# Introduction to Supersymmetry

A practical guide one year before the LHC

CERN Academic Lecture Series, 12-15 Feb, 2007

# What I hope to cover

Motivation for new physics at the LHC and
specific motivation for supersymmetry

Some basic useful supersymmetric formalism

The construction of the supersymmetric standard model

Specific models of supersymmetry

The phenomenology of supersymmetry

# What these lectures are NOT

No textbook derivation and description of supersymmetry

No numerical accuracy - only rough estimates of cross sections, branching ratios, etc. will be shown

No history of supersymmetry

No string-theory ‘predictions’ for supersymmetry

No sales pitch for (or derision of) supersymmetry

References will be given at the end for most of these

# I

# Motivation

# Ask the LHC

2008

$$p \longrightarrow \longleftarrow p$$

14 TeV

# Ask the LHC

2008

$$p \longrightarrow \longleftarrow p$$

- What is the "Higgs"?
- Is EWSB natural?
- What is dark matter?

14 TeV

# New Physics at (around) 1 TeV

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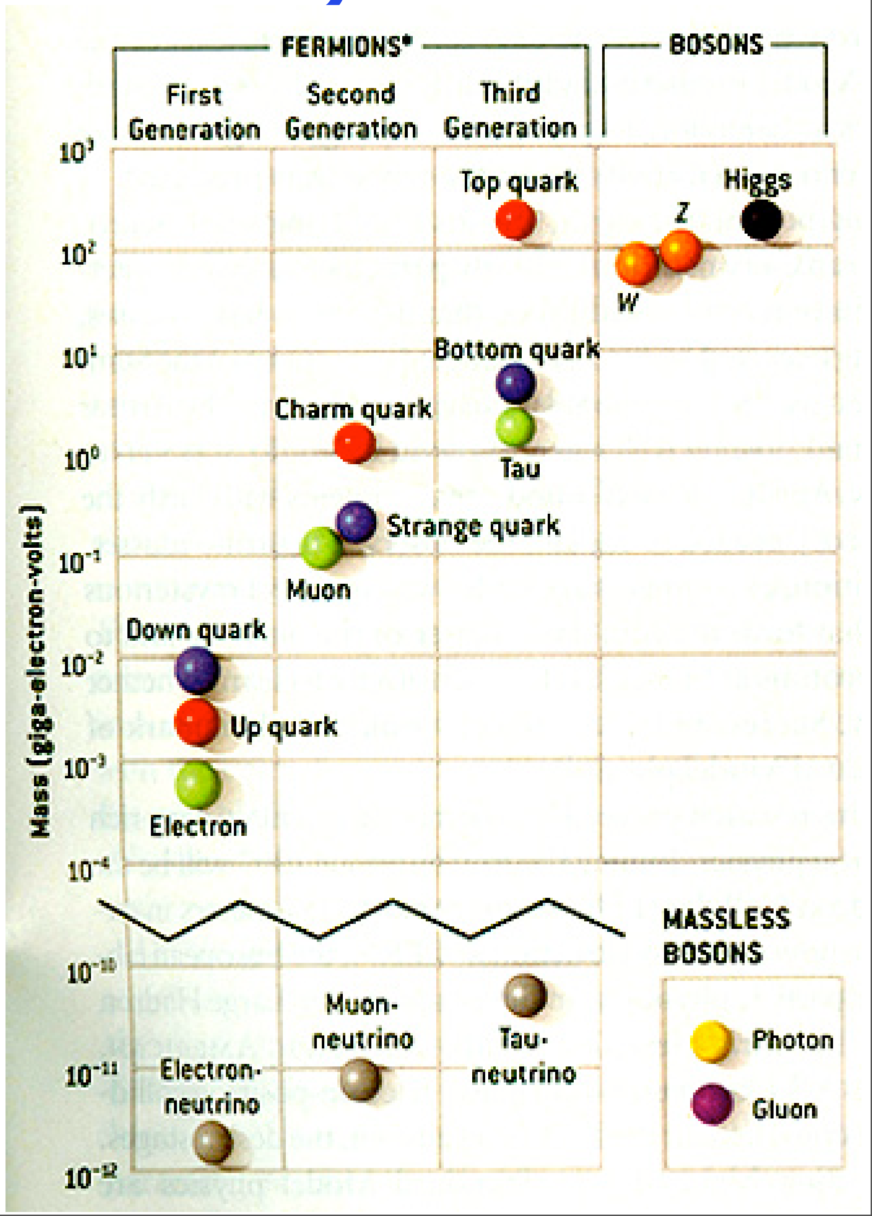
The Standard Model:



# New Physics at (around) 1 TeV

The Standard Model:

Particles we've seen, their interactions, *and the Higgs*.

Beyond the Standard Model:

"Weak Scale Physics"

Anything else

# New Physics at (around) 1 TeV

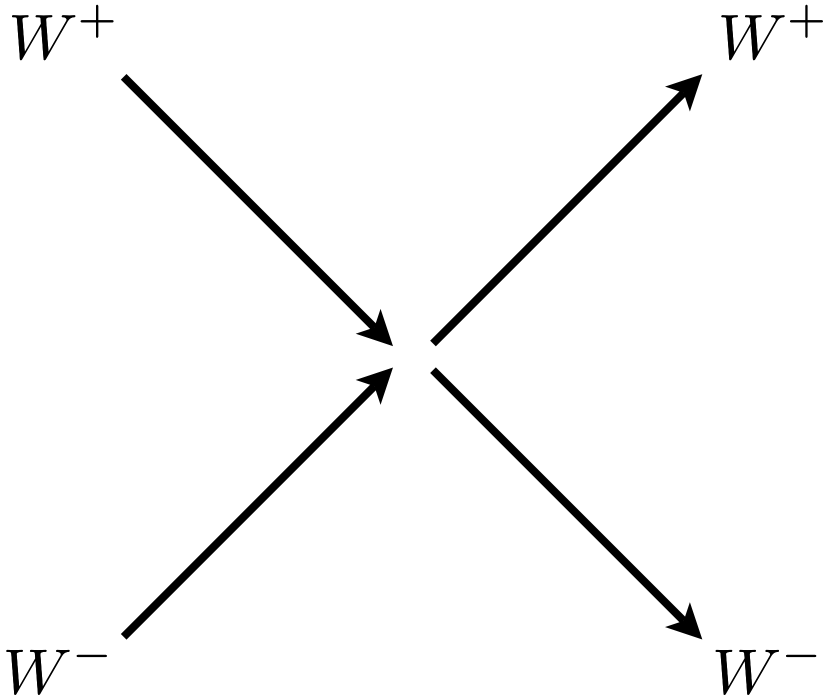
The Higgs:

★ The Unitarity Bound
★ Electroweak Precision

Beyond the Standard Model:

★ Naturalness (the Hierarchy Problem)
★ Dark Matter
★ Why Not?
★ (Unification)

# The Higgs Completes the Standard Model




$$\lim_{E\to\infty} \mathcal{A} \propto E^2$$

At high energies, the probability of scattering is greater than unity.

Theory breaks down at E ~ 1 TeV

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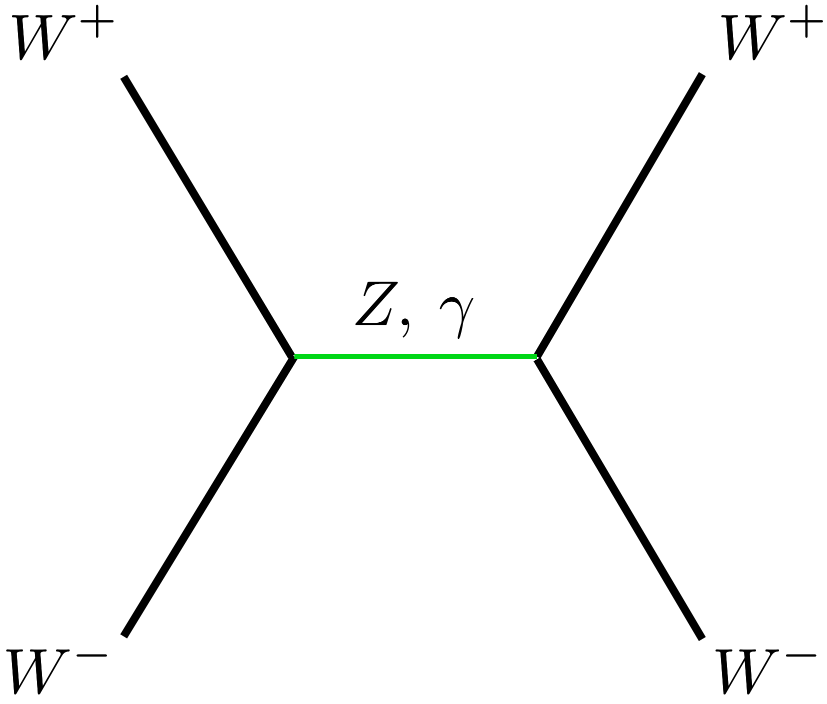


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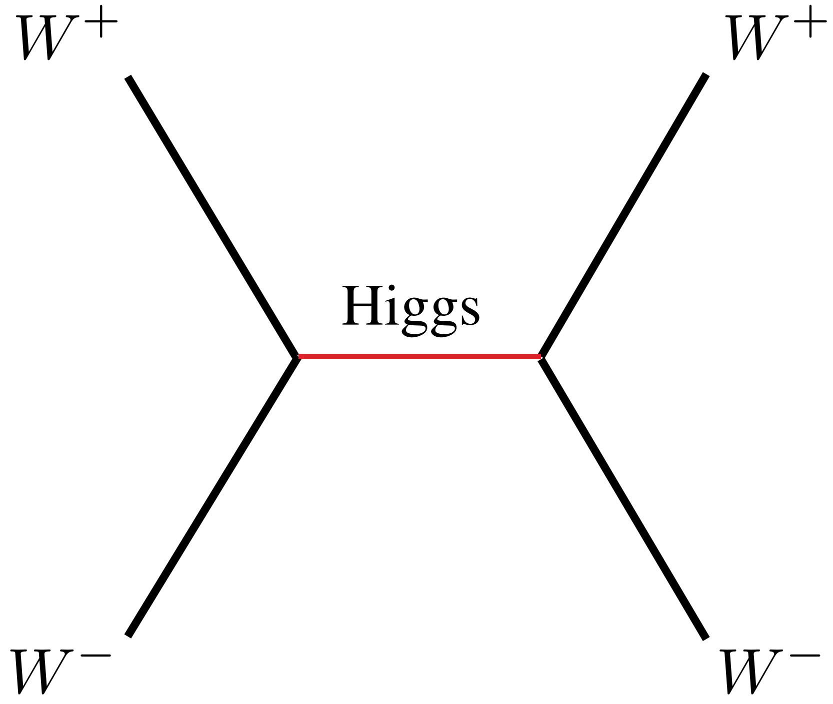


$$\lim_{E \to \infty} \mathcal{A} \propto \text{const.}$$

With the inclusion of the Higgs particle, the theory remains predictive.

