String Theory for Pedestrians

– CERN, Jan 29-31, 2007 –

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This series of 3 lecture series will cover the following topics

1. Introduction. The classical theory of strings. Application: physics of cosmic strings.

2. Quantum string theory. Applications:

   i) Systematics of hadronic spectra

   ii) Quark-antiquark potential (lattice simulations)

   iii) AdS/CFT: the quark-gluon plasma.

3. String models of particle physics. The string theory landscape. Alternatives: Loop quantum gravity?
   Formulations of string theory.

Introduction

For the last twenty years physicists have investigated String Theory rather vigorously.

Despite much progress, the basic features of the theory remain a mystery.

In the late 1960s, string theory attempted to describe strongly interacting particles. Along came Quantum Chromodynamics (QCD)– a theory of quarks and gluons – and despite their early promise, strings faded away.

This time string theory is a credible candidate for a theory of all interactions – **a unified theory of all forces and matter**. Additionally,

- Through the AdS/CFT correspondence, it is a valuable tool for the study of theories like QCD.

- It has helped understand the origin of the Bekenstein-Hawking entropy of black holes.

- Finally, it has inspired many of the scenarios for physics Beyond the Standard Model of Particle physics.

Greatest problem of twentieth century physics: the incompatibility of Einstein's General Relativity and the principles of Quantum Mechanics.

String theory appears to be the long-sought quantum mechanical theory of gravity and other interactions.

It is almost certain that string theory is a consistent theory.

It is less certain that it describes our real world.

Intense work has demonstrated that string theory incorporates many features of the physical universe.

Perhaps the most impressive feature of string theory is the appearance of **gravitons** as one of the quantum fluctuation modes of a closed string.

Considers a relativistic string, a string whose classical mechanics is consistent with Einstein's special theory of relativity.

Its classical vibrations, however, cannot be identified with physical particles.

Quantum theory comes to the rescue: the quantum vibrational modes of the relativistic string *can* be identified with elementary particles!

A particular quantum vibration mode of the closed string describes a graviton, the quantum of the gravitational field. A particular quantum vibration of an open string describes a photon, the quantum of the electromagnetic field.

In string theory all particles – matter particles and force carriers – arise as quantum fluctuations of the relativistic string.

Relativistic Particles and Strings

How do we describe the physics of a *free* particle?

Well, we say that it moves with constant velocity $\vec{v}$!

$$m \quad \bullet\!\!\longrightarrow$$

$$\vec{v}$$

This is true for both non-relativistic and relativistic particles, with an important caveat, $|\vec{v}| \leq c$ for the relativistic particle

In more detail we describe the energy E and momentum $\vec{p}$ of the particle in terms of the velocity.

Non-relativistic particle:

$$E = \frac{1}{2}mv^2\,, \qquad \vec{p} = m\vec{v}\,. \tag{1}$$

Relativistic particle:

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}\,, \qquad \vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}\,. \tag{2}$$

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = mc^2 + \underbrace{\frac{1}{2}mv^2}_{non-rel} + \mathcal{O}(v^4/c^4)\,. \tag{3}$$

How do we describe the physics of a *free* relativistic string ?

Not so easily!

The natural extension to relativity gives a string whose fundamental parameter is its tension T_0:

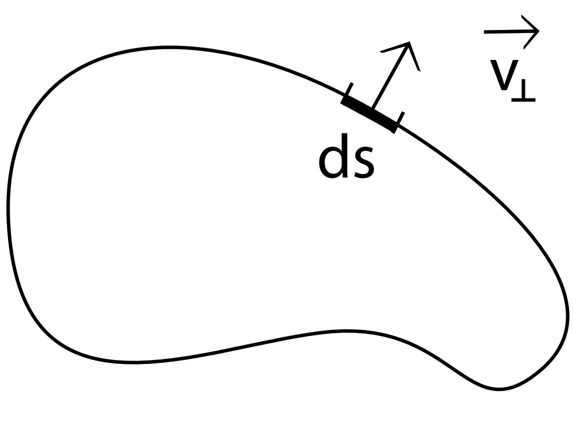
$$T_0 \qquad \text{has units of force}$$

A small static piece of string of length ds has an energy

$$dE = T_0\, ds\,.$$

A moving string has energy

$$\boxed{E = \int \frac{T_0 ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}}$$



Nonrelativistic strings are characterized by two independent parameters:

– a string tension T_0 ,

– a mass per unit length μ_0.

