

Measurements of radiative B meson decays at *Belle*

Hanjin Kim
(Yonsei Univ.)
for *Belle* Collaboration



Coverage of this talk

-Radiative Penguin decays of b quark

>> Reviews on recent works at *Belle*

- Search for $B_s^0 \rightarrow \gamma\gamma$ and $B_s^0 \rightarrow \phi\gamma$
- Search for $B^0 \rightarrow \phi\gamma$
- Semi-Inclusive $\mathcal{B}(b \rightarrow s\gamma)$
- Inclusive $A_{CP}(b \rightarrow (s+d)\gamma)$

>> **New result** at *Belle*

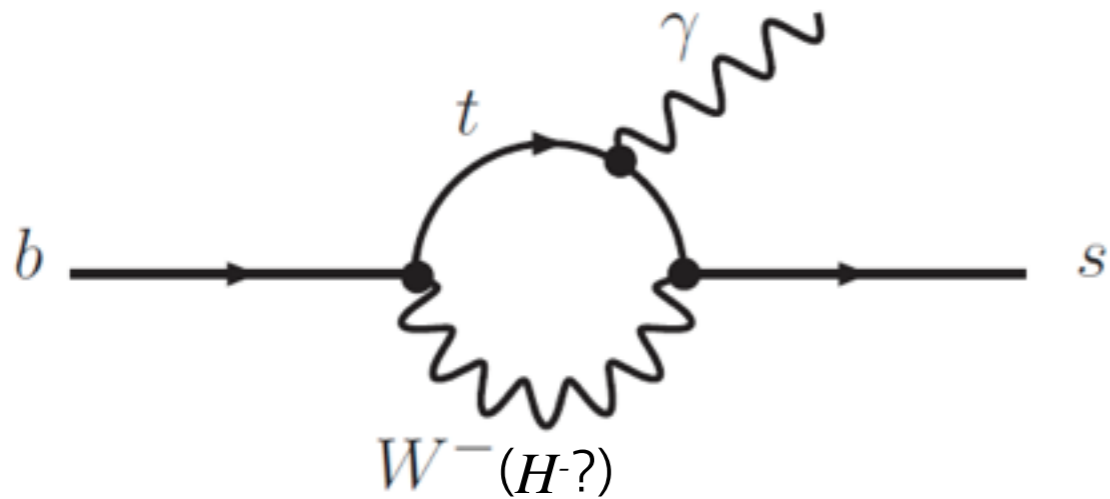
- Inclusive $b \rightarrow (s+d)\gamma$

Integrated Luminosity at *Belle*

711 fb⁻¹ for Y(4S)

121 fb⁻¹ for Y(5S)

Introduction to $b \rightarrow s\gamma$ decay



Electroweak penguin FCNC processes
highly suppressed in the tree level

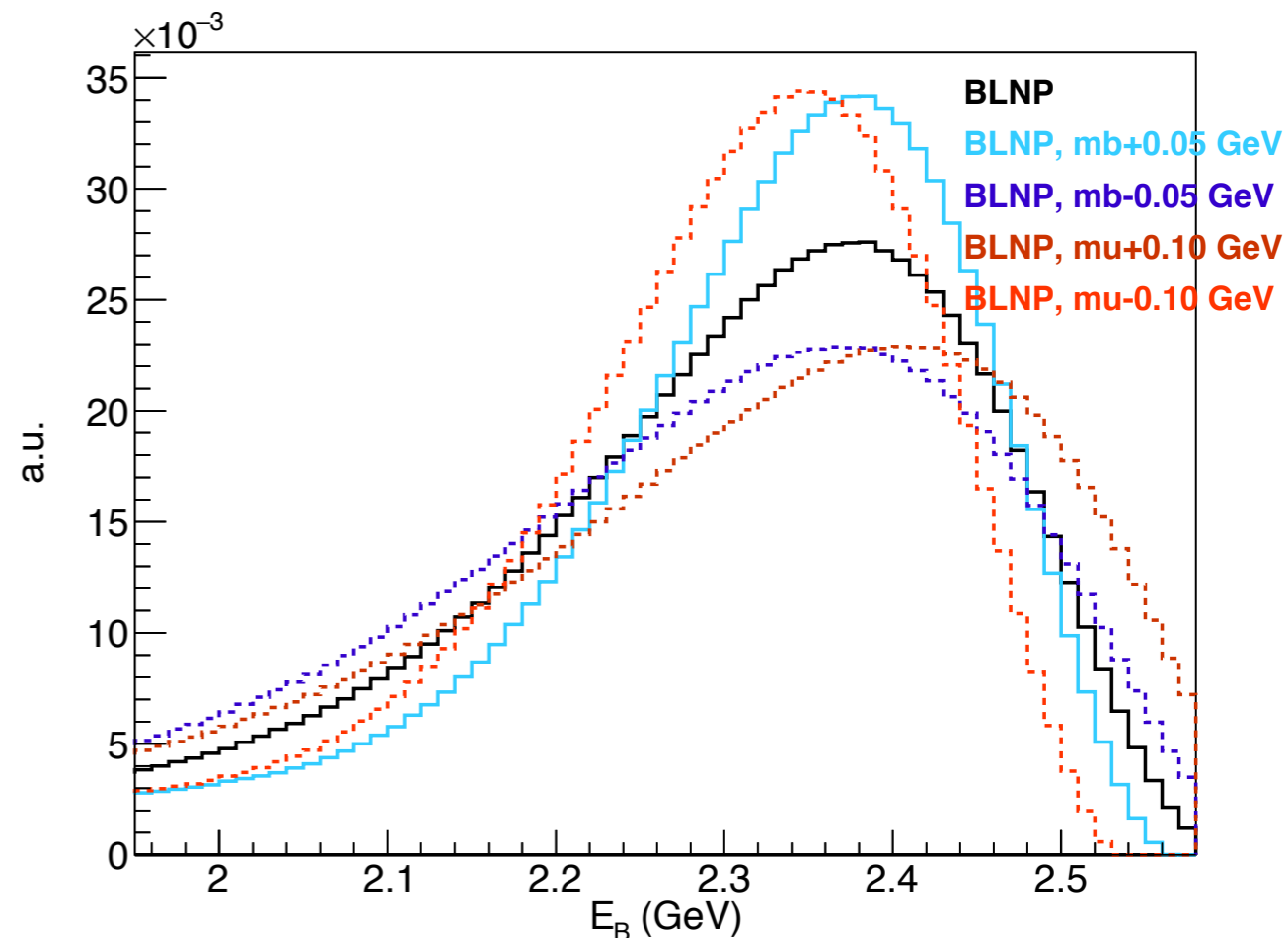
H^- in 2HDM Type-II or SUSY squarks
can enter the loop \rightarrow BF, A_{CP}

