Demystifying QUANTUM MECHANICS

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Will there be hints from LHC ?

CERN Colloquium
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The Standard Model

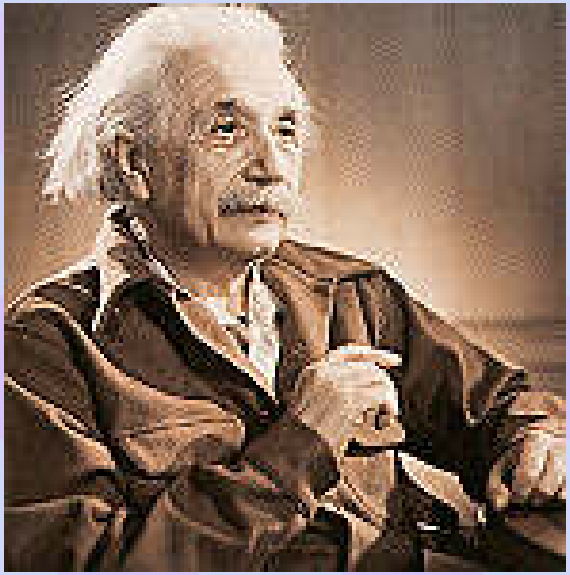
	Generation I	Generation II	Generation III
Leptons	ν_e e	ν_μ μ	ν_τ τ
Quarks	u u u d d d	c c c s s s	t t t b b b