Direction along a fixed static string: *longitudinal direction* .

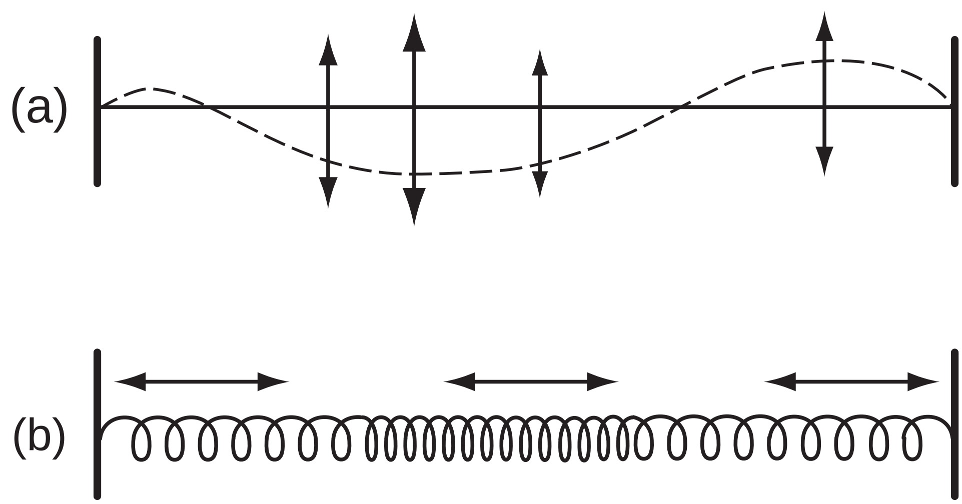
Transverse oscillations: the velocity of the string is orthogonal to the longitudinal direction.

The velocity v of a transverse *wave* is

$$\boxed{v = \sqrt{T_0/\mu_0}\,.}$$

A nonrelativistic string may support a **longitudinal oscillation** in which the velocity of any point on the string remains along the string.

A longitudinal wave requires the existence of "taggable" structure along the string. Otherwise, longitudinal oscillation is undetectable since, as a whole, the string does no move.



In order to detect longitudinal motion we must be able to tag the points along the string.

Four unusual properties of relativistic strings

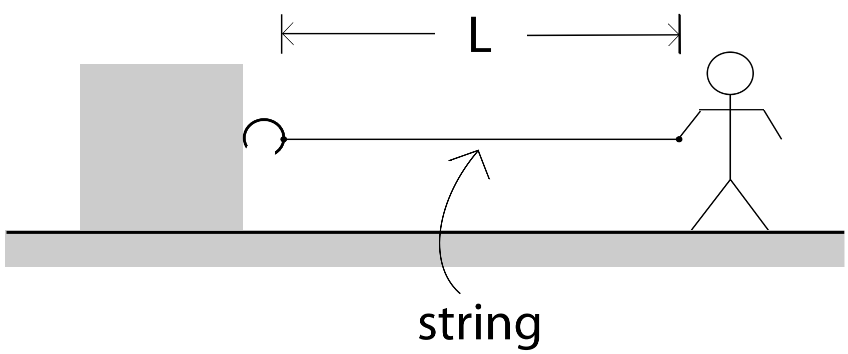
1. It is characterized by its tension T_0 alone !

   The velocity of transverse waves is c

   The mass density μ_0 is fixed once T_0 is fixed:

$$c = \sqrt{T_0/\mu_0} \quad \rightarrow \quad \mu_0 = T_0/c^2 \,.$$

   In the relativistic string energy/mass conversion occurs **classically**.