Theory requires a Higgs mass < 1 TeV

# Status of the Standard Model

W and Z bosons mediate forces:

Ratio of the strength of the weak force to the electromagnetic force: $\sin^2\theta_w$

The Standard Model predicts:

$$\rho \equiv \frac{M_W}{M_Z \cos\theta_w} = 1$$

1983:

$$\rho_{exp} = 1.0 \pm 0.2$$

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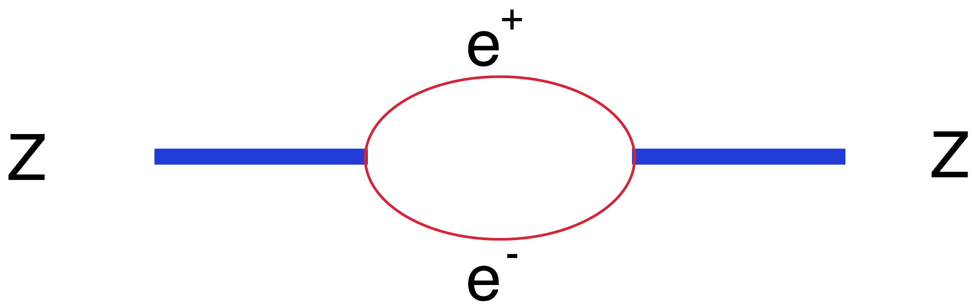
2006:

$$\rho_{exp} = 1.014 \pm 0.002$$

# Status of the Standard Model

Quantum effects:




$$\Delta E \Delta t \sim \hbar$$

The Standard Model predicts:

$$\rho = 1.017 \pm 0.003$$

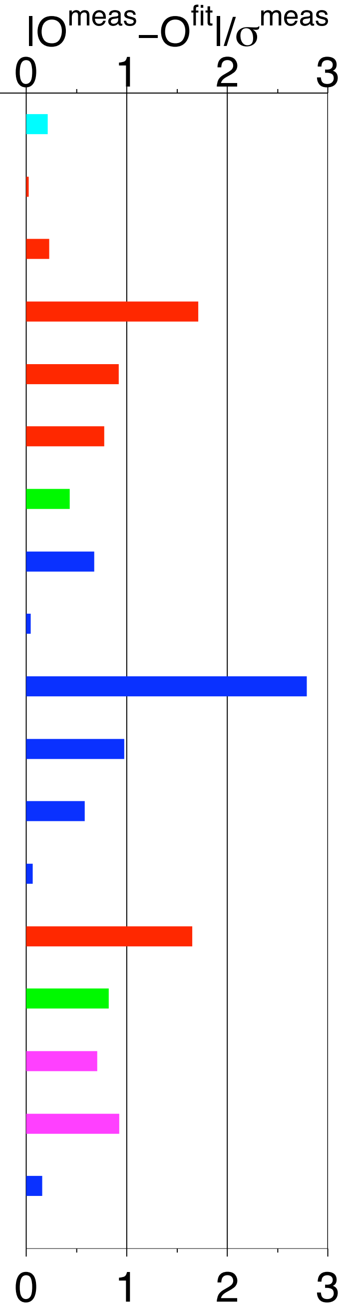
2006:

$$\rho_{exp} = 1.014 \pm 0.002$$

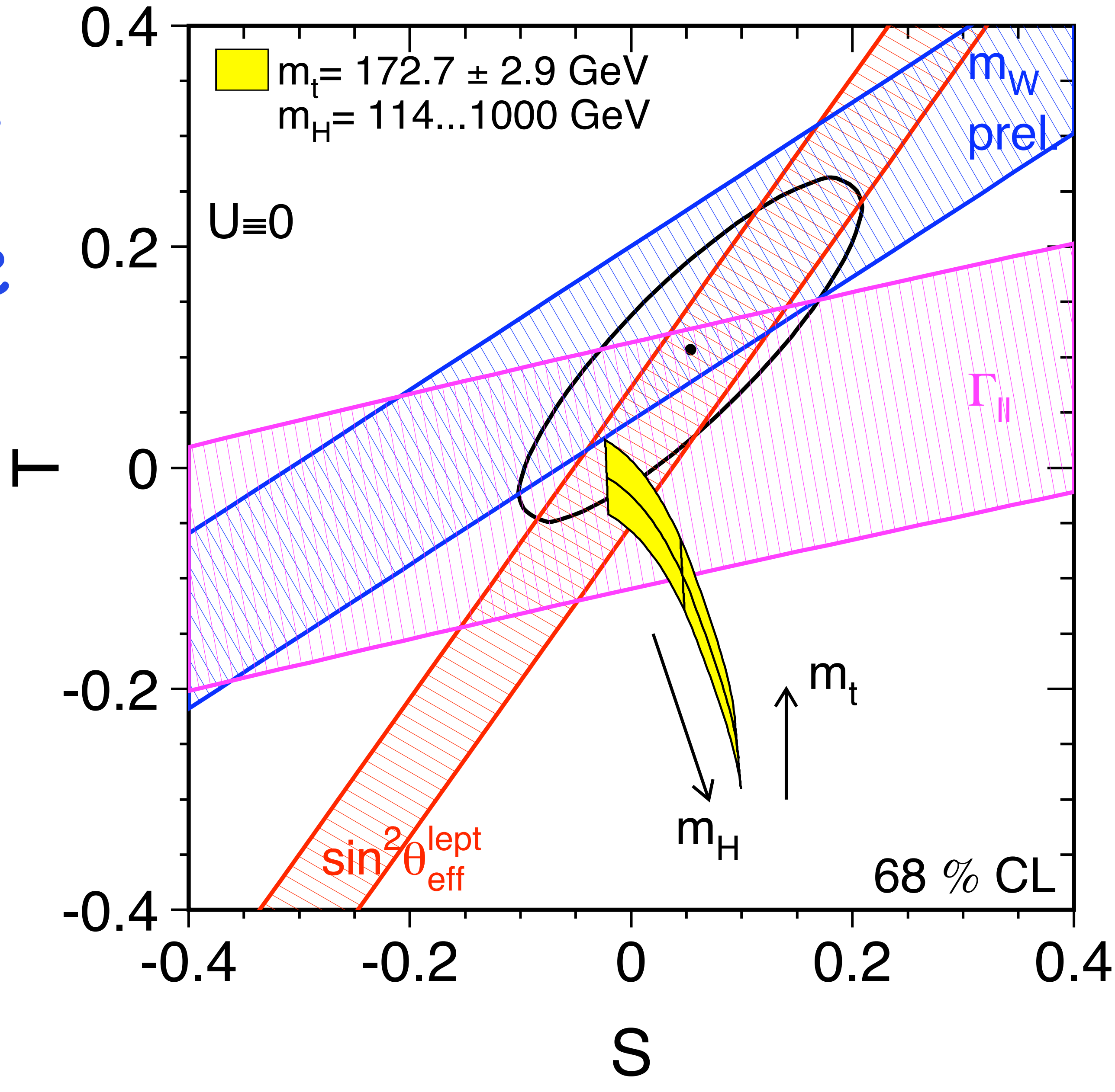
Precision electroweak tests confirm the standard model (with a Higgs!) up to a few parts per mil.

Summer, 2006

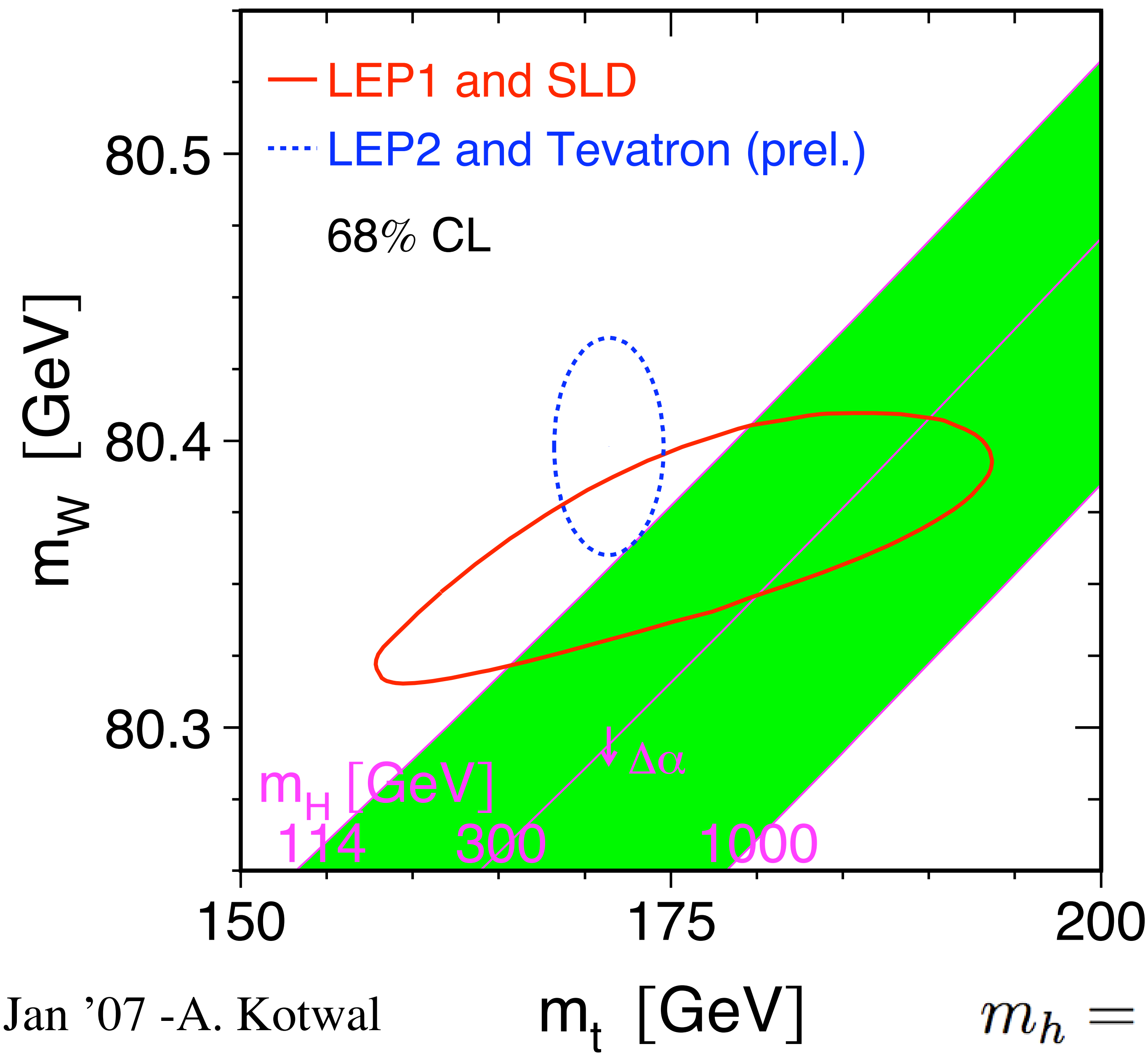
| | Measurement | Fit |
|---|---|---|
| $\Delta\alpha_{had}^{(5)}(m_Z)$ | $0.02758 \pm 0.00035$ | 0.02766 |
| $m_Z$ [GeV] | $91.1875 \pm 0.0021$ | 91.1874 |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | 2.4957 |
| $\sigma_{had}^{0}$ [nb] | $41.540 \pm 0.037$ | 41.477 |
| $R_l$ | $20.767 \pm 0.025$ | 20.744 |
| $A_{fb}^{0,l}$ | $0.01714 \pm 0.00095$ | 0.01640 |
| $A_l(P_\tau)$ | $0.1465 \pm 0.0032$ | 0.1479 |
| $R_b$ | $0.21629 \pm 0.00066$ | 0.21585 |
| $R_c$ | $0.1721 \pm 0.0030$ | 0.1722 |
| $A_{fb}^{0,b}$ | $0.0992 \pm 0.0016$ | 0.1037 |
| $A_{fb}^{0,c}$ | $0.0707 \pm 0.0035$ | 0.0741 |
| $A_b$ | $0.923 \pm 0.020$ | 0.935 |
| $A_c$ | $0.670 \pm 0.027$ | 0.668 |
| $A_l$(SLD) | $0.1513 \pm 0.0021$ | 0.1479 |
| $\sin^2\theta_{eff}^{lept}(Q_{fb})$ | $0.2324 \pm 0.0012$ | 0.2314 |
| $m_W$ [GeV] | $80.392 \pm 0.029$ | 80.371 |
| $\Gamma_W$ [GeV] | $2.147 \pm 0.060$ | 2.091 |
| $m_t$ [GeV] | $171.4 \pm 2.1$ | 171.7 |



# (Old) Indirect Evidence for the Higgs




The mass of the Higgs can even be fit.



