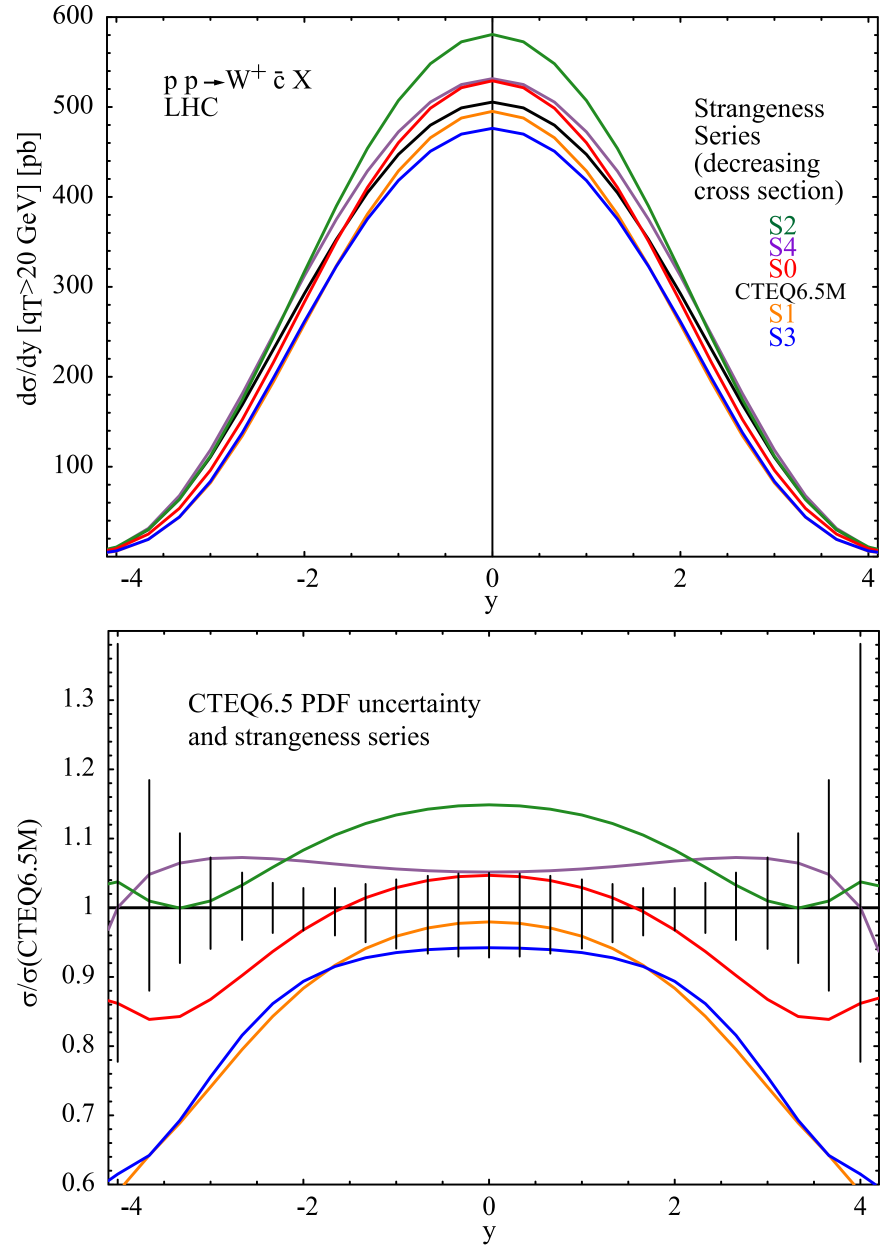
NNPDF also obtain $|V_{cd}| = 0.244 \pm 0.019$ and $|V_{cs}| = 0.96 \pm 0.07$.

Impact of Strange Uncertainty.

CTEQ look at special sets with fits to dimuon data and possible (generous) variations.

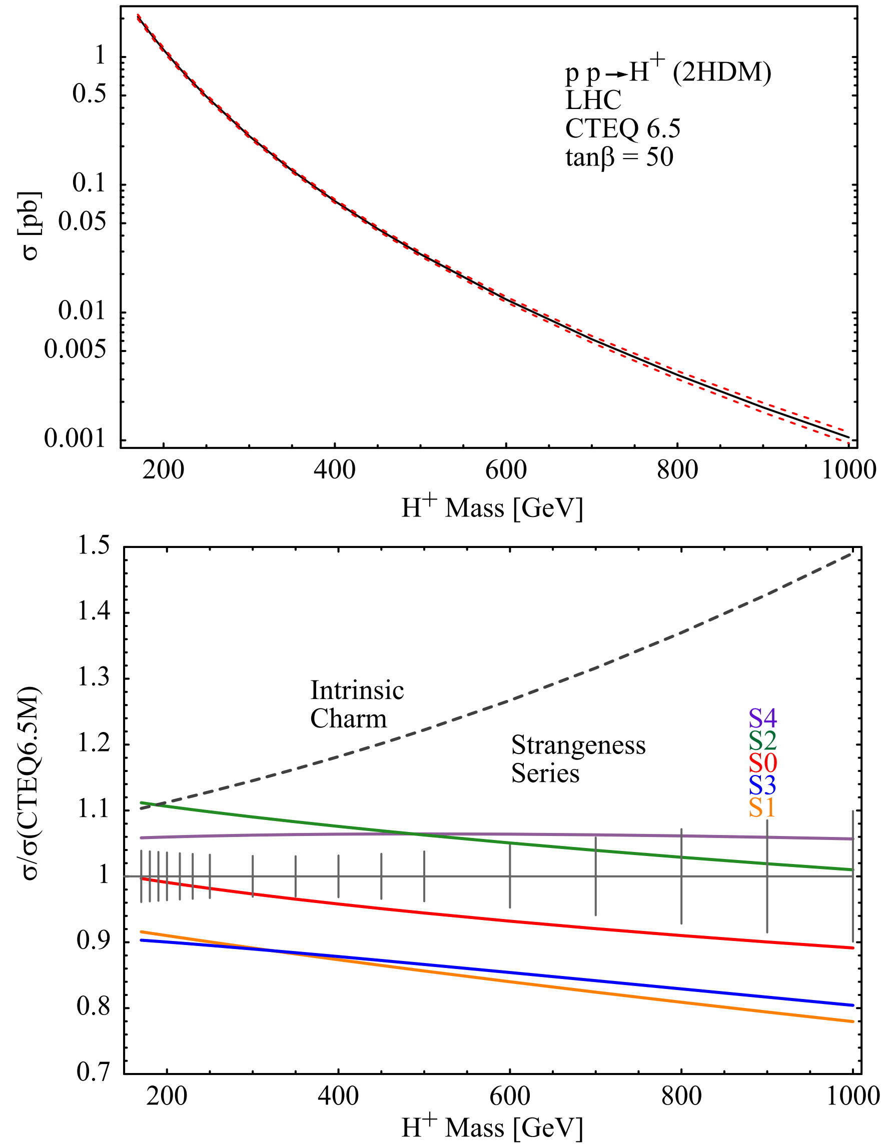
Band represents uncertainty of default CTEQ6.5 set.

Look at implications for strange sensitive final state, i.e. $W + c$ at LHC.



Also examine uncertainty of predictions for BSM physics, e.g. $c + \bar{s} \rightarrow H^+$.

Again allowed sets give wider range of predictions than default uncertainty.



Heavy Quarks – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\ln^n(Q^2/m_H^2)$ terms, and not calculated for many processes beyond LO. Still occasionally used. Sometimes final state details in this scheme only.

Alternative, at high scales $Q^2 \gg m_H^2$ heavy quarks like massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Normal assumption in calculations. Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

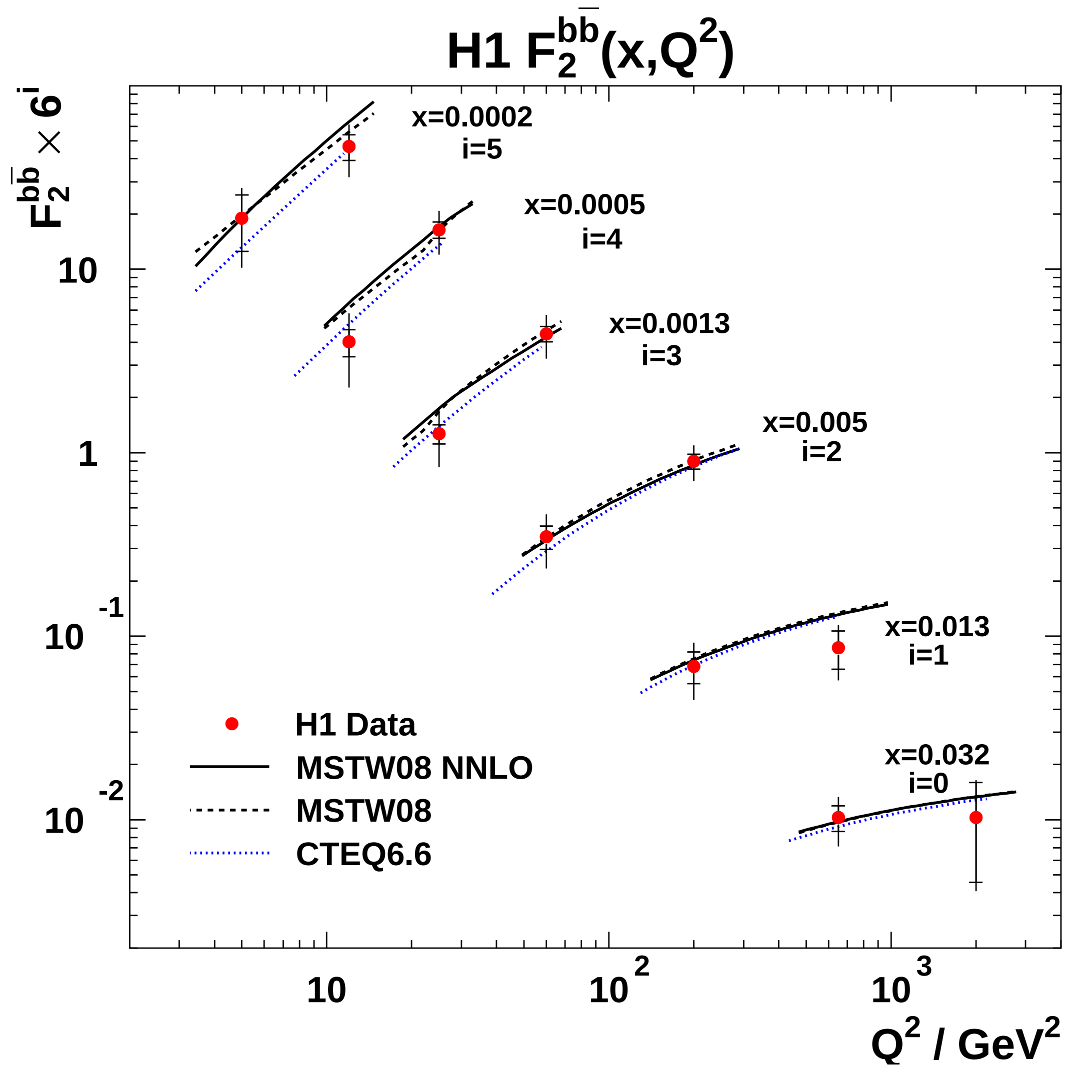
Need a **General Mass Variable Flavour Number Scheme** (GM-VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$. Used by MRST/MSTW and more recently (as default) by CTEQ, and now also more regularly by H1,ZEUS.

Various definitions possible. Versions used by MSTW (RT) and CTEQ (ACOT) have converged somewhat.

Freedom in choices and consistency of kinematic limits (heavy quark pair produced in final state) introduced in RT scheme.

Simplest choice in heavy flavour coefficient function now commonly based on ACOT(χ) prescription, i.e. scaling variable x replaced by $\chi \equiv x(1+4m_H^2/Q^2)$.

Various significant differences still exist as illustrated by comparison to most recent H1 data on bottom production.



Check effect of change in flavour prescription for NLO.

Compare MRST2004 (with 2001 uncertainties) to unofficial "MRST2006 NLO".

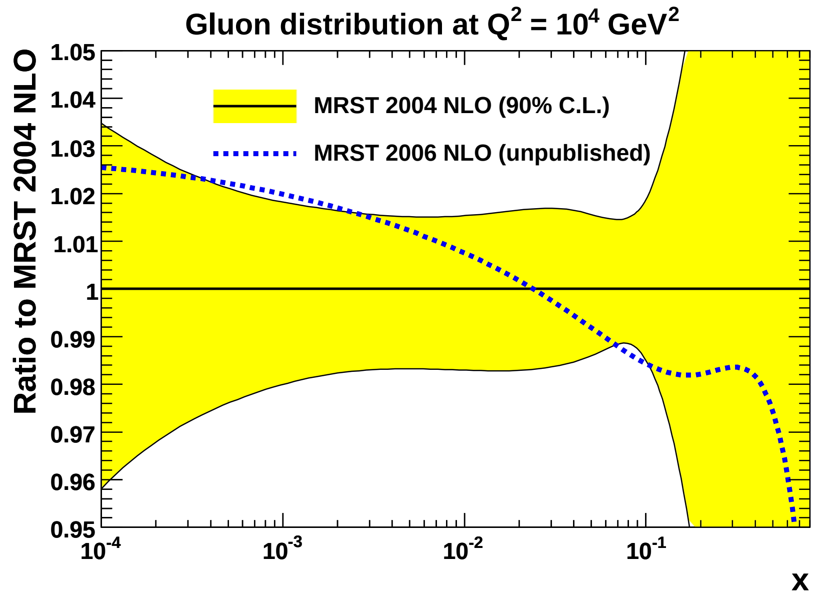
Only difference in flavour schemes (both well-defined).

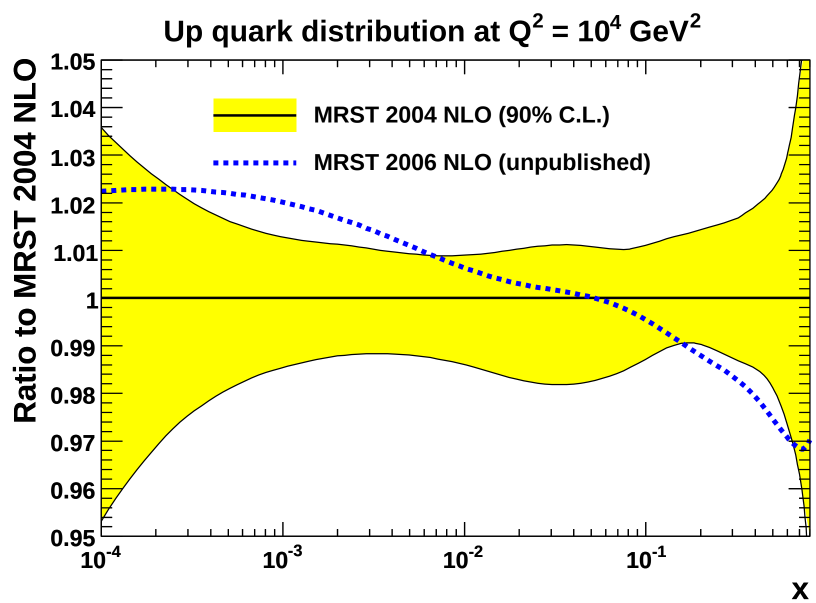
Changes of up to 2% in PDFs.

Up to 3% increase in σ_W and σ_Z at the LHC.

This is a genuine theory uncertainty due to competing but equally valid choices. Ambiguity decreases at higher orders.

Some – but probably quite little – anti-correlation with PDF uncertainties.