   Stretching a string out to length L:

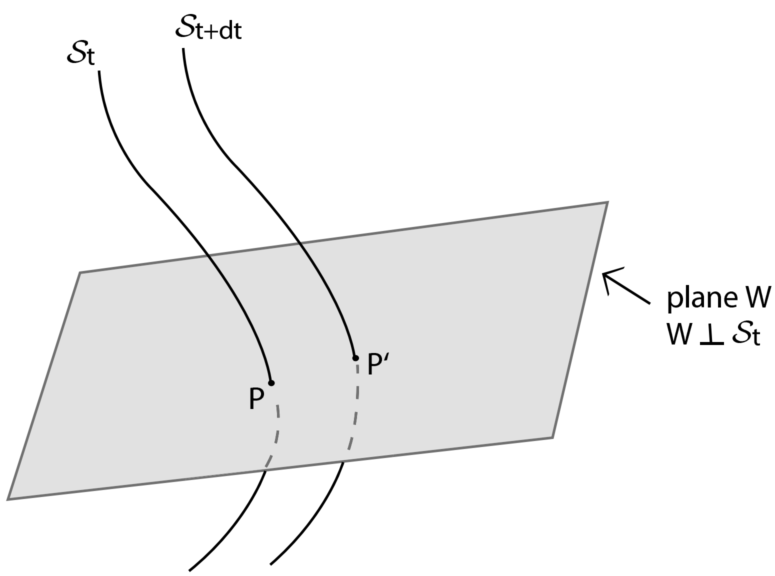
$$E_{st} = W = T_0 L \,, \quad M = \frac{T_0 L}{c^2}, \quad \mu_0 = \frac{M}{L} = \frac{T_0}{c^2}$$

   Energy is converted into rest mass by stretching the string!

2. The relativistic string does not support longitudinal oscillations.

$$\boxed{\text{the string has no substructure.}}$$

Moreover, when a string moves, we cannot really tell which point went where.

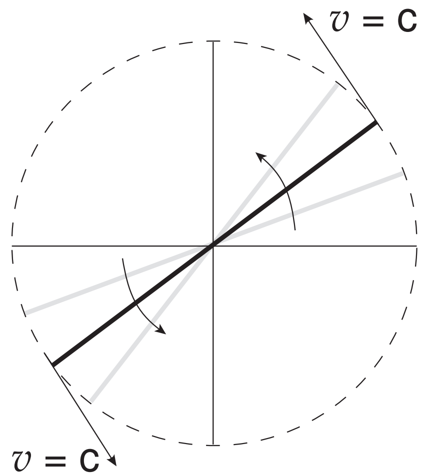


People ask: what is the string made of?

No meaningful answer can be provided: the classical relativistic string has no identifiable constituents.

3. The endpoints of a free relativistic open string move with the speed of light.

Simplest open string motion: A string of length ℓ rotating with angular velocity ω.



$$\omega\,\ell/2 = c\,.$$

Unusual property: the angular momentum J of this string is linearly proportional to the square of the energy E of the string:

$$\boxed{J = \alpha' E^2\,, \quad \alpha' = \text{slope parameter}}$$

Good fit to hadronic resonances (more later)

$J \sim E^2$ is very unusual !

For a rigid non-relativistic bar

$$J = I\omega, \quad E = \frac{1}{2} I\omega^2 .$$

Since the moment of inertia I is a constant,

$$J \sim \sqrt{E} .$$

$J \sim E^2$ can be understood roughly as follows

$$J \sim I\omega \sim (ML^2)\frac{1}{L} \sim E^2$$

since $M \sim L \sim E$.

More quantitatively: $v(s) = c(s/(\ell/2)) = 2cs/\ell$.

The energy E of the rotating string is:

$$E = \int_{-\ell/2}^{\ell/2} \frac{T_0 ds}{\sqrt{1 - 4s^2/\ell^2}} = (T_0 \ell)\frac{\pi}{2} .$$

This is 1.57 times the rest energy of a string of length ℓ.

The angular momentum J of the rotating string is

$$J = 2\int_0^{\ell/2} \frac{s\,(T_0 ds/c^2)v(s)}{\sqrt{1 - 4s^2/\ell^2}} = \frac{4T_0}{\ell c}\int_0^{\ell/2} \frac{s^2 ds}{\sqrt{1 - 4s^2/\ell^2}}$$

$$J = \frac{T_0 \ell^2 \pi}{8c} = \frac{1}{2\pi T_0\, c} E^2 \quad \rightarrow \quad \boxed{\frac{J}{\hbar} = \frac{1}{2\pi T_0\, \hbar c} E^2 = \alpha' E^2 .}$$