Probe to New Physics

Inclusive spectrum described by m_b and μ_π^2
Significantly dependent on the parameters
Low cutoff necessary for a good prediction
on the inclusive BF

$$E_\gamma^{cutoff} = 1.6 \text{ GeV}$$

Measured spectrum can be used to
constrain HQE parameters **Later in this talk!**



Introduction to $b \rightarrow s\gamma$ decay

Current SM **NNLO BF** [PRL 114, 221801, 2015]

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{NNLO} = (3.36 \pm 0.23) \times 10^{-4}$$

HFAG 2016 / PDG 2015 Average

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.49 \pm 0.19) \times 10^{-4}$$

BF used to constrain the *new physics parameters* [arXiv:1412.7515]

$$M(H^-) > 480 \text{ GeV at 95\% CL}$$

$$A_{CP} = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}$$

$f = X_{s,d}\gamma$

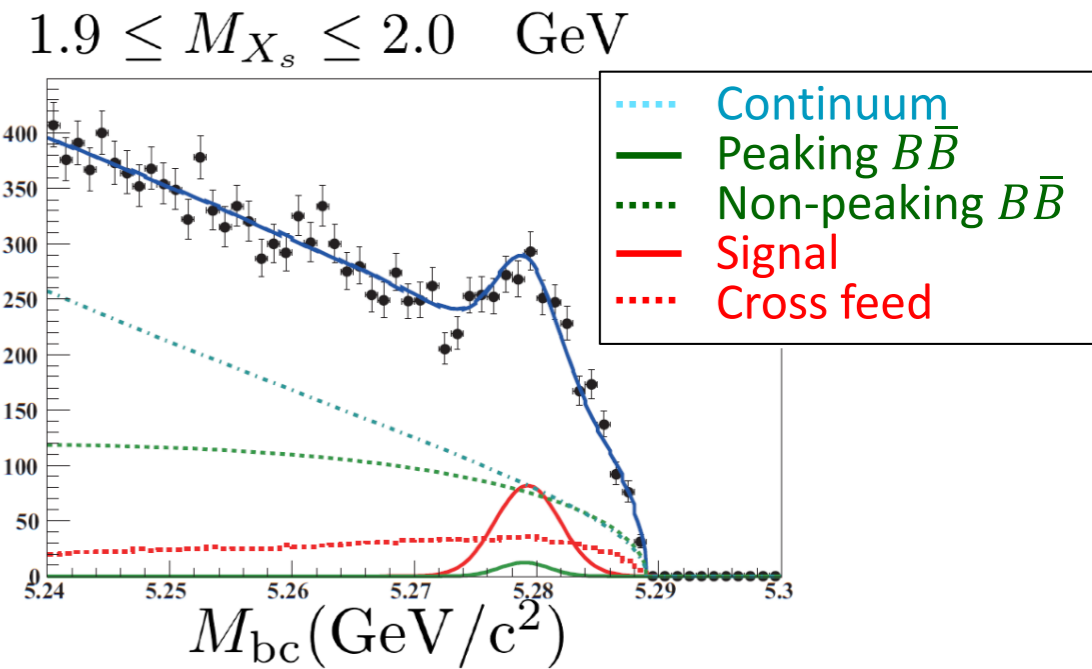
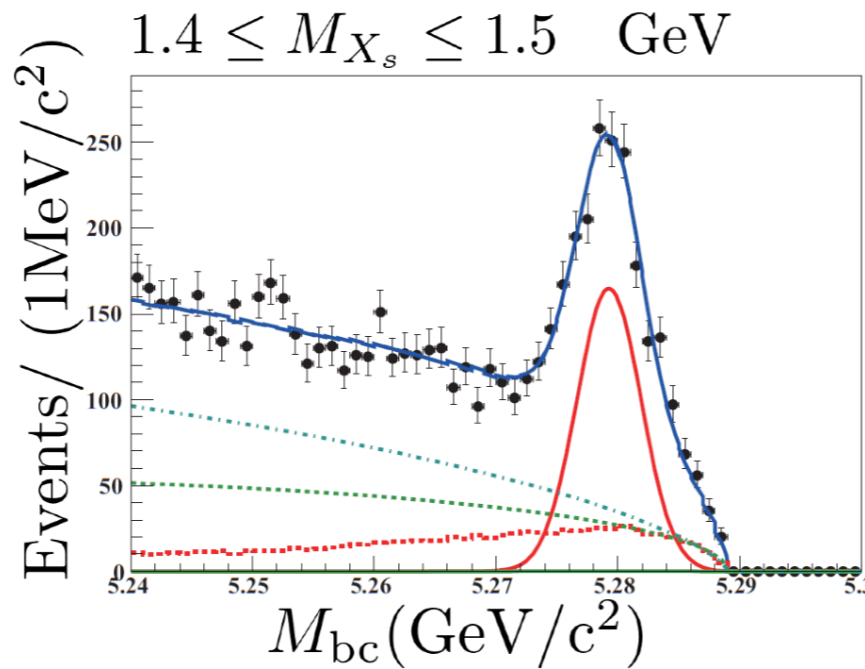
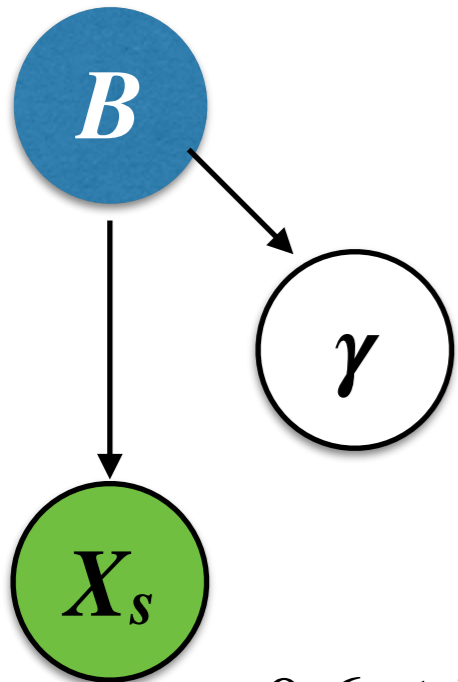
Zero asymmetry predicted by SM for s+d
(cancellation due to CKM unitarity)

Channel	$A_{CP}(\text{SM})$
$B \rightarrow X_s \gamma$	$[-0.6\% , +2.8\%]$
$B \rightarrow X_d \gamma$	$[-62\% , +14\%]$
$B \rightarrow X_{s+d} \gamma$	0

$$A_{CP}^{HFAG} = -0.008 \pm 0.029$$

$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ with semi-inclusive method

T.Saito, A.ishikawa, H.Yamamoto, et al. (Belle Collaboration), published in [PRD 91, 052004 \(2015\)](#)



Signal Extraction : M_{bc} Maximum Likelihood fit on 19 $M(X_s)$ bins

$0.6 \leq M(X_s) \leq 2.8 \text{ GeV}/c^2$

1 or 3 K/ K_s
(1 K_s at most)

up to 4 π/π^0
(2 π^0 at most)

at most 1 η

In total 38 exclusive X_s states (**70% of total BF**)

- continuum suppressed by neural network trained with topological variables
- Peaking D background veto using invariant mass

$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ with semi-inclusive method