LEP1 and SLD
LEP2 and Tevatron (prel.)
68% CL
80.5
80.4
80.3
$m_W$ [GeV]
$\Delta\alpha$
$m_H$ [GeV]
114
300
1000
150
175
200
$m_t$ [GeV]

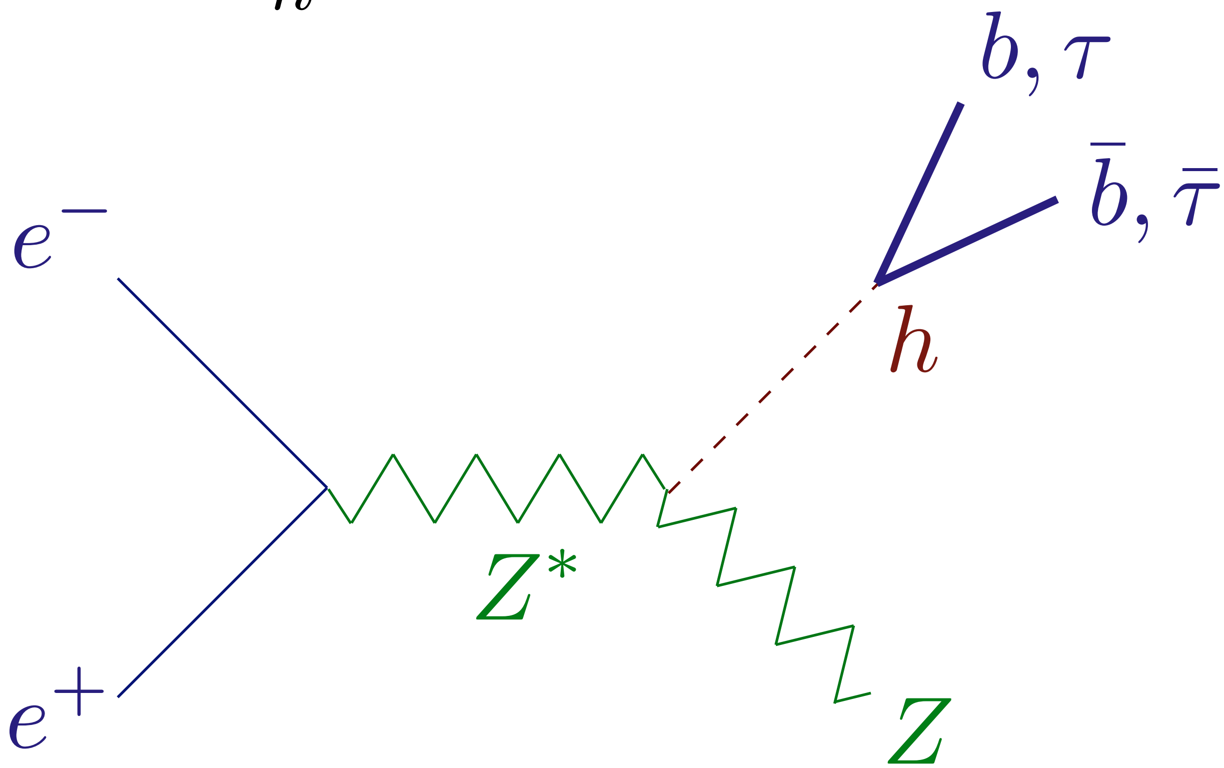



$m_h = 80^{+36}_{-26}$

# Higgs: Direct Search

Dominant bound from LEP II

SM: $$m_h > 114.4\,\mathrm{GeV}$$



# Decay modes of the Higgs

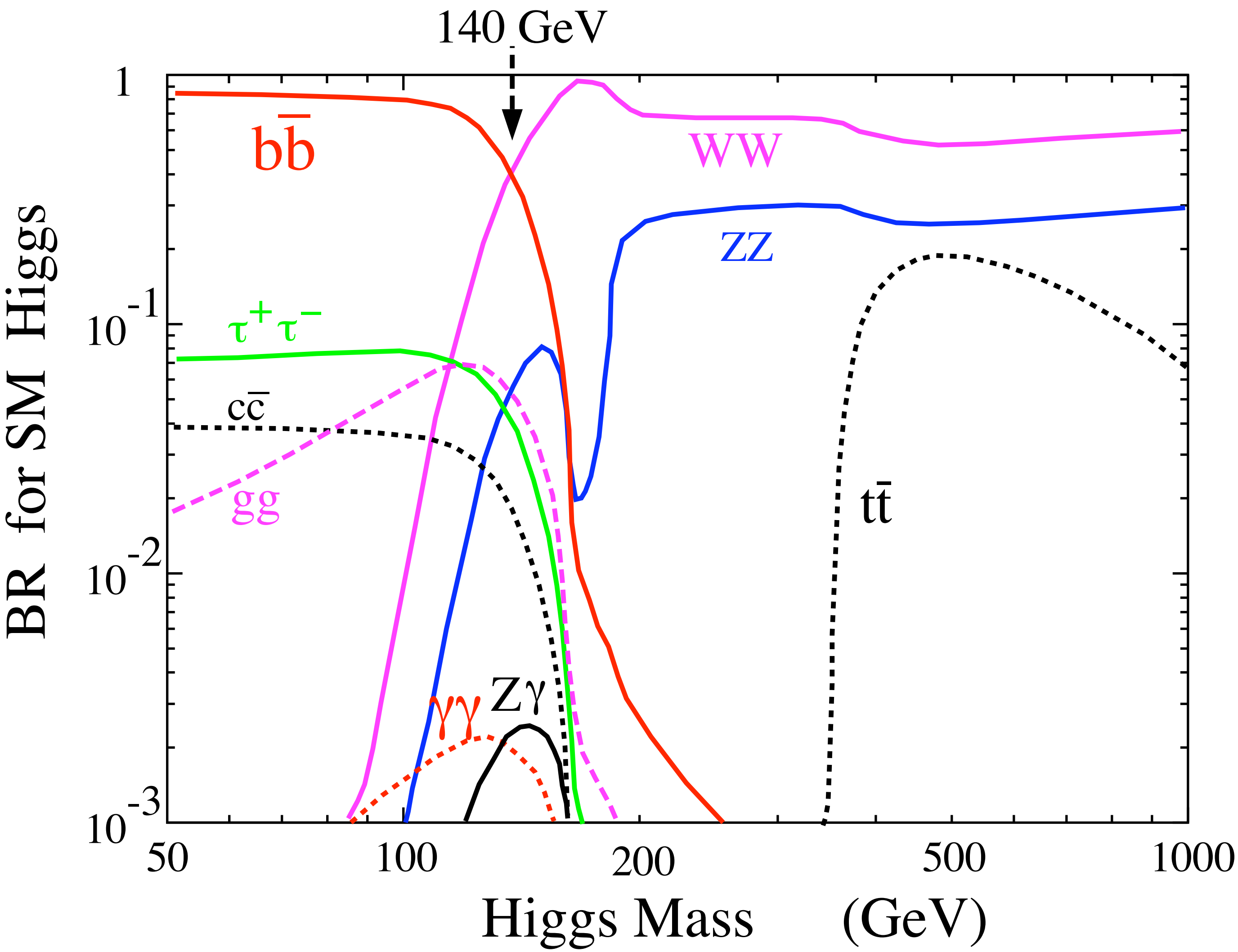
$$\mathcal{H} = \lambda h f \bar{f} + \ldots \qquad\qquad h = v + \delta h$$

$$m_f = \lambda v$$

$$m_W = g v$$

$$\Gamma_{h \to f\bar{f}} \propto \lambda^2 m_h, \quad 2m_f \ll m_h$$

# Decay modes of the Higgs



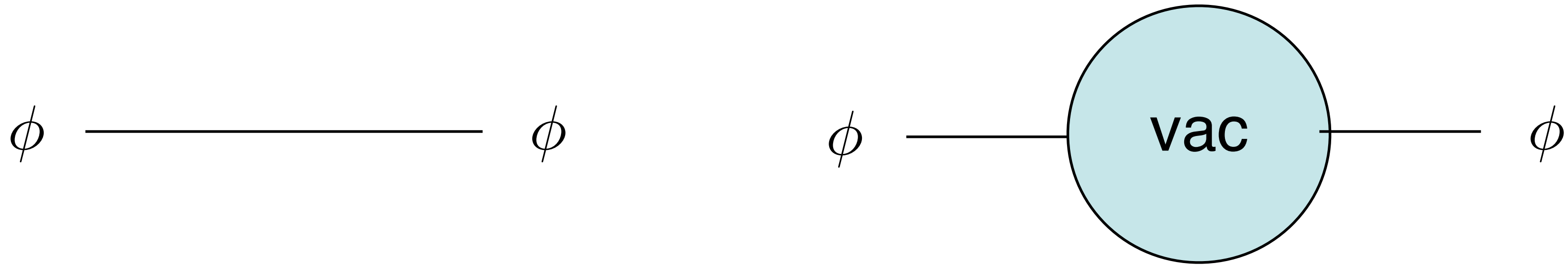
# Naturalness
# (the Hierarchy Problem)

$$m_h^2 = m_{cl}^2 + m_{qt}^2$$

# Quantum Instability

A spin 0 particle in a “dirty” quantum vacuum:

classical quantum




Doing perturbation theory in a quantum field theory

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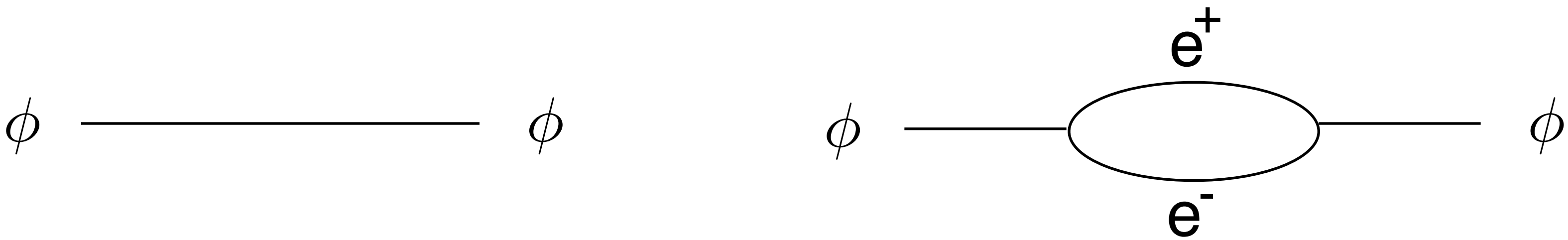


