Lecture 2

1. Introduction. The classical theory of strings. Application: physics of cosmic strings.

2. Quantum string theory. Applications:

   i) Systematics of hadronic spectra

   ii) Quark-antiquark potential (lattice simulations)

   iii) AdS/CFT correspondence.

   iv) AdS/CFT and the quark-gluon plasma.

3. String models of particle physics. The string theory landscape. Alternatives: Loop quantum gravity?
   Formulations of string theory.

Quantum Relativistic Strings

Physicists struggled to invent a quantum theory of gravity during much of the 20th century, and the answer came from the quantization of classical relativistic strings!

We are reminded of a statement by Dirac (1966):

> "The only value of the classical theory is to provide us with hints for getting a quantum theory; the quantum theory is then something that has to stand in its own right. If we were sufficiently clever to be able to think of a good quantum theory straight away, we could manage without classical theory at all. But we're not that clever, and we have to get all the hints that we can to help us in setting up a good quantum theory."

"Lectures in Quantum Field Theory"

Yeshiva University.

Quantization of Relativistic Strings

The dynamical variables of a string are its position

$$X^\mu(\tau,\sigma), \quad \mu = 0,1,2,\ldots,d\,, \quad D = d+1$$

and its (conjugate) momentum density $\mathcal{P}_\nu(\tau,\sigma)$.

This is a subtle gauge system, and quantization is most straightforward in: the light-cone gauge.

In light-cone coordinates:

$$X^\mu \;\to\; \{X^+, X^-, X^I\},\; I = 2,3\ldots,D\,, \qquad X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^1)\,.$$

In light-cone gauge:

$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1) = c\tau$$

It turns out that one can solve for X^- in terms of the X^I, which become the $D-2$ independent variables.

$$\left[X^I(\tau,\sigma),\, \mathcal{P}_J(\tau,\sigma')\right] = i\hbar\,\delta^I_J\,\delta(\sigma-\sigma')\,.$$

For superstrings there are additional fermionic dynamical variables $\psi^I s(\tau,\sigma)$.

Upon quantization we learn that:

1. Consistency fixes the dimensionality of Minkowski spacetime.

   For bosonic strings $D = 26$.

   For superstrings $D = 10$.

2. Strings have quantum states of oscillation that can be identified with particles.

3. While classical closed string oscillations that resemble gravitons have positive mass, their quantum counterparts have zero mass. This is perfect: gravitons are massless particles.

4. Quantum open string oscillations describe massless gauge fields (like the photon), massive gauge fields (like the $W^{\pm}, Z$), and fermion fields (like quarks and leptons).

5. We learn of the existence of D-branes.

Open strings parameterized by $\sigma \in [0, \sigma_1]$.

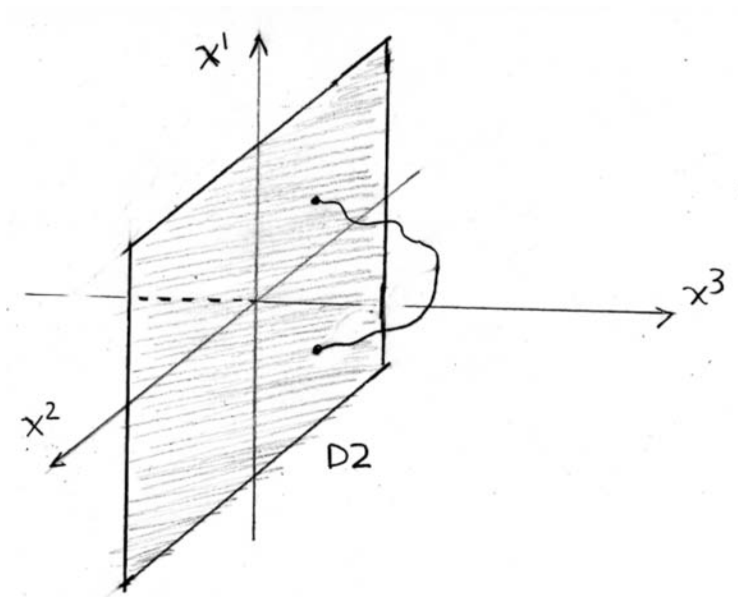
$\sigma = 0$ and $\sigma = \sigma_1$ are the endpoints of the open string.