Note that $[\alpha'] = M^{-2} = L^2$, so one can define a string length:

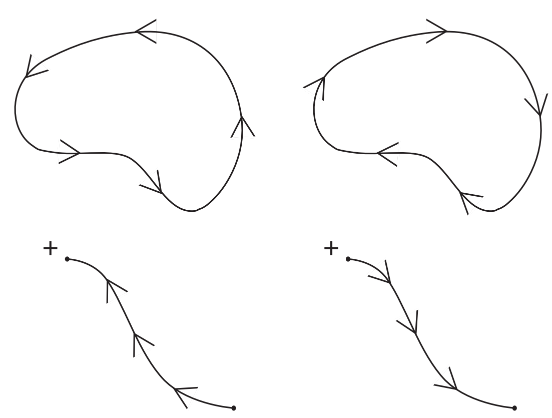
$$\ell_s = \hbar c \sqrt{\alpha'}$$

4. A relativistic string has an **orientation** which determines the sign of the string charge.

For zero size particles there is no intrinsic geometrical property represents charge.

This is different for strings! The orientation of a string is an arrow that defines a preferred direction along the string. Oppositely oriented strings have opposite *string* charges.

A surprising effect: the string charge forces the open string endpoints to acquire opposite *electric* charges! String charge transmutes into electric charge.



Since open strings carry electric charges, we may identify charged particles with open strings.

Classical String Dynamics

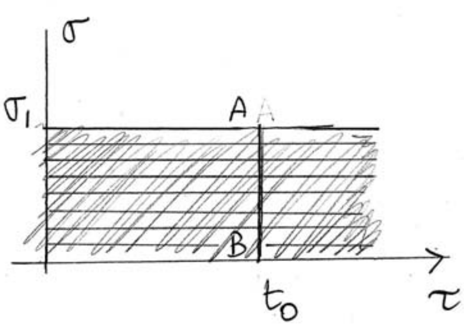
Dynamical variables $X^\mu(\tau,\sigma)$ (maps from (τ,σ) space to spacetime) governed by the Nambu-Goto action:

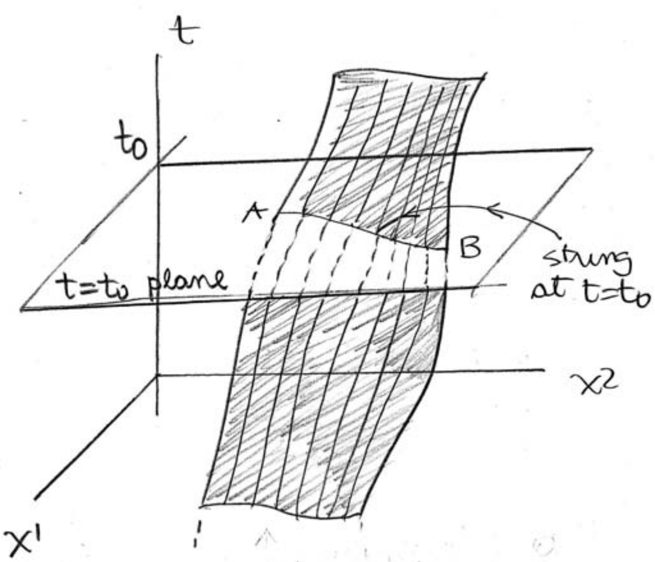
$$S = -\frac{T_0}{c}\int d\tau \int d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2}$$

Subtle action principle. Hard to analyze without gauge conditions. Learning how to draw the lines of constant τ and σ on the physical string surface.

Static gauge: $X^0(\tau,\sigma) \equiv ct = c\tau$. This identifies "strings".

$$X^\mu(\tau,\sigma) = \{ct, \vec{X}(\tau,\sigma)\}$$





Orthonormality: Choose the lines of constant σ orthogonal to the ones of constant $\tau(=t)$:

$$\frac{\partial \vec{X}}{\partial \sigma}\cdot\frac{\partial \vec{X}}{\partial t} = 0$$

σ-parameterization: Choose sigma such that equal intervals carry equal energy:

$$d\sigma = \frac{dE}{T_0}$$

With the 3 conditions above the dynamics reduces to:

Wave equations:

$$\boxed{\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = \vec{0}\,.}$$

and Virasoro constraints

$$\frac{\partial \vec{X}}{\partial \sigma} \cdot \frac{\partial \vec{X}}{\partial t} = 0$$

$$\left(\frac{\partial \vec{X}}{\partial \sigma}\right)^2 + \frac{1}{c^2}\left(\frac{\partial \vec{X}}{\partial t}\right)^2 = 1\,.$$

which can be summarized as

$$\boxed{\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1\,.}$$

The dynamics of relativistic strings is determined by the above boxed equations and some boundary (and initial) conditions.