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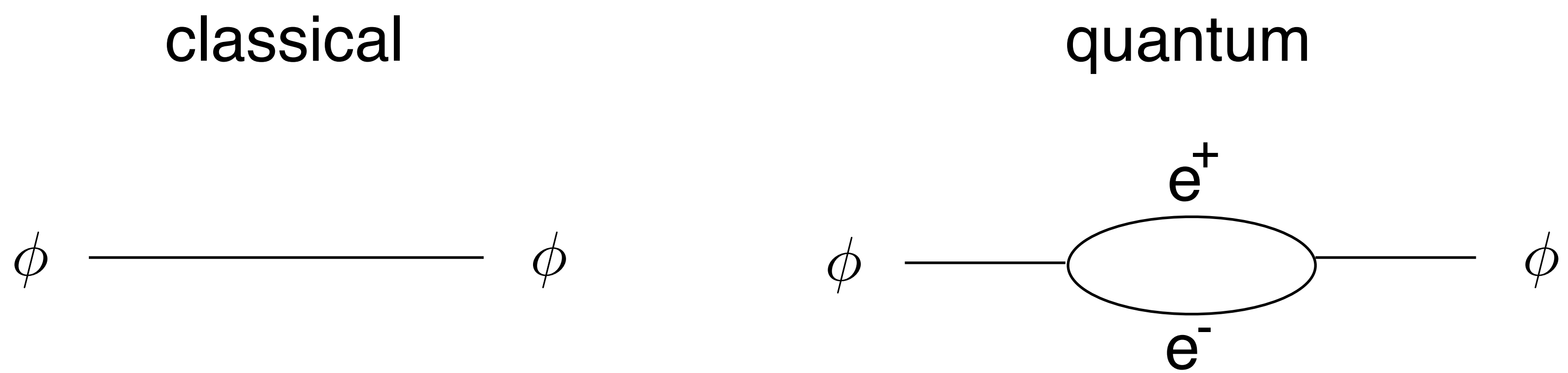
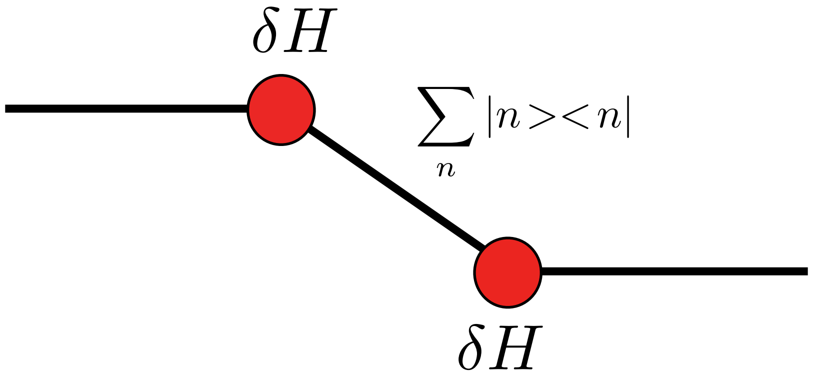


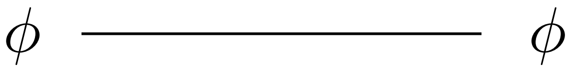




In quantum mechanics, we sum over intermediate states

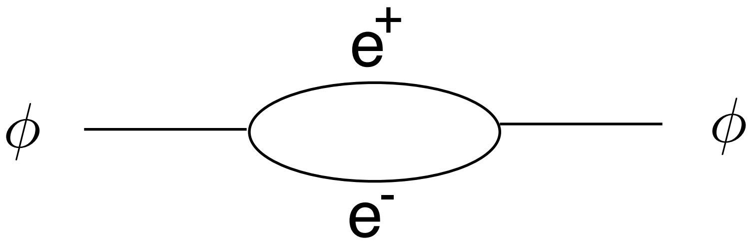
# Quantum Instability

A spin 0 particle in a “dirty” quantum vacuum:

classical

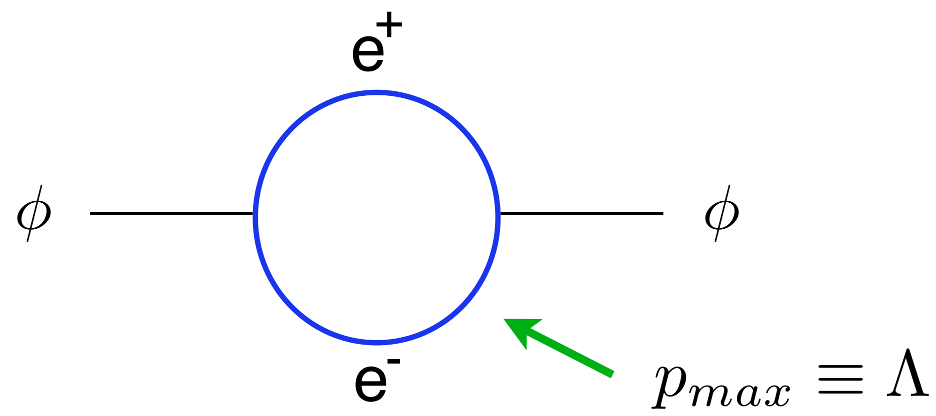



quantum



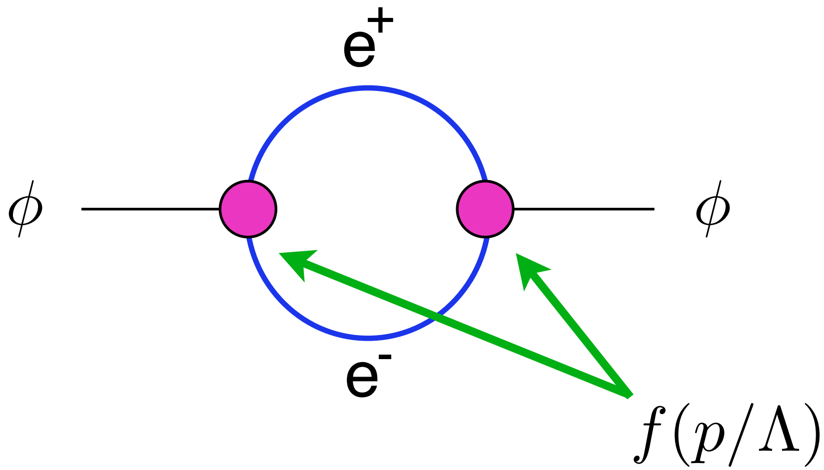

In field theory, we sum over the intermediate energies of the particles in the loop... and get infinity.

# Regulating the Theory




Whatever makes this finite becomes important at energies of order $\Lambda$

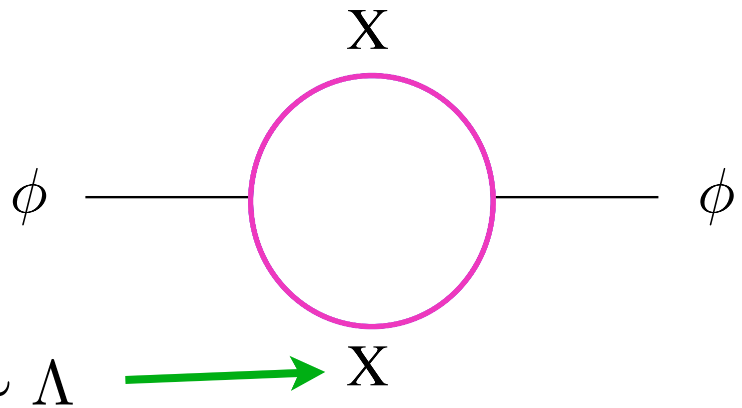
# Regulating the Theory




Whatever makes this finite becomes important at energies of order $\Lambda$

momentum-dependent couplings (compositeness)

# Regulating the Theory

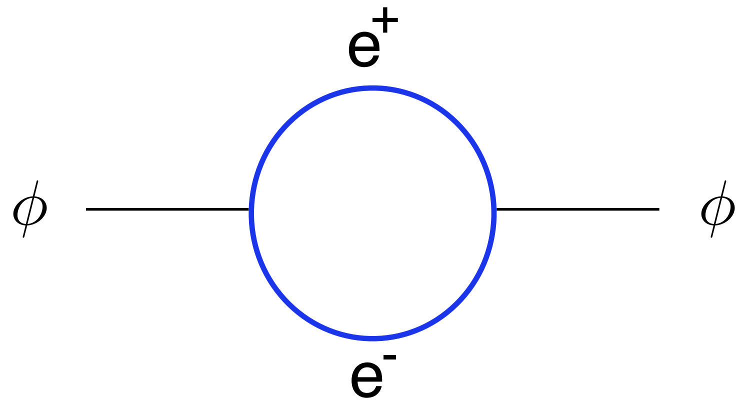



$m_x \sim \Lambda$

Whatever makes this finite becomes important at energies of order $\Lambda$

New particles in the loop

# Regulating the Theory

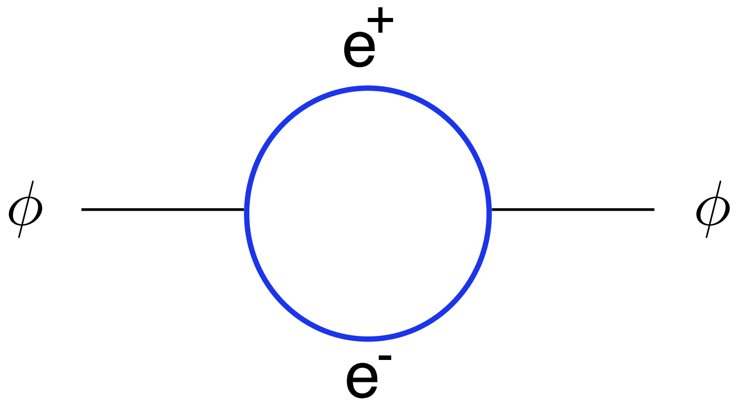



Whatever makes this finite becomes important at energies of order $\Lambda$

$$m_\phi \sim \epsilon\Lambda$$

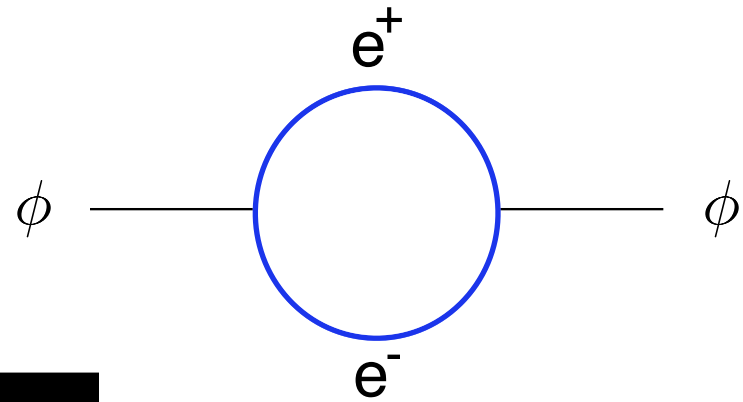
$1/3 > \epsilon > 1/10$, and so the cutoff is $\Lambda \sim 1\ \mathrm{TeV}$