Open strings can be quantized with Dirichlet boundary conditions:

Mathematically: specify constant values of $X^i(\tau, 0)$ and $X^i(\tau, \sigma_1)$

Physically: constrain the motion of the endpoints. The locus of the endpoints is a real dynamical object: a D-brane!

In 3 space dimensions, if we impose:
$X^3(\tau, 0) = X^3(\tau, \sigma_1) = 0$ we get a D2-brane lying on the plane $x^3 = 0$.



A Dp-brane is an extended object with p spatial dimensions.

For a string, tension times length is mass. For a D-brane, tension times "volume" is mass. The tension T_p of a Dp-brane goes like

$$\boxed{T_p \sim \frac{1}{g_s}\frac{1}{(\alpha')^{(p+1)/2}}}\,, \qquad \text{units: } [T_p] = M/L^p = 1/L^{p+1}\,.$$

g_s is the string coupling constant.

Since $g_s \sim g_0^2$, with g_0 the open string coupling, D-branes can be thought to be open string solitons!

The ground states of strings that begin and end on a D-brane represent quantum excitations of a Maxwell field. They are massless photon states that live on the brane.

A string that has only one end on a D-brane represents a charged particle.

If we have N coincident D-branes there are N^2 types of open strings that begin and end on the branes.

Ground states of those strings describe the massless non-abelian gauge fields of a $U(N)$ gauge theory that lives on the brane.

Spectrum of Hadrons:

The relation $J = \alpha' E^2$ for a classical rotating string becomes in the quantum theory

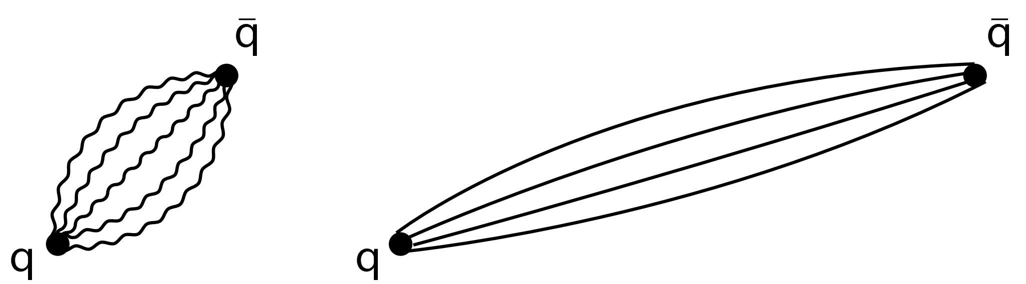
$$J = \alpha' E^2 + \text{const.}$$

The scale of α' is for us to choose. To fit hadrons:

$$\alpha' \simeq 0.95 \text{ Gev}^{-2}, \quad \ell_s = \hbar c \sqrt{\alpha'} \simeq 0.2 \text{ fm},$$

$$T_0 = \frac{1}{2\pi\alpha'\hbar c} \simeq \frac{1}{(0.48\,\text{fm})^2} \simeq 13.8 \text{ ton}.$$

We have the picture of a meson as a quark/anti-quark pair joined by a string made of lines of the "color" field.