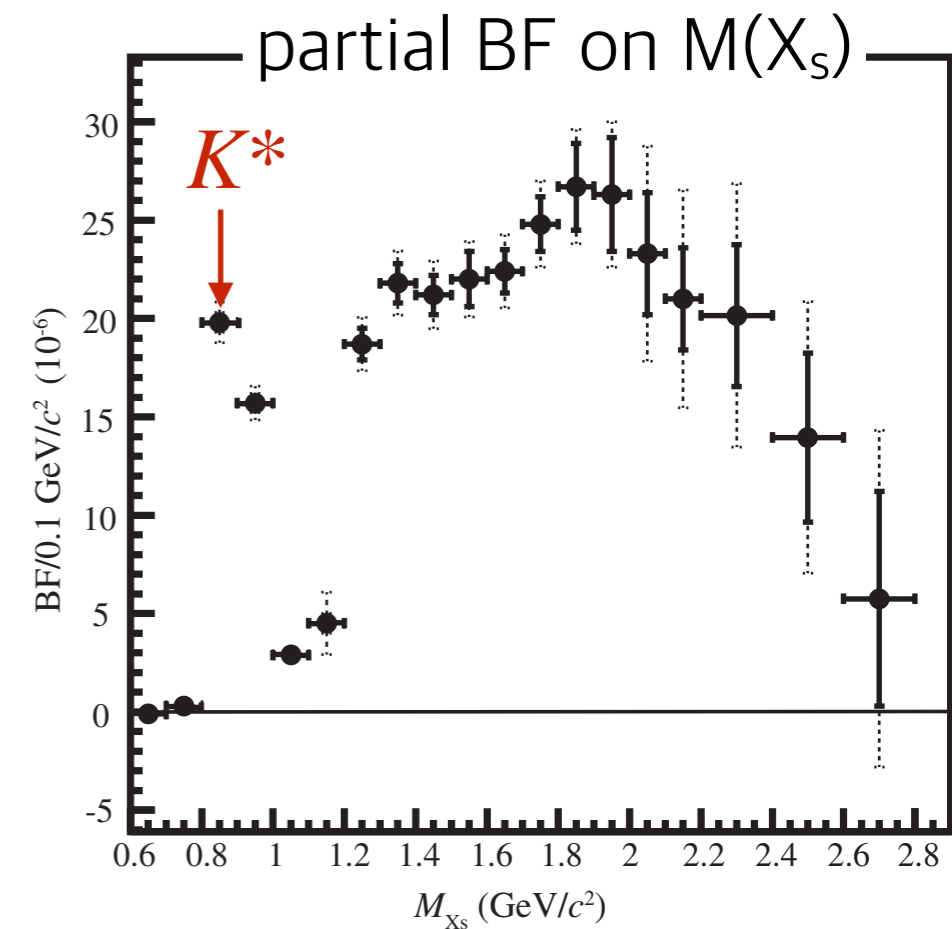
T.Saito, A.ishikawa, H.Yamamoto, et al. (Belle Collaboration), published in [PRD 91, 052004 \(2015\)](#)

>> Result

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.51 \pm 0.17_{stat} \pm 0.33_{syst}) \times 10^{-4}$$

- The largest systematic uncertainty is associated Hadronization Model (~7%)
- The most precise measurement** ever implemented with this method

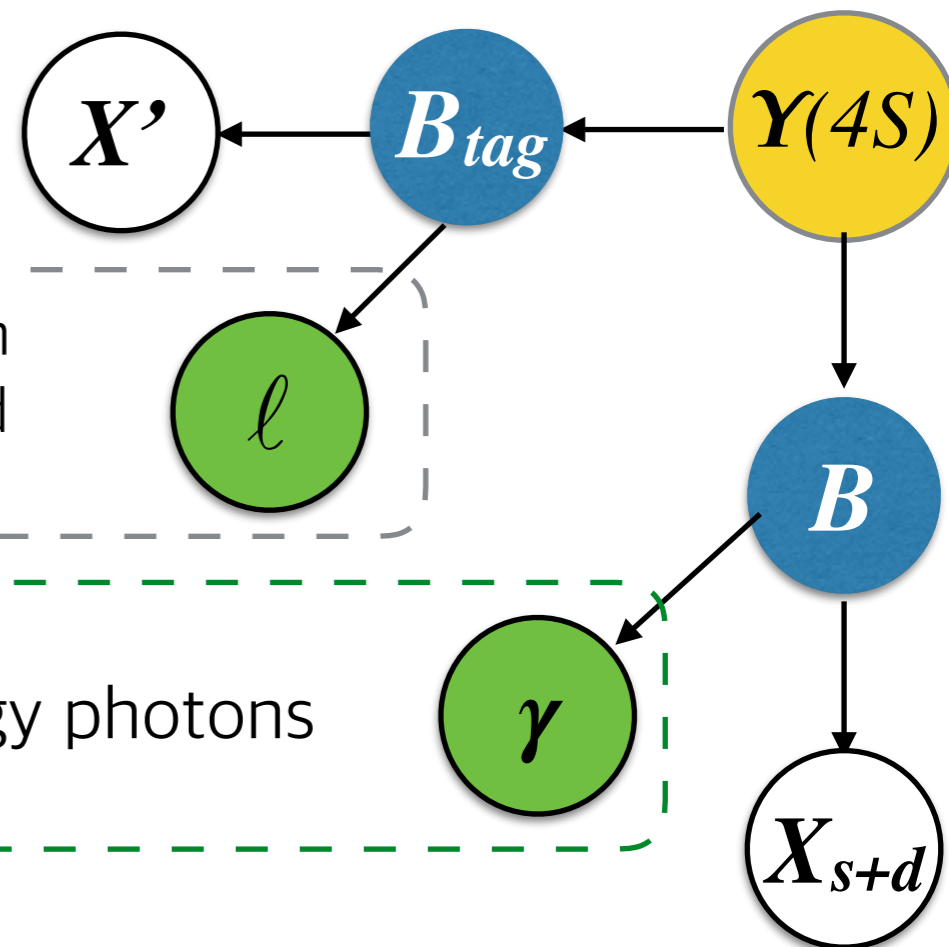
Source	Systematic uncertainty (%)
$B\bar{B}$ counting	1.37
Detector response	2.98
Background rejection	3.38
M_{bc} PDF	5.06
Hadronization model	6.66
Missing mode	1.59
Total	9.3