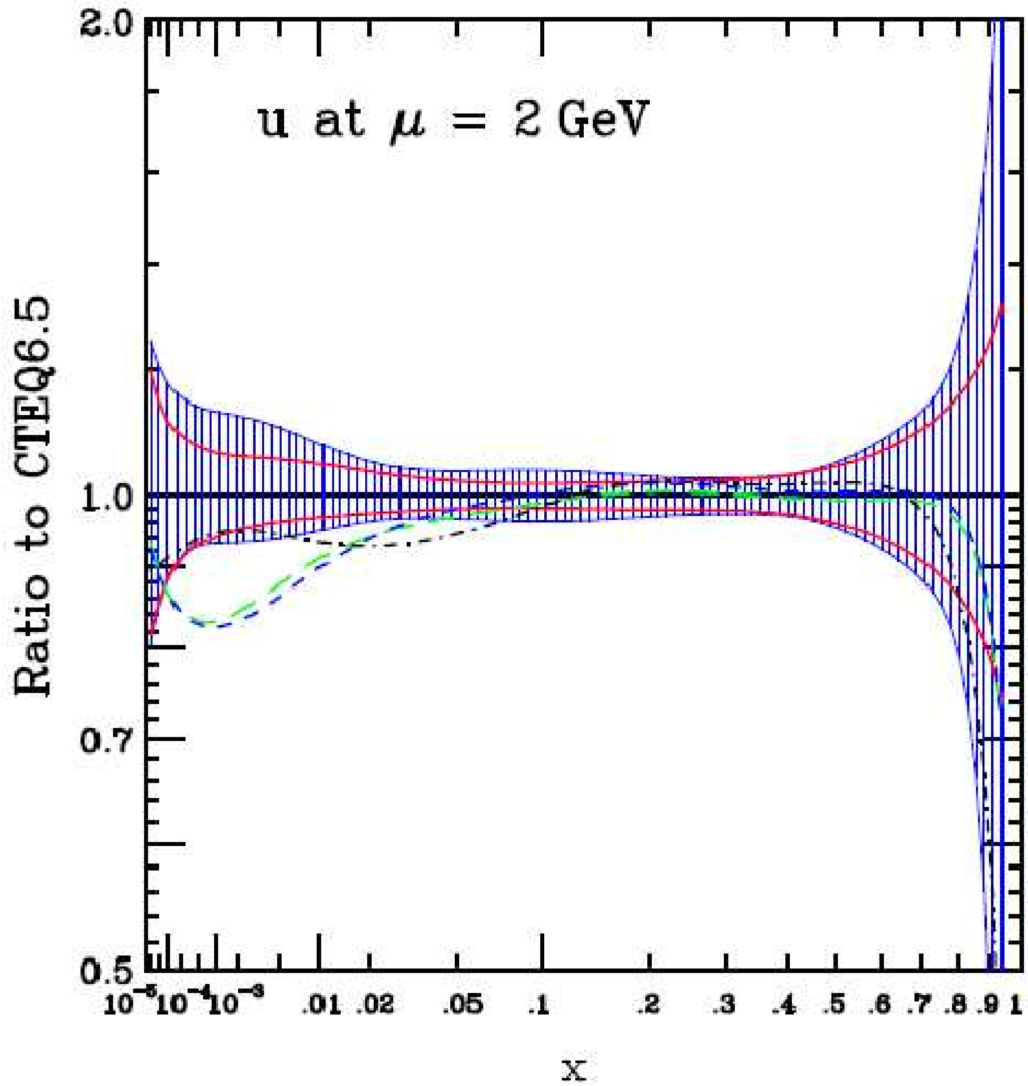




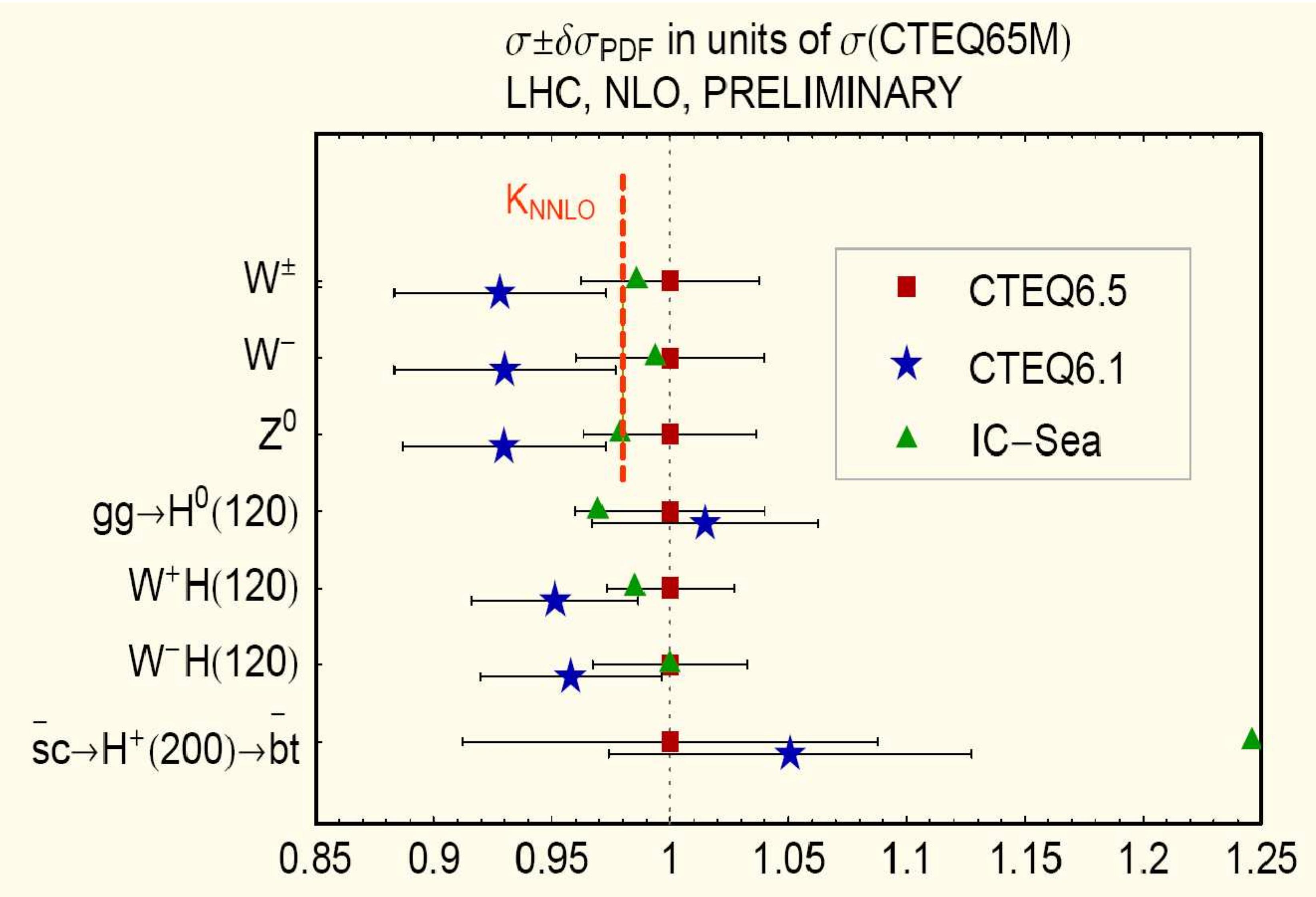
Importance of using GM-VFNS instead of massless approach illustrated by CTEQ6.5 up quark with uncertainties compared with previous versions, e.g. CTEQ6 in green.

Can be $> 8\%$ error in PDFs. Much more than scheme uncertainty.

MRST in dash-dot line. Reasonable agreement. Already used heavy flavour treatment in default sets.



Leads to large change in predictions using CTEQ partons at LHC of $5 - 10\%$.



Note effects of *intrinsic charm* in final case.

Could also be nonperturbative (intrinsic) heavy flavour.

Suppressed by $\frac{\Lambda^2_{QCD}}{Q^2}$ or possibly $\frac{\Lambda^2_{QCD}}{W^2} \sim \frac{\Lambda^2_{QCD}}{Q^2(1-x)}$.

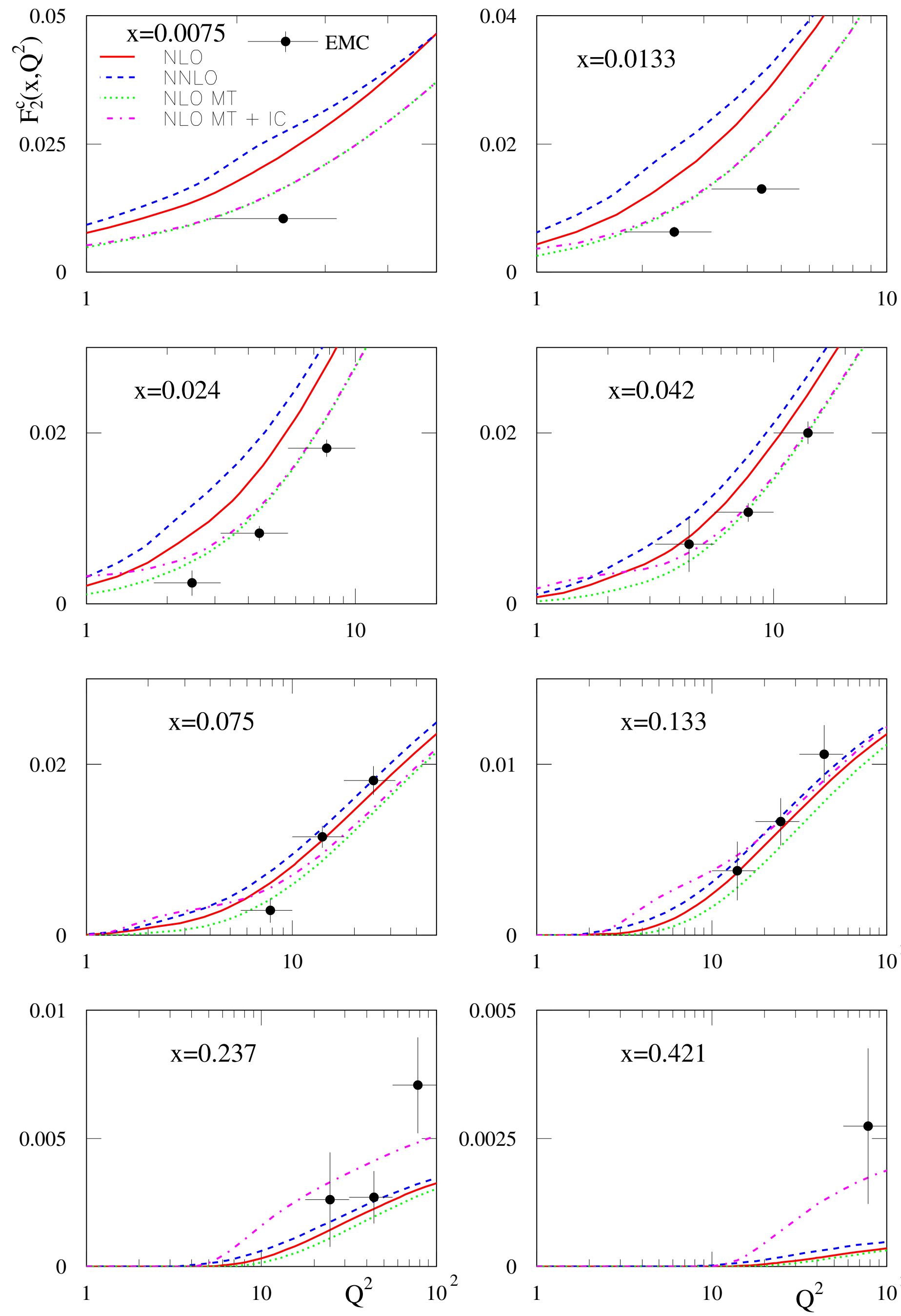
Enhanced at high x (Brodsky *et al*).

CTEQ constrain from normal global fit (and consider large effect at all x).

Check against old EMC data. Suggest at most $\frac{1}{10}th$ this value.

Need to modify threshold physics for good fit.

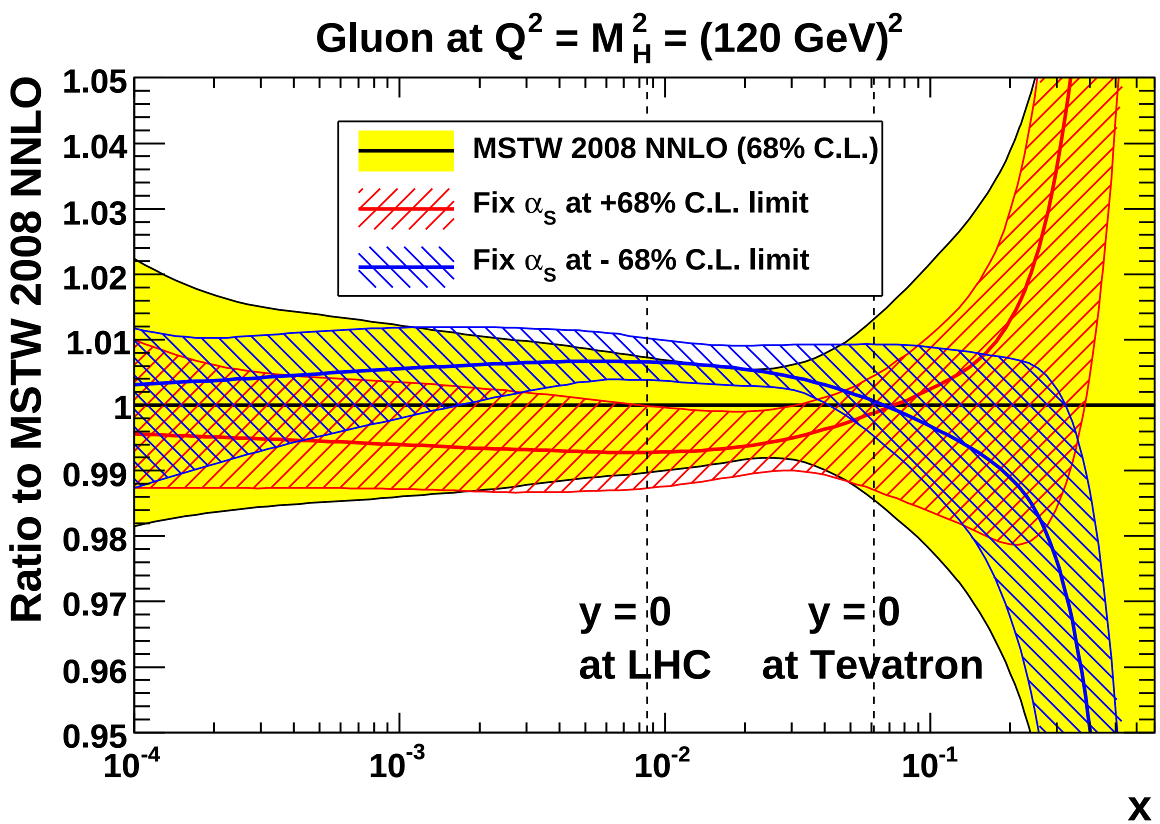
Large intrinsic $b+\bar{b}$ could dominate Higgs production at $y \geq 5$ at LHC (Brodsky *et al*).



PDF correlation with α_S.

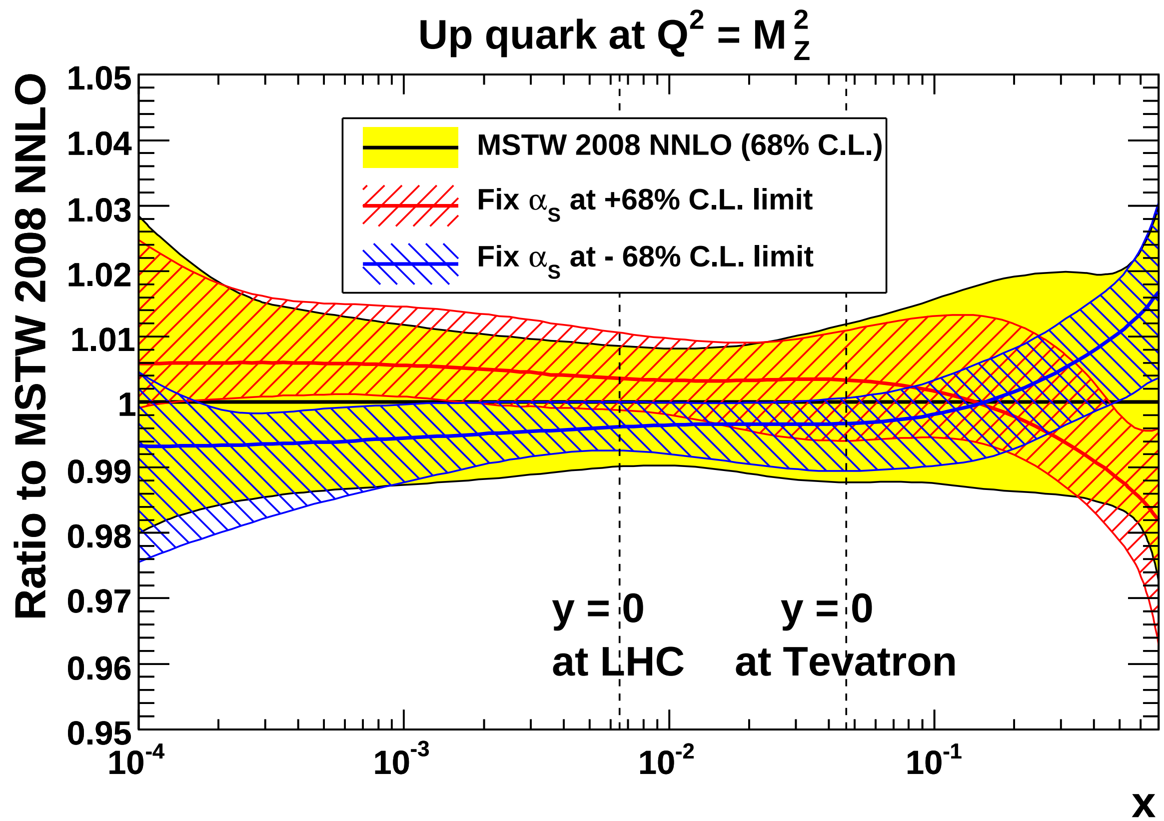
Can also look at PDF changes and uncertainties at different $\alpha_S(M_Z^2)$. Latter usually only for one fixed $\alpha_S(M_Z^2)$. Can be determined from fit, e.g. $\alpha_S(M_Z^2) = 0.1202^{+0.0012}_{-0.0015}$ at NLO and $\alpha_S(M_Z^2) = 0.1171^{+0.0014}_{-0.0014}$ at NNLO from MSTW.

PDF uncertainties reduced since quality of fit already worse than best fit.



Expected gluon–$\alpha_S(M_Z^2)$ small–x anti-correlation $\rightarrow$ high-x correlation from sum rule.

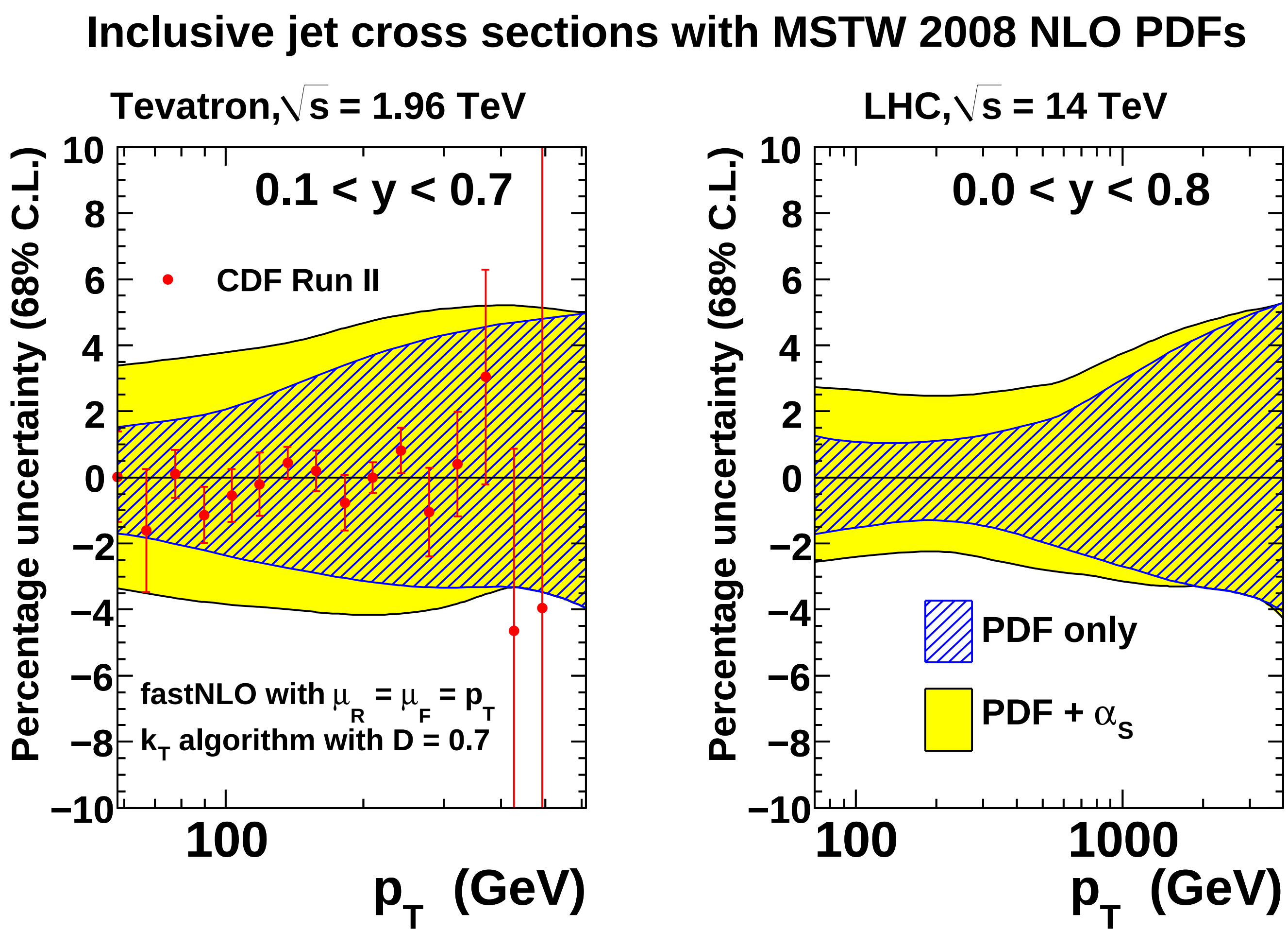
Gluon feeds into evolution of quarks, but change in $\alpha_S(M_Z^2)$ just outweighs gluon change, i.e. larger $\alpha_S(M_Z^2) \rightarrow$ slightly more evolution.