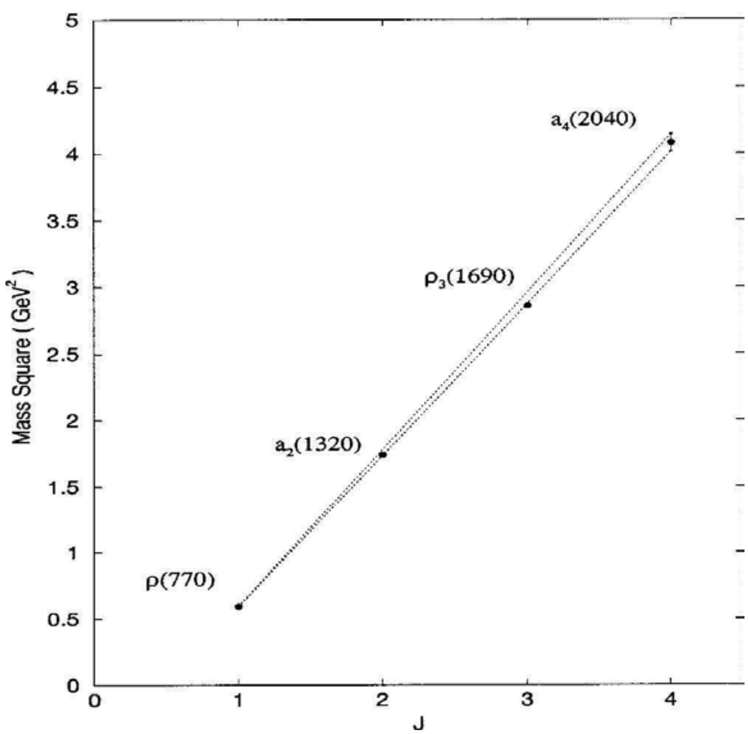
$$J = \alpha' E^2 + \text{const.}$$

is a good fit to meson excited states.

The ρ vector meson or $\rho(770)$, is a spin-one $S = 1$ combination of u and d quarks (and antiquarks) with orbital angular momentum $L = 0$:

$$\rho(770): \quad J = S + L = 1$$

$\rho(770)$ and the excited versions with $L = 1, 2,$ and 3 fit:



$$M^2 = \left(1.0629\ J - 0.4831 + 0.0191 J^2\right) \text{GeV}^2$$

Tang and Norbury, Phys.Rev., **D62**, 016006 (2000).

Selem and Wilczek, Hadron systematics and emergent di-quarks (hep-ph/0602128)

Quark-Antiquark Potential

Strategy:

1. Use an effective string theory, valid for arbitrary spacetime dimension D to calculate the energy $V(r)$ of a stretched string with Dirichlet boundary conditions. Get a prediction for $V(r)$.

2. Compare the results of 1 with lattice gauge theory computations of $V(r)$ for a $q\bar{q}$ pair separated a distance r.

Step 1: Quantize open strings stretched between two D-branes separated a distance L. The mass M of the stretched string, in its ground state is

$$M^2 = \left(\frac{L}{2\pi\alpha'}\right)^2 - \frac{1}{\alpha'}\frac{(D-2)}{24}$$

Recall $T_0 = \dfrac{1}{2\pi\alpha'}$, and let $\boxed{\gamma_D \equiv -\dfrac{\pi}{24}(D-2)}$

$$\rightarrow \qquad M = \sqrt{(T_0 L)^2 + 2T_0\gamma_D}$$

or, in terms of potential energy,

$$V(r) = \sqrt{(T_0 r)^2 + 2T_0\gamma_D}$$

expanding for large r:

$$\boxed{V(r) = T_0\, r + \gamma_D \cdot \frac{1}{r} - \frac{\gamma_D^2}{2}\frac{1}{T_0\, r^3} + \ldots}$$

We find corrections to the naive $V(r) = T_0 r$!!

Question: Why can this formula be trusted except in the critical dimension ($D = 26$) where it is useless ?

Answer: It is possible to formulate an effective string theory that is Poincare invariant in arbitrary D dimensions (Polchinski and Strominger, PRD **67** (1991) 1681.) The theory is nonlocal and has infinitely many effective interactions. Its quantization reproduces the above result, at least to terms of order $1/r^3$. See also Luscher, Symanzik and Weisz, Nucl. Phys. **B173**(1980)385.