$\bar{B} \rightarrow X_{(s+d)}\gamma$ with inclusive method

L. Pesantez, P. Urquijo, J. Dingfelder, et al. (Belle Collaboration), published in [PRL 114, 151601 \(2015\)](https://arxiv.org/abs/151601)

Full-inclusive Method with lepton tagging



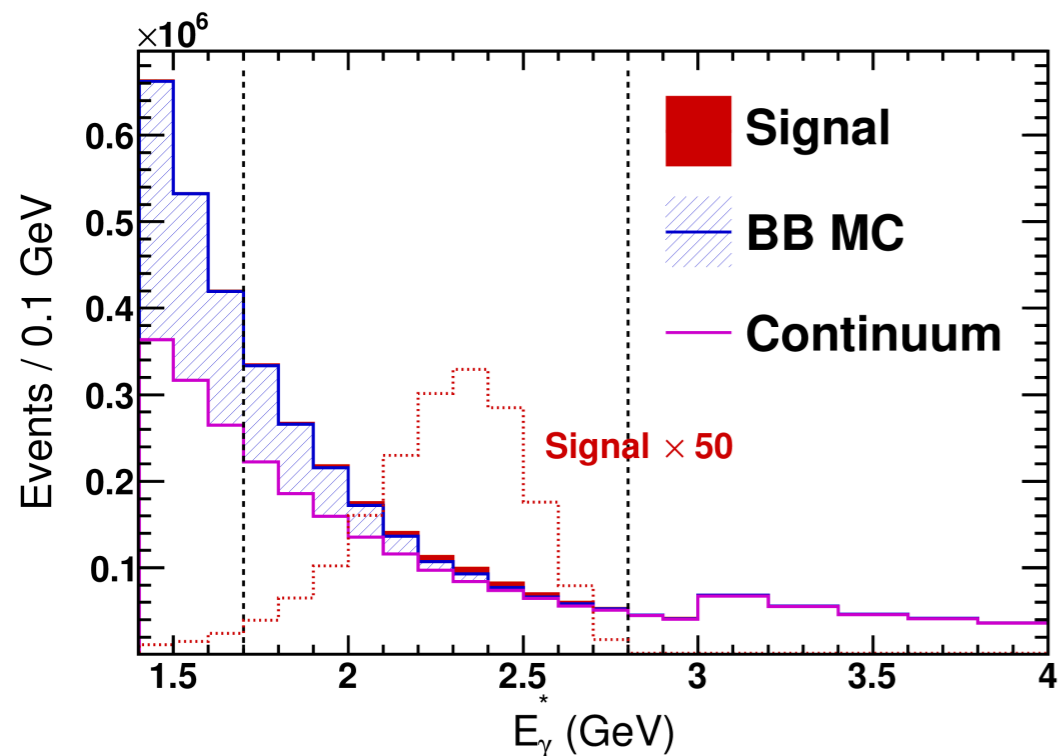
High momentum lepton :

Continuum suppression
B mesons flavor tagged



A_{CP} measurement

Inclusive measurement with high energy photons



Large contribution from π^0 and η

→ calibrated using control sample

Large background suppressed by Multivariate analysis

Statistical limit from continuum subtraction

Inclusive $A_{CP}(\bar{B} \rightarrow X_{(s+d)}\gamma)$ with lepton tagging

L. Pesantez, P. Urquijo, J. Dingfelder, et al. (Belle Collaboration), published in [PRL 114, 151601 \(2015\)](https://arxiv.org/abs/151601)

After bkg subtraction $A_{CP}^{meas} = \frac{N(\ell^+) - N(\ell^-)}{N(\ell^+) + N(\ell^-)}$