Motion of a closed string:

Solution of the wave equation:

$$\vec{X}(t,\sigma) = \frac{1}{2}\Big(\vec{F}(ct+\sigma) + \vec{G}(ct-\sigma)\Big)$$

We introduce two independent parameters u and v"

$$u = ct + \sigma\,, \quad v = ct - \sigma$$

We then find:

$$\frac{\partial \vec{X}}{\partial \sigma} = \frac{1}{2}\Big(\vec{F}'(u) - \vec{G}'(v)\Big)$$

$$\frac{1}{c}\frac{\partial \vec{X}}{\partial t} = \frac{1}{2}\Big(\vec{F}'(u) + \vec{G}'(v)\Big)$$

The Virasoro conditions become

$$|\vec{F}'(u)| = |\vec{G}'(v)| = 1$$

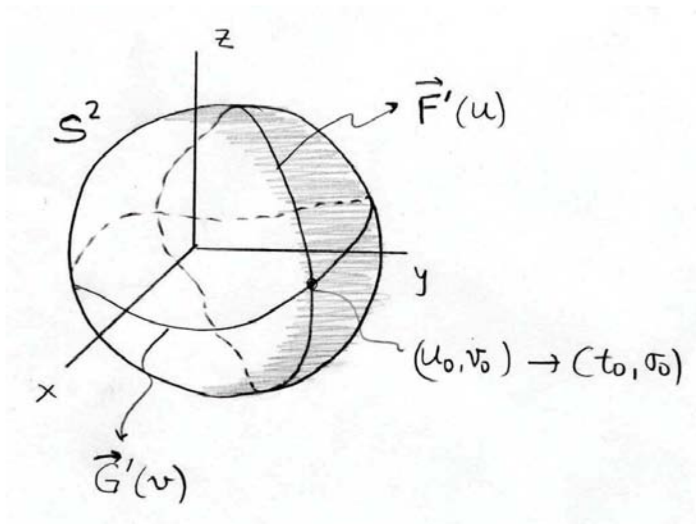
Periodicity condition: Parameterize the string with $\sigma \in [0, \sigma_1]$, with $\sigma_1 = E/T_0$ and E the energy of the string. Then, $\vec{X}(t, \sigma + \sigma_1) = \vec{X}(t,\sigma)$ leads to

$$\vec{F}(u+\sigma_1) + \vec{G}(v-\sigma_1) = \vec{F}(u) + \vec{G}(v)$$

Differentiating with respect to u and then with respect to v we get

$$\vec{F}'(u+\sigma_1) = \vec{F}'(u)\,, \quad \vec{G}'(v+\sigma_1) = \vec{G}'(v)$$

Geometrical interpretation: Can view the tips of the vectors $\vec{F}'(u)$ and $\vec{G}'(v)$ as points of the unit sphere that trace closed curves with periodicity σ_1.



If the paths intersect at $u = u_0$ and $v = v_0$ we have

$$\vec{F}'(u_0) = \vec{G}'(v_0)$$

The values u_0 and v_0 determine a t_0 and a σ_0. Recall that

$$\frac{1}{c}\frac{\partial \vec{X}}{\partial t} = \frac{1}{2}\Big(\vec{F}'(u) + \vec{G}'(v)\Big), \quad \frac{\partial \vec{X}}{\partial \sigma} = \frac{1}{2}\Big(\vec{F}'(u) - \vec{G}'(v)\Big).$$

The first equation gives

$$\left|\frac{1}{c}\frac{\partial \vec{X}}{\partial t}(t_0, \sigma_0)\right| = \frac{1}{2}\left|\Big(\vec{F}'(u_0) + \vec{G}'(v_0)\Big)\right| = \left|\vec{F}'(u_0)\right| = 1\,.$$

The point σ_0 moves with the speed of light at this $t = t_0$.