Strong anti-correlation at high-x due to evolution and positive coefficient functions.

Quarks roughly opposite to gluons.

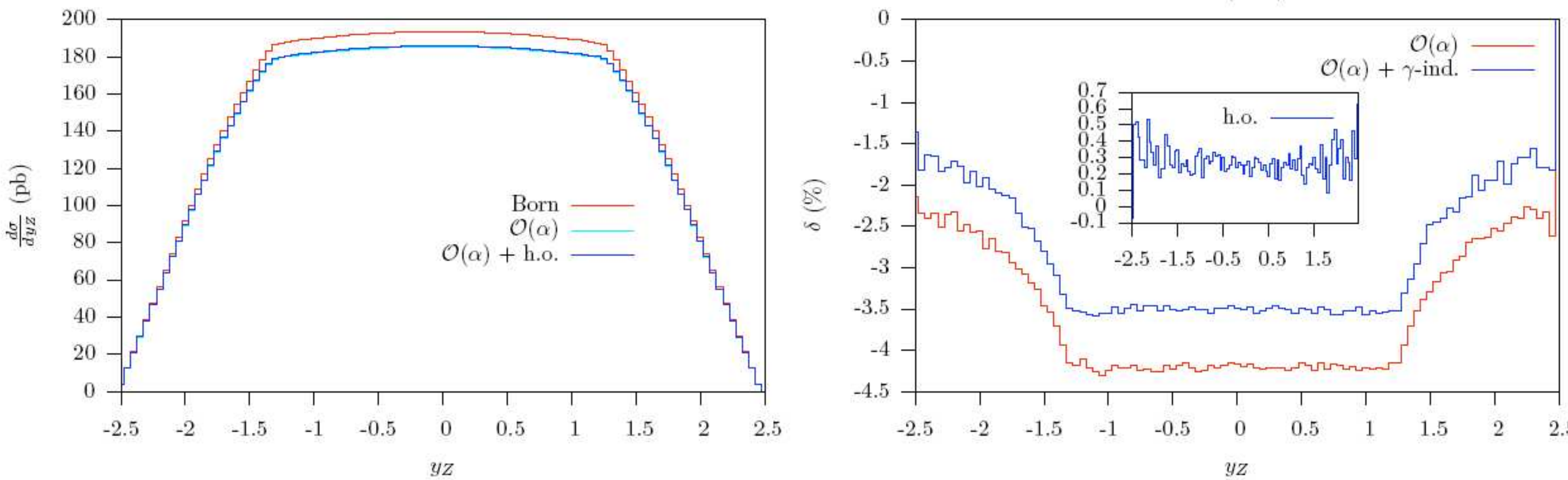
Can also investigate uncertainties for inclusive jets at Tevatron and at LHC.



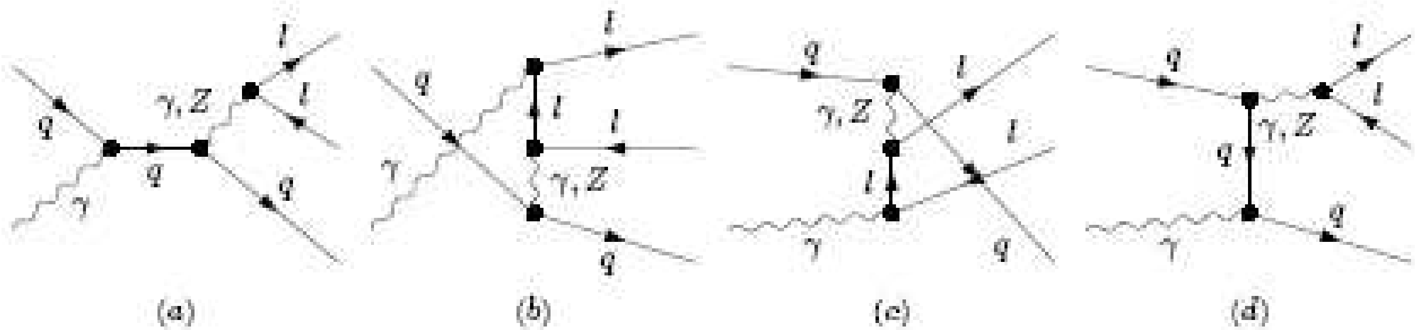
At lower p_T gluons dominate and α_S correlated. At higher p_T quarks become more important and high-x quarks anti-correlated to α_S so no additional α_S uncertainty.

Electroweak corrections

Typically a few percent, e.g. Calone Calame *et al* who look at Drell-Yan processes.



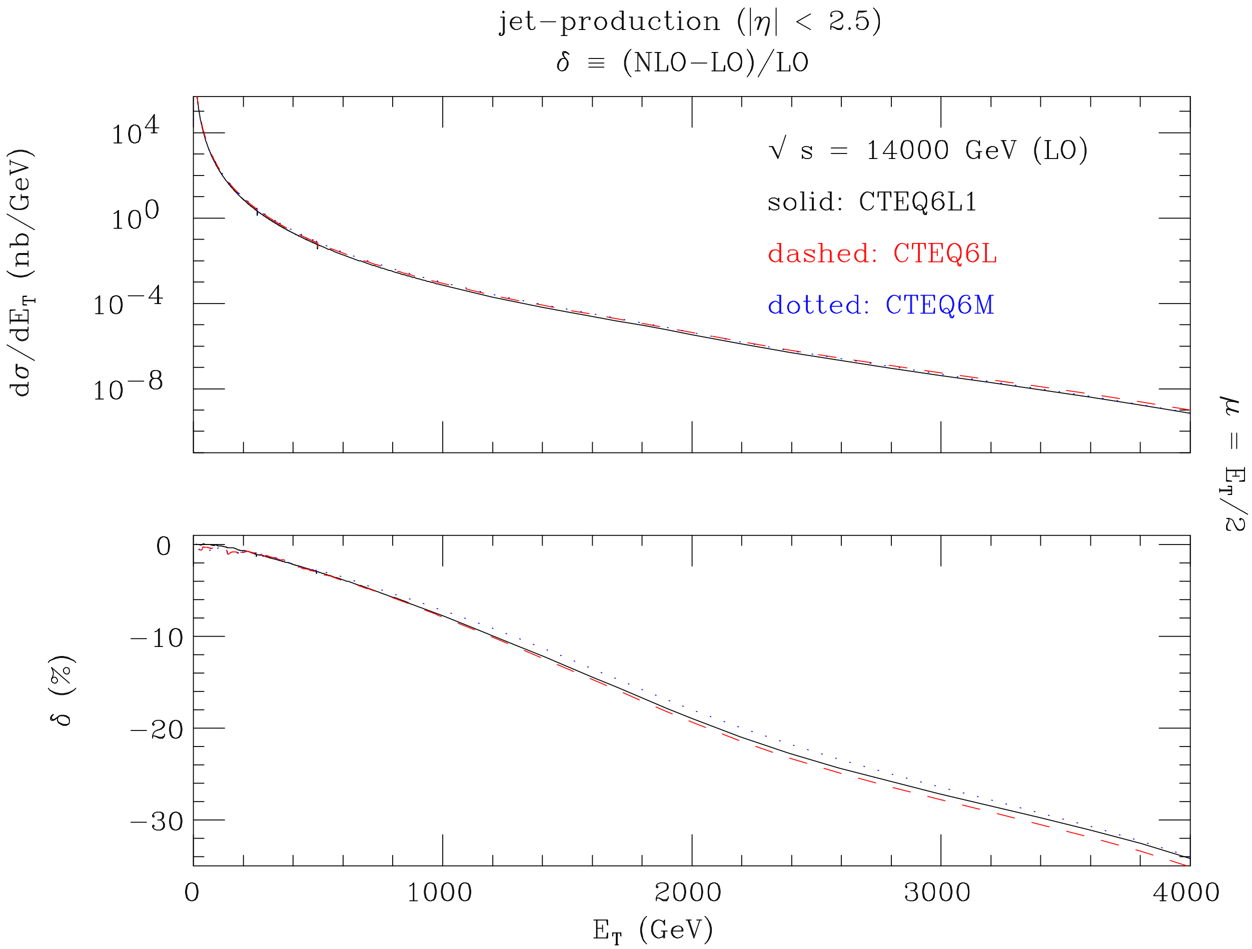
Also consider photon-induced processes. Requires the photon distribution of the proton. Currently only one QED-corrected pdf (MRST2004) set (leads to automatic isospin violation - reduces NuTeV anomaly).



Can also be a couple of percent (here in opposite direction).

Large Electroweak corrections

Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{1}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2))$.



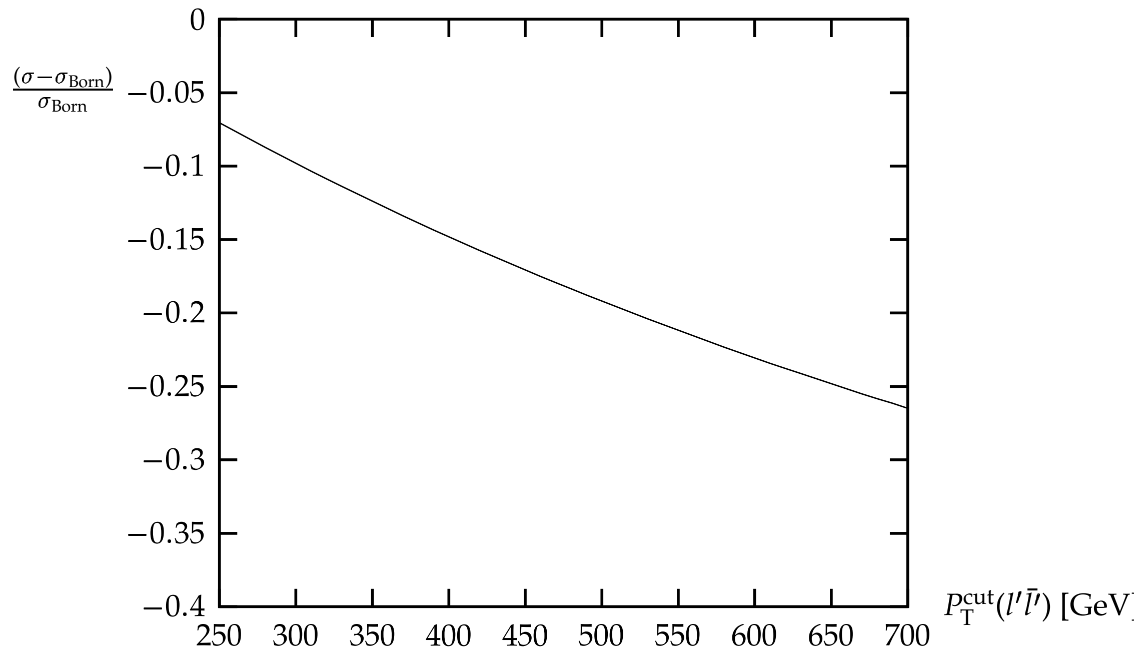
Big effect at LHC energies – $\log^2(E_T^2/M_W^2)$ a very large number. Up to 30%. Bigger than NLO QCD.

Similar results for corrections to other processes with a hard scale, e.g. Di-boson production (Accomando *et al*).

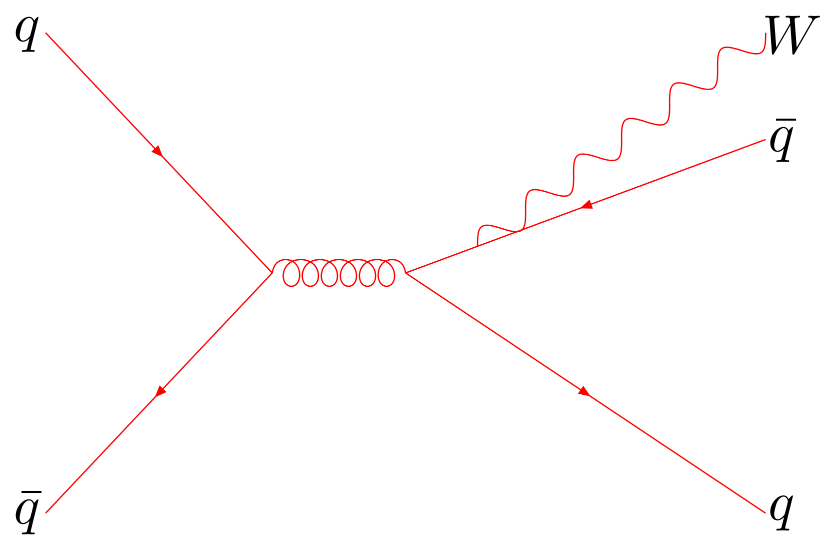
Plot shows fractional corrections as function of reconstructed Z transverse momentum in WZ production.

Same sort of corrections in large-p_T vector bosons in conjunction with jets (Kühn *et al*, Maina *et al*)...

$\ln(s/m_W^2)$ terms can also affect Γ_W extraction from the transverse mass distribution.



Only virtual corrections for W, Z. Must have contributions of the form



Some electroweak bosons included with jets – some almost collinear with quark, and many decaying into hadrons.

Opposite sign, potentially large contribution. However, perfect cancellation will not happen. Total effect very possibly still large. Similar situation in variety of processes.

Needs calculation and decisions on experimental definitions. Calculations by Baur suggests opposite, but somewhat smaller contributions. Very sensitive to jet veto in di-boson production.

Perhaps want partons with Weak as well as QED corrections, (splitting functions derived – P Ciafaloni and Comelli).

NNLO

Default has long been NLO. Essentially well understood. Now starting to go further.

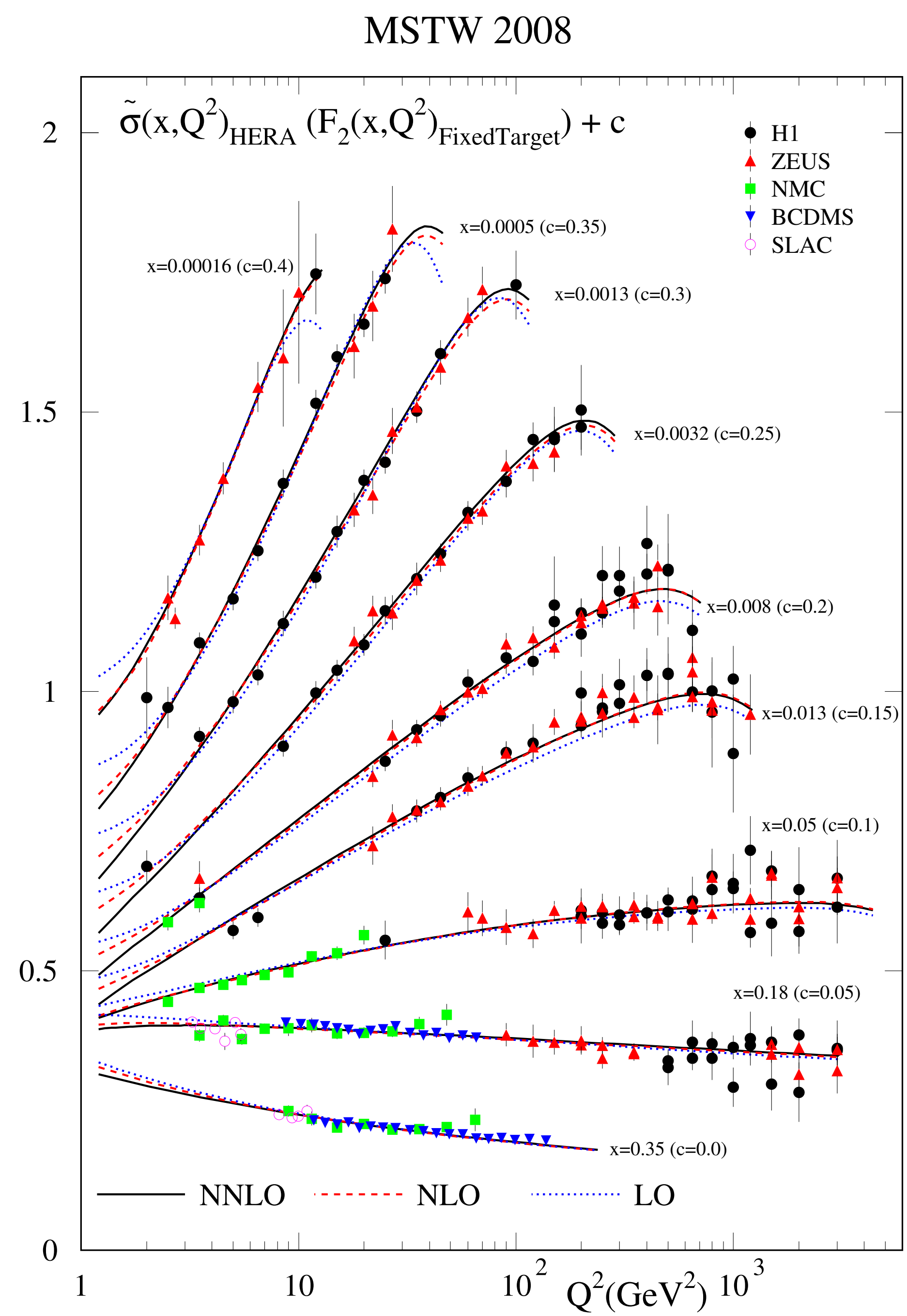
NNLO coefficient functions for structure functions know for many years.

Splitting functions now complete. (Moch, Vermaseren and Vogt). Improve consistency of fit very slightly (MSTW), and reduces α_S.

Fit to $F_2(x,Q^2)$ data.

Slope poor (too flat) at LO, ok at NLO and better at NNLO.

Some slight room for improvements.



Essentially full NNLO determination of partons now being performed (MRST/MSTW, Alekhin), though heavy flavour not fully worked out in pre-2006 sets or in the fixed-flavour number scheme (FFNS) PDFs.

Surely this is best, i.e. most accurate.

Yes, but ...... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

$pp(\bar{p}) \rightarrow \gamma^{\star}, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

NNLO tells us more about the convergence of perturbation theory.

Resummations may be important even beyond NNLO in some regions.