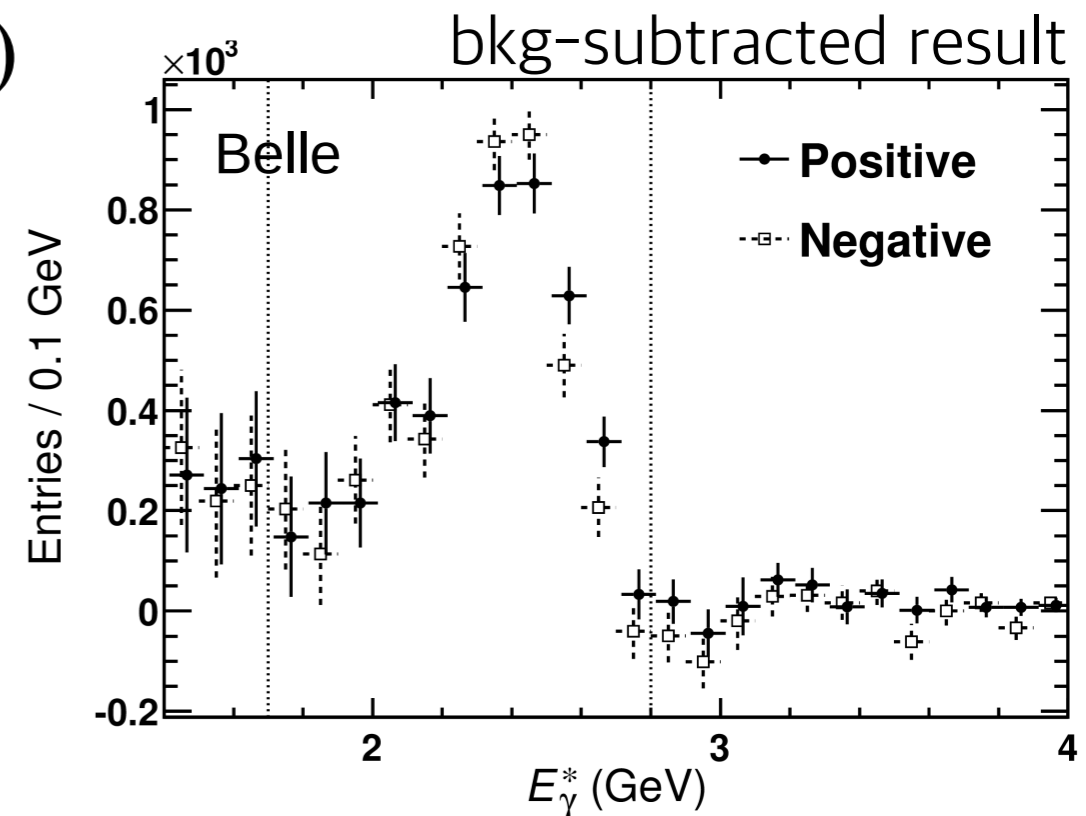
Corrected A_{CP}

$$A_{CP}^{true} = \frac{1}{1 - 2\omega} (A_{CP}^{meas} - A_{bkg} - A_{det})$$

Wrong tag factor ω : $\sim 14\%$ Oscillation, secondary, fakes

Bkg Asymmetry A_{bkg} : $\sim (0.0 \pm 0.7)\%$ in $E_\gamma < 1.7$ GeV

Detector induced A_{det} : $\sim (0.0 \pm 0.3)\%$ from $B \rightarrow XJ / \psi(\ell^+\ell^-)$