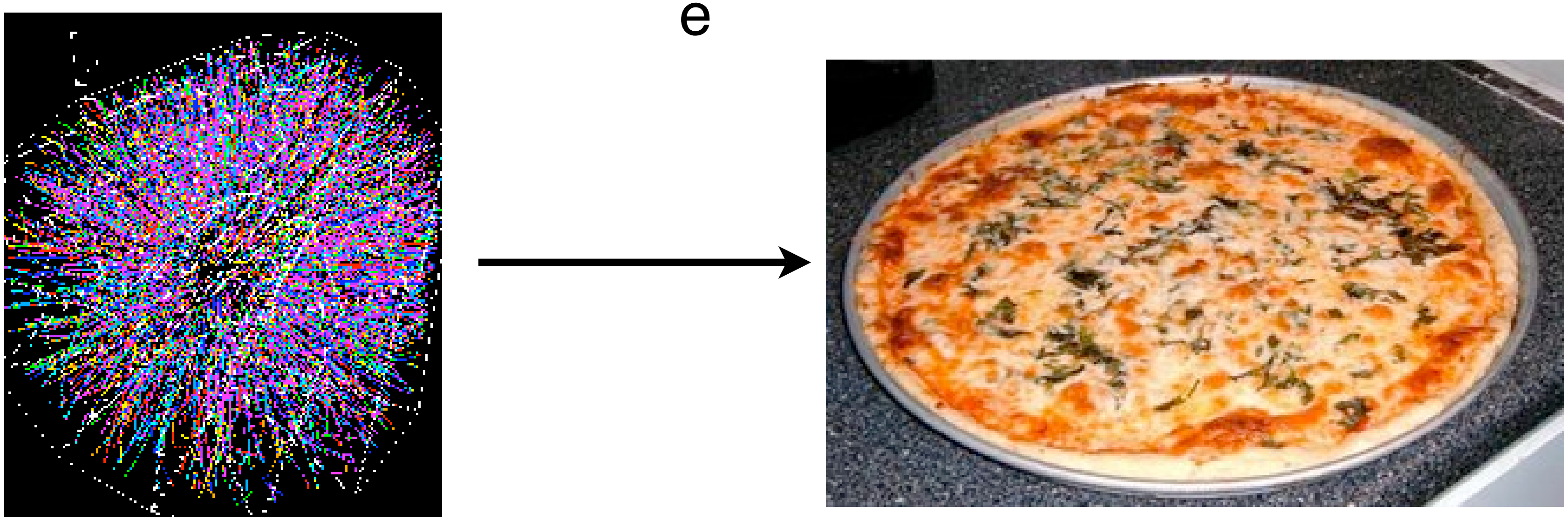
# Regulating the Theory



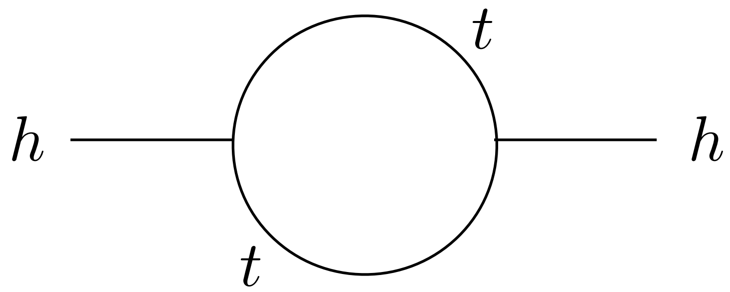

If the cutoff is >> 1 TeV, you are ‘fine tuning’

# Regulating the Theory



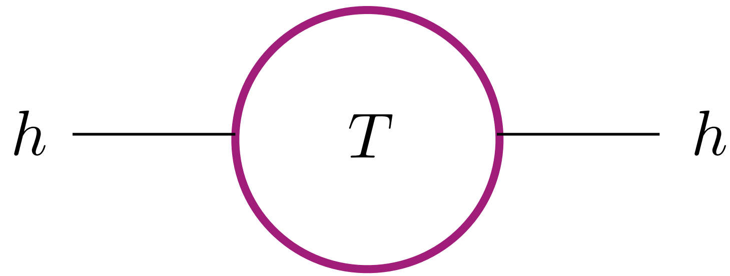



# Model Independent: Weak Coupling




$$\delta m_h^2 \sim -\frac{3}{8\pi^2}\lambda_t^2 \Lambda^2$$

# Model Independent:
# Weak Coupling




$$\delta m_{h(t)} + \delta m_{h(T)} \sim \frac{3}{8\pi^2} \lambda_T^2 M_T^2$$

$$M_T \sim 5 m_h$$

(about 8x and 12x for gauge and Higgs)

# New particles must abide by the experimental constraints

## Preliminary

| | Measurement | Fit |
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| $\Delta\alpha_{had}^{(5)}(m_Z)$ | $0.02758 \pm 0.00035$ | 0.02768 |
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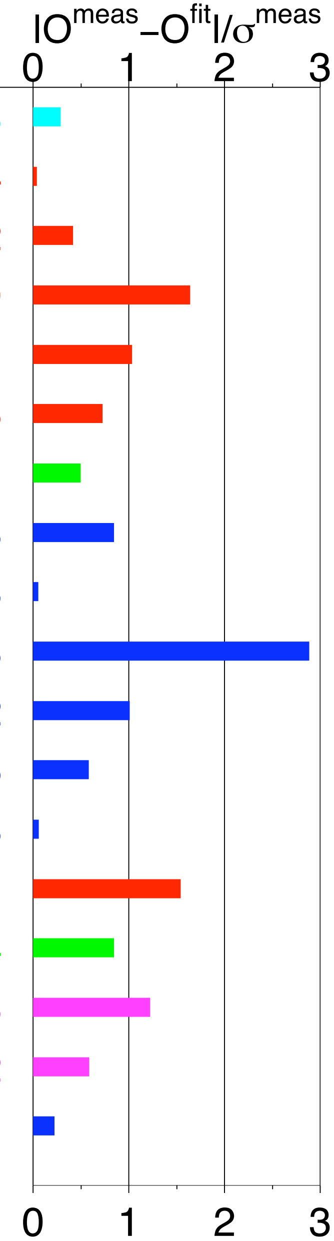
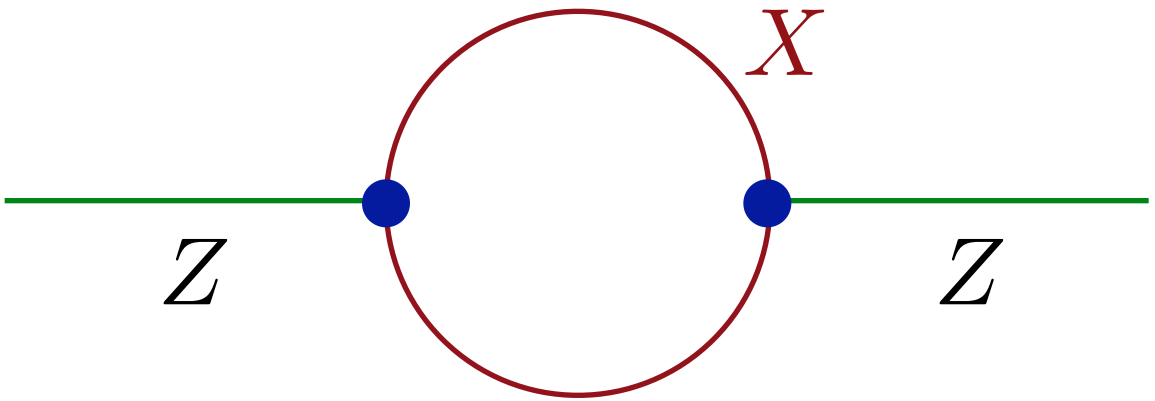


# Few Per Mil Corrections...


$$\delta M_Z^2 \sim \delta (M_Z^0)^2 \left(1 + \frac{M_Z^2}{M_{Z'}^2}\right)$$

$$M_{Z'} > 1 \text{ - } 2 \text{ TeV}$$

# Few Per Mil Corrections...

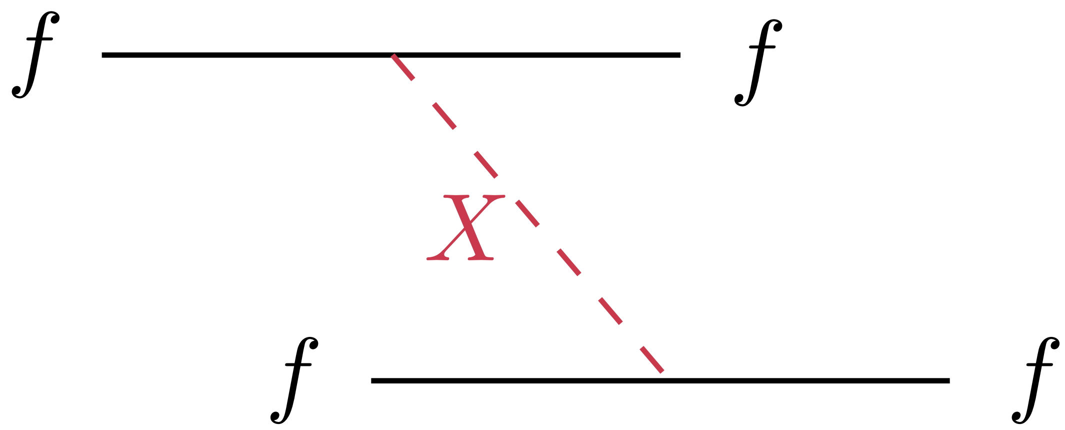



$$\delta M_Z^2 \sim \delta (M_Z^0)^2 \left(1 + \frac{g^2}{16\pi^2} \frac{M_Z^n}{M_X^n}\right)$$

$$M_X > 100 \text{ GeV}$$

# Scalar field exchange

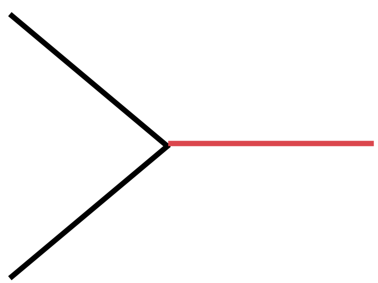
Can mediate flavor-changing neutral currents, or even proton decay.



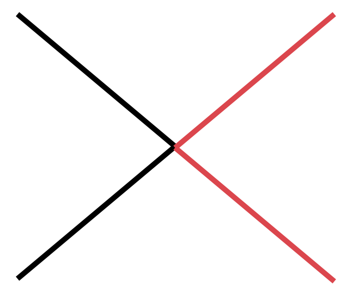

Eliminating single field couplings again can avoid these problems.

# Low Energy Tests
# -> A New Parity?

Single couplings can be avoided if a parity exists under which some or all new fields are “odd” under this symmetry.



FORBIDDED



ALLOWED

The lightest parity odd particle cannot decay.

# Weak Coupling Solution: Implications

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New particle masses at 100s of GeV

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Large missing transverse momentum.

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Large missing transverse momentum.