γ_D is the *universal* Luscher coefficient

$$\gamma_D = -\frac{\pi}{24}(D-2): \quad \gamma_4 = -\frac{\pi}{12} = -0.262, \quad \gamma_3 = -\frac{\pi}{24} = -0.1309\,.$$

The magnitude of the force

$$F(r) = \frac{\partial V}{\partial r} = T_0 - \gamma_D \cdot \frac{1}{r^2} + \frac{3\gamma_D^2}{2}\frac{1}{T_0\, r^4} + \ldots$$

The force $F(r)$ is roughly linear in $1/r^2$.

The Casimir term $C_{eff}(r)$ is given by

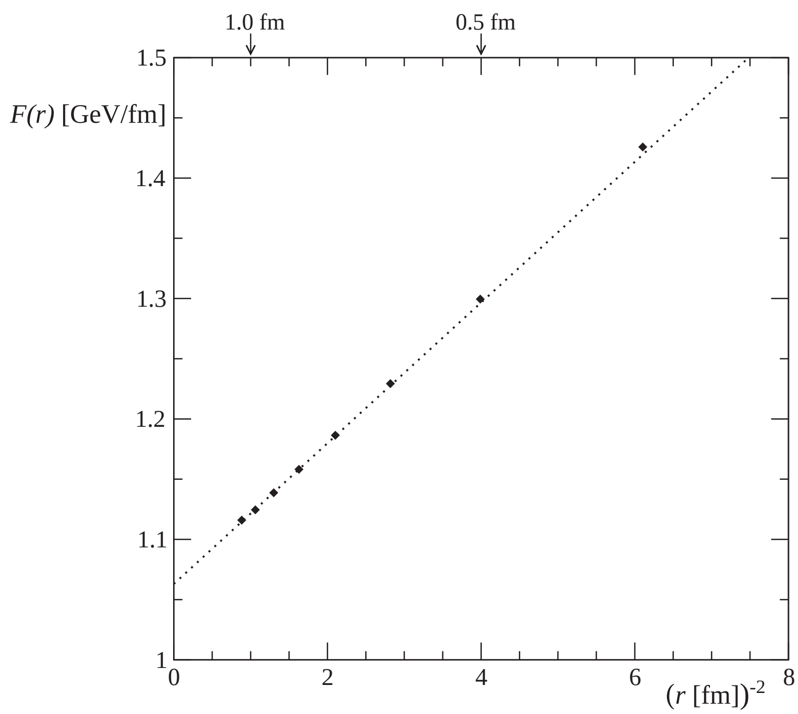
$$C_{eff}(r) \equiv -\frac{1}{2}r^3\,\frac{\partial F}{\partial r} = -\gamma_D\Big(1 - 3\gamma_D\,\frac{1}{T_0 r^2}\Big) + \ldots\,.$$

Note that $C_{eff}(r) \sim -\gamma_D$, with small corrections for large r.

Step 2: Lattice calculations in gauge theory

$$\text{Recall prediction:} \quad F(r) = T_0 - \gamma_D \cdot \frac{1}{r^2} + \ldots$$

The linearity of the force against $1/r^2$ sets in accurately for $r \gtrsim 0.5\,\text{fm}$.