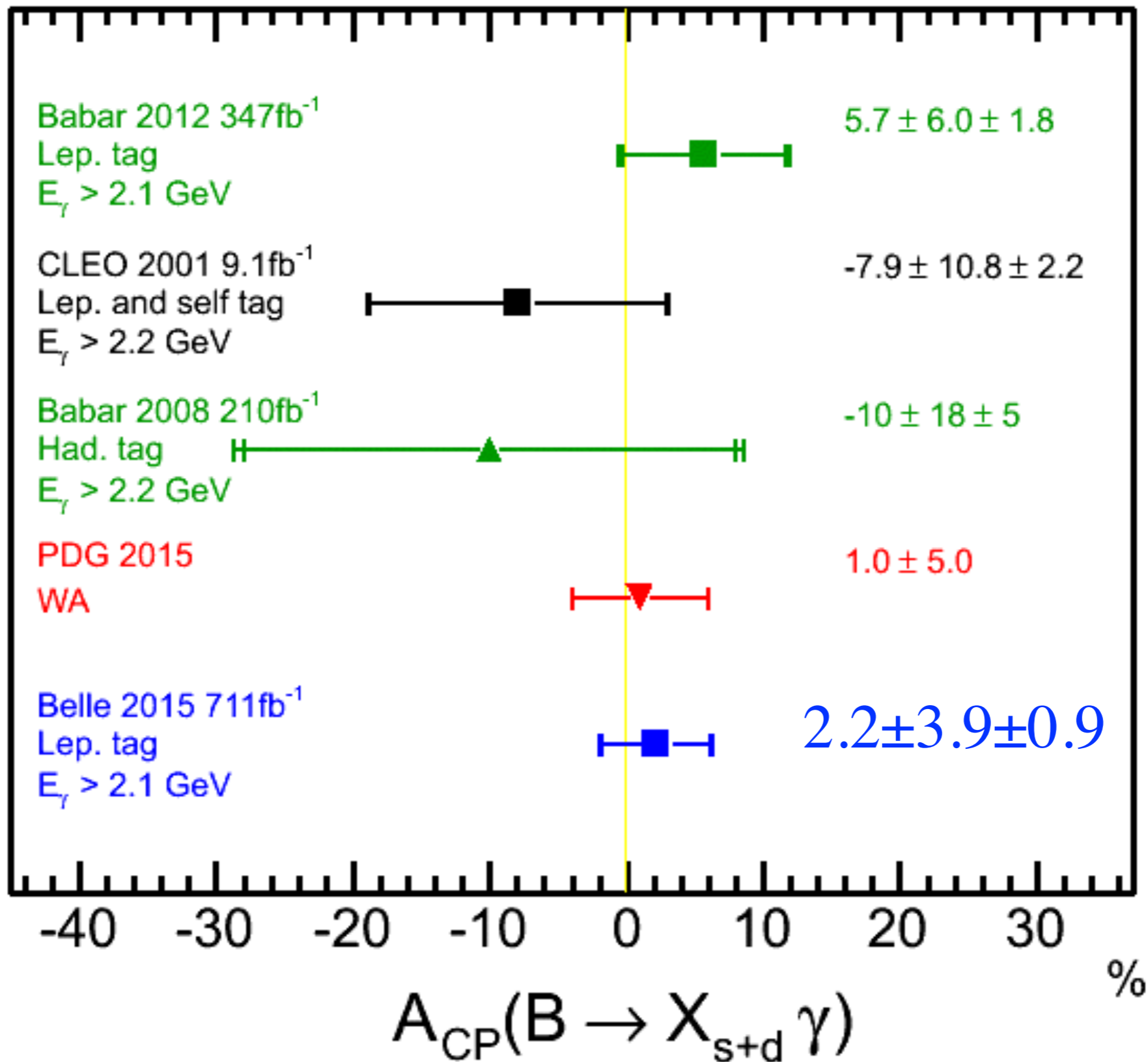


>> Result

$$A_{CP}(X_{s+d}\gamma) = (2.2 \pm 3.9_{stat} \pm 0.9_{syst})\% \\ E_\gamma^* > 2.1 \text{ GeV}$$

The most precise measurement

Summary of Full-inclusive $A_{CP}(\bar{B} \rightarrow X_{(s+d)}\gamma)$



HQE parameters from $\bar{B} \rightarrow X_{(s+d)}\gamma$ with lepton tagging

NEW Result!

>> *Bkg-corrected spectrum*

MC Bkg yields corrected with sideband events and control samples (π^0, η , mis-identified hadrons)

The largest systematic uncertainty is from bkg subtraction

>> *HQE parameters measurement*

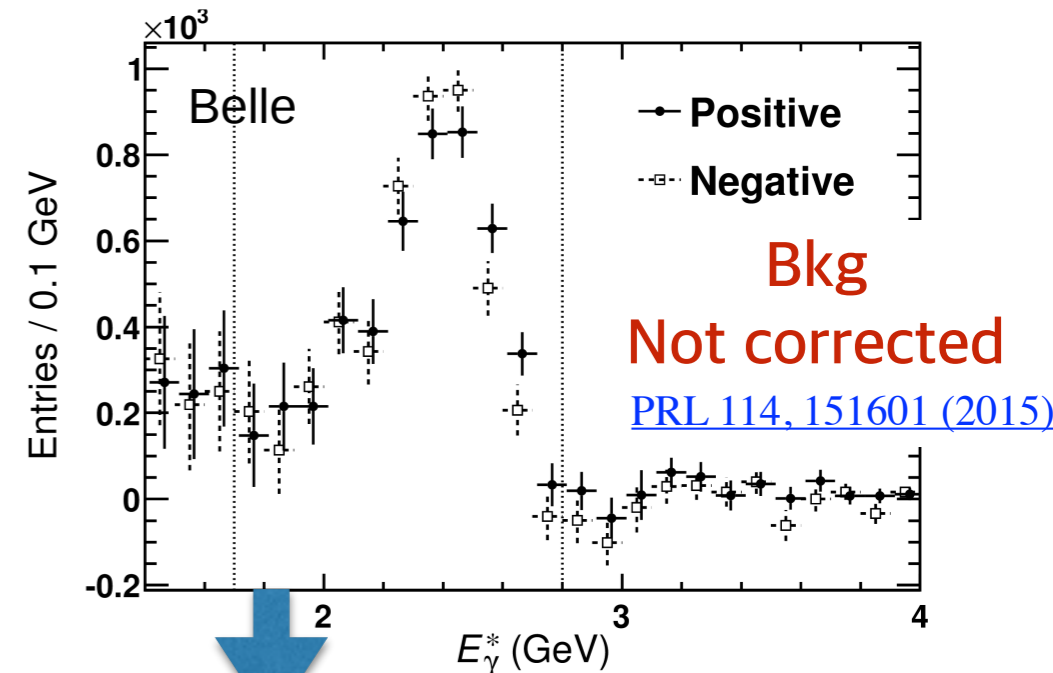
Theoretical spectrum folded :
(in BLNP-SF scheme [[PRD 72, 073006 \(2005\)](#)])