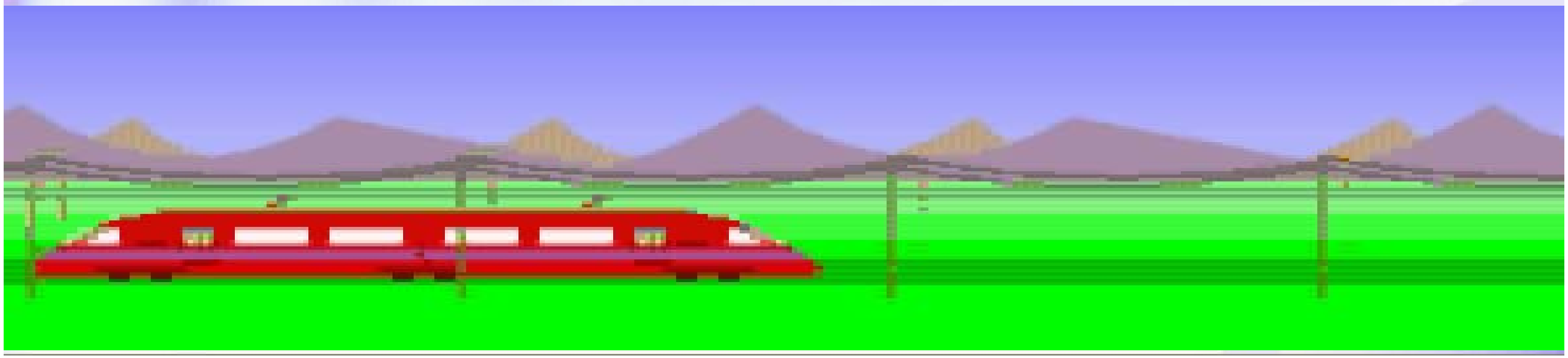
Gauge Bosons: Z^0 W^+ W^- γ | g

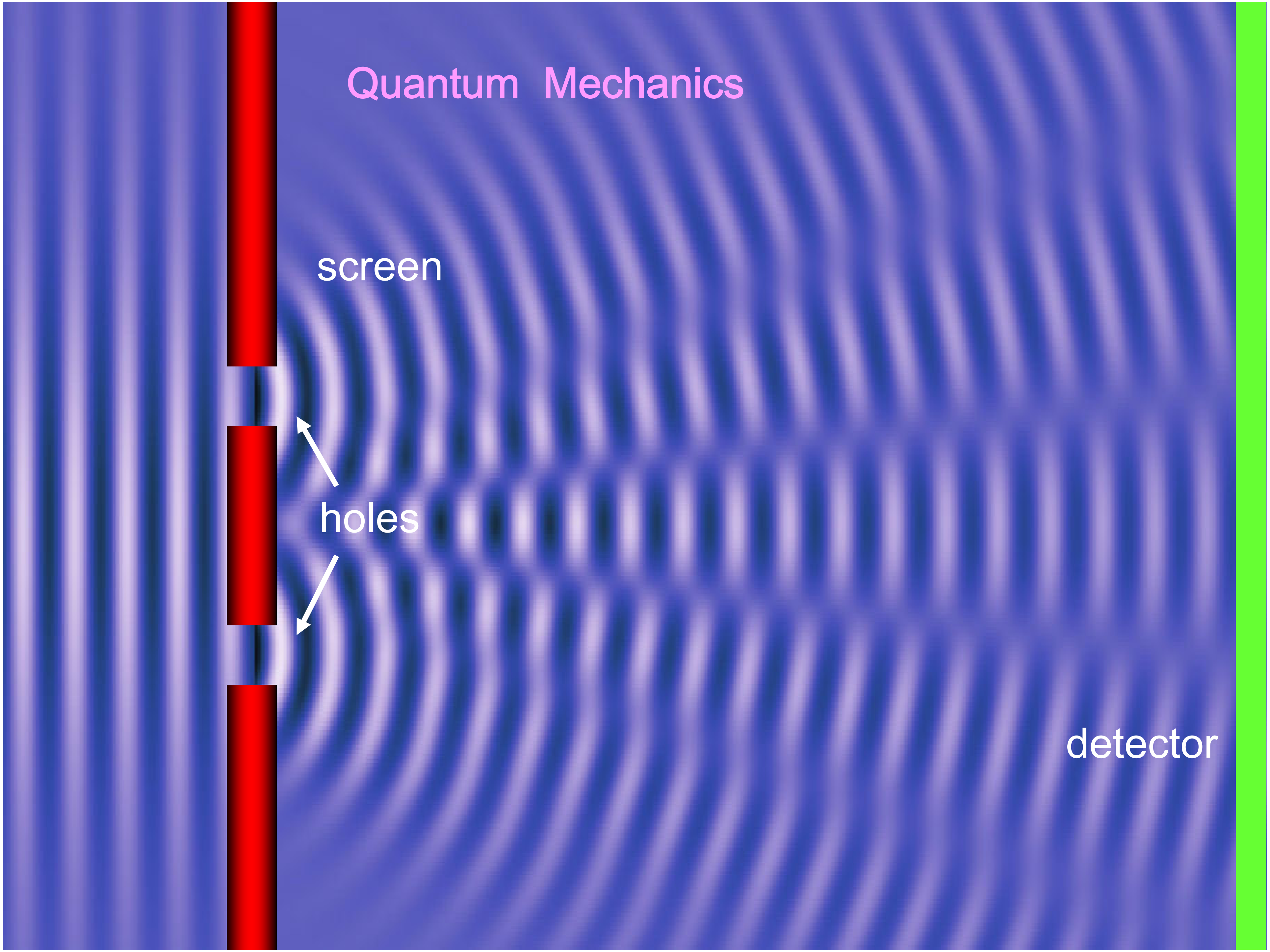
Higgs

Special Relativity

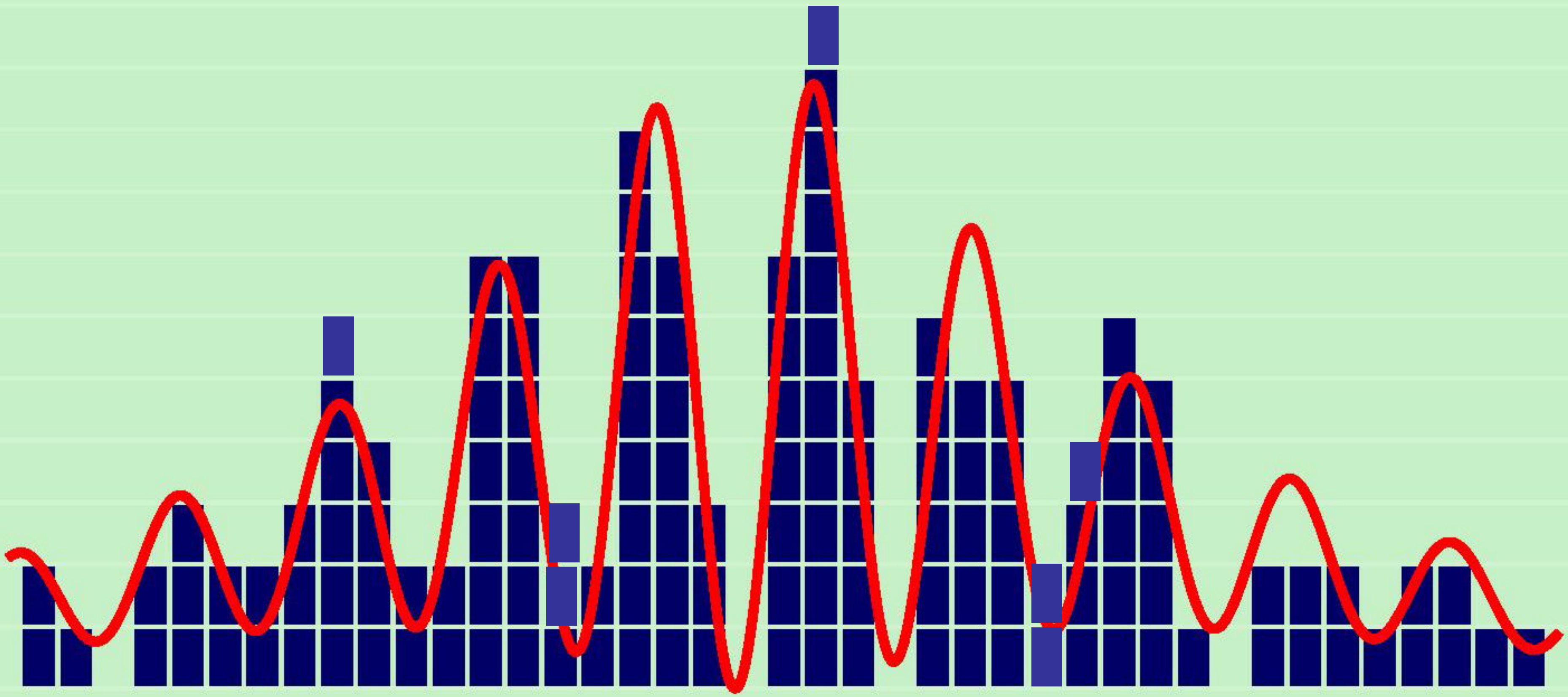
$v / c = 7 / 8$

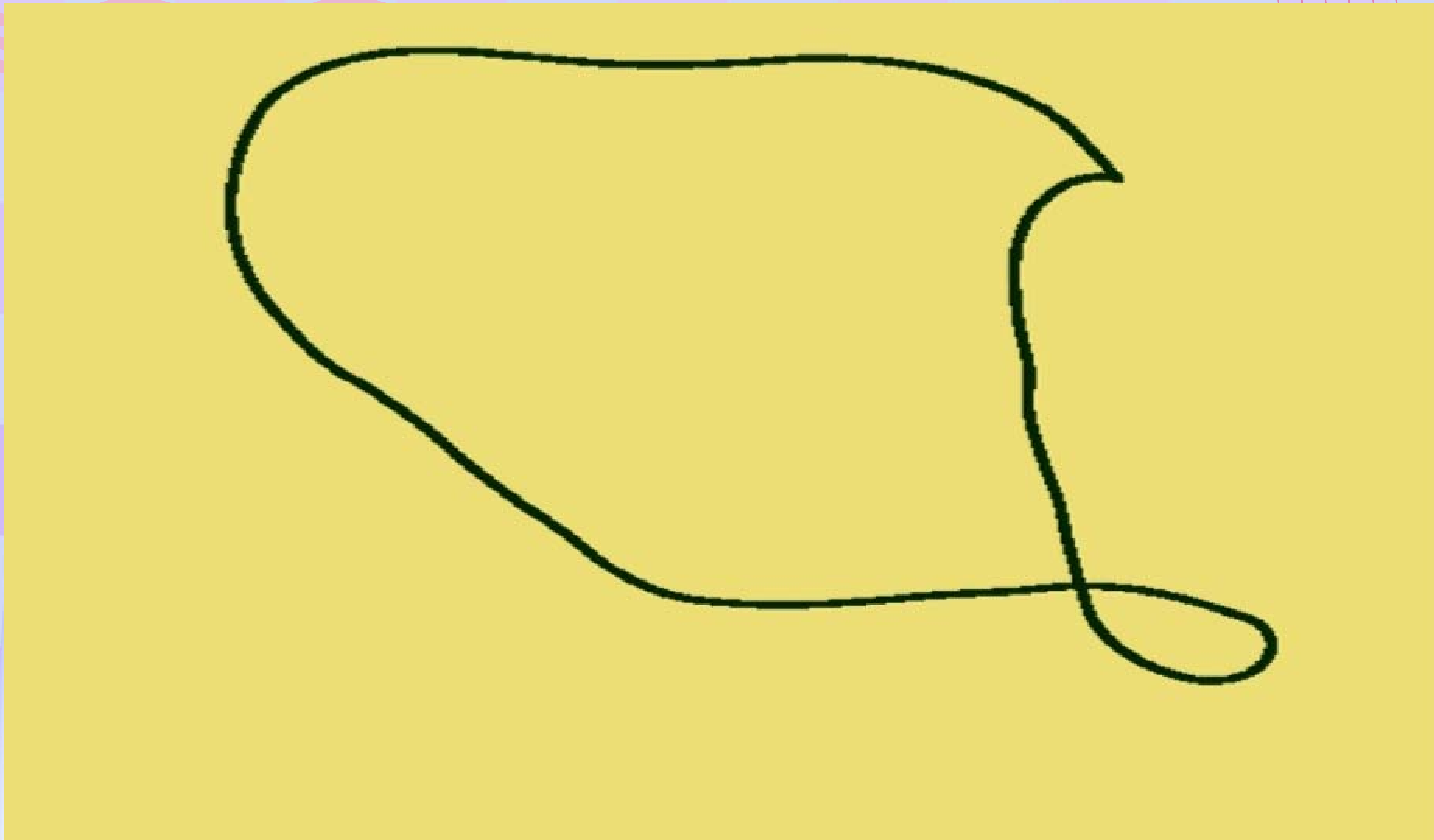




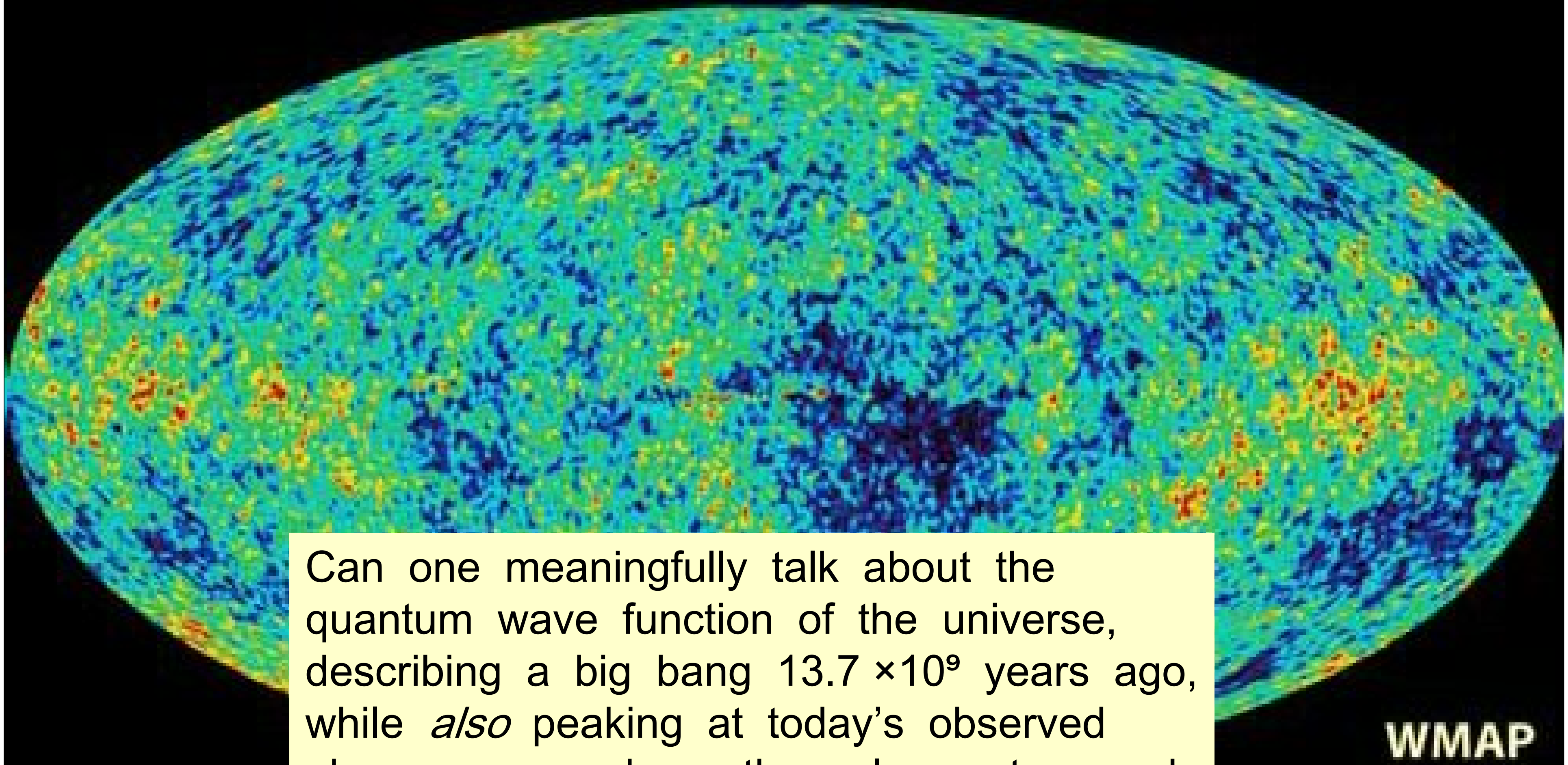

Quantum Mechanics
screen
holes
detector

Quantum Mechanics gives us statistical probabilities with fluctuations in the outcome:





String theory



Can one meaningfully talk about the quantum wave function of the universe, describing a big bang 13.7×10^9 years ago, while *also* peaking at today's observed phenomena such as the solar system and the distribution of the continents on earth?