This repeats every $t_0 + k\sigma_1/c$, with k integer.

The second equation gives:

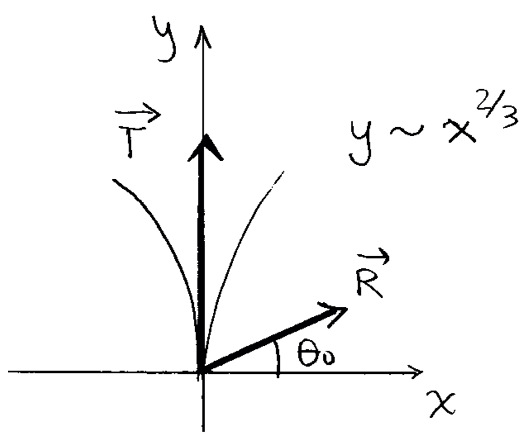
$$\frac{\partial \vec{X}}{\partial \sigma}(t_0, \sigma_0) = 0$$

This implies that, in general,

$$\vec{X}(t_0, \sigma) = (\sigma - \sigma_0)^2 \vec{T} + (\sigma - \sigma_0)^3 \vec{R} + \ldots$$

for some constant vectors $\vec{T}$ and $\vec{R}$.

Align the y-axis with $\vec{T}$ and the x-axis such that $\vec{R}$ is on the (x, y)-plane, say at an angle θ_0.



We then get

$$y(\sigma) = (\sigma - \sigma_0)^2 |\vec{T}| + (\sigma - \sigma_0)^3 \sin\theta_0 |\vec{R}| + \ldots$$

$$x(\sigma) = (\sigma - \sigma_0)^3 \cos\theta_0 |\vec{R}| + \ldots$$

which means

$$\boxed{y \sim x^{2/3}}$$ local cusp singularity

The Energy Momentum tensor of a String

With spacetime signature $(-,++\ldots+)$ a fluid has

$$T_{00} = \rho c^2\,, \text{ and } \; T_{ij} = p\delta_{ij}\,,$$

with ρ the mass density and $p > 0$ the pressure.

Consider a string stretched along the x-axis.

The mass per unit length is T_0/c^2, so we get

$$T_{00} = c^2\rho = c^2\frac{T_0}{c^2}\delta(y)\delta(z) = T_0\,\delta(y)\delta(z)$$

The pressure is negative (stretches a cubic box, rather than compressing it)

$$T_{11} = -T_0\delta(y)\delta(z)$$

So, we have

$$T_{\mu\nu} = -T_0\delta(y)\delta(z)\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

This is boost invariant along the x axis, as expected physically.

In the Newtonian limit the gravitational potential Φ is sourced by

$$\nabla^2\Phi = 4\pi G(T_{00} + T_{11} + T_{22} + T_{33})$$

For the string the right-hand side gives zero.

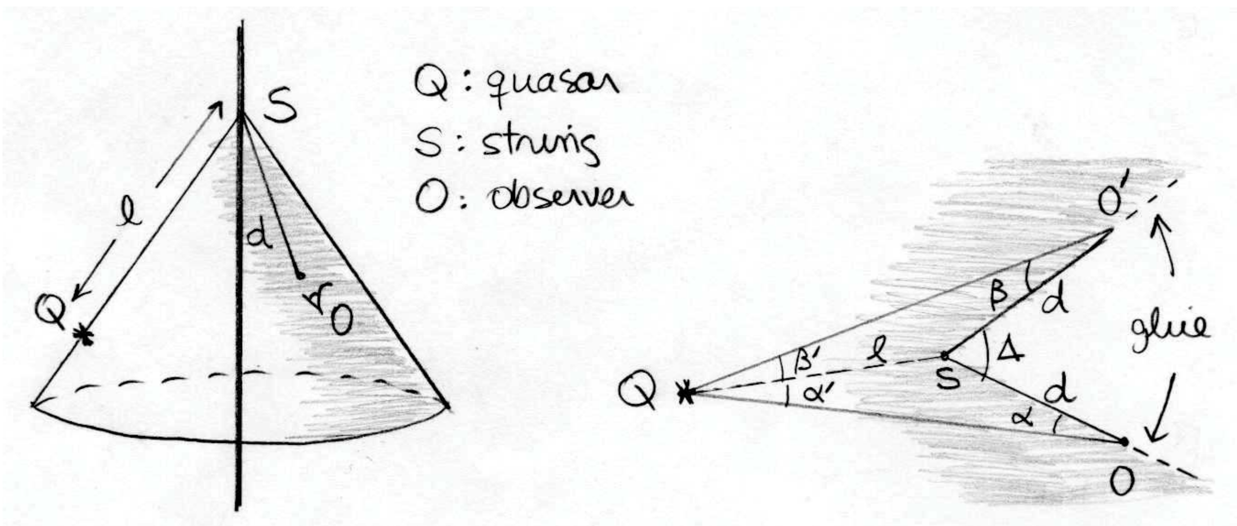
The string does not attract objects gravitationally!