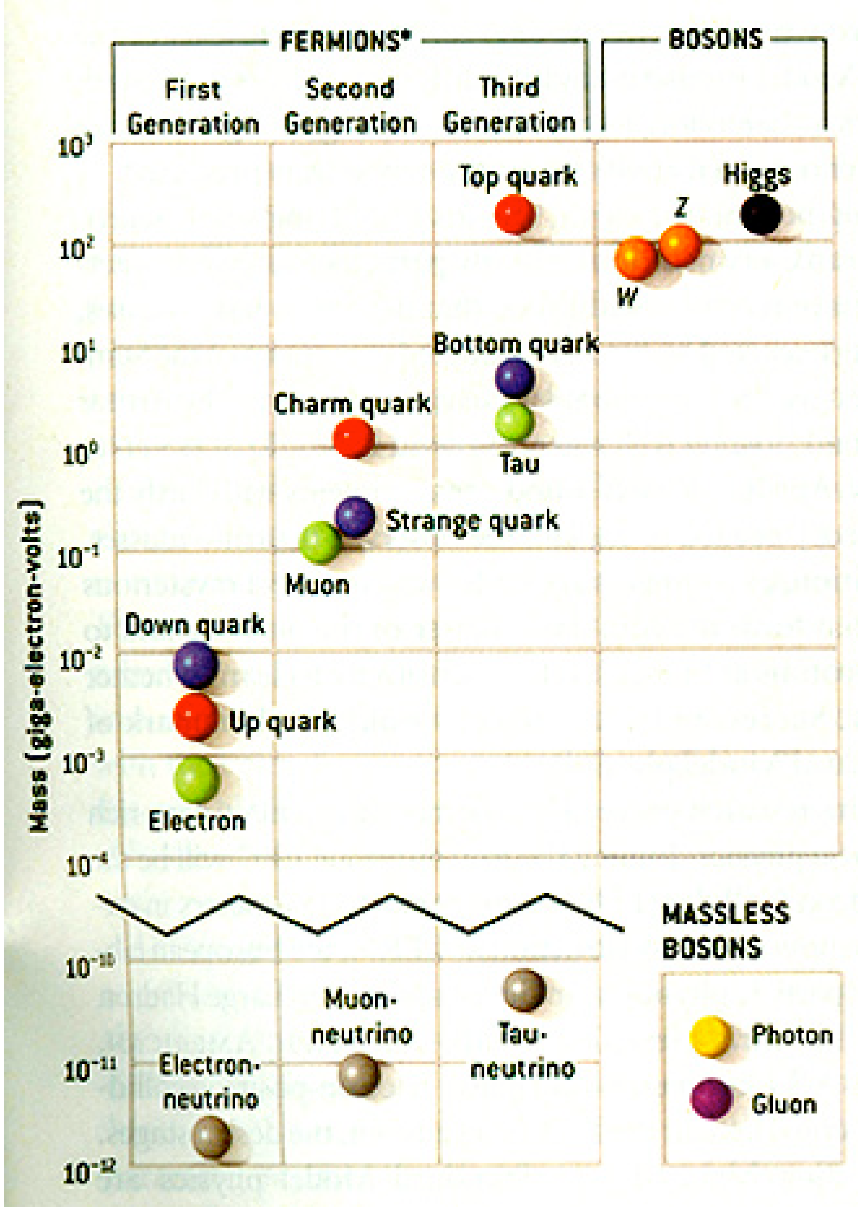
Dark Matter Particle?

# Naturalness
# (the Hierarchy Problem)

It all starts with the Higgs being a scalar particle (spin 0)

# The Standard Model

We have never seen a fundamental spin-0 particle



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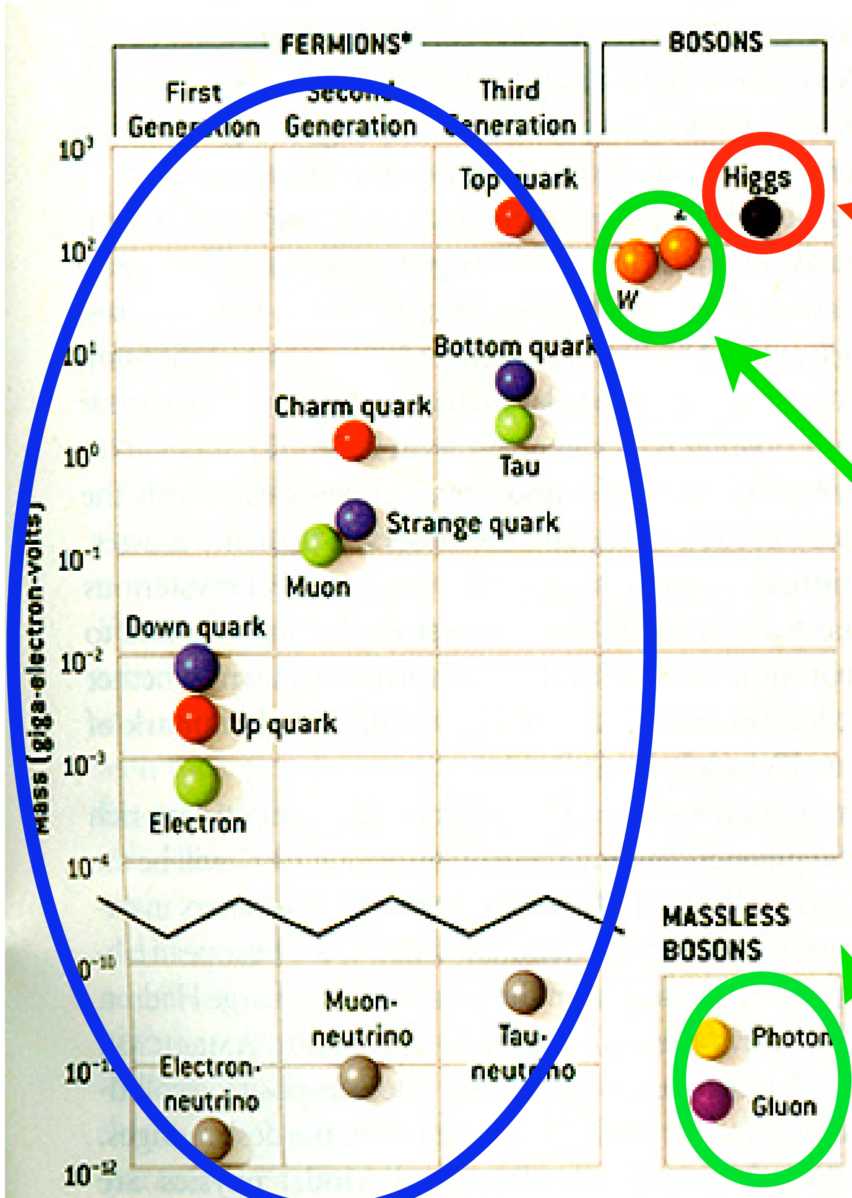
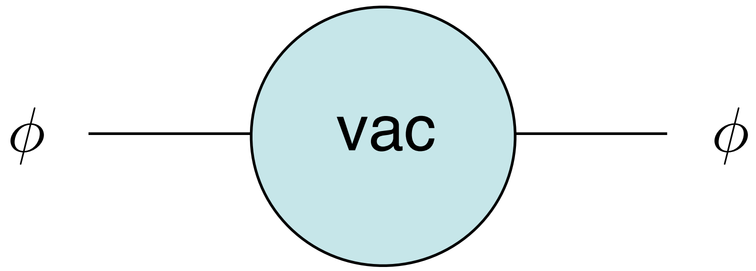



Spin 1/2

Spin 0 (Unseen)

Spin 1

# "Scalar" - Spin 0




$$m_\phi \sim \Lambda$$

This is the energy scale where the theory breaks down.

Expansion parameter: $E/\Lambda$

When we have the energy to produce the Higgs, we are already probing the underlying theory.

# Non-zero Spin Particles

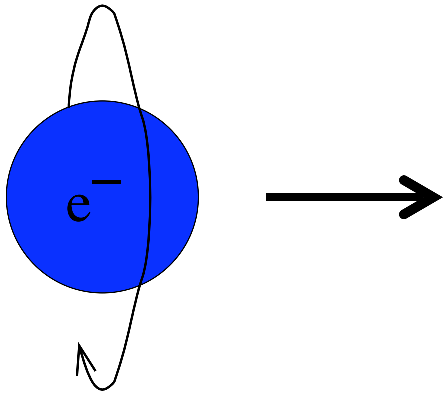
What about other SM particles?

$$m_e = .511\,\mathrm{MeV}$$

And there is nothing new happening right above the electron mass $(m_\mu = 106\,\mathrm{MeV})$

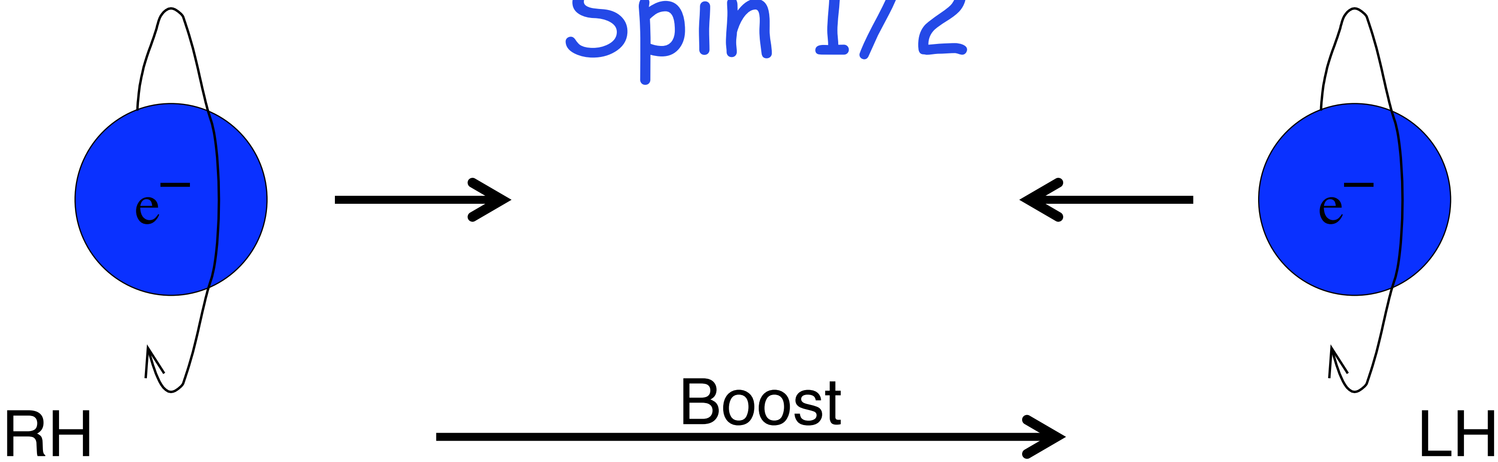
Why is the electron mass much smaller than the cutoff?