Quantum mechanics was designed to be a theory to describe the outcome of an experiment that will only be accurate ...

if repeated many times!

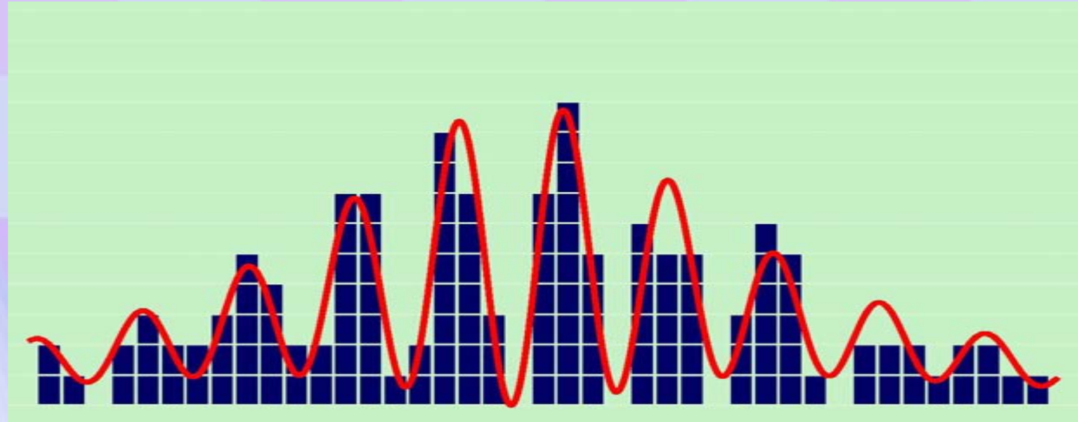
Thus, QM refers to events that happen in tiny subsections of the universe, in space and in time.

But today, theories such as superstring theory attempt to describe the entire universe

Can one meaningfully talk about "Quantum cosmology"?

More precise statements:

Quantum mechanics is a prescription to obtain the *best possible prediction* for the future, given the past, in any given experimental setup.

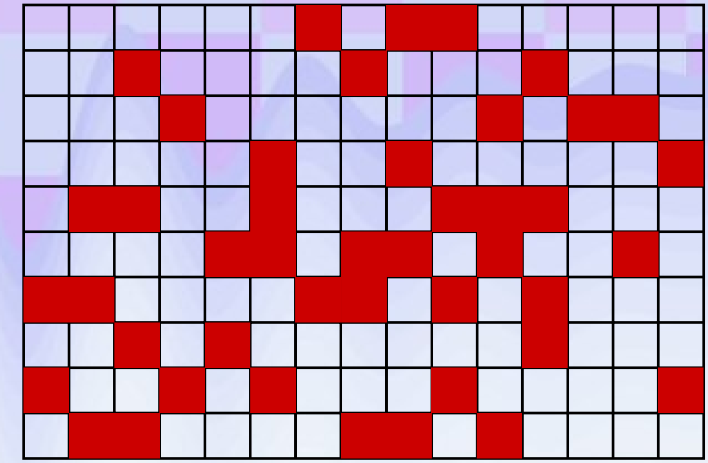


In numerous experiments it has been verified that better predictions are not possible.

Quantum mechanics is *not* a description of the *actual course of events* between past and future.

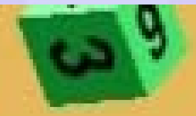
One might imagine that there are equations of Nature that can only be solved in a statistical sense. Quantum Mechanics appears to be a magnificent mathematical scheme to do such calculations.

Example of such a system: the ISING MODEL



L. Onsager,
B. Kaufman
1949

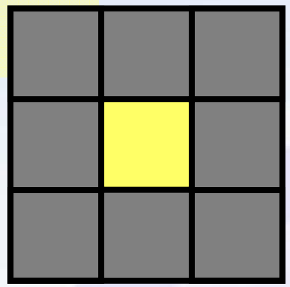
In short: QM appears to be the *solution* of a mathematical *problem*.
We know the solution, but what was the problem ?
Or, ...



Conway's Game of Life

1. Any live cell with fewer than two neighbours dies, as if by loneliness.

2. Any live cell with more than three neighbours dies, as if by overcrowding.



3. Any live cell with two or three neighbours lives, unchanged, to the next generation.

4. Any dead cell with exactly three neighbours comes to life.

The use of **Hilbert Space Techniques** as technical devices for the treatment of the statistics of chaos ...

A “state” of the universe:

í $\vec{x}$, ... , $\vec{p}$, ..., i, ...,  , anything ... ý

A simple model universe: í 1ý → í 2ý → í 3ý → í 1ý

“Beable”

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$|\Psi\rangle = \alpha \quad \ldots \quad \gamma|3\rangle;$$

$$P_1 = |\alpha|^2, \quad P_2 = |\beta|^2, \quad P_3 = |\gamma|^2$$

Di onalize:

$$U \to \begin{pmatrix} 1 & & \\ & e^{2\pi i/3} & \\ & & e^{-2\pi i/3} \end{pmatrix} = e^{-iH}$$

“Changeable”

$\frac{2}{3}\pi$

0

$-\frac{2}{3}\pi$

Emergent quantum mechanics in a deterministic system

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x})$$

$$\hat{\vec{p}} = -i\frac{\partial}{\partial \vec{x}}$$

$$\hat{H} = \hat{\vec{p}} \cdot \vec{f}(\vec{x}) + \vec{g}(\vec{x})$$

$$\frac{d}{dt}\vec{x}(t) = -i\left[\vec{x}(t),\ \hat{H}\right] = \vec{f}(\vec{x})$$

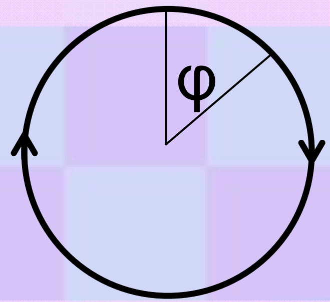
$$\hat{H} - \hat{H}^{\dagger} = i\left(-\vec{\nabla}\cdot\vec{f} + 2\ \mathrm{Im}(\vec{g})\right) \to 0$$

but $\langle \hat{H} \rangle \not\geq 0$??

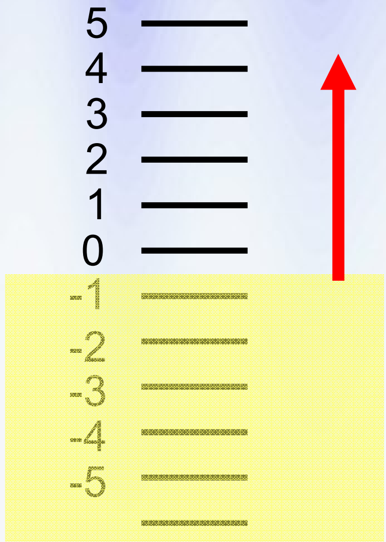
The
POSITIVITY
Problem

In *any* periodic system, the Hamiltonian can be written as

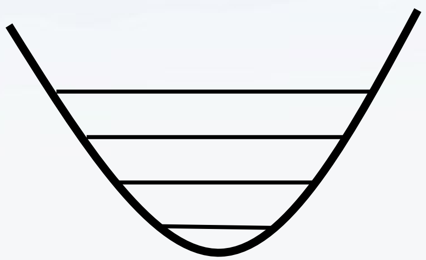
$$H = \omega p \; ; \qquad \omega = \frac{2\pi}{T}; \qquad p = -i\frac{\partial}{\partial \varphi}$$



$$e^{-iHT} = 1 \quad \rightarrow \quad H = \frac{2\pi n}{T} = \omega n \quad ; \qquad n = 0, \pm 1, \pm 2, \ldots$$



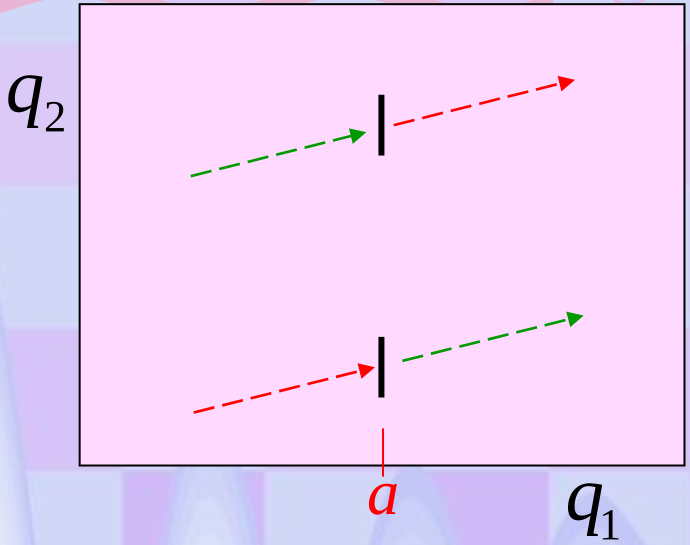
This is the spectrum of a harmonic oscillator !!