Stability order-by-order.

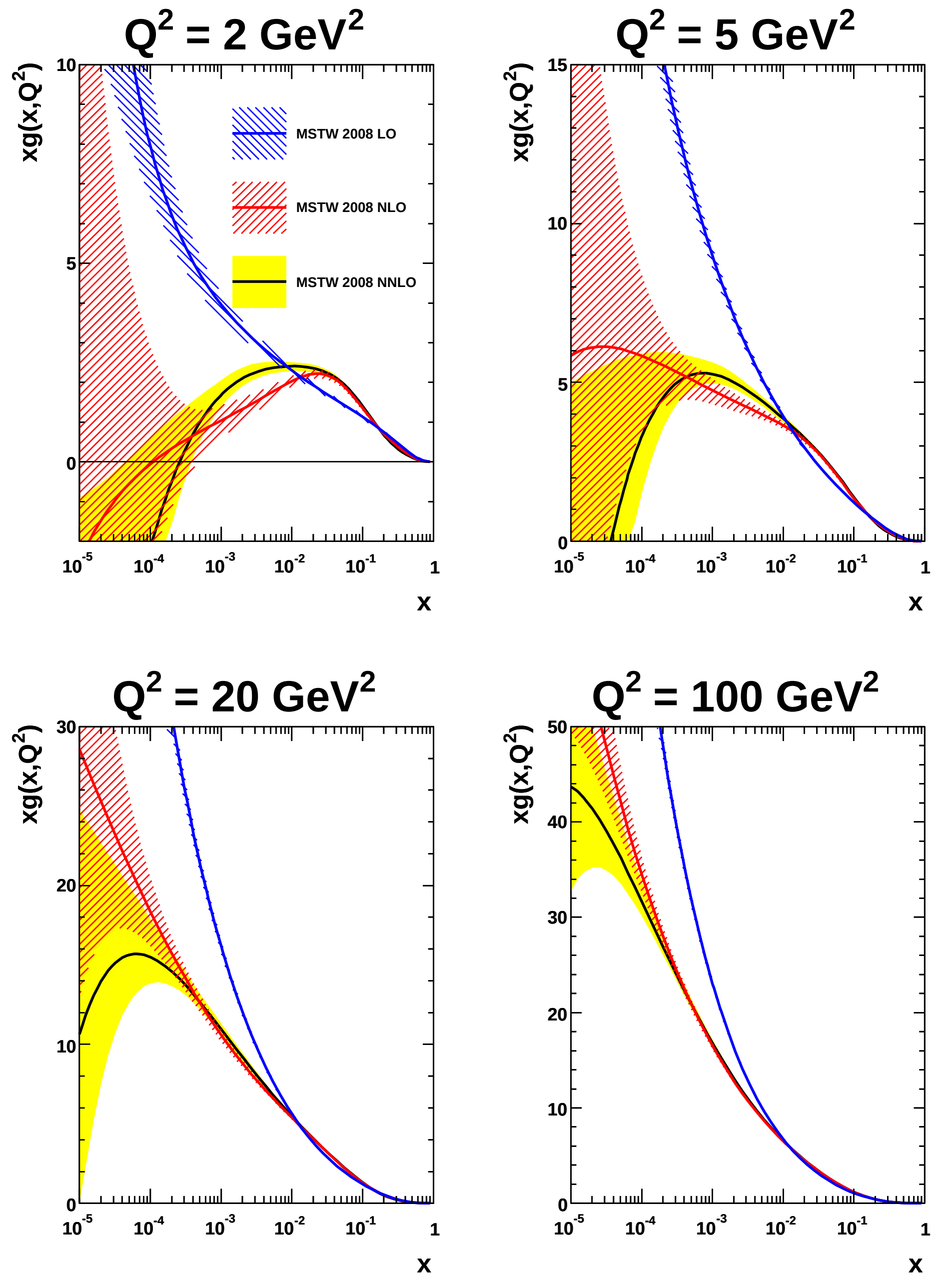
Start by looking at fixed order QCD.

The gluon extracted from the global fit at LO, NLO and NNLO.

Additional and positive small-x contributions in P_{qg} at each order leads to smaller small-x gluon at each order.

Clearly poor stability.

Similar for $F_L(x, Q^2)$

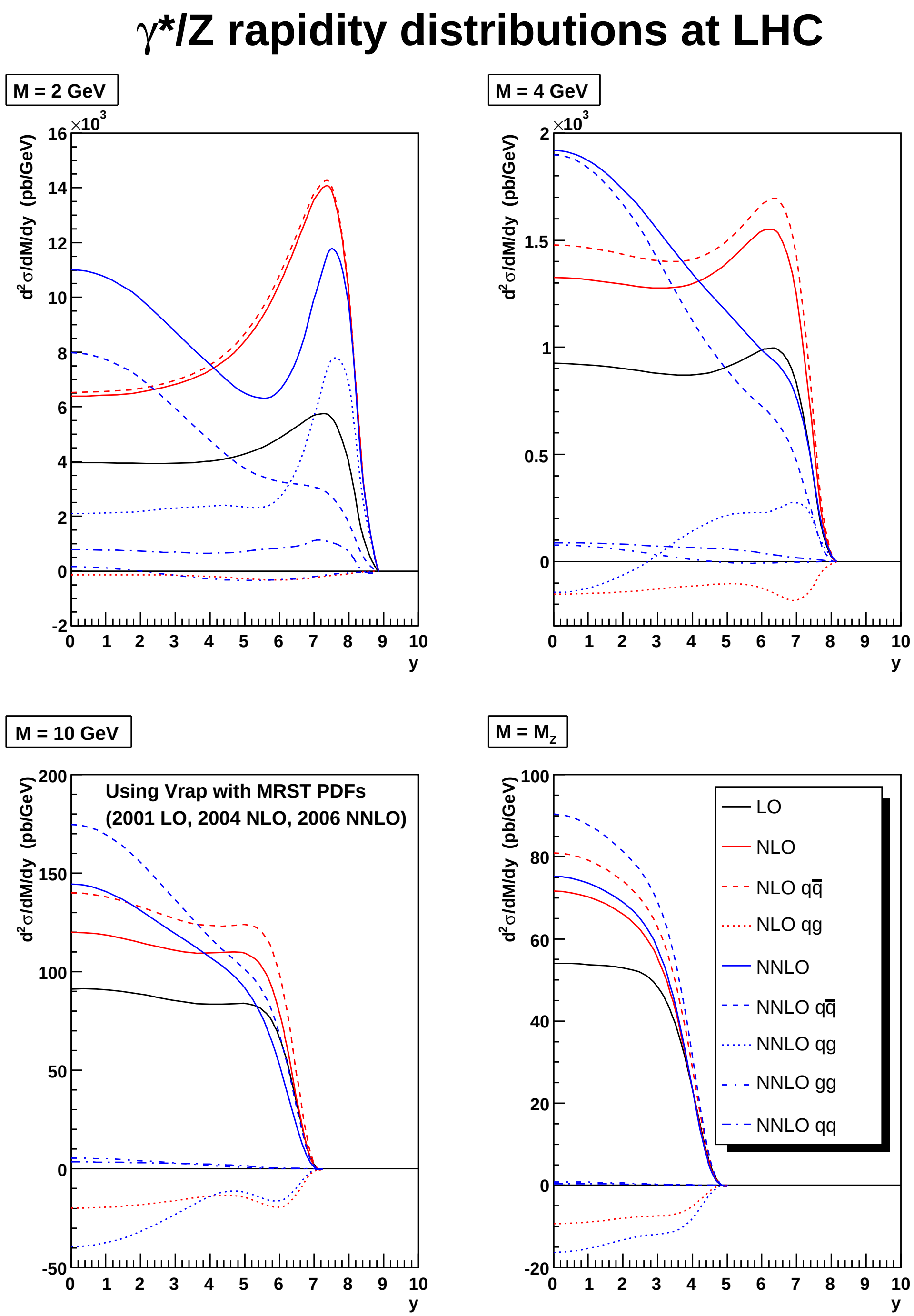


Consequences for LHC

Now have QCD calculations at LO, NLO and NNLO in the coupling constant α_S for Z, W and $\gamma^\star$ production Anastasiou, Dixon, Melnikov, Petriello).

Good stability in predictions for e.g. Z and $\gamma^\star$ cross-sections for very high virtuality.

Becomes worse at lower scales where α_S larger and large $\ln(s/M^2)$ terms appear in expansion (equivalent to $\ln(1/x)$).

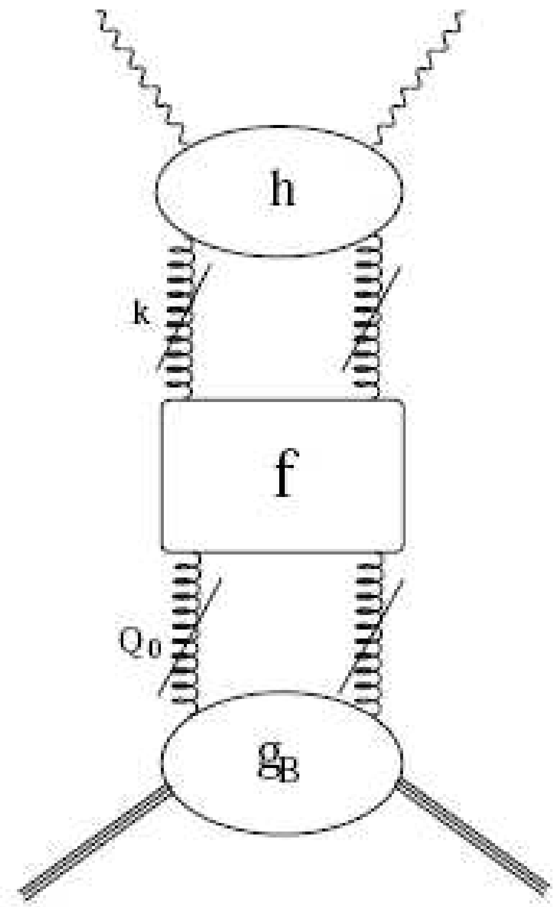


Small-x Theory

Reason for this instability – at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e. $P_{ij}(x,\alpha_s(Q^2)), C_i^P(x,\alpha_s(Q^2)) \sim \alpha_s^m(Q^2)\ln^{m-1}(1/x)$.

BFKL equation for high-energy limit

$$f(k^2,x) = f_I(Q_0^2) + \int_x^1 \frac{dx'}{x'}\bar{\alpha}_S \int_0^\infty \frac{dq^2}{q^2} K(q^2,k^2) f(q^2,x),$$



where $f(k^2,x)$ is the unintegrated gluon distribution $g(x,Q^2) = \int_0^{Q^2} (dk^2/k^2) f(x,k^2)$, and $K(q^2,k^2)$ is a calculated kernel known to NLO.

Physical structure functions obtained from

$$\sigma(Q^2,x) = \int (dk^2/k^2)\, h(k^2/Q^2) f(k^2,x)$$

where $h(k^2/Q^2)$ is a calculable impact factor.

The global fits usually assume that this is unimportant in practice, and proceed regardless.

Fits work well at small x, but could improve.

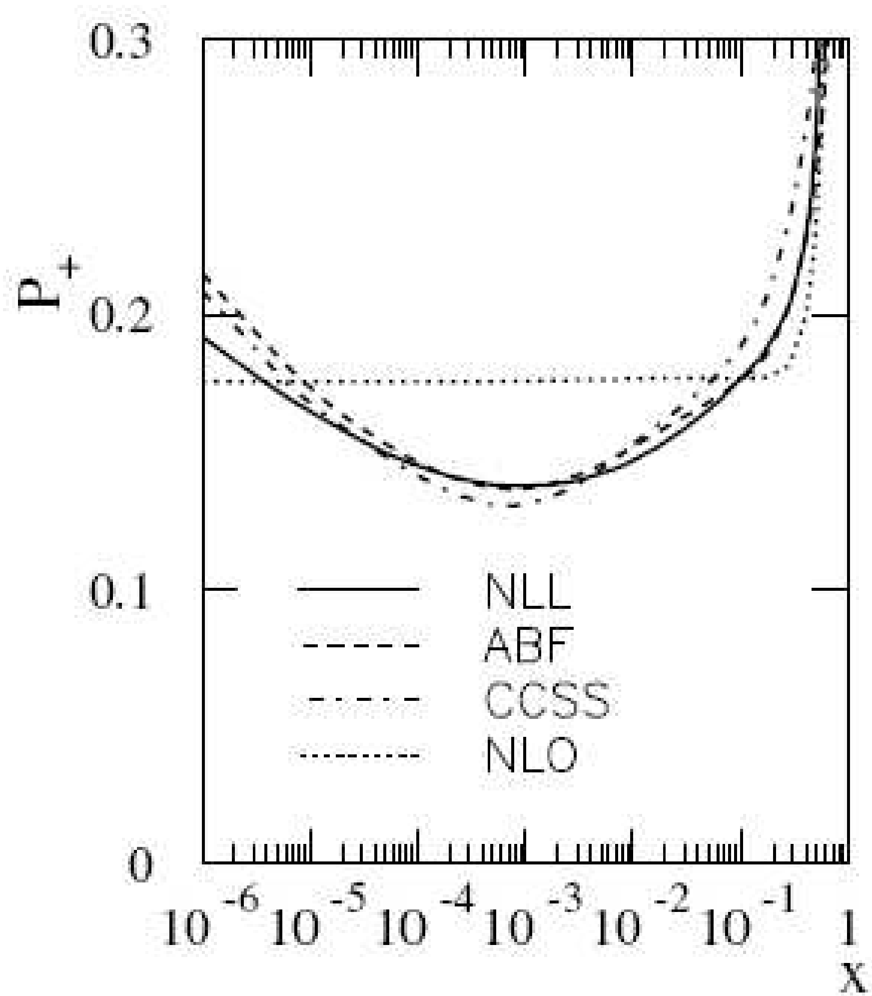
Good recent progress in incorporating $\ln(1/x)$ resummation Altarelli-Ball-Forte, Ciafaloni-Colferai-Salam-Stasto and White-RT.

Include running coupling effects and variety (depending on group) of other corrections

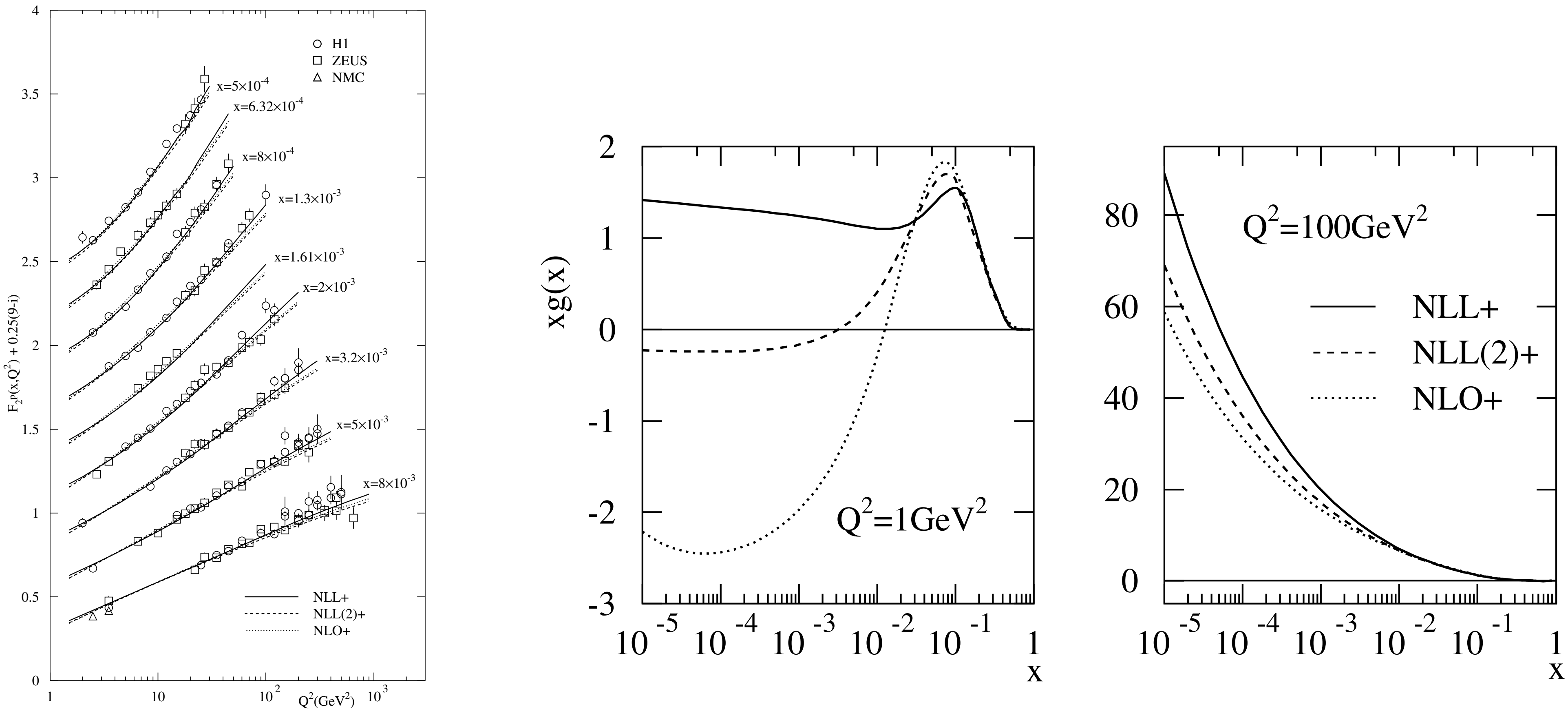
By 2008 very similar results coming from the competing procedures, despite some differences in technique.

Full set of coefficient functions still to come in some cases, but splitting functions comparable.

Note, in all cases NLO corrections lead to dip in functions below fixed order values until slower growth (running coupling effect) at very small x.