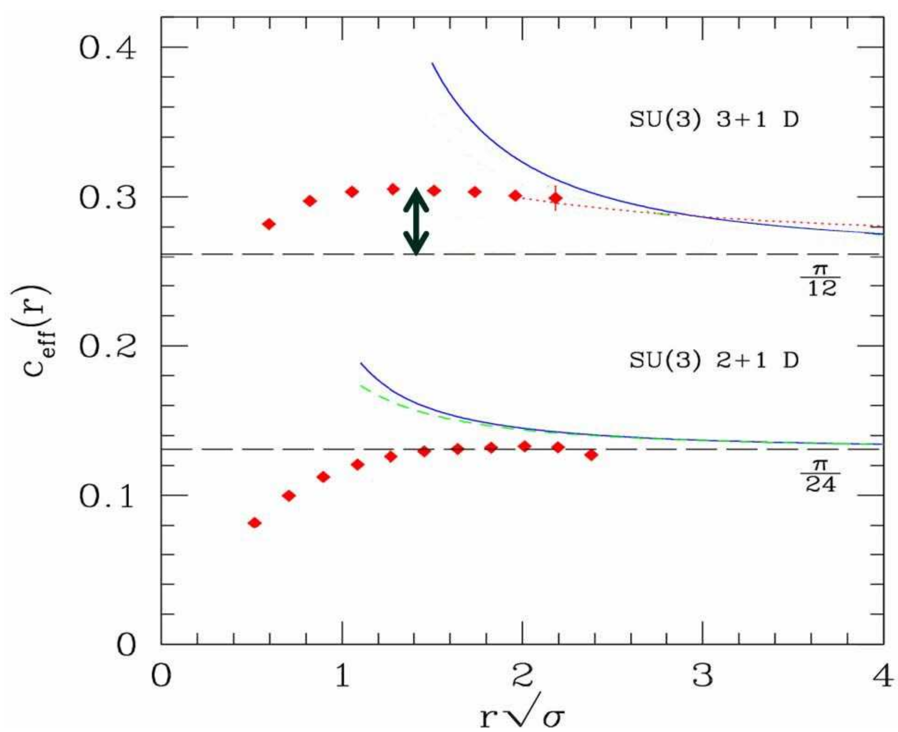


Luscher and Weisz, JHEP 0207:049,2002 (hep-lat/0207003).

The lattice simulation of quark-antiquark C_{eff} for $SU(3)$ gauge theory.

Recall prediction: $C_{eff}(r) \sim -\gamma_D + \mathcal{O}(1/r^2)$

$$\gamma_4 = -\frac{\pi}{12} = -0.262, \quad \gamma_3 = -\frac{\pi}{24} = -0.1309\,.$$



J. Kuty, PoS LAT2005:001,2005 (hep-lat/0511023).

AdS/CFT correspondence

A surprising reformulation of certain non-abelian gauge theories, theories of the kind used to describe strong interactions

In fact, it seems very likely that:

Every non-Abelian gauge theory is equivalent to a theory of quantum gravity – a string theory on a given background spacetime.

Two key examples:

- Maximally supersymmetric $SU(N)$ Yang-Mills theory is equivalent to type IIB superstrings on $\mathrm{AdS}_5 \times S^5$.

- The above $SU(N)$ gauge theory at finite temperature T is equivalent to the same type IIB superstring with a Black Hole of Hawking temperature T on the AdS_5 part of the spacetime.

How about QCD itself ?? Presumably some 5-dimensional superstring with nontrivial tachyon and dilaton profiles ....

Let us understand the first correspondence:

Maximally supersymmetric $SU(N)$ Yang-Mills theory is equivalent to type IIB superstrings on $\mathrm{AdS}_5 \times S^5$.

The parameters that define the gauge theory are

$$\boxed{\text{Gauge theory parameters:} \quad N\,, \ g_{YM}\,.}$$

Let us look at the parameters of the superstring theory on on $\mathrm{AdS}_5 \times S^5$

The space S^5 is a five-dimensional sphere of radius R. The AdS_5 spacetime is a five-dimensional negatively curved space, with radius of curvature R. It contains the time dimension and it has infinite volume.

$$\boxed{\text{Superstring parameters:} \quad g_s\,, \ \frac{R}{\sqrt{\alpha'}}\,.}$$

How are they related by the correspondence ??