The string creates a conical singularity with a deficit angle Δ

$$\Delta = \frac{8\pi G T_0}{c^4} = \frac{8\pi G\mu}{c^2} \simeq 5.2'' \left(\frac{G\mu}{10^{-6}}\right).$$

where μ is the mass per unit length of the string.

The string S creates a perfect lens with two identical images of the quasar Q separated by an angle $\delta\varphi$ as seen by the observer O:



$$\delta\varphi = \alpha + \beta$$

$$\frac{\sin\beta}{\ell} = \frac{\sin\beta'}{d}, \qquad \frac{\sin\alpha}{\ell} = \frac{\sin\alpha'}{d}$$

For small angles

$$\frac{\beta}{\ell} = \frac{\beta'}{d}, \qquad \frac{\alpha}{\ell} = \frac{\alpha'}{d}$$

We then have

$$\Delta = \alpha + \beta + \alpha' + \beta' = \left(1 + \frac{d}{\ell}\right)(\alpha + \beta)$$

$$\rightarrow \quad \boxed{\delta\varphi = \frac{\Delta}{\left(1 + \frac{d}{\ell}\right)}.} \quad \delta\varphi \rightarrow \Delta, \text{ if } \ell \rightarrow \infty.$$

The dimensionless number $G\mu$ governs the behavior of strings. With $\hbar = c = 1$ units,

$$\mu \sim (M_s)^2 \,, \quad M_s \text{ is the string mass}\,.$$

$$G \sim 1/(M_p)^2 \,, \quad M_p \text{ is the Planck mass}\,.$$

$$\rightarrow \qquad G\,\mu \sim \left(\frac{M_s}{M_p}\right)^2$$

If the string mass would arise from a GUT scale, then $M_s \sim 10^{-3} M_p$ and one find $G\mu \sim 10^{-6}$.

Cosmic strings were originally proposed as the main sources of primordial density fluctuations

$$\frac{\delta\rho}{\rho} \sim G\mu \,,$$

but this possibility (which requires $G\mu \sim 10^{-5.5}$) has been rejected. It fails to reproduce the COBE results. Density fluctuations are thought to arise as quantum fluctuations at an inflationary period.

From WMAP analysis [Jeong and Smoot, astro-ph/0406432] searching for string contributions to density fluctuations:

$$G\mu \lesssim 10^{-7} \,.$$

There are also some limits from gravitational wave radiation from cosmic strings. Such a gravitational wave background would affect the regularity of pulsars [astro-ph/0208572], suggesting also that $G\mu \lesssim 10^{-7}$.

Damour and Vilenkin (Phys. Rev. D**64** , 064008 (2001)) have pointed out that cusps radiate efficiently and may be detected by LIGO (Laser Interferometric Gravitation Observatory) and LISA (Laser Interference Space Antenna) even for $G\mu \sim 10^{-13}$.

Most uncertainties arise in the analysis and simulation of networks of strings, and their evolution as the universe expands and the strings collide. This is still an active area of research (see, Polchinski and Rocha, hep-ph/0606205).

The observation of a cosmic string would be a very exciting event

A lot of work would follow to decide if this is a string theory string, or a string arising from another, more conventional field theory.