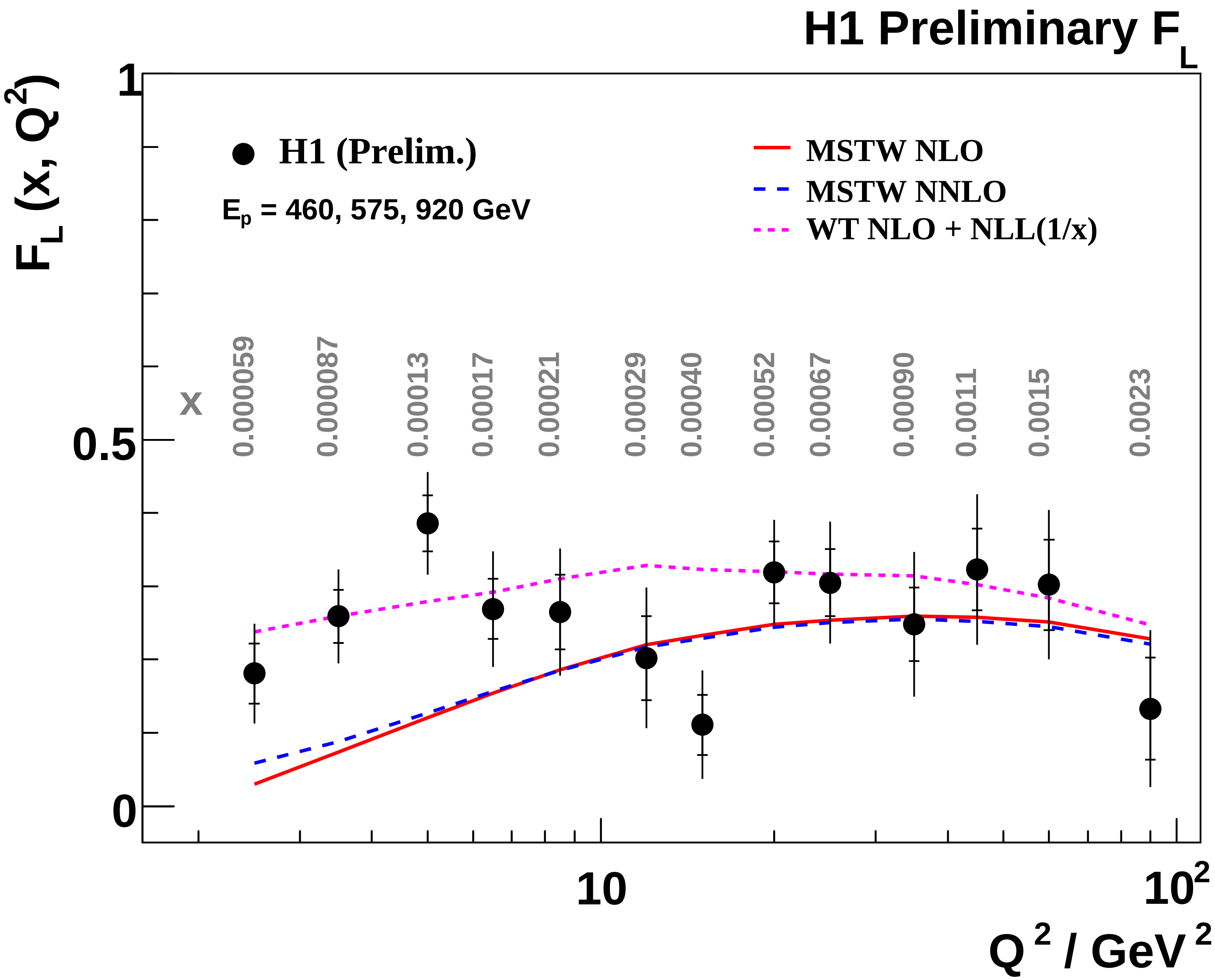
A fit to data with NLO plus NLO resummation, with heavy quarks included (White,RT) performed.



$\rightarrow$ moderate improvement in fit to HERA data within global fit, and change in extracted gluon (more like quarks at low Q^2).

Together with indications from Drell Yan resummation calculations (Marzani, Ball) few percent effect quite possible.

Comparison to H1 prelim data on $F_L(x,Q^2)$ at low Q^2 suggests resummations may be important.

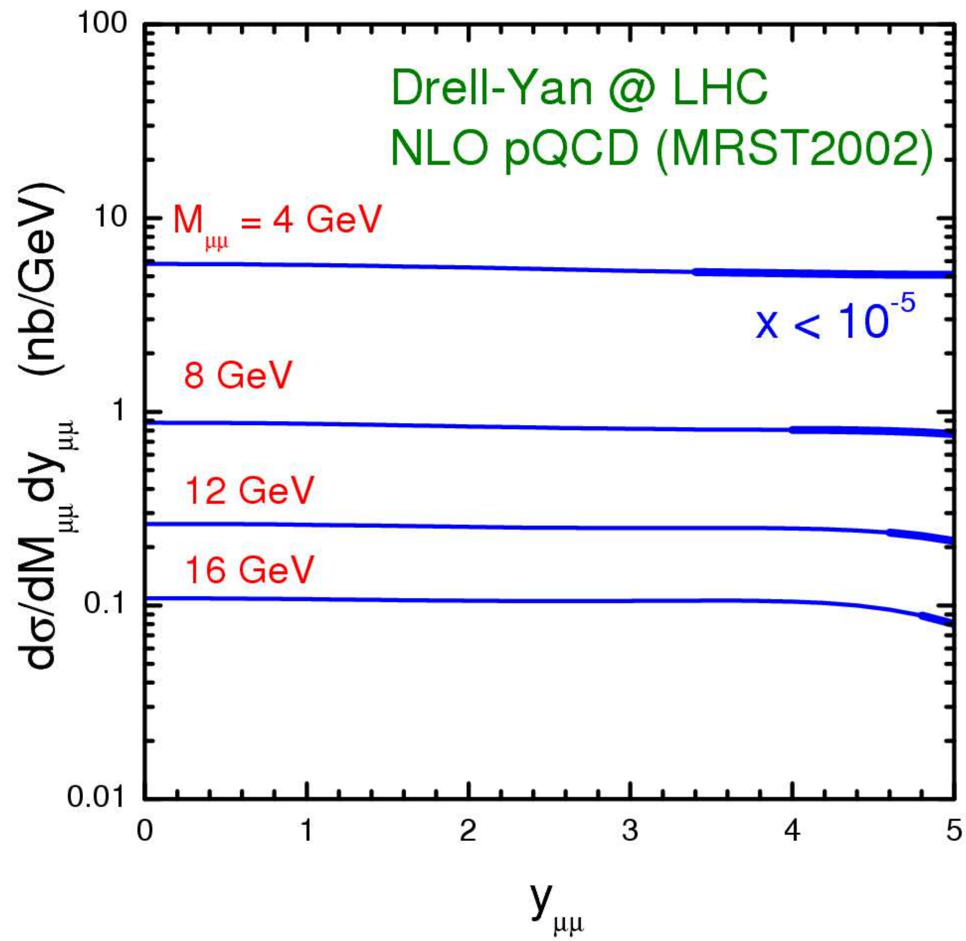


Other possible (sometimes related) explanations.

Possible to get to very low values of x at the LHC, particularly LHCb.

Can probe below $x = 10^{-5}$ - beyond range tested at HERA.

Effects possibly much larger here.



PDFs for LO Monte Carlo generators.

Often need to use generators which calculate only at LO in QCD.

LO matrix elements + LO PDFs often very inaccurate.

Using NLO PDFS suggested – sometimes better, sometimes even worse (particularly small x, important for underlying event etc).

Leads to introduction of new type of LO* PDF.

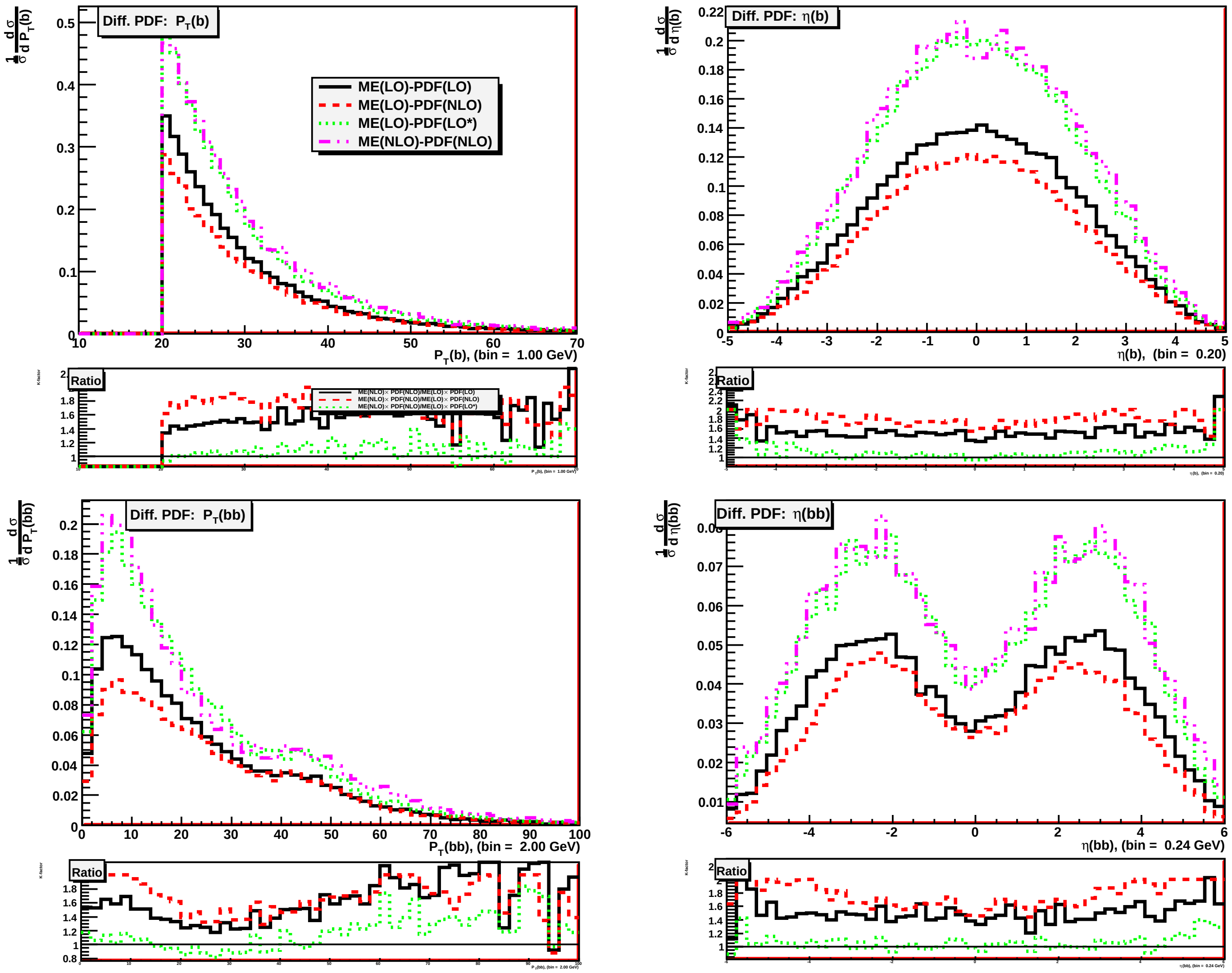
NLO corrections to cross-section usually positive $\rightarrow$ LO PDFs bigger by allowing momentum violation in global fits, using NLO α_S, fit LHC pseudo-data ......

Can also make evolution more "Monte Carlo like", e.g. change of scale in coupling.

LO* PDFs from MRST/MSTW followed by imminent ones from CTEQ.

Also work on fits using Monte Carlo generators directly (Jung $et\ al$).

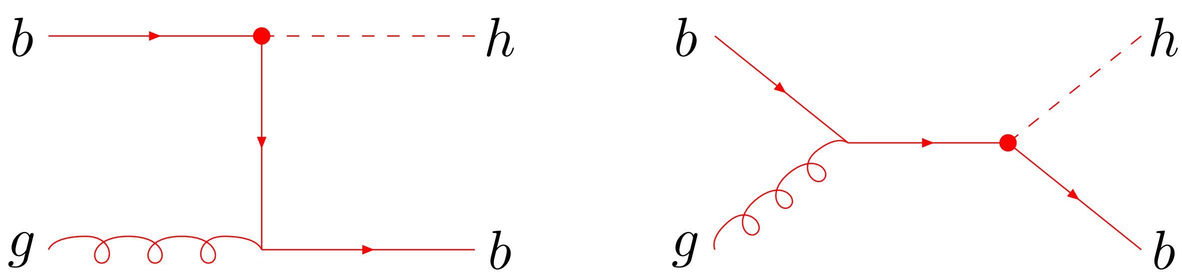
Look at e.g. distributions for single b and $b\bar{b}$ pair (Shertsnev, RT).



Results using LO* partons clearly best in normalization. NLO worst and problems with shape at low scales (i.e. small x).

Final Example

Consider bottom production along with a Higgs boson.



In Standard Model tiny since Higgs-bottom coupling $g_{b\bar{b}h} = m_b/v$, (v Higgs vacuum expectation value.) $m_b = 4.5\mathrm{GeV}$, $v = 246\mathrm{GeV}$.

In Minimal Supersymmetric Standard Model two Higgs doublets coupling separately to d-type and u-type quarks. Expectation values v_d and v_u.

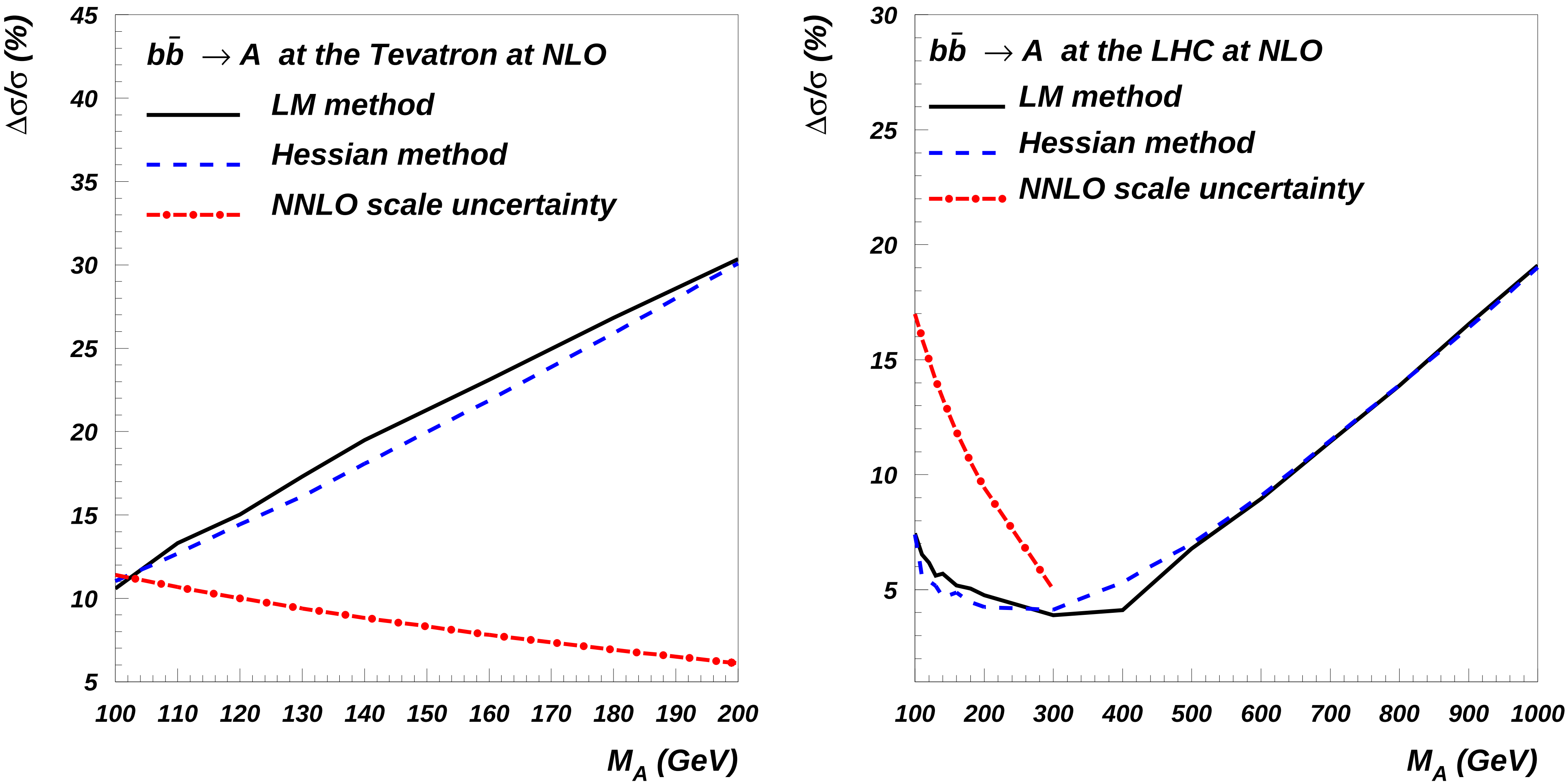
Ratio $\tan\beta = v_u/v_d \to$ enhancement of Higgs-bottom coupling

$$g_{b\bar{b}h} \propto \frac{g^{SM}_{b\bar{b}h}}{\cos\beta}.$$

Bounds from LEP, $\tan\beta$ large $\to$ $\cos\beta$ small. Enhancement of Higgs-bottom coupling.

Example of need to understand both heavy flavours and small-x physics for LHC.

Production of supersymmetric Higgs depends on parton uncertainties (Belyaev, Pumplin, Tung and Yuan), heavy flavour procedure and high-energy (small-x) treatment.



Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. Uncertainties naively rather small – $\sim 1-5\%$ for most LHC quantities. Ratios, e.g. W^+/W^- tight constraint on partons.

Effects from input assumptions e.g. selection of data fitted, cuts and input parameterisation can shift central values of predictions significantly. Also affect size of uncertainties. Want balance between freedom and sensible constraints.

Complete heavy flavour treatments essential in extraction and use of PDFs.

PDFs and α_S heavily correlated.

Electroweak corrections potentially large at very high energies – $\ln^2(E^2/M_W^2)$.

Errors from higher orders/resummation potentially large. Direct measurement of $F_L(x,Q^2)$ at HERA now testing this. At LHC measurement at high rapidities, e.g. W, Z would be useful in testing understanding of QCD, and particularly quantities sensitive to low x at low scales, e.g. low mass Drell-Yan.

Extraction of PDFs from existing data and use for LHC far from a straightforward procedure. Lots of theoretical issues to consider for real precision. Relatively few cases where Standard Model discrepancies will not require some significant input from PDF physics to determine real significance.