# Spin 1/2




RH

# Spin 1/2



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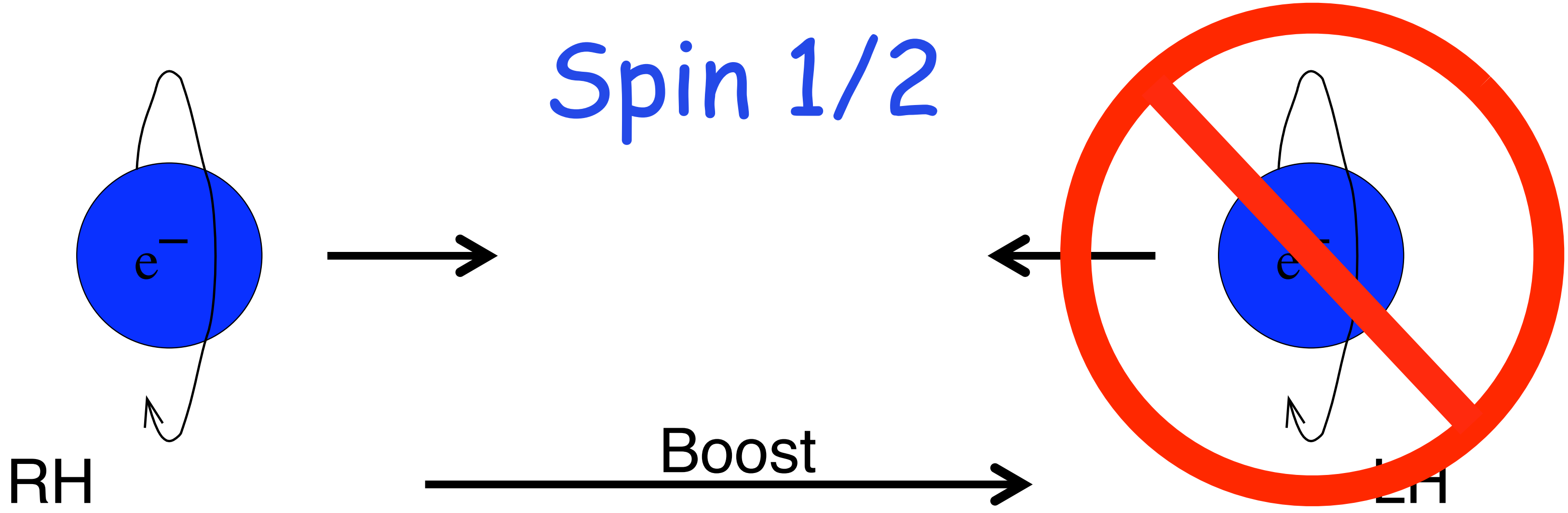
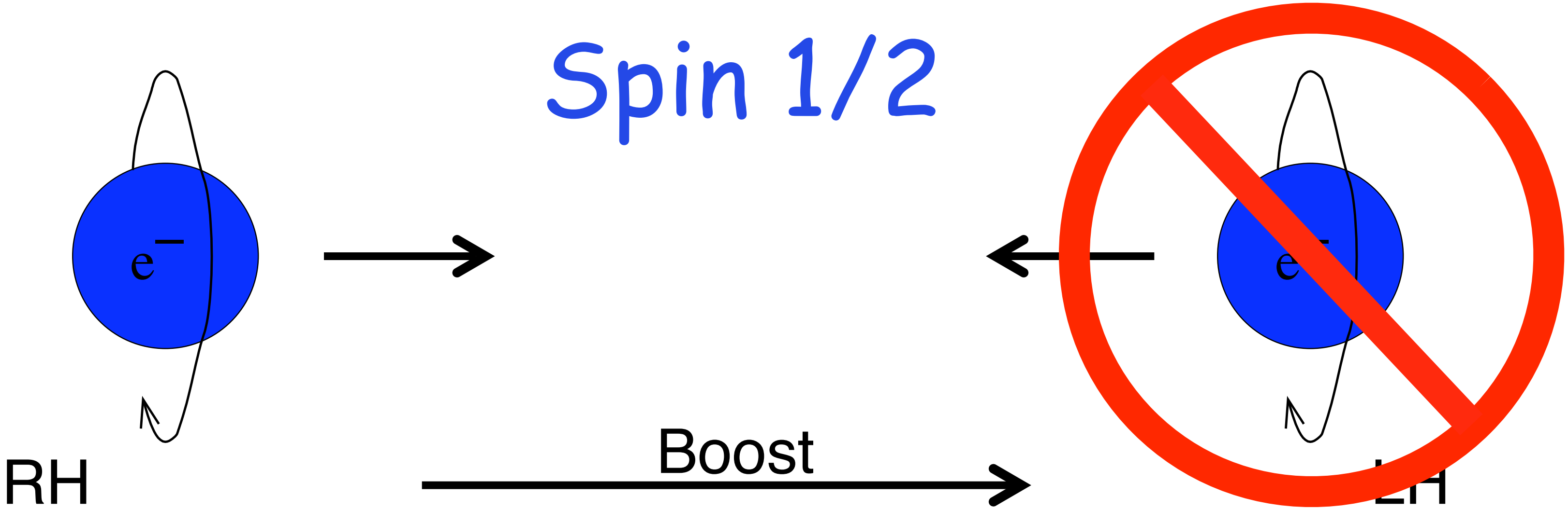



If the electron was massless, it would travel the speed of light and the two would be independent states.

# Spin 1/2

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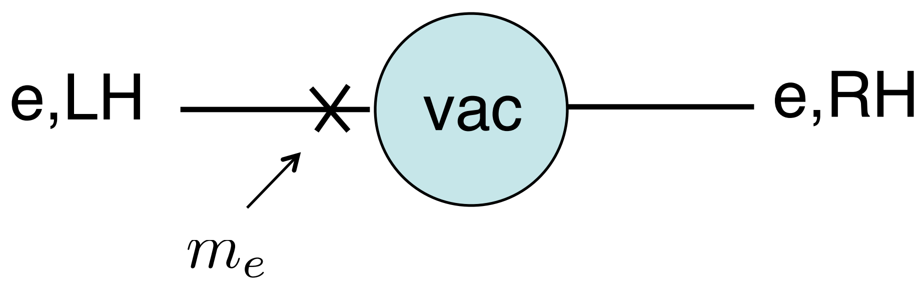
If $m_e \ll \Lambda$, then one can use perturbation theory



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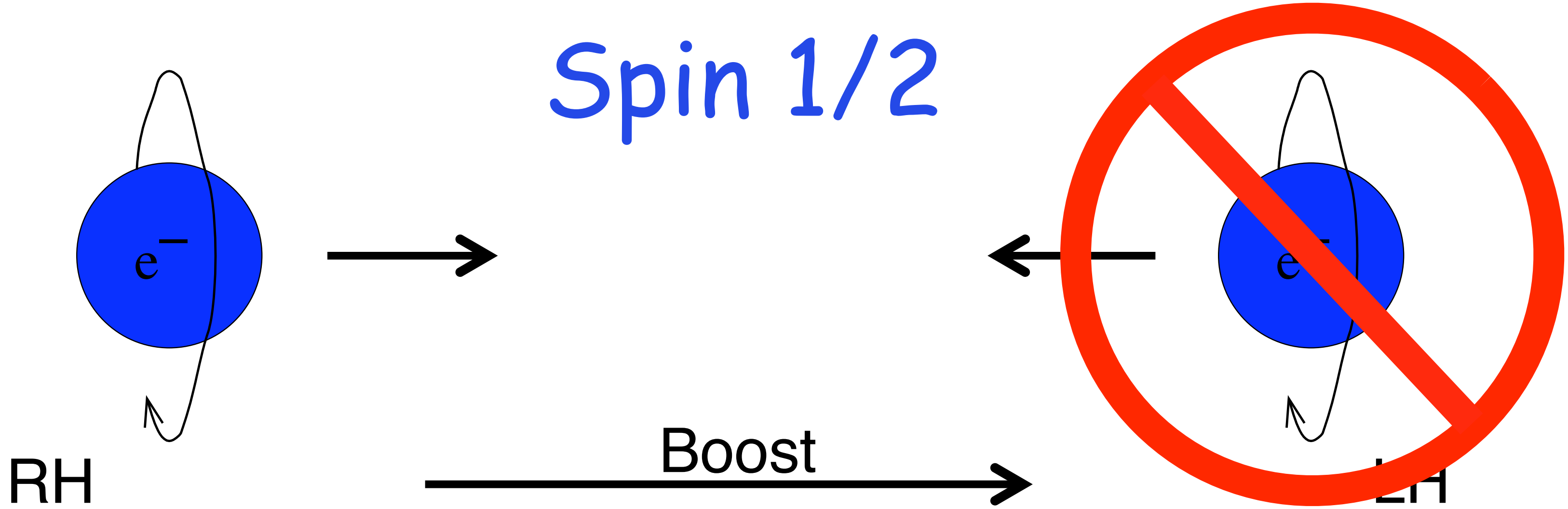


"Chiral Symmetry"

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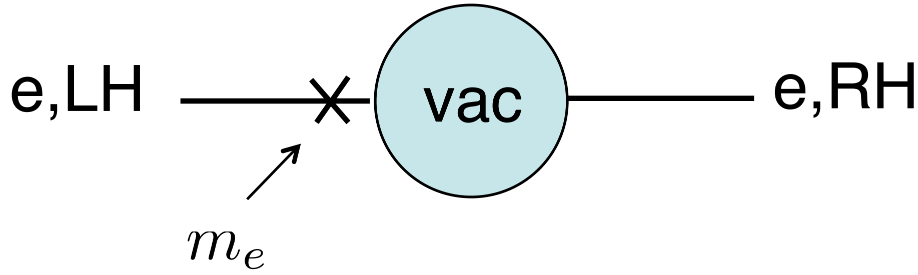

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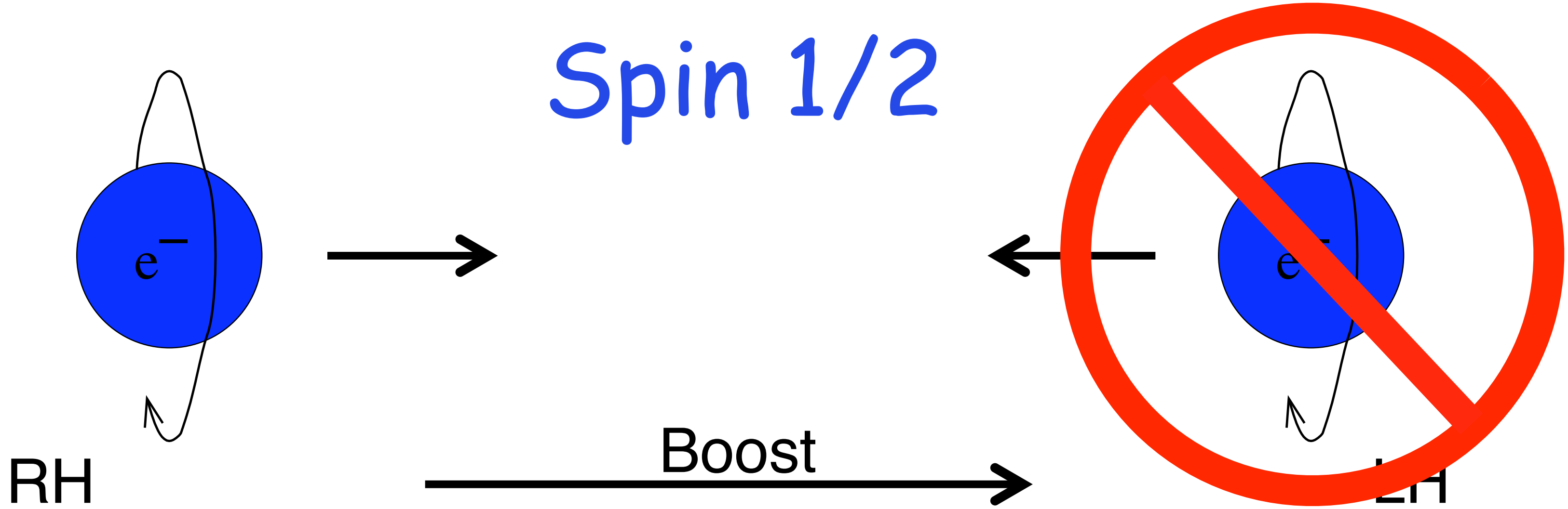
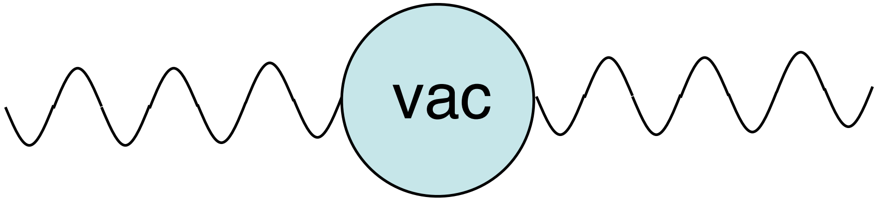



“Chiral Symmetry”

If $m_e \ll \Lambda$, then one can use perturbation theory

$$\delta m_e \sim \epsilon\, m_e \ln \Lambda \qquad \text{(dimensional analysis)}$$

# The Photon




Quantum corrections,

yet massless!

