T′ as a Family Symmetry at the GUT and EW Scales

Mu-Chun Chen
University of California at Irvine

Based on work done in collaboration
with K.T. Mahanthappa, arXiv:0904.1721; PLB652 (2007) 34
with K.T. Mahanthappa, F. Yu, arXiv:0907.3963

Introduction

- Experimental measurements for neutrino oscillation parameters
  $\Rightarrow$ Tri-bimaximal neutrino mixing:

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\rm atm,\,TBM} = 1/2$$

$$\sin^2\theta_{\odot,\rm TBM} = 1/3$$

$$\sin\theta_{13,\rm TBM} = 0.$$

- TBM neutrino mixing can arise from underlying A4 family symmetry
  - even permutations of four objects
    S: (1234) → (4321)   T: (1234) → (2314)
  - geometrically -- invariant group of tetrahedron
  - does NOT give rise to CKM mixing: $V_{ckm} = 1$
  - all CG coefficients real

T′ Group Theory

- Double covering of tetrahedral group A4:
  - in-equivalent representations of T':

$$\begin{array}{ll} \text{A4:} & 1,\ 1',\ 1'',\ 3 \longrightarrow \\ \text{other:} & 2,\ 2',\ 2'' \longrightarrow \end{array}$$

TBM for neutrinos

2 +1 assignments for quarks

★ complex CG coefficients in T′

complexity cannot be avoided by different basis choice

- spinorial x spinorial ⊃ vector:

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1 \qquad 3 = \begin{pmatrix} \left(\frac{1-i}{2}\right)(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

- spinorial x vector ⊃ spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2'' \qquad 2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$

⇒ group theoretical origin of CP violation

M.-C.C, K.T. Mahanthappa, arXiv:0904.1721

- real Yukawa coupling constants
- real VEVs of scalar fields

The Model

M.-C.C, K.T. Mahanthappa, PLB652 (2007) 34

- Symmetry: SU(5) x T´
- Particle Content: $10(Q,\, u^c,\, e^c)_L \qquad \overline{5}(d^c, \ell)_L$

	T_3	T_a	$\overline{F}$	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
$^{(d)}T$	1	2	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

$$\omega = e^{i\pi/6}.$$

- Lagrangian: only 9 operators allowed!!

$$\begin{aligned}
\mathcal{L}_{\rm Yuk} &= \mathcal{L}_{\rm TT} + \mathcal{L}_{\rm TF} + \mathcal{L}_{\rm FF} \\
\mathcal{L}_{\rm TT} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\
\mathcal{L}_{\rm TF} &= \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi\zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi\psi N + y_d H'_{\overline{5}} \overline{F} T_a \phi^2 \psi' \right] \\
\mathcal{L}_{\rm FF} &= \frac{1}{M_x \Lambda}\left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F}\xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F}\eta \right],
\end{aligned}$$

Neutrino Sector

- Operators: $$\mathcal{L}_{\rm FF} = \frac{1}{M_x\Lambda}\left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F}\xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F}\eta\right]$$
- Symmetry breaking:

$$\langle\xi\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\xi_0\Lambda \qquad\qquad T'-\text{invariant:}\ \ \langle\eta\rangle = u_0\Lambda$$

- Resulting mass matrix:

$$M_\nu = \begin{pmatrix} 2\xi_0 + u_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0 + u_0 \\ -\xi_0 & -\xi_0 + u_0 & 2\xi_0 \end{pmatrix}\frac{\lambda v^2}{M_x}$$

only vector representations involved
⇒ all CG are real

⇒ Majorana phases either 0 or π

$$U^T_{\rm TBM} M_\nu U_{\rm TBM} = {\rm diag}(u_0 + 3\xi_0, u_0, -u_0 + 3\xi_0)\frac{v_u^2}{M_X}$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

Charged Fermion Sector

- Operators: $$\begin{aligned}\mathcal{L}_{\rm TT} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\ \mathcal{L}_{\rm TF} &= \frac{1}{\Lambda^2} y_b H'_{\bar{5}} \overline{F} T_3 \phi\zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi\psi N + y_d H'_{\bar{5}} \overline{F} T_a \phi^2 \psi' \right]\end{aligned}$$

- top mass: allowed by T′; lighter family masses at higher dimensionalities

- symmetry breaking:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \; , \; \langle\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda \qquad \langle\phi'\rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\psi'\rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- Mass matrices:

$$M_u = \begin{pmatrix} i\phi'^3_0 & \frac{1-i}{2}\phi'^3_0 & 0 \\ \frac{1-i}{2}\phi'^3_0 & \phi'^3_0 + (1-\frac{i}{2})\phi^2_0 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

both vector and spinorial reps involved
⇒ complex CG

⇒ CPV in quark & lepton sectors

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0 N_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0, \qquad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0 N_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

- Georgi-Jarlskog relations ⇒ corrections to TBM pattern

Quark and Lepton Mixing Matrices

- CKM mixing matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u \qquad M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