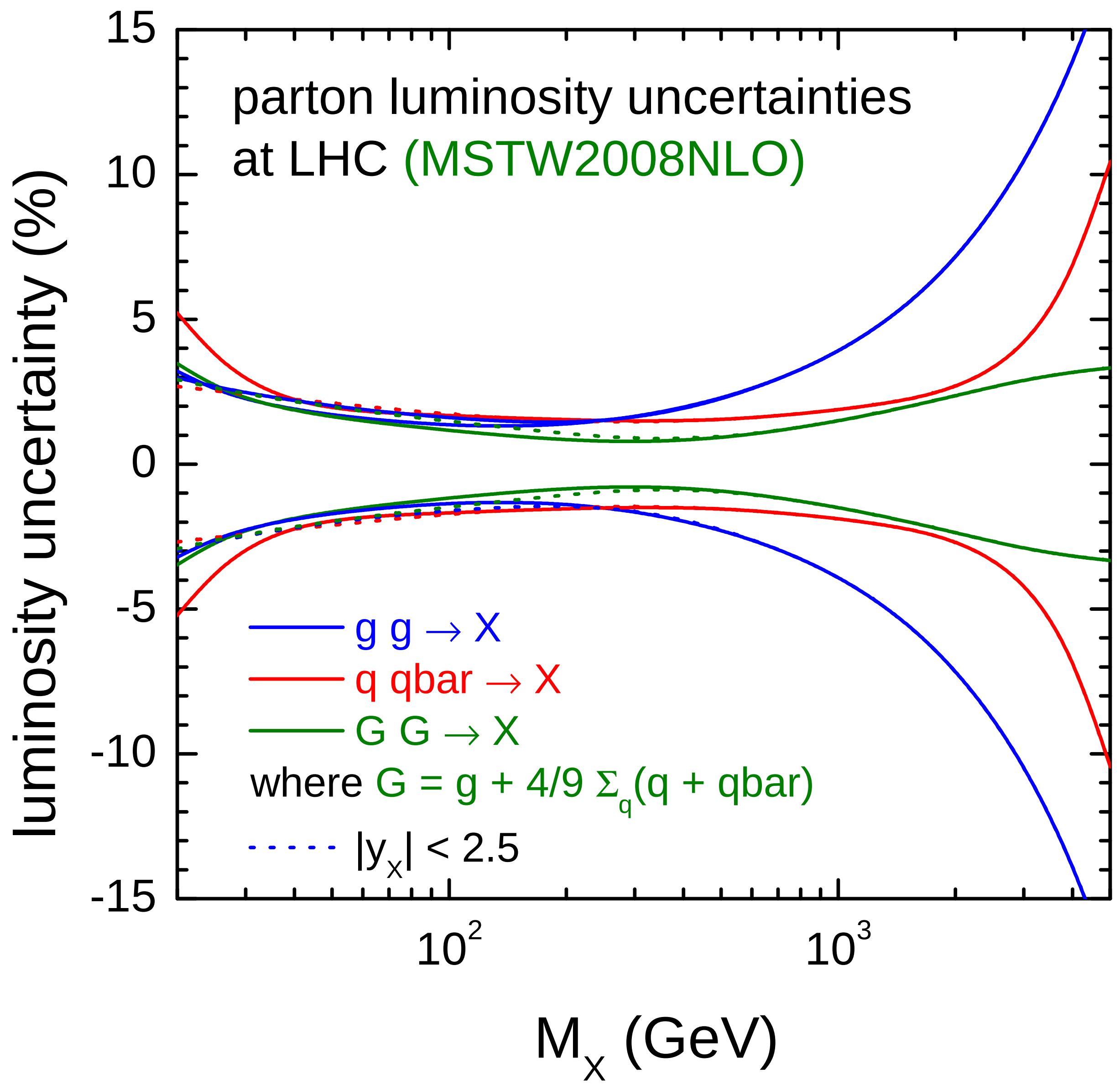
We will be discussing all of the types of issues briefly introduced in this talk in the dedicated PDF4LHC session at this institute for the next two days.

Would welcome the input of those unfamiliar to the details of this field if their interest has been piqued.

Parton Luminosity Uncertainies

Uncertainties on parton luminosities, i.e. of fundamental rates for creation processes, are optimum for standard model particle production.

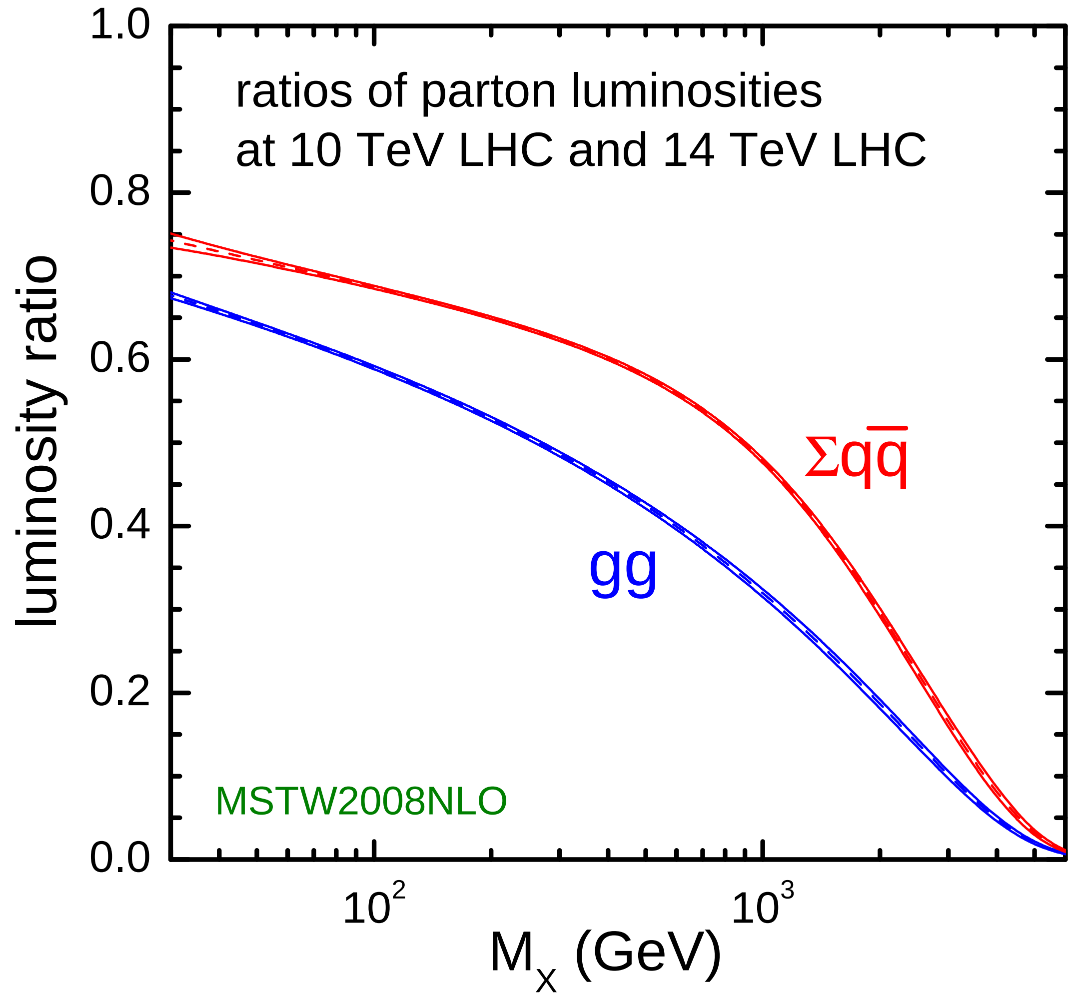
Start to worsten at highest masses where sensitive to large-x PDFs.



Initial Running

Of course, will be starting the LHC running at 10 TeV rather than the full 14 TeV.

Roughly $60 - 70\%$ the full cross-sections for most standard model (including light Higgs) processes.



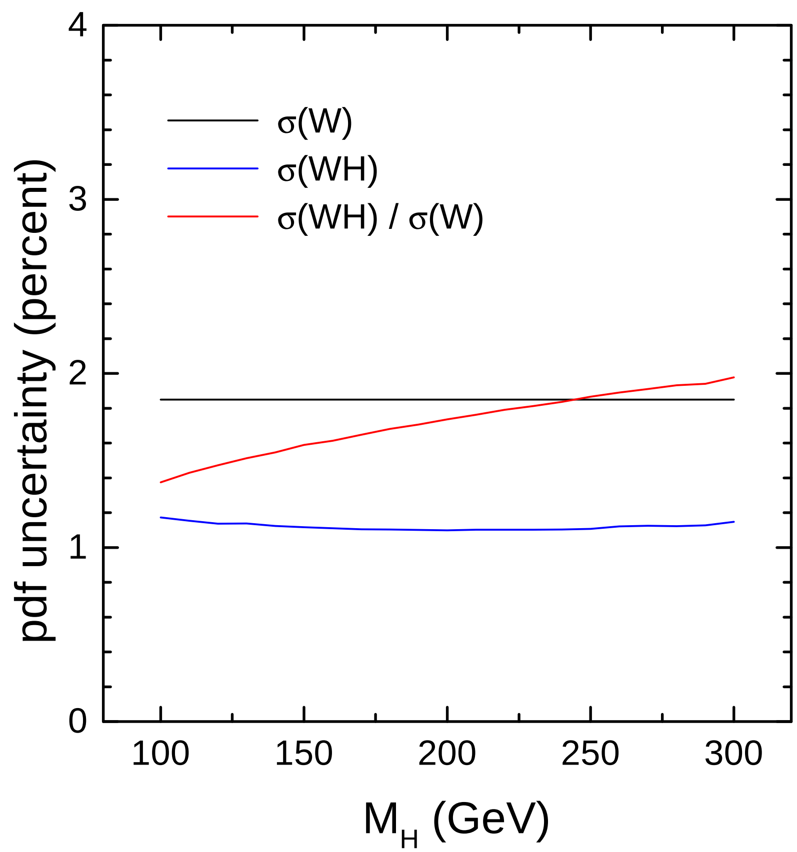
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of x are often anti-correlated rather than correlated, partially due to sum rules.

pdf uncertainties on W, WH
cross sections at LHC (MRST2001E)



Importance of treating heavy flavour correctly illustrated at NNLO with MRST2006 partons.

Previous approximate NNLO sets used (declared) approximate VFNS at flavour thresholds.

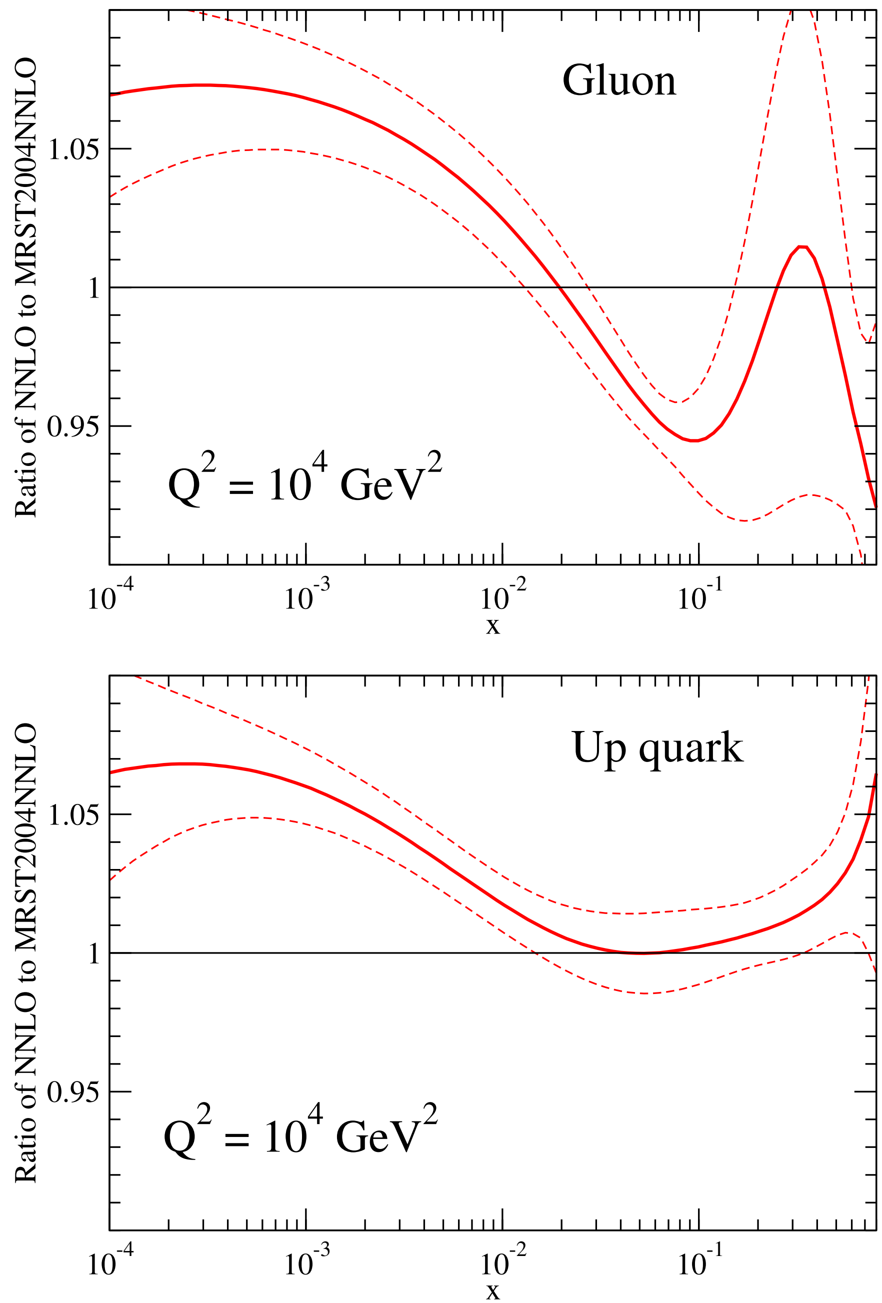
Full VFNS $\rightarrow$ flatter evolution of charm

$\rightarrow$ bigger gluon and more evolution of light sea and bigger α_S.

$\rightarrow$ 6% increase in σ_W and σ_Z at the LHC.

This is a correction not uncertainty.

Very important changes nonetheless.



Treatment of errors.

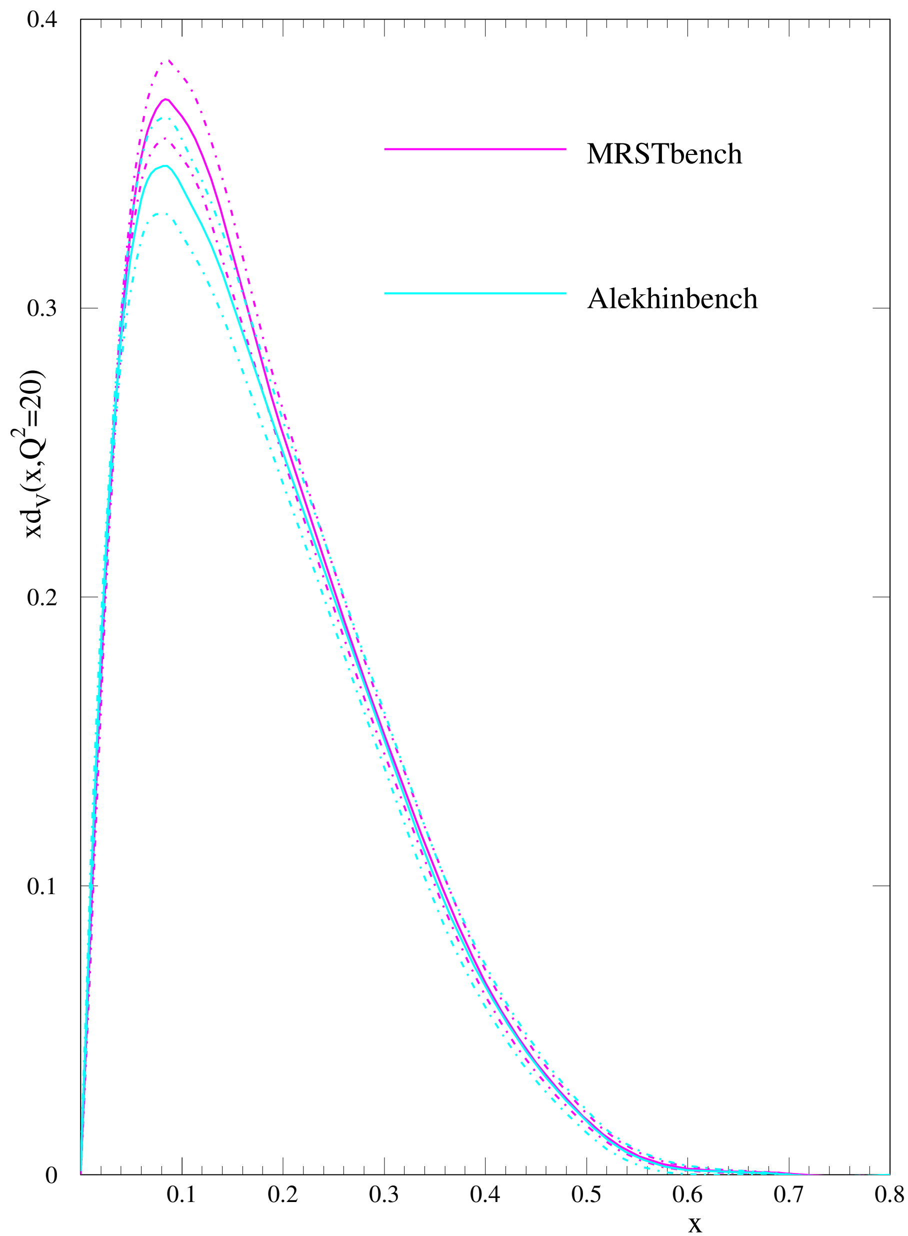
Exercise for $HERA-LHC$ meeting. Fit proton and deuteron structure function data from H1, ZEUS, NMC and BCDMS, for $Q^2 > 9\mathrm{GeV}^2$ using $ZM-VFNS$ and same form of parton inputs at same $Q_0^2 = 1\mathrm{GeV}^2$.

Very conservative fit.

Compare rigorous treatment of all systematic errors (Alekhin) with simple quadratures approach (MRST), both with $\Delta\chi^2 = 1$.

$\longrightarrow$ some difference in central values (other possible reasons) and similar errors.

Fairly consistent.