Interactions can take place in two ways. Consider two (or more) periodic variables.
1:

$$\frac{dq_i}{dt} = \omega_i q_i + \varepsilon f_i(\vec{q}); \qquad H^{\text{int}} = \varepsilon f_i p_i$$

Do perturbation theory in the usual way by computing $\langle n | H^{\text{int}} | m \rangle$.



2: Write $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for the hopping operator.

$$q_1 = q_1(0) + \omega_1 t$$

$$H = \omega_1 p_1 + \omega_2 p_2 + \boxed{A\,(\sigma_x - 1)\;\delta(q_1 - a)\;\theta(q_2)}$$

Use $\sigma_x = \pm 1$; $e^{\pm\frac{1}{2}\pi i} = \pm i$; $e^{\frac{1}{2}\pi i \sigma_x} = i\sigma_x$; $e^{\frac{1}{2}\pi i(\sigma_x - 1)} = \sigma_x$

to derive

$$\exp\left(-i\int_{t_1}^{t_2} H\,dt\right) = \exp\left(-i\,A\theta(q_2)(\sigma_x - 1)\tfrac{1}{\omega_1}\right)$$

$$= \sigma_x \quad \text{if} \quad \begin{cases} A = \frac{1}{2}\pi\,\omega_1 \\ \theta(q_2) = 1 \end{cases}$$

H^{int}

However, in both cases, $\langle n | H^{\text{int}} | m \rangle$

will take values over the *entire* range of values for *n* and *m*.

Positive and negative values for *n* and *m* are mixed !

→ negative energy states cannot be projected out !

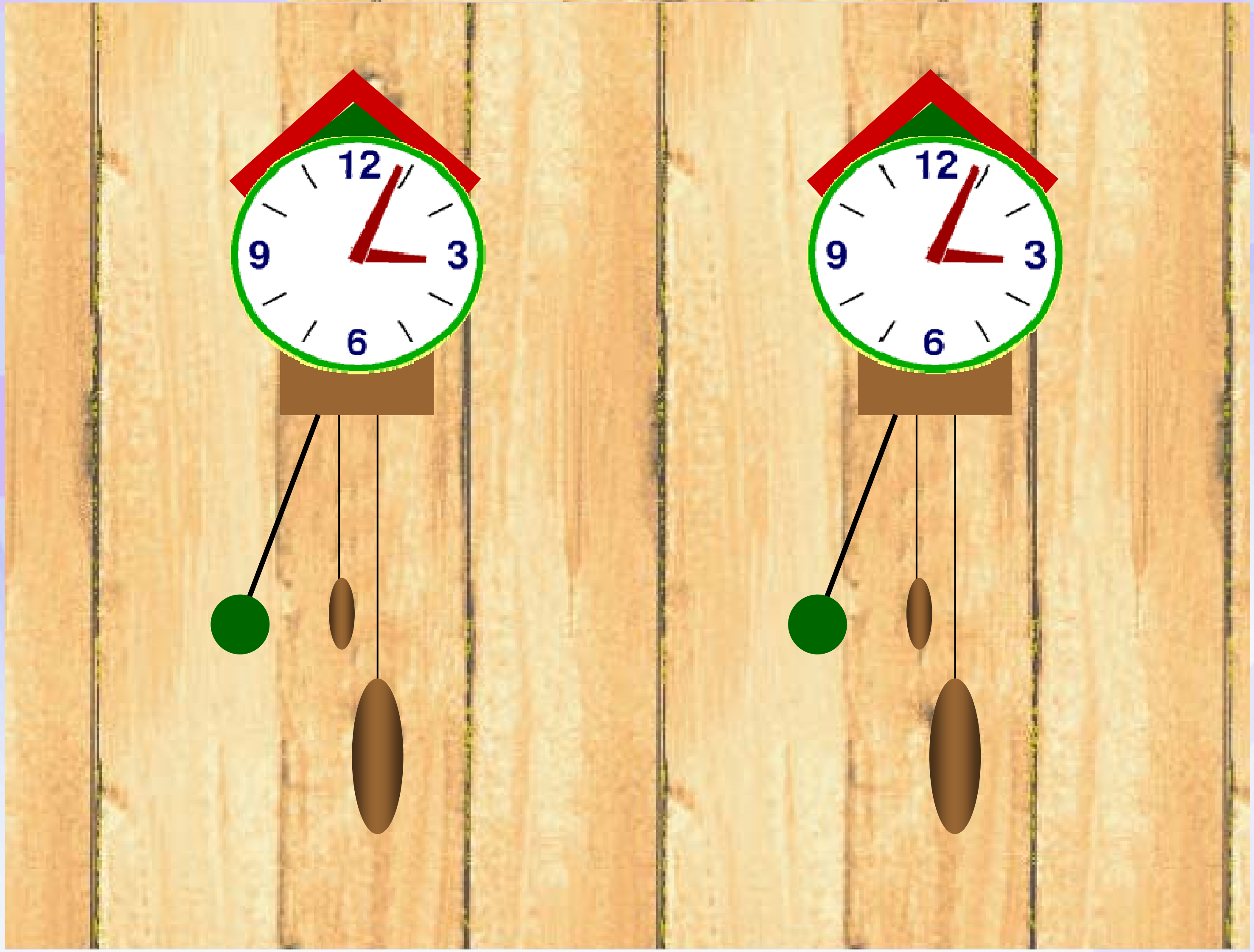
But it can "nearly" be done! suppose we take many slits, and average: $\langle \theta(q_2) \rangle \rightarrow f(q_2)$

Then we can choose to have the desired Fourier coefficients for $\langle n | H^{\text{int}} | m \rangle$

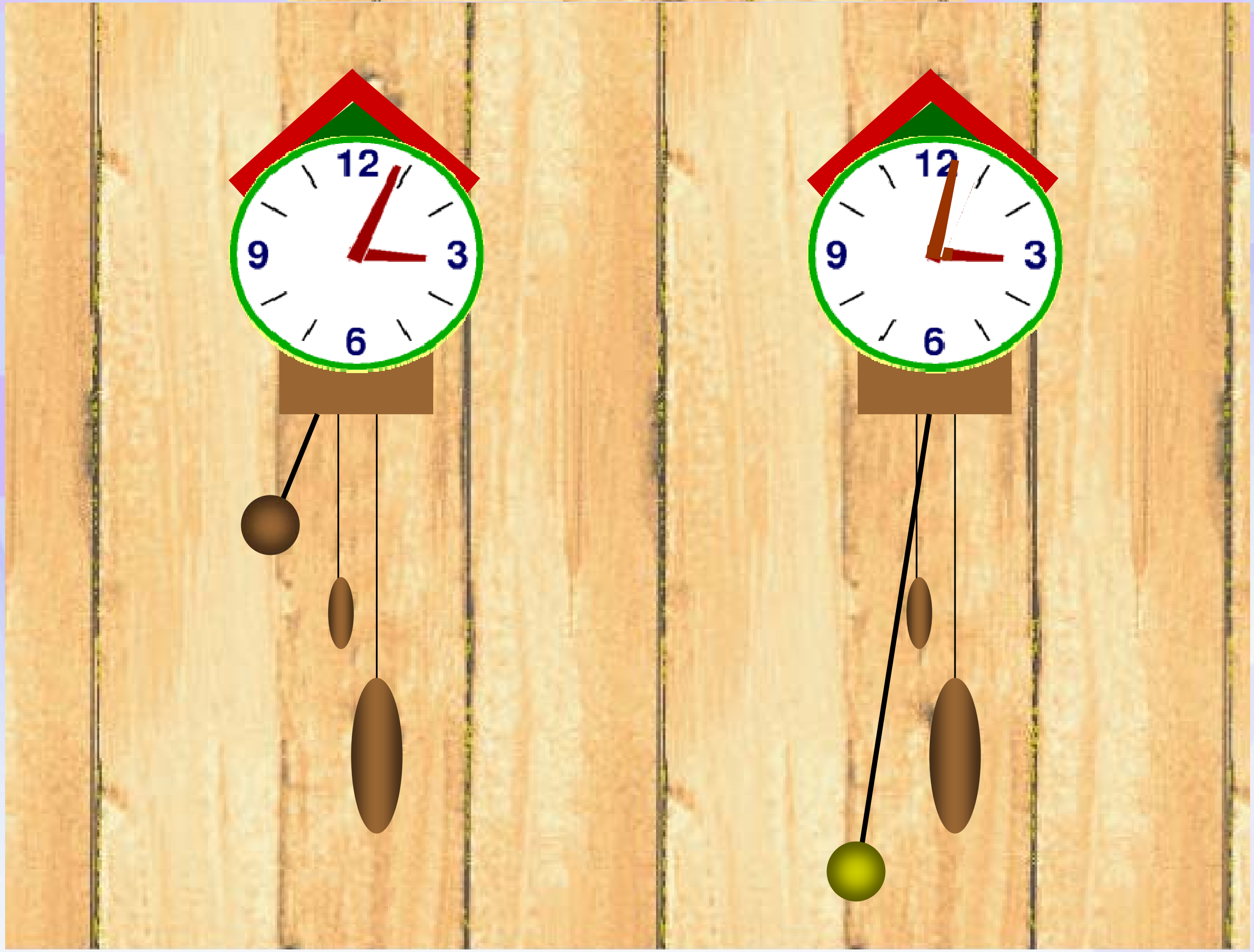
But this leads to decoherence !