$$\boxed{g_s = g_{YM}^2\,, \qquad \Big(\frac{R}{\sqrt{\alpha'}}\Big)^4 = g_{YM}^2 N\,.}$$

Let us try to understand the second relation:

Horizon size: The Schwarzchild radius R of a black hole of mass M can be estimated as the distance at which the rest energy plus the gravitational potential energy are zero :

$$m - \frac{G^{(4)} M m}{R} = 0 \quad \to \quad R \sim G^{(4)} M$$

For a black hole of mass M in 6 space dimensions

$$m - \frac{G^{(7)} M m}{R^4} \sim 0 \quad \to \quad \boxed{R^4 \sim G^{(7)} M \, .}$$

Consider a $D = 10$ spacetime and N D3 branes wrapped around a 3-dimensional space of volume V.

The $D = 7$ observer sees a point mass M of value:

$$M = N T_3 V = \frac{NV}{g_s \alpha'^2} \, .$$

In string theory $G^{(10)} \sim g_s^2 (\alpha')^4$. So, compactification gives

$$G^{(7)} = \frac{G^{(10)}}{V} \sim \frac{g_s^2 \alpha'^4}{V}$$

Combining into the boxed equation:

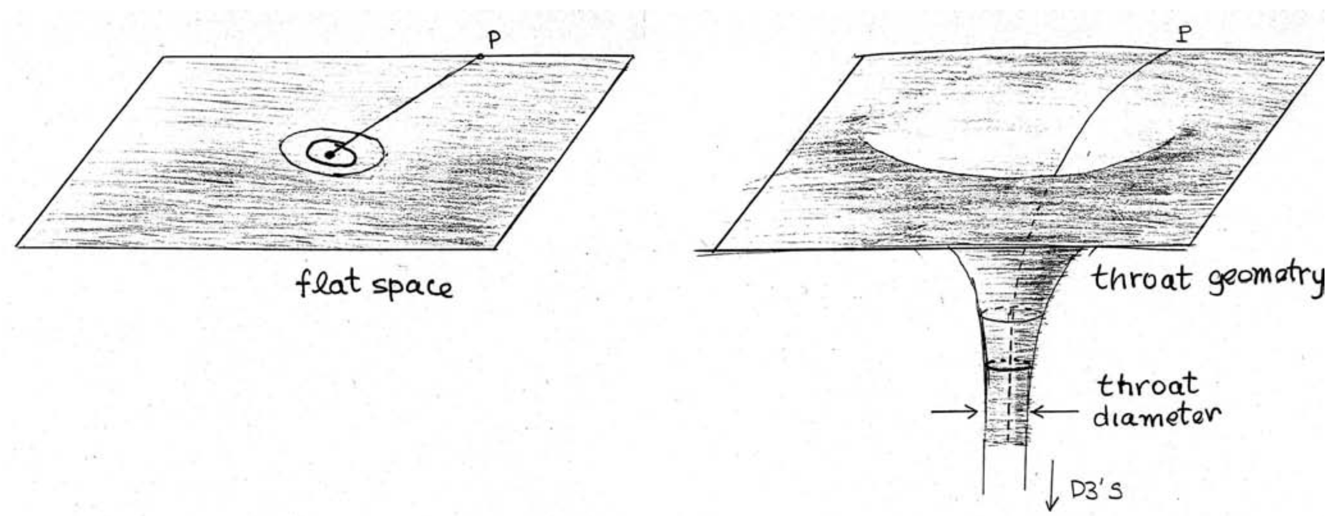
$$R^4 \sim \frac{g_s^2 \alpha'^4}{V} \cdot \frac{NV}{g_s \alpha'^2} = g_s N \, \alpha'^2 \quad \text{success!}$$

The dependence on V cancelled out!

What we said above is not exactly right, there is an interesting twist to the story!

D3-branes will not produce a black hole!

D3-branes produce a throat !!