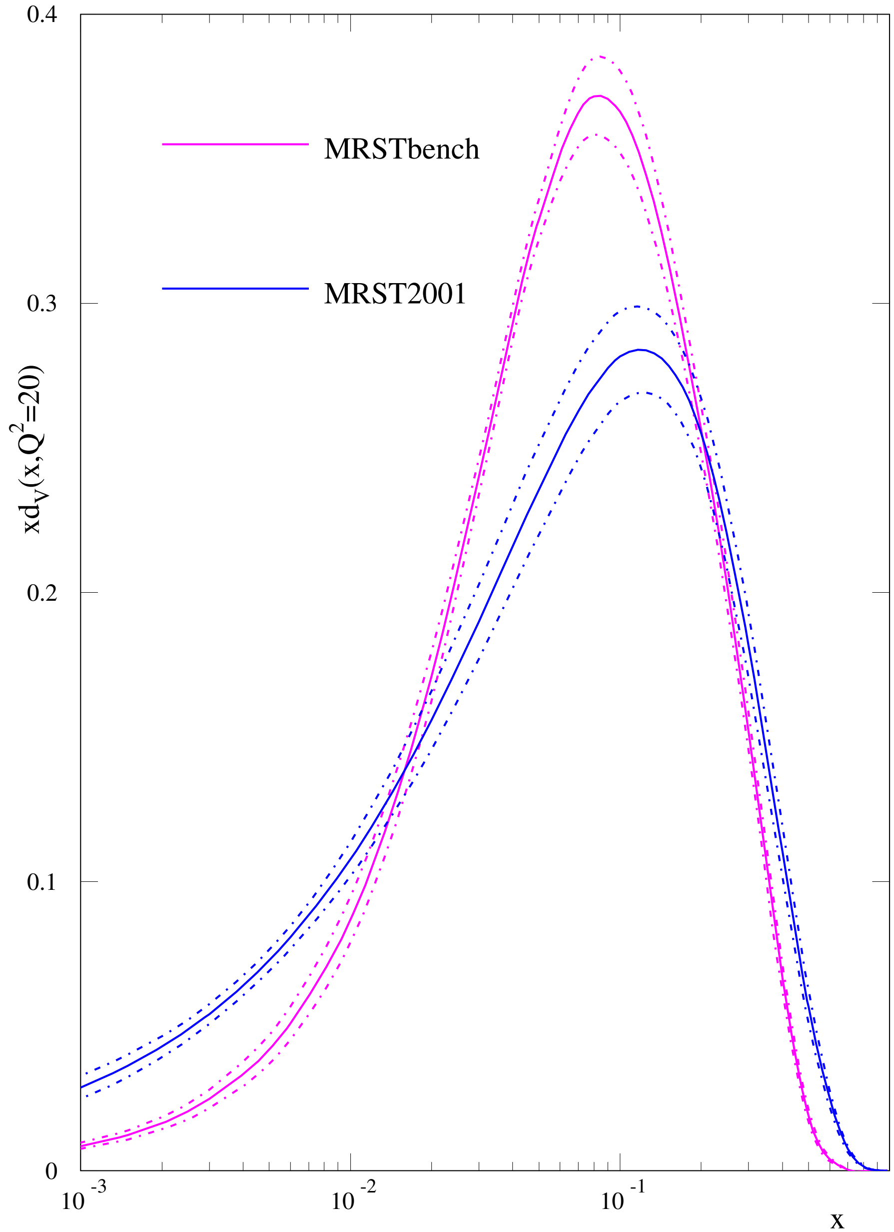
Back to HERA-LHC benchmark partons.

How do partons from very conservative, structure function only data compare to global partons?

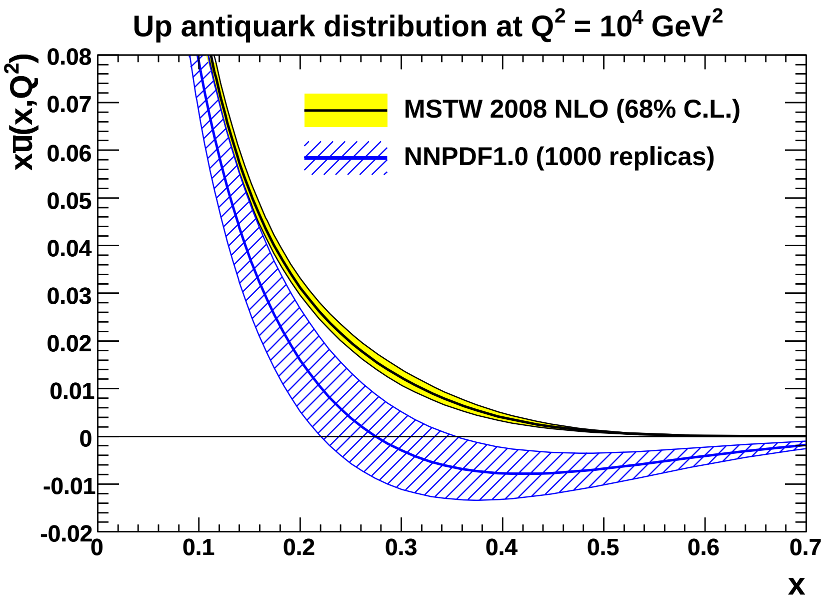
Compare to MRST01 partons with uncertainty from $\Delta\chi^2 = 50$.

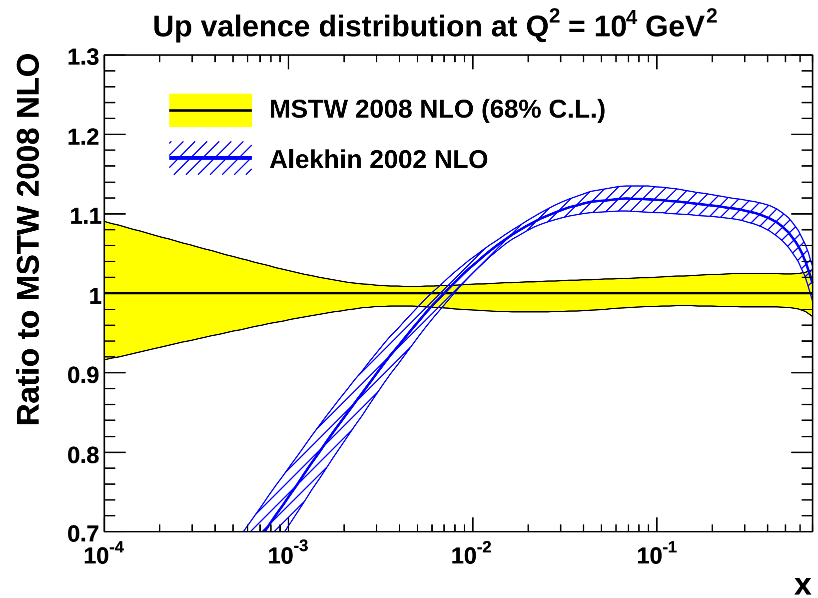
Enormous difference in central values.

Errors similar.



PDF sets sometimes differ significantly in central values though. Largely due to data fit

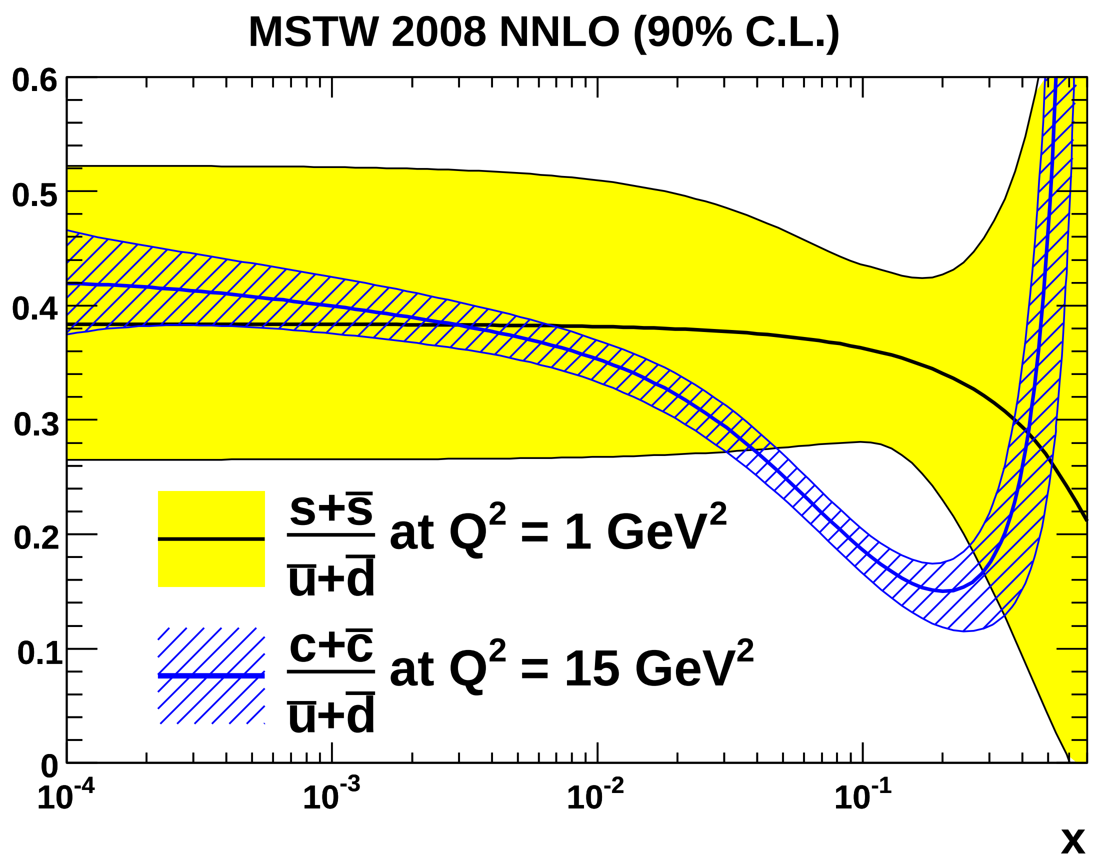




Clearly seen comparing up sea for MSTW and NNPDF and up valence for MSTW and Alekhin.

Strange itself has some non-insignificant mass, and this should qualitatively lead to suppression compared to light sea quarks up and down.

When c and b turn on they evolve like massless quarks, but always lag behind. $\rightarrow$ some suppression at all x for finite Q^2.



$c+\bar{c}$ evolved through $\sim 7-8$ times input scale similar to $s+\bar{s}$ at $Q^2 = 1\mathrm{GeV}^2$. Do not expect exact correspondence, but very good except $c+\bar{c}$ more suppressed at $x \sim 0.1$. (Implication for $s+\bar{s}$ from recent HERMES $K^{\pm}$ data).

No obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross-section, except maybe for very particular, and rather large, M_H.

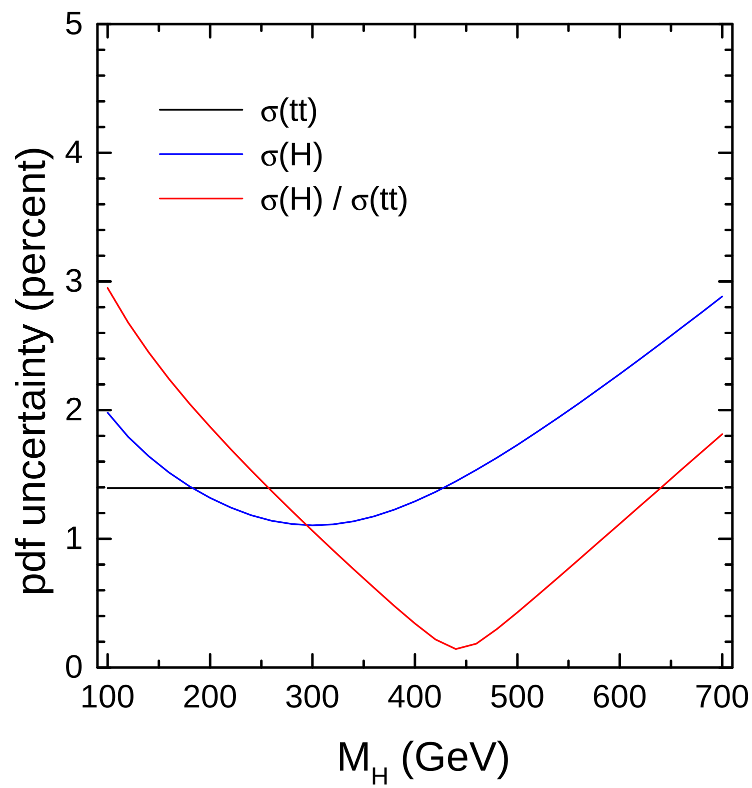
However, a light (SM or MSSM) Higgs dominantly produced via $gg \to H$ and the cross-section has small pdf uncertainty because $g(x)$ at small x is well constrained by HERA DIS data.

Current best (MRST) estimate, for $M_H = 120$ GeV: $\delta\sigma_H^{\rm NLO}(\text{expt pdf}) = \pm 2 - 3\%$ with less sensitivity to small x than $\sigma(W)$.

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

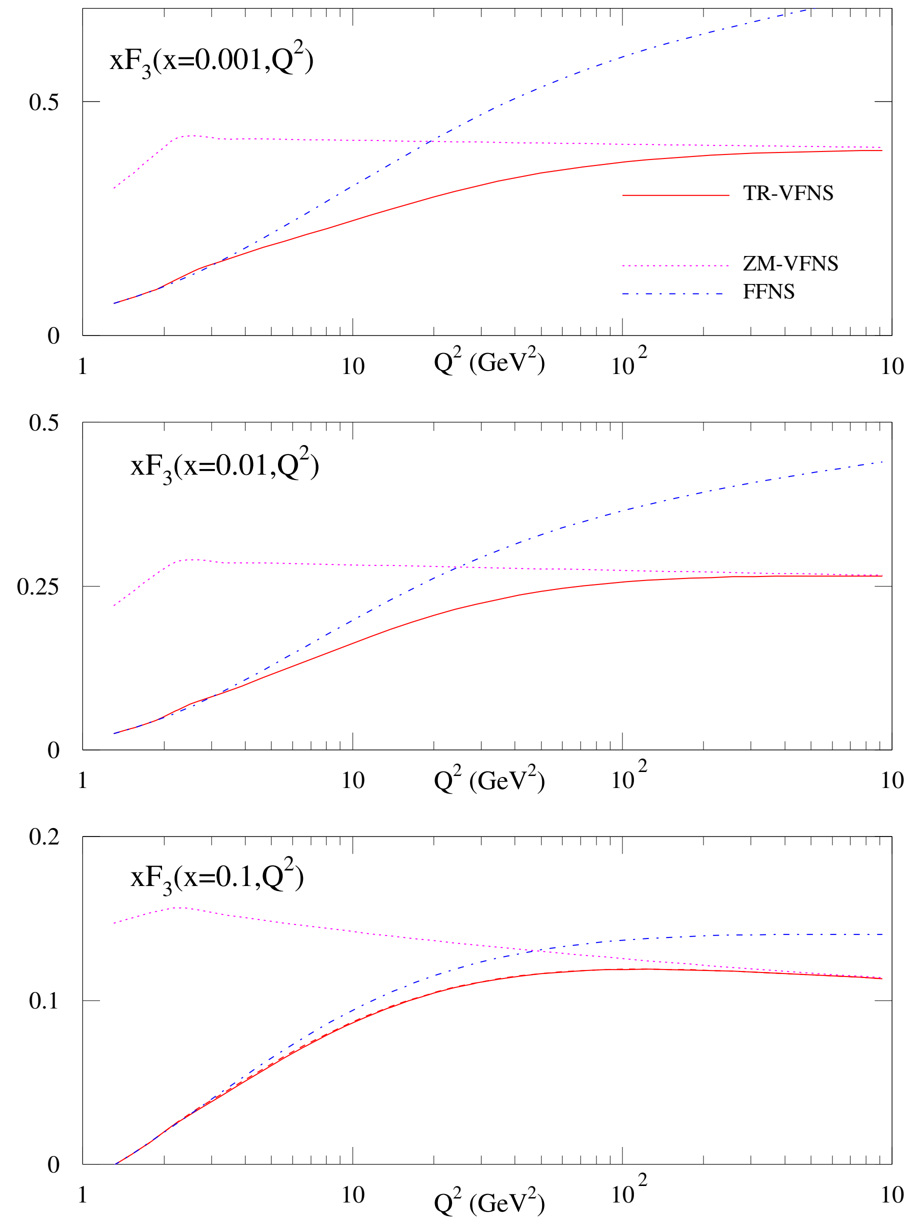
$\delta\sigma_H^{\rm NNLL}(\text{scale variation}) = \pm 8\%$

pdf uncertainties on top, (gg→) H cross sections at LHC (MRST2001E)



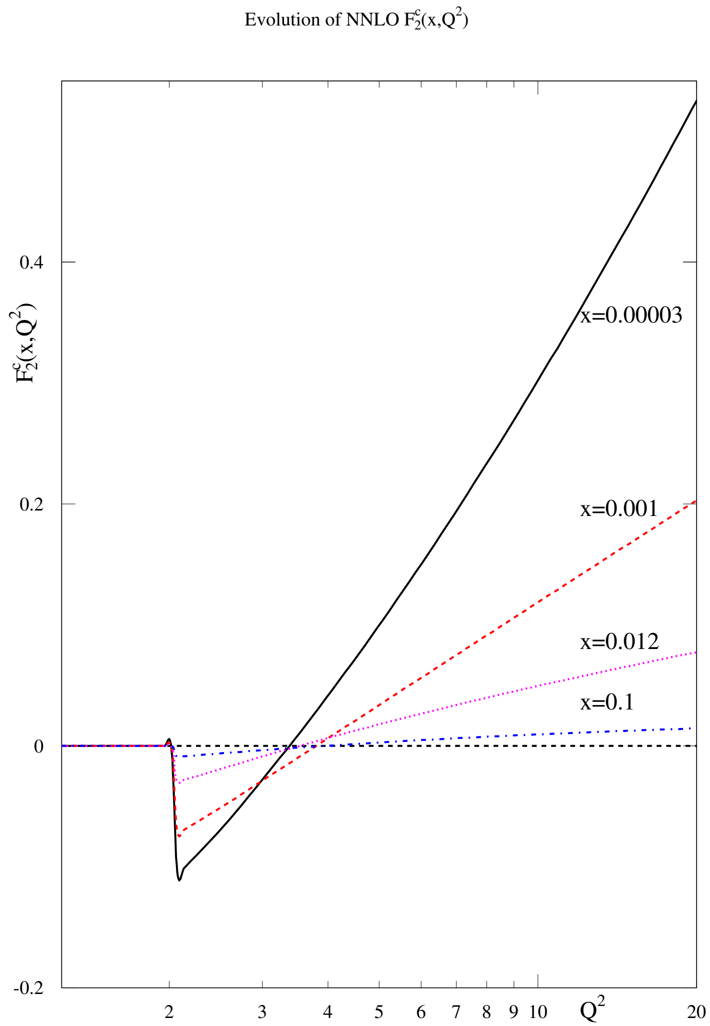
Need a general **Variable Flavour Number Scheme** (VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

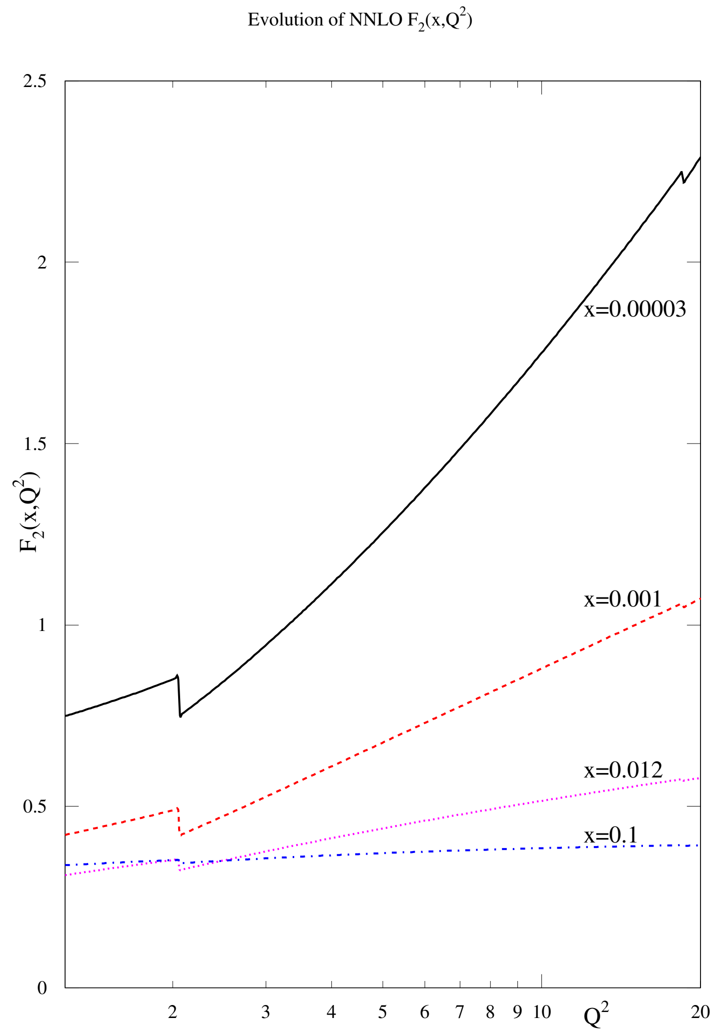
Conclusion easily reached by looking at the extrapolation between the two simple kinematic regimes for xF_3, measured using neutrino scattering at NuTeV



At NNLO additional complications – partons become discontinuous Buza *et al*.