In search for a

Lock-in mechanism

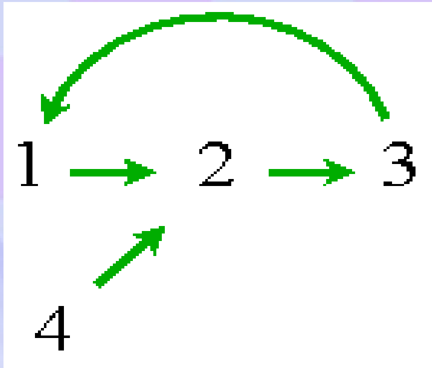


Lock-in mechanism



A key ingredient for an ontological theory:

Information loss

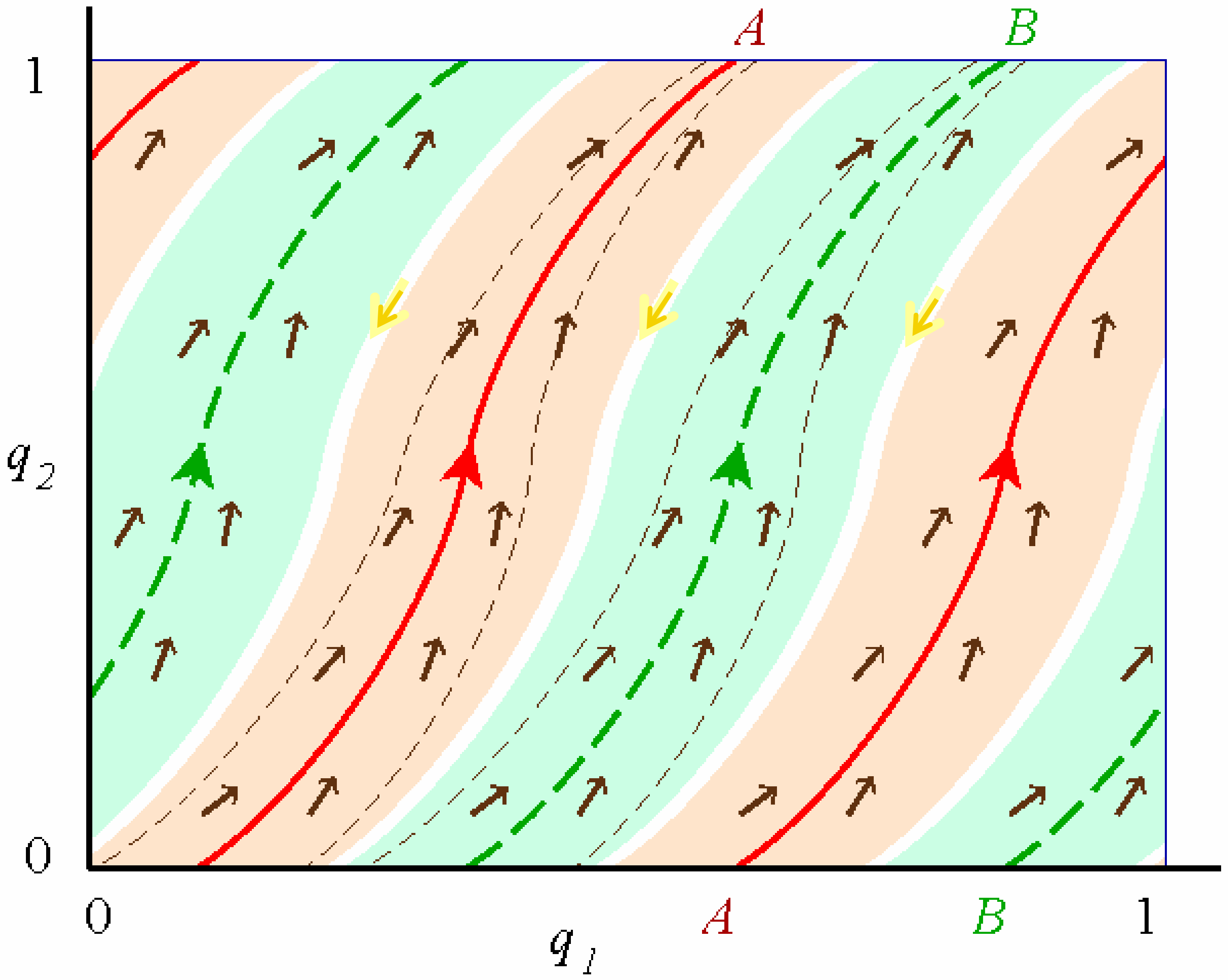


Introduce equivalence classes

$$|\text{í } 1\text{ý}, \text{í } 4\text{ý}\rangle \rightarrow |\text{í } 2\text{ý}\rangle \rightarrow |\text{í } 3\text{ý}\rangle$$

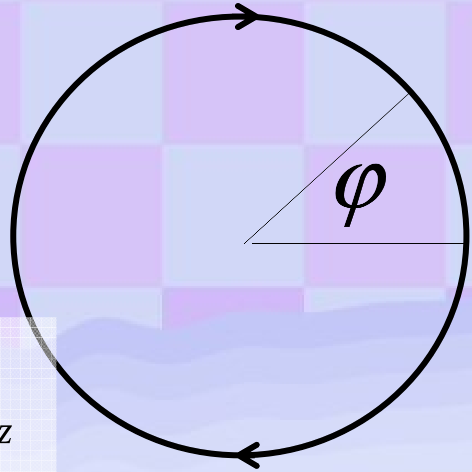
With (virtual) **black holes**, information loss will be very large! →
Large equivalence classes !

Two (weakly) coupled degrees of freedom



Consider a periodic variable:

$$\frac{d\varphi}{dt} = \omega\,; \qquad \varphi \in [0,\ 2\pi]$$

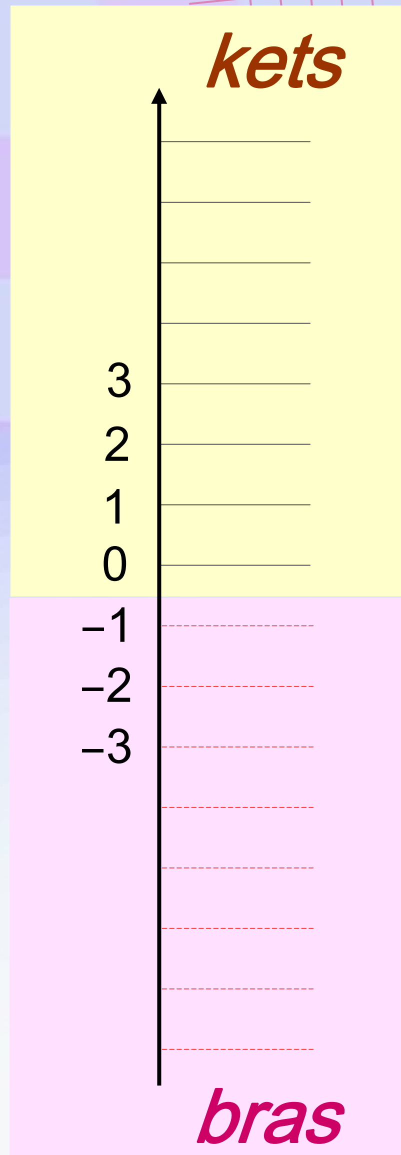


$$H = \omega p = -i\omega \frac{\partial}{\partial \varphi} = \omega L_z$$

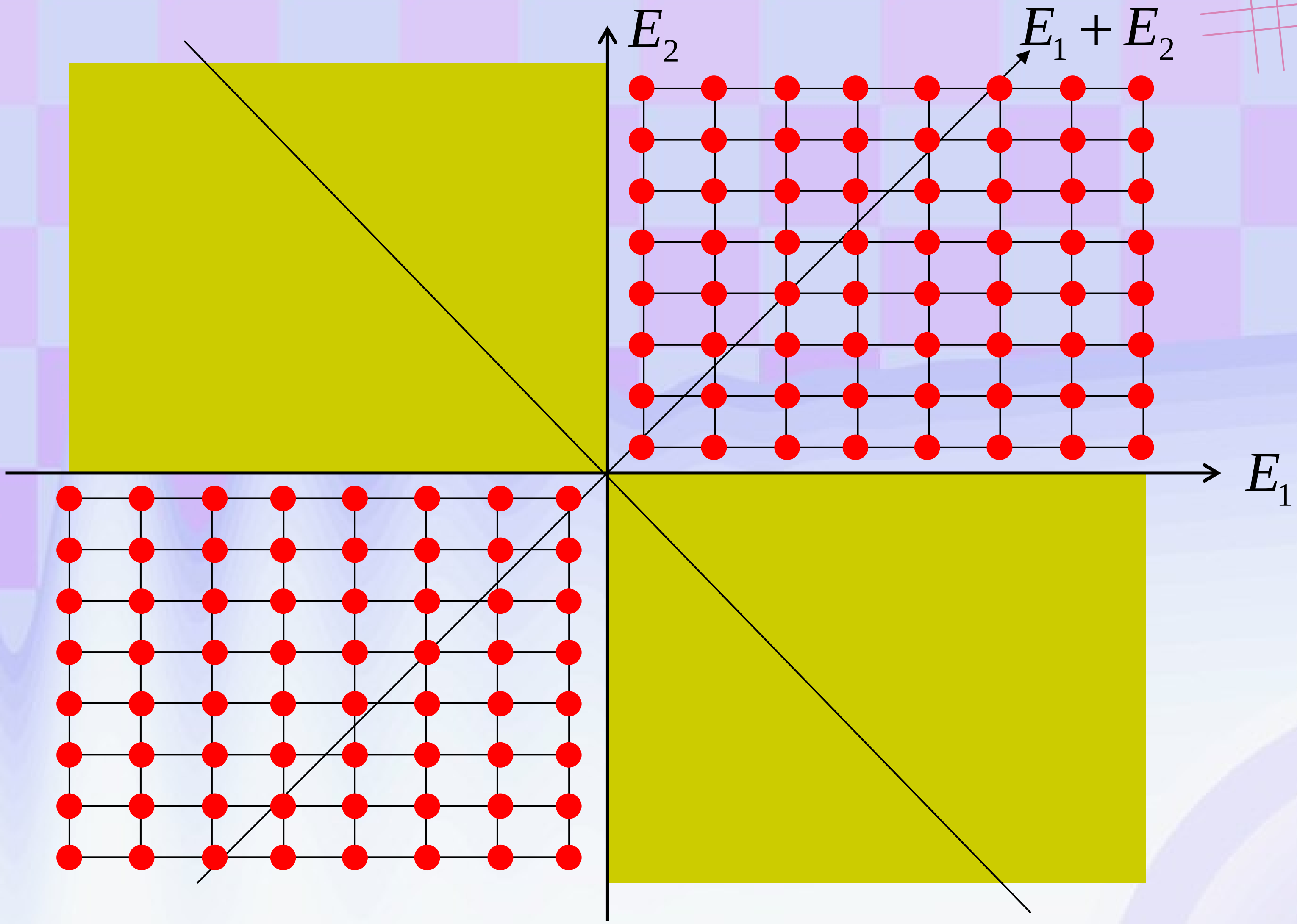
$$= \omega m\,, \qquad m = 0, \pm 1, \pm 2, \cdots$$

The *quantum* harmonic oscillator has only:

$$H = \omega n\,, \qquad m = 0, +1, +2, \cdots$$



Consider two *non* - interacting systems:



$$H = \omega_1 n_1 + \omega_2 n_2$$

The allowed states have "*kets*" with

$$H = E_1 + E_2 = \omega_1 (n_1 + \tfrac{1}{2}) + \omega_2 (n_2 + \tfrac{1}{2}) \ , \quad n_{1,2} \geq 0$$

and "*bras*" with

$$-H = E_1 + E_2 = \omega_1 (n_1 + \tfrac{1}{2}) + \omega_2 (n_2 + \tfrac{1}{2}) \ , \quad n_{1,2} \leq -1$$

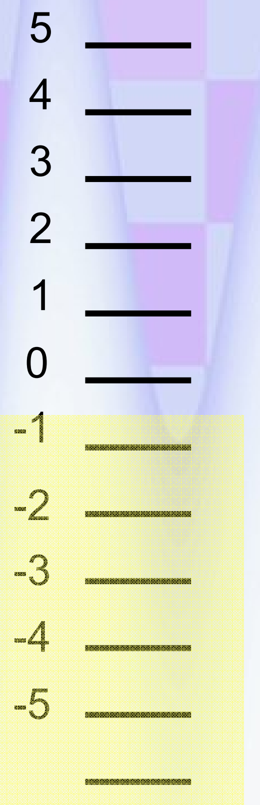
$$E_1 \cdot E_2 \geq 0 \quad \Rightarrow \quad |E_1 + E_2| \geq |E_1 - E_2|$$

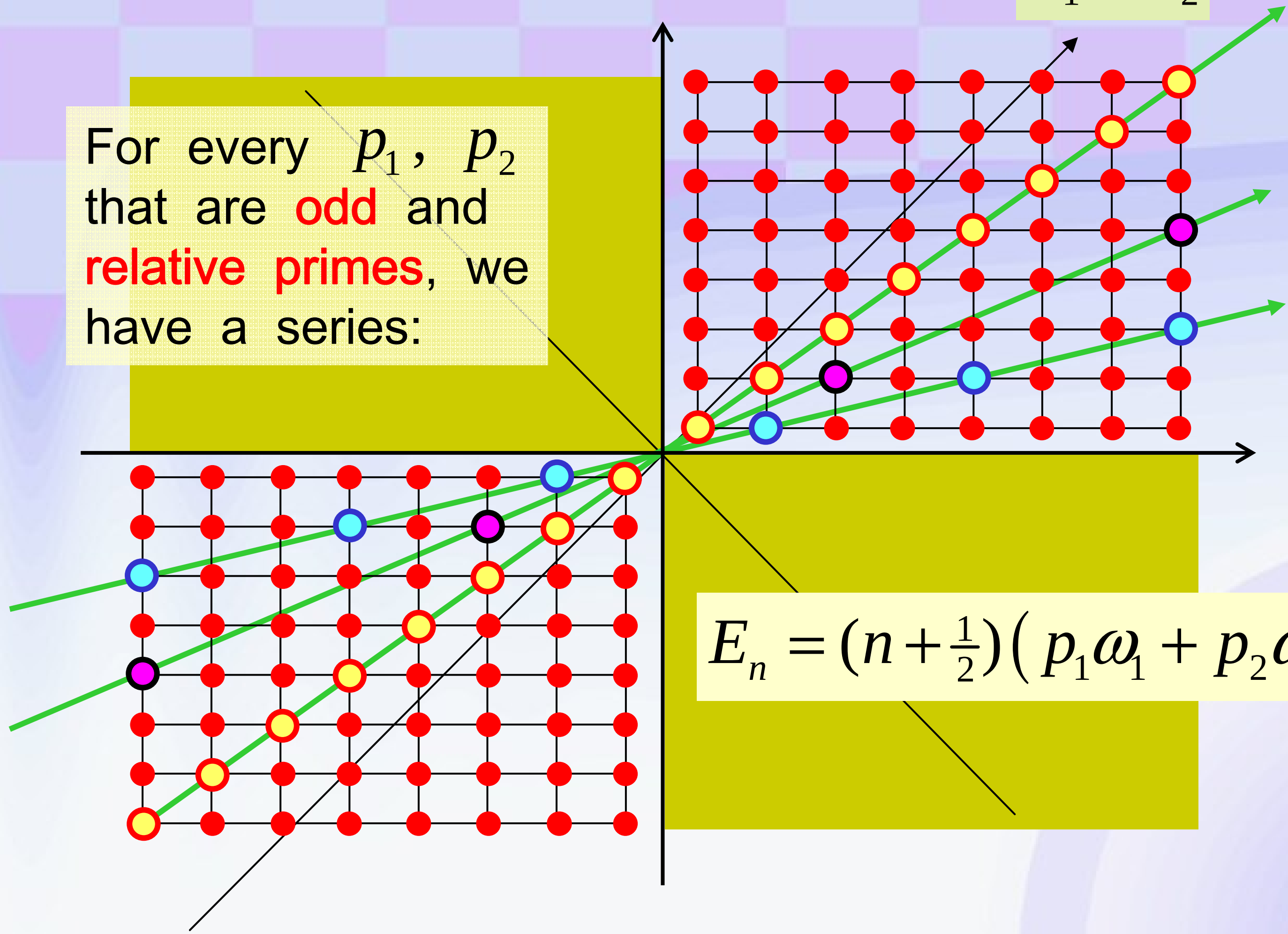
Now, $\delta E \cdot \delta t = \tfrac{1}{2}$ and

$$E_1 \cdot t_1 + E_2 \cdot t_2 = \tfrac{1}{2}\left((E_1 + E_2)(t_1 + t_2) + (E_1 - E_2)(t_1 - t_2)\right)$$

So we also have: $\delta(t_1 + t_2) \leq \delta(t_1 - t_2)$

The combined system is expected again to behave as a periodic unit, so, its energy spectrum must be some combination of series of integers:





For every p_1, p_2 that are odd and relative primes, we have a series:

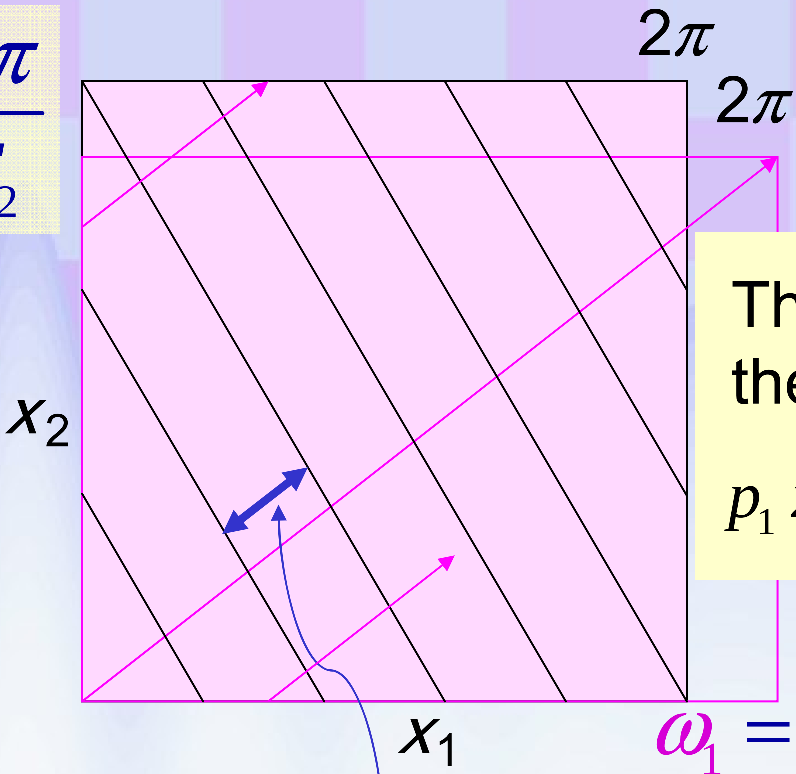
$$E_n = (n + \tfrac{1}{2})\left(p_1\omega_1 + p_2\omega_2\right)$$

$$E_n = (n+\tfrac{1}{2})\left(p_1\omega_1 + p_2\omega_2\right)$$

The case

$$p_1 = 5, \quad p_2 = 3$$

$$\omega_2 = \frac{2\pi}{T_2}$$



This is the periodicity of the *equivalence class*:

$$p_1\, x_1 + p_2\, x_2 = \mathrm{Cnst}\,(\mathrm{Mod}\ 2\pi)$$

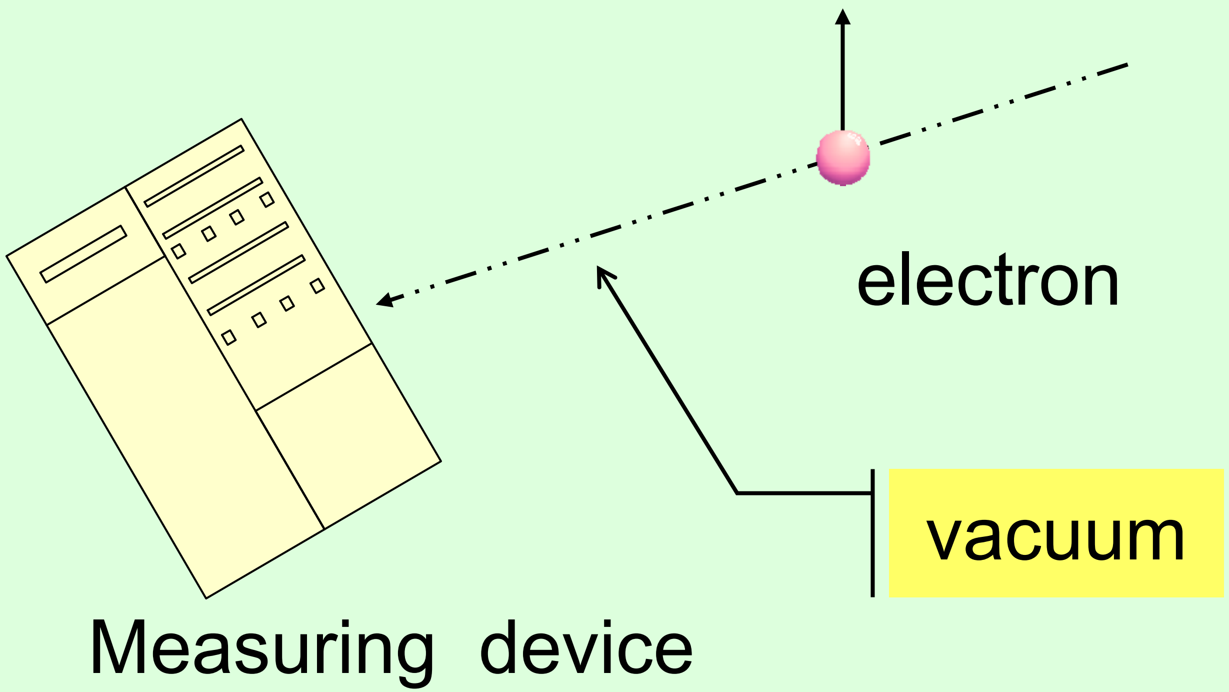
$$\omega_1 = \frac{2\pi}{T_1}$$

$$T_{12} = \frac{2\pi}{p_1\omega_1 + p_2\omega_2}$$

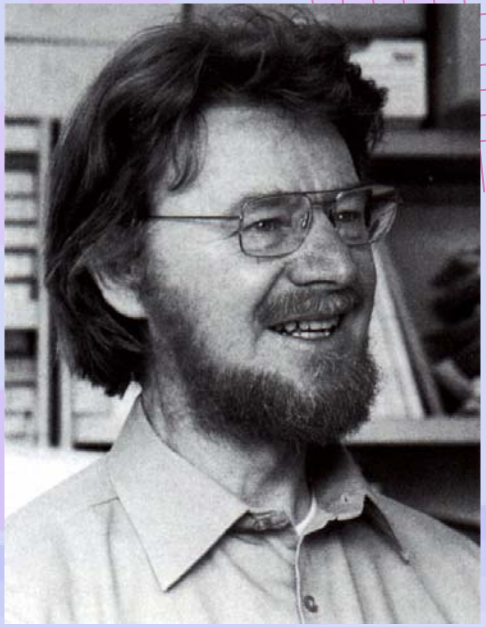
And what about the

Bell inequalities

?



Free Will : John S. Bell



"Any observer can *freely choose* which feature of a system he/she wishes to measure or observe."

Is that so, in a deterministic theory ?

In a deterministic theory, one cannot change the present without also changing the past.

Changing the past might well affect the correlation functions of the physical degrees of freedom in the present – if not the beables, then at least the *phases* of the wave functions, may well be modified by the observer's "change of mind".

Do we have a **FREE WILL** , *that does not even affect the phases?*

Using this concept, physicists "prove" that deterministic theories for QM are impossible.

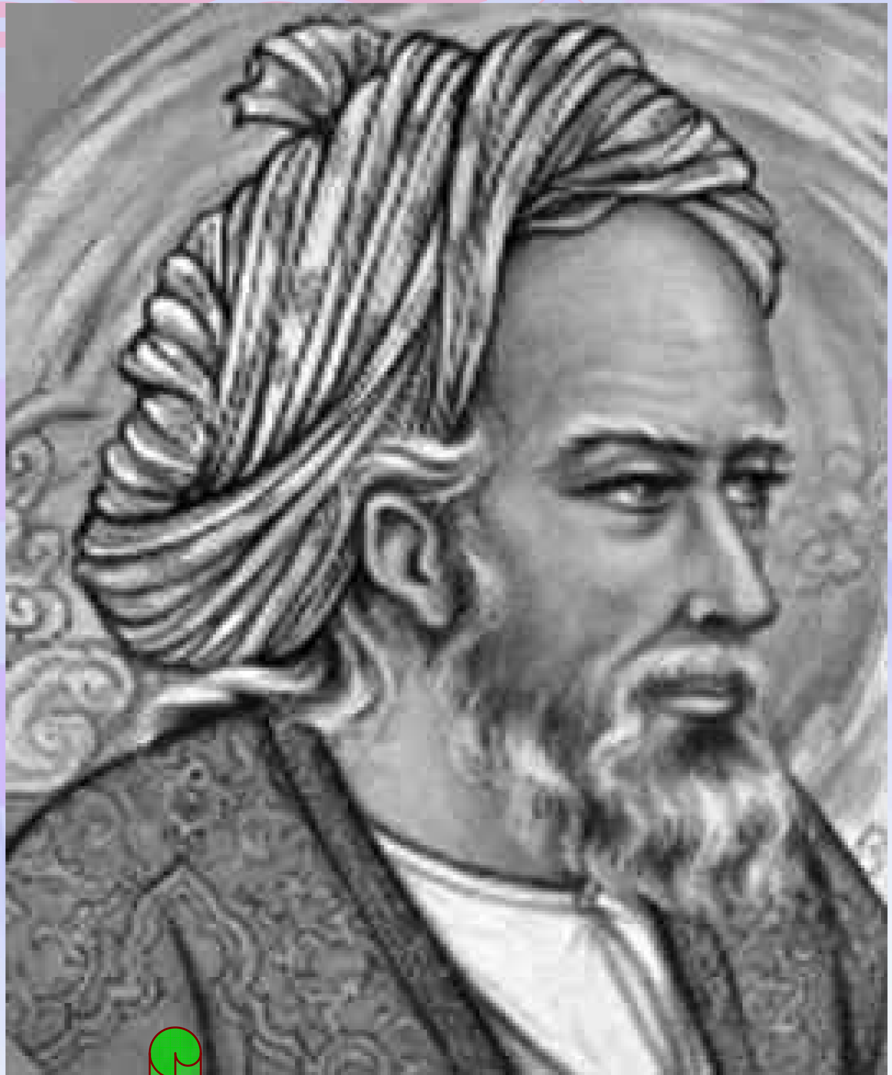
The existence of this "free will" seems to be indisputable.