$\mathrm{V_{cb}}$

$\mathrm{V_{ub}}$

$$\boxed{\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},}$$

Georgi-Jarlskog relations $\Rightarrow V_{d,L} \neq I$

SU(5) $\Rightarrow M_d = (M_e)^T$

$\Rightarrow$ corrections to TBM related to θ_c

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \Longrightarrow \boxed{\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c}$$

$$U_{\mathrm{MNS}} = V_{e,L}^\dagger U_{\mathrm{TBM}} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta}$$

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2}}$$

leptonic Dirac CP phase

Predictions

- quark sector:

charged fermion sector: 7 parameters

$$m_u : m_c : m_t \;=\; \theta_c^{7.5} : \theta_c^{3.7} : 1 \qquad m_d : m_s : m_b \;=\; \theta_c^{4.6} : \theta_c^{2.7} : 1$$

  - CKM mixing angles & CPV measures

$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.227 & 0.00412 \\ 0.227 & 0.973 & 0.0412 \\ 0.00718 & 0.0408 & 0.999 \end{pmatrix}$$

$$\begin{aligned} \beta &\equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 23.6^o, \ \sin 2\beta = 0.734 \ , \\ \alpha &\equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 110^o \ , \\ \gamma &\equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 45.6^o \ , \\ J &\equiv \mathrm{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 2.69 \times 10^{-5} \ , \end{aligned}$$

- Neutrino sector

$$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_\odot = 0.419, \quad |U_{e3}| = 0.0583$$

$$|m_1| = 0.0156 \text{ eV}, \quad |m_2| = 0.0179 \text{ eV}, \quad |m_3| = 0.0514 \text{ eV}$$

2 parameters in neutrino sector

  - Majorana phases $\alpha_{21} = \pi \qquad \alpha_{31} = 0.$

prediction for Dirac CP phase: δ = 227 degrees

$$J_\ell = -0.00967$$

⇒ connection between leptogenesis & CPV in neutrino oscillation

Flavor Violation in RS

- Two sources of flavor violation in RS:

$$\mathcal{L}_{5\mathrm{D}} \supset \overline{\Psi} C_\Psi \Psi + \overline{\psi_u} C_{\psi_u} \psi_u + \overline{\psi_d} C_{\psi_d} \psi_d + H\,\overline{\Psi} \lambda_U \psi_u + \overline{H}\,\overline{\Psi} \lambda_D \psi_d$$

  - 5D Yukawa coupling constants
  - 5D bulk mass terms

  $\Big\}$ generically independent

- gauge-fermion couplings in fermion mass eigenstates:

$$\sum_n G^n ({\Psi^0}^\dagger V^\dagger f^2_{\Psi^0} V \Psi^0 + {\psi_u^0}^\dagger W_u^\dagger f^2_{\psi_u^0} W_u \psi_u^0 + {\psi_d^0}^\dagger W_d^\dagger f^2_{\psi_d^0} W_d \psi_d^0)$$

  - non-universal bulk mass terms ⇒ tree-level FCNCs

- quark sector:
  - most stringent constraint from 1st & 2nd generations
- lepton sector:
  - presence even in the limit of massless neutrinos
  - μ → 3 e, μ-e conversion
- Generally: FCNC constraints ⇒ Λ > O(10) TeV

Neutrino Mass Spectrum & Mixing

- Minimal Flavor Violation
  - RS Quark sector: Fitzpatrick, Perez, Randall, 2007

$$C_{u,d} = Y^\dagger_{u,d} Y_{u,d} \qquad C_Q = r Y_u Y_u^\dagger + Y_d Y_d^\dagger$$

  - RS Lepton sector: M.-C. C, H.B. Yu, PLB 2008

$$C_e = a Y_e^\dagger Y_e, \quad C_N = d Y_\nu^\dagger Y_\nu, \quad C_L = c(\xi Y_\nu Y_\nu^\dagger + Y_e Y_e^\dagger)$$

- mild hierarchy among 5D parameters: tuning needed for large neutrino mixing

generic anarchy case: $V_{ij} \sim f_{L_i}/f_{L_j}$

large atm & solar mixing angles: $f_{L_1}/f_{L_2} \sim 1$ and $f_{L_2}/f_{L_3} \sim 1.$

- MFV with $\xi = 0,$ f_{L_i}/f_{L_j} is fixed by $\sqrt{m_i/m_j}$

$$f_{L_1}/f_{L_2} \simeq 0.07 \qquad f_{L_2}/f_{L_3} \simeq 0.24.$$

  - some structure in 5D Yukawa needed to accommodate mixing angles and mass ratios simultaneously

T′ Family Symmetry as a Solution

M.-C.C., K.T. Mahanthappa, F. Yu, arXiv:0907.3963

- Family Symmetry based on double tetrahedral group, T′ :
  - TBM neutrino mixing

    ⇒ 3 families of lepton doublets transform as 3

    ⇒ common bulk mass term for all lepton doublets

    ⇒ absence of tree-level FCNCs

    ⇒ neutrino mixing from CG coefficients (decouple from masses)

    ⇒ alleviate tuning in Yukawa couplings
  - Realistic quark masses & mixing

    ⇒ 3 families of quarks transform as (1+2)

    ⇒ common bulk mass term for 1st & 2nd generations of quarks

    ⇒ absence of FCNCs in 1st & 2nd generations