ZM-VFNS leads to peculiar, unphysical results. FFNS not known at this order.





Makes need for **Variable Flavour Number Scheme** more vital but also more difficult to implement.

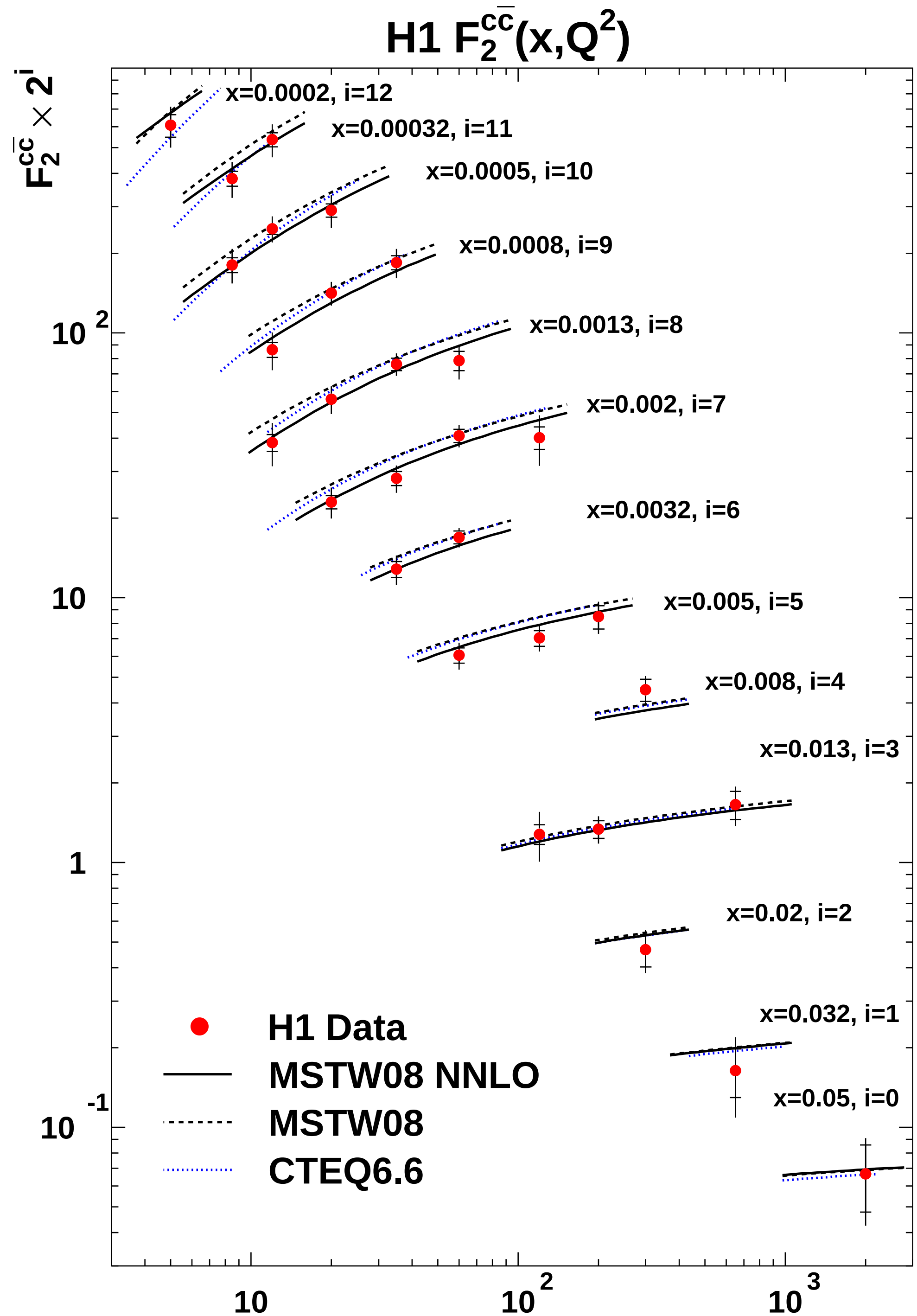
Heavy Quarks – Essential to treat these correctly.

Need to go from threshold $Q^2 \sim m_H^2$ to high scales $Q^2 \gg m_H^2$ where heavy quarks behave like massless partons while including all $\mathcal{O}(m_H^2/Q^2)$ corrections.

Work on **General-Mass Variable-Flavour-Number Scheme** (GM-VFNS) which achieves this.

Results shown compared to H1 charm data.

Note big predicted change at NNLO.



Low Q^2.

Perform fits with the known NNNLO large $\ln(1-x)$ terms included explicitly.

Also parameterize higher twist contributions by

$$F_i^{\mathrm{HT}}(x,Q^2) = F_i^{\mathrm{LT}}(x,Q^2)\left(1+\frac{D_i(x)}{Q^2}\right)$$

where i spans bins of x.

No evidence for any higher twist except at low W^2.

x	LO	NLO	NNLO	NNNLO
0–0.0005	-0.07	-0.02	-0.02	-0.03
0.0005–0.005	-0.03	-0.01	0.03	0.03
0.005–0.01	-0.13	-0.09	-0.04	-0.03
0.01–0.06	-0.09	-0.08	-0.04	-0.03
0.06–0.1	-0.02	0.02	0.03	0.04
0.1–0.2	-0.07	-0.03	-0.00	0.01
0.2–0.3	-0.11	-0.09	-0.04	0.00
0.3–0.4	-0.06	-0.13	-0.06	-0.01
0.4–0.5	0.22	0.01	0.07	0.11
0.5–0.6	0.85	0.40	0.41	0.39
0.6–0.7	2.6	1.7	1.6	1.4
0.7–0.8	7.3	5.5	5.1	4.4
0.8–0.9	20.2	16.7	16.1	13.4

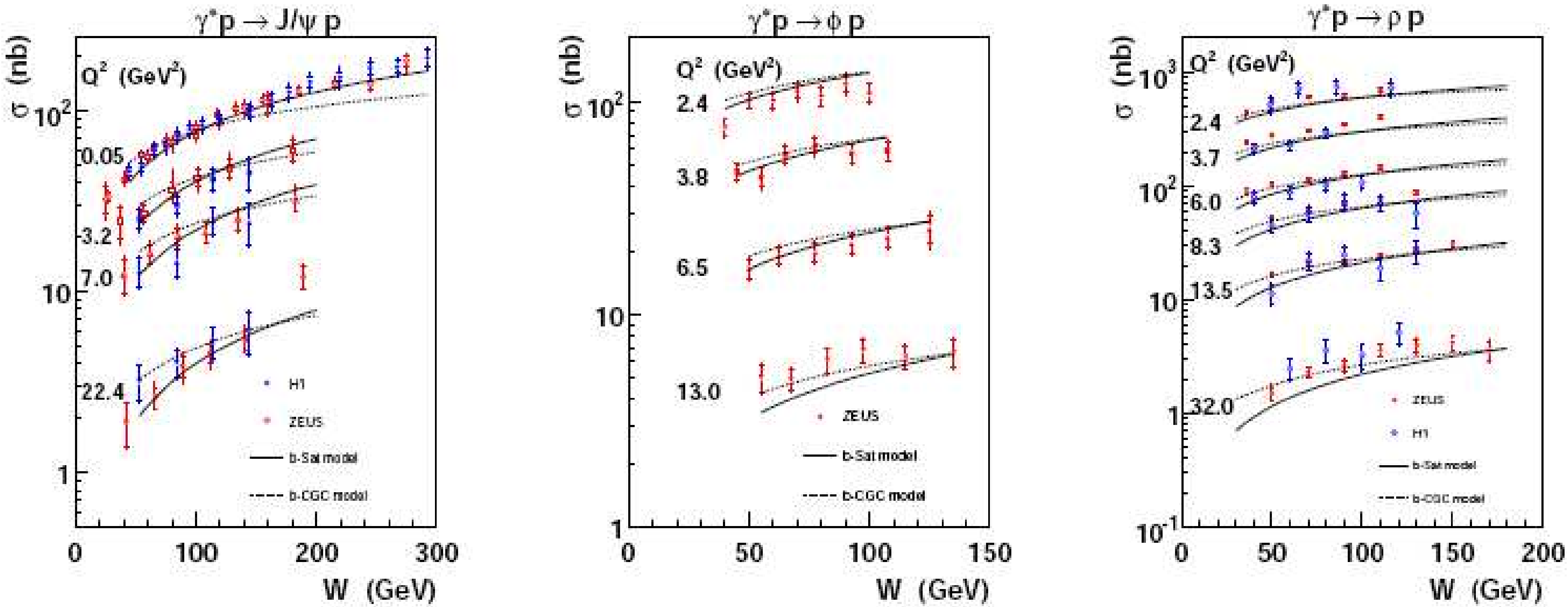
Table 1: The values of the higher-twist coefficients D_i, in the chosen bins of x, extracted from the LO, NLO, NNLO and NNNLO (NNLO with the approximate NNNLO non-singlet quark coefficient function) global fits.

Dipole Models, Saturation

Small-x automatically takes us to dipole models, saturation and overlap with diffraction.

Example, (Watt) extended dipole model with impact parameter b dependence,

Free parameters determined by fit to $F_2(x, Q^2)$ and results compared to variety of exclusive processes with good results.

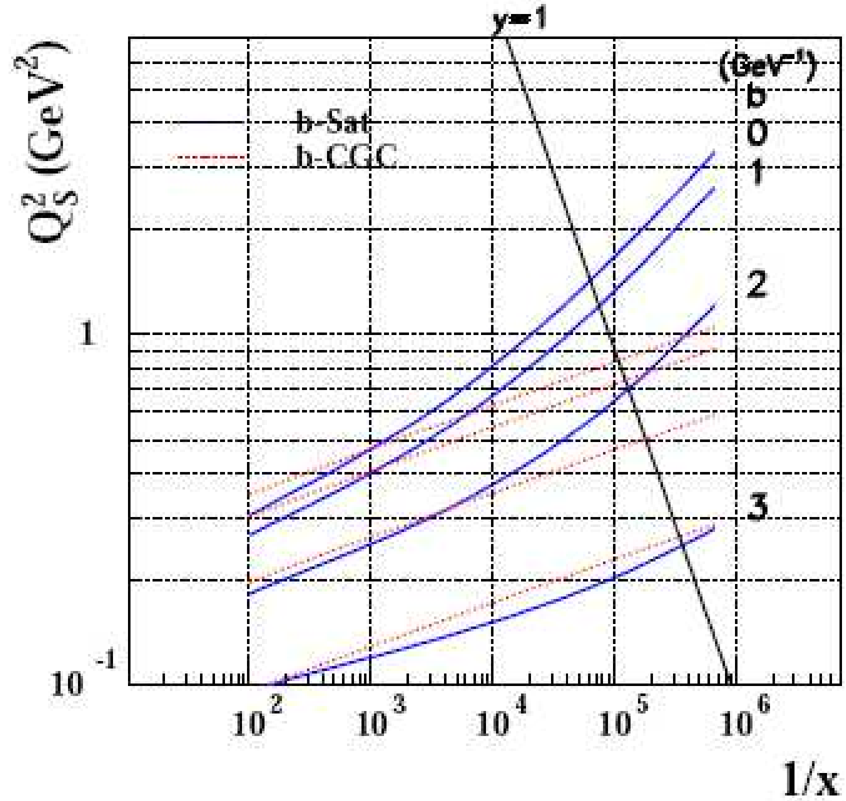


Saturation scale at very low x even for $b = 0$ – falling to lower x as b rises.

Average for inclusive processes $b \sim 2 - 3\mathrm{GeV}^{-1}$.

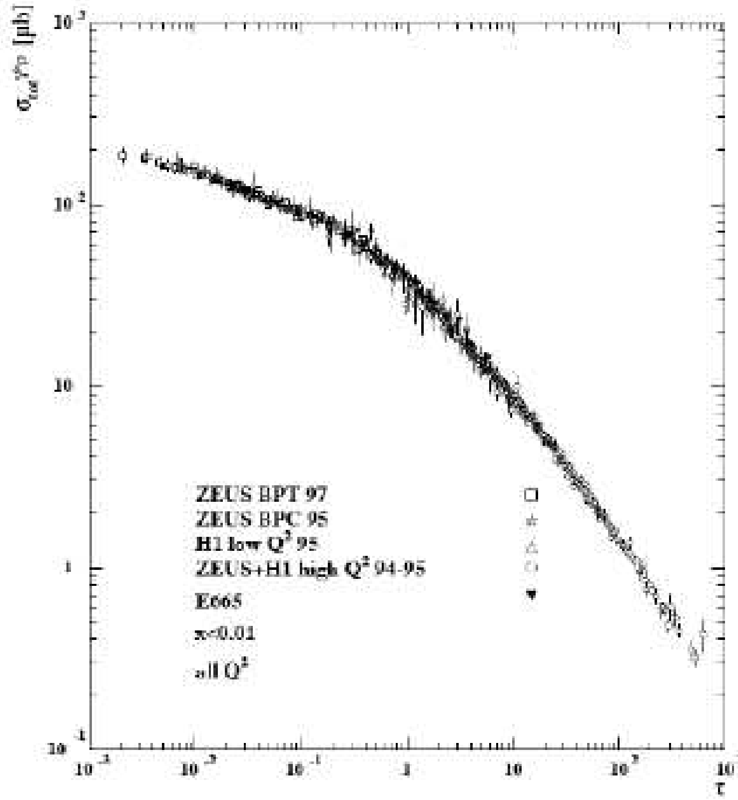
Similar results from most sophisticated and recent determinations of parameters using saturation based dipole models.

At HERA impact of saturation on inclusive quantities seems minimal.



Geometric scaling often cited as *evidence* for saturation effects.

- $\sigma(\gamma^* p)$ as a function of τ
- A. M. Stasto, K. Golec-Biernat, J. Kwiecinski, Phys. Rev. Let. 86 (2001) 596



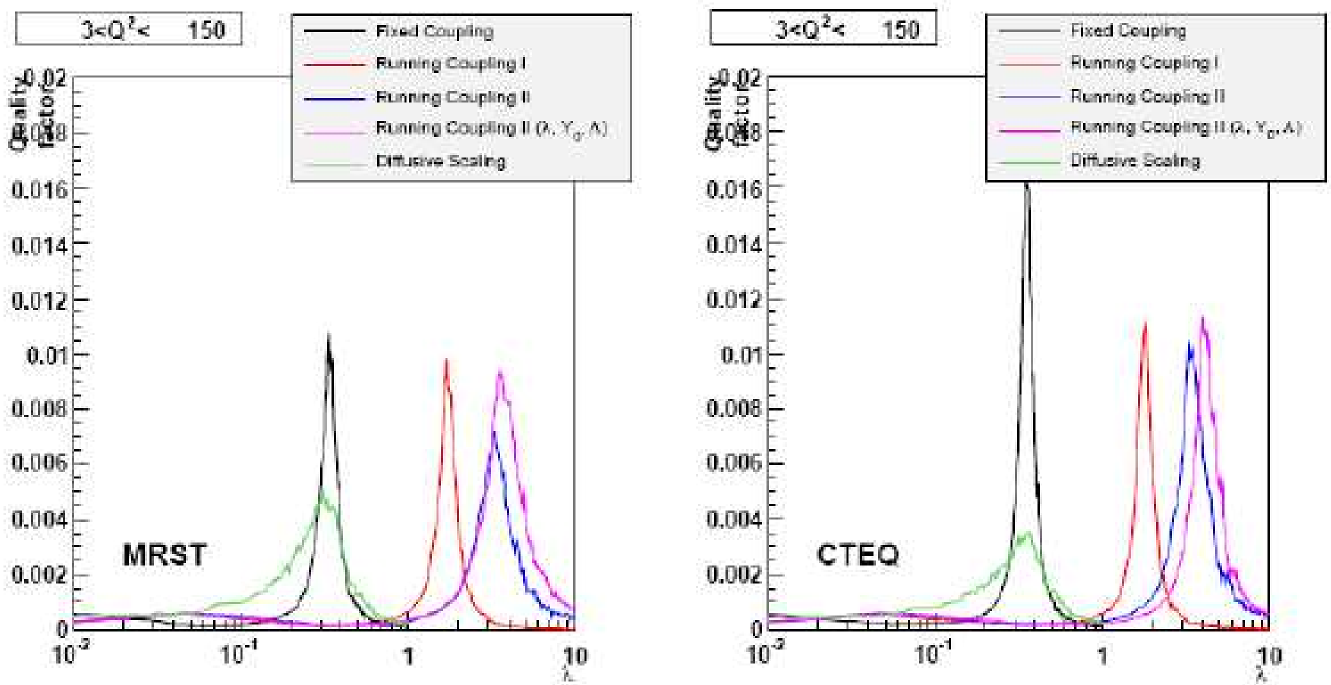
$$\boxed{\tau = Q^2 (x/x_0)^\lambda}$$

Simplest model shown. However, always going to be broken (higher orders, quark masses, ...) by more than size of error bars.

Now lots of variations on the type of geometric scaling depending on sophistication.

Theoretical evidence that geometric scaling appears in DGLAP evolution (Forte et al).

More revealingly demonstrated that $F_2(x, Q^2)$ generated from MRST and CTEQ PDFS display all types of geometric scaling with good quality factors (Salek).



In this case there is no saturation in the input at all, yet is displayed by output.

All results suggest saturation effects in inclusive quantities at HERA are at very low scales. Dipole approach and proabaly satauration more important for understyanding exclusive quantities. Does not appear that geometric scaling (which type?) is evidence.