Citations:

Conway, Kochen: *free will is just that the experimenter can freely choose to make any one of a small number of observations* ... *this failure* [of QM] *to predict is a merit rather than a defect, since these results involve free decisions that the universe has not yet made.*

must be true, thus B is free to measure along any triple of directions. ...

free will). A theory seems unsatisfactory if somehow the initial conditions of the universe are so contrived that EPR pairs always know in advance which magnetic fields the experimenters will choose.



Determinism

Omar Khayyam
(1048-1131)

in his *robāʻīyāt* :

"And the first Morning of creation wrote /
What the Last Dawn of Reckoning shall read."

The most questionable element in the usual discussions concerning Bell's inequalities, is the assumption of

FREE WILL

It has to be replaced with

Unconstrained Initial State

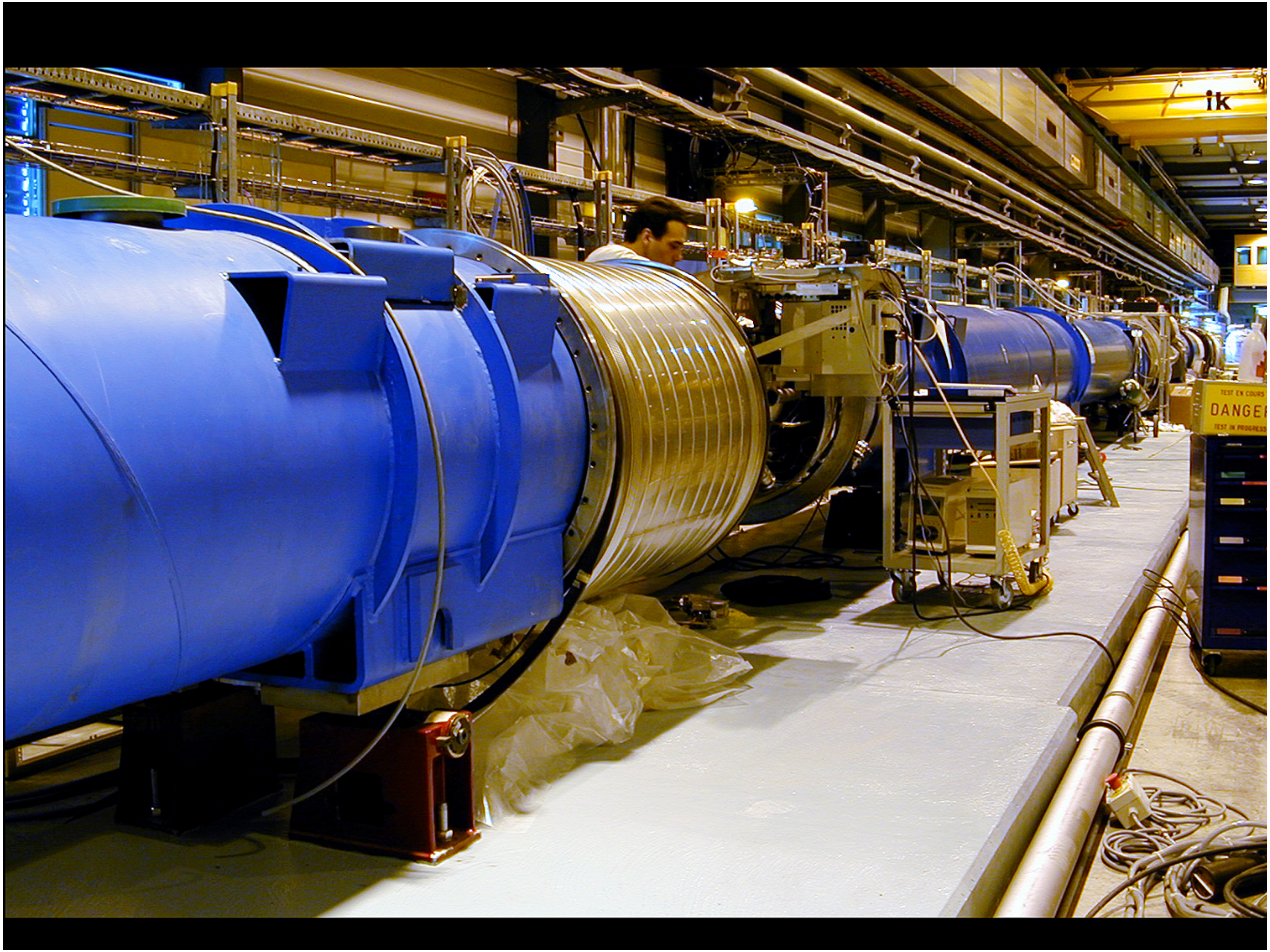
General conclusions

At the Planck scale, Quantum Mechanics is *not* wrong, but its interpretation may have to be revised, *not* for philosophical reasons, but to enable us to construct more concise theories, recovering e.g. locality (which appears to have been lost in string theory).

The “random numbers”, inherent in the usual statistical interpretation of the wave functions, may well find their origins at the Planck scale, so that, there, we have an ontological (deterministic) mechanics

For this to work, this deterministic system must feature *information loss* at a vast scale

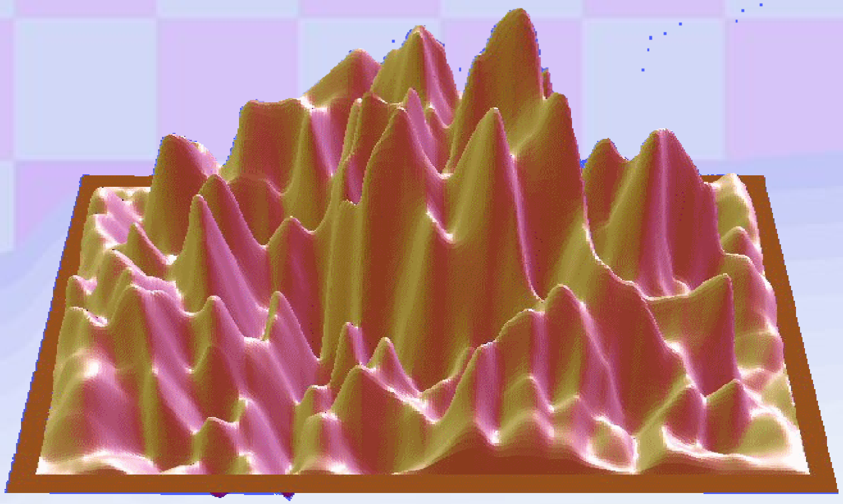
Any isolated system, if left by itself for long enough time, will go into a *limit cycle*, with a very short period.
Energy is defined to be the inverse of that period: $E = h\nu$


TEST EN COURS
TEST IN PROGRESS

Are there any consequences for particle physics?

Understanding QM is essential for the construction of the "Ultimate theory", which must be *more* than Superstring theory.

the LANDSCAPE



Gauge theories

The equivalence classes have so much in common with the gauge orbits in a local gauge theory, that one might suspect these actually to be the same, in many cases
(→ Future speculation)

Gravity 1: $H = (n + \tfrac{1}{2}) \sum \omega_i$

Since only the overall *n* variable is a changeable, whereas the rest of the Hamiltonian, ω_i , are beables, our theory will allow to couple the Hamiltonian to gravity such that the gravitational field is a beable.

(Indeed, total *momentum* can also be argued to be a beable ...)

Gravity 2:

General coordinate transformations might also connect members of an equivalence class ... maybe the ultimate “ontological” theory does have a preferred (rectangular ?) coordinate frame !

Predictions (which may well be wrong) :

Gauge invariance(s) play an important role in these theories .... → plenty of new gauge particles !

No obvious role is found for super symmetry
(a disappointment, in spite of attempts ...)

G. ’t Hooft
demystifying Quantum Mechanics

The End