Left: 2-dimensional space with a mass at the origin. The space is built by a radial coordinate and circles S^1 at each point on the radial coordinate. The circles shrink as we approach the mass, which is at finite distance.

Right: 2-dimensional space (left) with a source that produces a throat. The circles approach a finite value R and the mass has been sent to infinity.

D3 branes on 10-dimensional spacetime. The space transverse to the D3-branes is $9-3=6$-dimensional, with the D3-branes at the end of a throat. The space is built by a radial coordinate and S^5's at each point. The radius of the S^5 approaches R as we move down the throat.

Amplifying the region near the throat:

$$\Big(\underbrace{\text{Time coord.} + \text{Radial coord.} + \text{3D3-coords.}}_{\text{AdS}_5}\Big) \times S^5$$

Conventional description of the N D3-branes

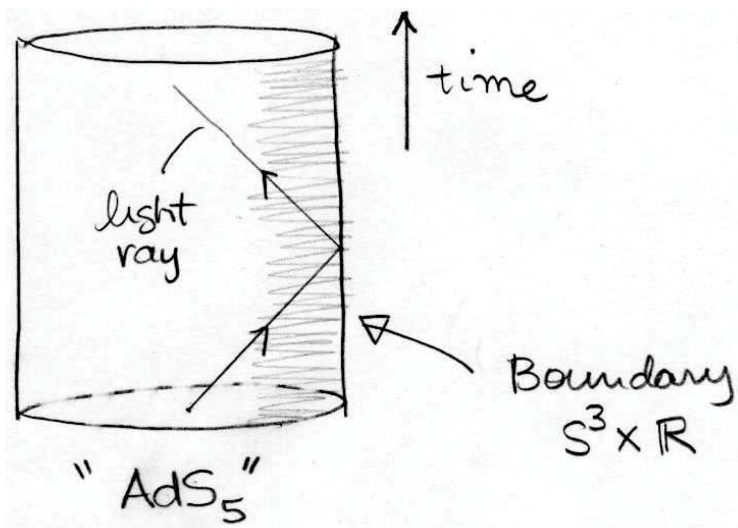
On the N D3-branes, at low energies the effective field theory is $SU(N)$ gauge theory on the branes on flat spacetime, gravity decouples from the branes.

Gravitational description of the N D3-branes

A curved spacetime with a throat. The region near the throat decouples from the gravity that defines a flat spacetime far away from the throat.

Natural Conjecture: The $SU(N)$ gauge theory is equivalent to gravity in the throat region AdS$_5 \times S^5$.

Caveat: the $SU(N)$ theory is supersymmetric (QCD is not!)



Intuition: AdS$_5$ is infinite but somewhat reminiscent of a finite cavity. Light takes finite time to get to the boundary.

The 4-dimensional world of the gauge theory can be imagined as the boundary of AdS$_5$.

A realization of holography: Describe the full physics of the interior using a theory on the boundary.

SU(N) Calculation $\rightleftarrows$ $\boxed{\text{DICTIONARY}}$ $\rightleftarrows$ Gravity Calculation

Recall:

$$\boxed{g_s = g^2_{YM}\,, \qquad \left(\frac{R}{\sqrt{\alpha'}}\right)^4 = g^2_{YM} N \equiv \lambda\,.}$$

λ is the t'Hooft coupling. It is the relevant coupling in the large N limit.

String theory is easy when:

$$g_s \ll 1\,, \; \frac{R}{\sqrt{\alpha'}} \gg 1 \tag{1}$$

The gauge theory is easy when

$$\lambda \ll 1 \quad \text{(small t'Hooft coupling)}$$

Easy computations in gravity translate into difficult, strong coupling computations in the gauge theory.

The conditions in (1) correspond to

$$g_{YM} \ll 1\,, \quad g^2_{YM} N \gg 1\,, \quad N \text{ very large} \tag{2}$$

.

This is the famous large N limit of t'Hooft.