A massless photon has 2

polarizations (left- and right-circular)

$$\begin{pmatrix} k \\ 0 \\ 0 \\ k \end{pmatrix}$$

A massive photon (spin-1 particle)

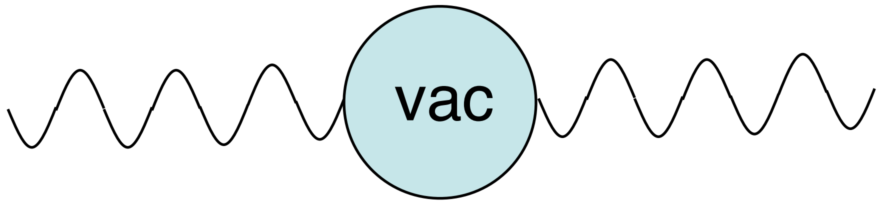
Has 3 spin states: $s_z = +1,\ 0,\ -1$

$$\begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Something dramatic happens in the vacuum to add an

additional degree of freedom.

# The Photon




Quantum corrections,

yet massless!

"Gauge Invariance"

Something dramatic happens in the vacuum to add an additional degree of freedom.

# Symmetries for the Spinless

- Extending spacetime symmetries (2 possibilities)

- Spontaneous symmetry breaking (Goldstone's Theorem)

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Supersymmetry is one of these.

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# Symmetries Between Particles with Different Spins

Chiral

Symmetry

Spin 0 $\longleftrightarrow$ Spin 1/2

# Symmetries Between Particles with Different Spins

Chiral Symmetry

Supersymmetry

Spin 0 $\longleftrightarrow$ Spin 1/2

# Symmetries Between Particles with Different Spins

Chiral Symmetry

Supersymmetry

Spin 0 ⟷ Spin 1/2

Bosons ⟷ Fermions

# How to Add Supersymmetry

$$e^- \longrightarrow \widetilde{e}^-$$

$$\gamma \longrightarrow \widetilde{\gamma}$$

$$q \longrightarrow \widetilde{q}$$

$$W \longrightarrow \widetilde{W}$$

$$h \longrightarrow \widetilde{h}$$

# Simple Example

$$\mathcal{L} \;=\; -|\partial\phi|^2 - i\psi^\dagger\bar{\sigma}\cdot\partial\psi - |m\phi|^2 - \frac{1}{2}(m\psi\psi + \mathrm{h.c.})$$

Invariant under:

$$\delta\phi = \epsilon\psi \qquad \delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi$$

# Simple Example

$$\mathcal{L} \;=\; -|\partial\phi|^2 - i\psi^\dagger\bar{\sigma}\cdot\partial\psi - |m\phi|^2$$

$$-\frac{1}{2}(m + y\phi)\psi\psi - \text{h.c.} - \frac{1}{2}m^* y|\phi|^2\phi - \text{h.c.} + \frac{1}{4}|y\phi^2|^2$$

Invariant under:

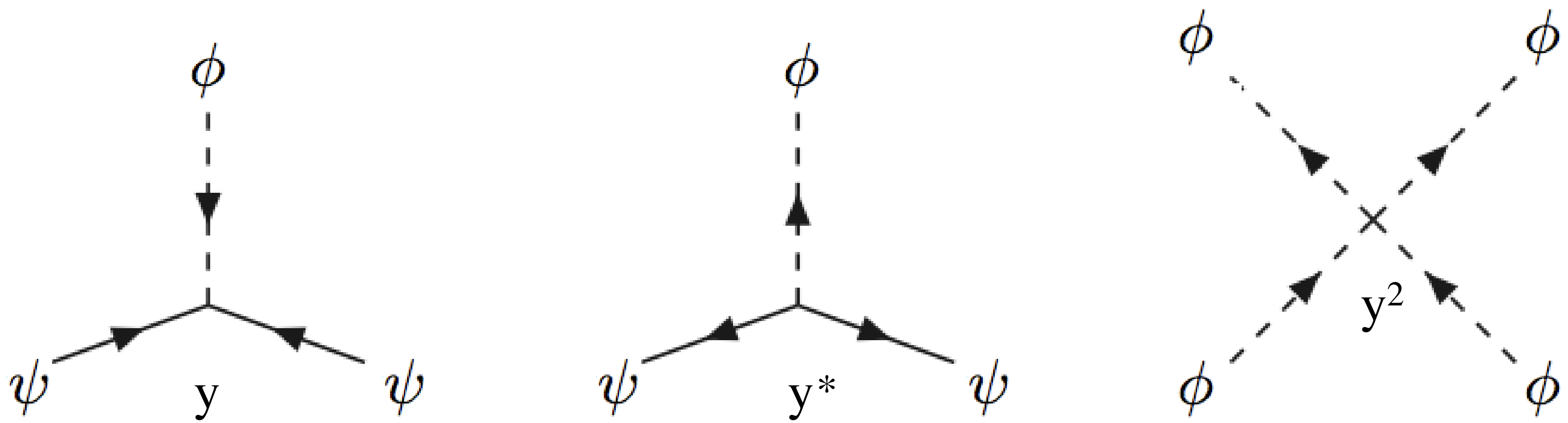
$$\delta\phi = \epsilon\psi \qquad\qquad \delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi$$

# Implications

$\phi$ complex - two bosons, spin 0

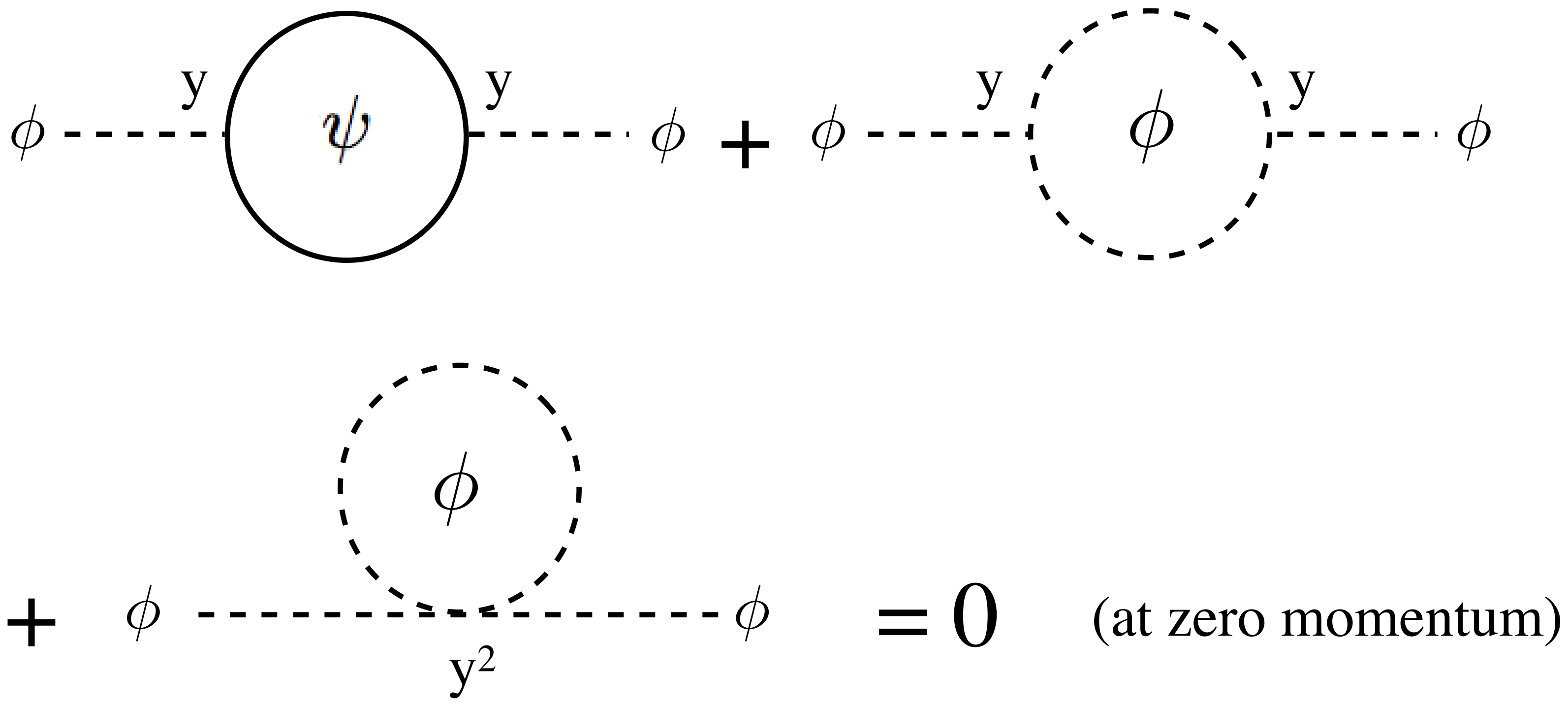
$\psi$ Weyl spinor - two fermions, spin 1/2

All particles of mass *m* - degenerate




couplings related

# Canceling contributions



# Getting a supersymmetric Lagrangian

$$W = \frac{1}{2} m\phi^2 + \frac{1}{6} y\phi^3$$

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# Getting a supersymmetric Lagrangian

"Superpotential"

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fermion partner of phi

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# The Superpotential

You pick a superpotential W, generate the potential V using the rules below, and you have a supersymmetric theory.

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$$

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# Homework

Two pairs of particles, A and B, with superpotential:

$$W = \frac{1}{2} m A^2 + \frac{1}{2} M B^2 + \frac{1}{6} y A^2 B$$

with M>>m

What are the Feynman rules (in terms of couplings only)?

How does the B-fermion decay? How does the B-boson decay? Extra credit: are the widths the same?

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# Constructing the Supersymmetric SM

| Names | | spin 0 | spin 1/2 | $SU(3)_C, SU(2)_L, U(1)_Y$ |
|---|---|---|---|---|
| squarks, quarks | $Q$ | $(\widetilde{u}_L \;\; \widetilde{d}_L)$ | $(u_L \;\; d_L)$ | $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ |
| (×3 families) | $\overline{u}$ | $\widetilde{u}^*_R$ | $u^\dagger_R$ | $(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ |
| | $\overline{d}$ | $\widetilde{d}^*_R$ | $d^\dagger_R$ | $(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$ |
| sleptons, leptons | $L$ | $(\widetilde{\nu} \;\; \widetilde{e}_L)$ | $(\nu \;\; e_L)$ | $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ |
| (×3 families) | $\overline{e}$ | $\widetilde{e}^*_R$ | $e^\dagger_R$ | $(\mathbf{1}, \mathbf{1}, 1)$ |
| Higgs, higgsinos | $H_u$ | $(H^+_u \;\; H^0_u)$ | $(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$ | $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ |
| | $H_d$ | $(H^0_d \;\; H^-_d)$ | $(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$ | $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ |

| Names | spin 1/2 | spin 1 | $SU(3)_C, SU(2)_L, U(1)_Y$ |
|---|---|---|---|
| gluino, gluon | $\widetilde{g}$ | $g$ | $(\mathbf{8}, \mathbf{1}, 0)$ |
| winos, W bosons | $\widetilde{W}^\pm \;\; \widetilde{W}^0$ | $W^\pm \;\; W^0$ | $(\mathbf{1}, \mathbf{3}, 0)$ |
| bino, B boson | $\widetilde{B}^0$ | $B^0$ | $(\mathbf{1}, \mathbf{1}, 0)$ |