ECAL resolution

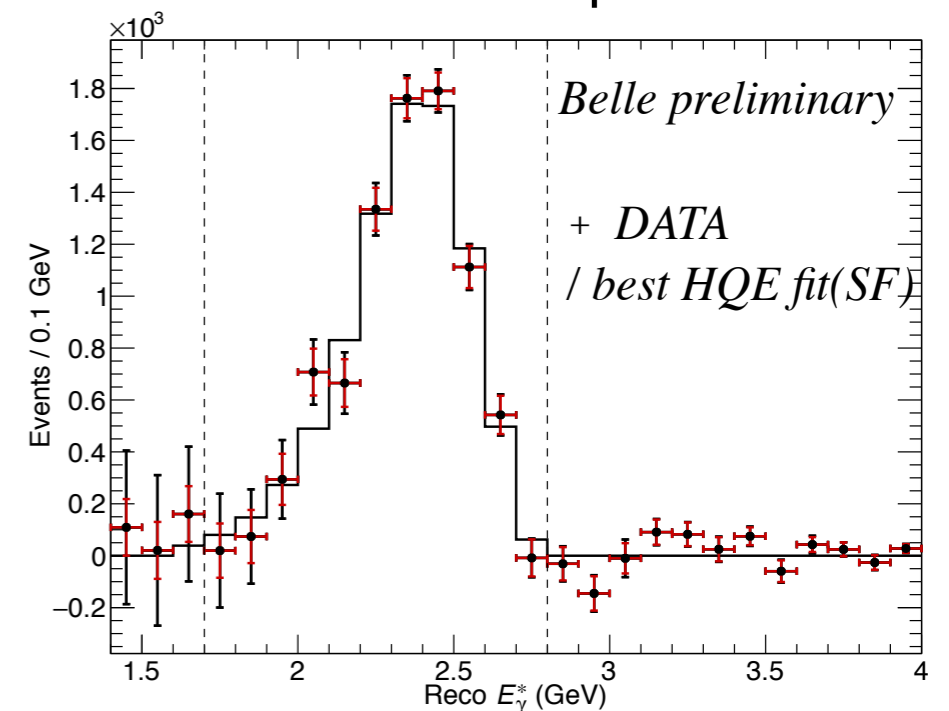
Doppler smearing (B-frame to CM-frame)

HQE parameters fitted to minimize χ^2 between the folded theoretical spectrum to the data spectrum.

BELLE-CONF-1606 [arXiv:1608.02344]



Corrected spectrum



Model-averaged selection efficiency $\sim 2.5\%$

HQE parameters from $\bar{B} \rightarrow X_{(s+d)}\gamma$ with lepton tagging

BELLE-CONF-1606 [arXiv:1608.02344]

NEW Result!

>> HQE parameters fit result

HFAG 2014

$$m_b(SF) = 4.626 \pm 0.028 \text{ GeV}/c^2$$

$$m_b(SF) = 4.569 \pm 0.023 \pm 0.018 \text{ GeV}/c^2$$

$$\mu_\pi^2(SF) = 0.301 \pm 0.063 \text{ GeV}^2$$

$$\mu_\pi^2(SF) = 0.145 \pm 0.089^{+0.020}_{-0.040} \text{ GeV}^2$$

$$(\rho = -0.701)$$

$$(\rho = -0.311)$$

Belle preliminary

→ **Good precisions achieved!**

For example, [PRD 72, 073006 \(2005\)](#)

these values may be used to obtain $|V_{ub}|$ in BLNP-SF scheme lowering it by $\sim 6\%$ (3%) for endpoint analysis with $E_{lepton} > 2.0(1.0) \text{ GeV}$,

compared to the HFAG2014 value : $|V_{ub}|_{BLNP-SF}^{HFAG} = (4.45 \pm 0.15^{+0.20}_{-0.21}) \times 10^{-3}$

>> Moments of the spectrum

Threshold	Mean (GeV)	Variance $\times 10^2$ (GeV ²)
1.8	$2.320 \pm 0.034 \pm 0.110 \pm 0.003$	$4.258 \pm 1.118 \pm 3.612 \pm 0.108$
1.9	$2.338 \pm 0.022 \pm 0.046 \pm 0.003$	$3.563 \pm 0.530 \pm 1.156 \pm 0.065$
2.0	$2.360 \pm 0.015 \pm 0.021 \pm 0.003$	$2.869 \pm 0.290 \pm 0.379 \pm 0.047$

Belle preliminary

Inclusive $\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma)$ with lepton tagging

NEW Result!

BELLE-CONF-1606 [arXiv:1608.02344]

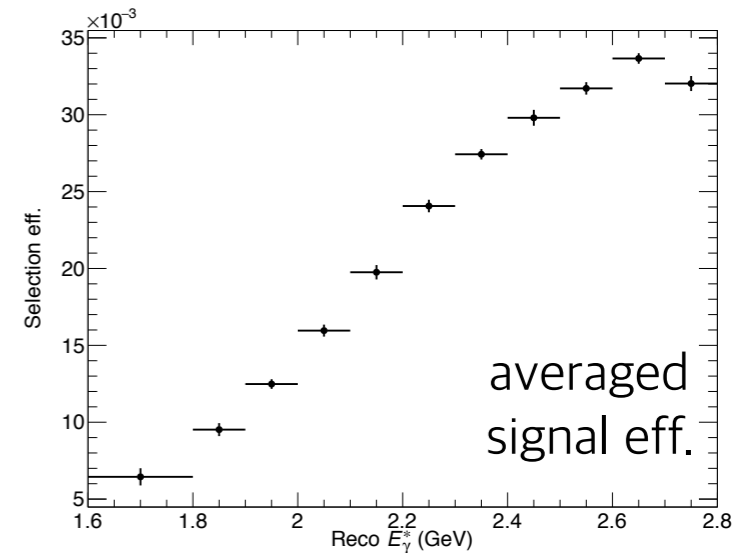
Detector resolution effect unfolded by SVD method

Signal efficiency obtained in average of 3 signal models

Threshold $E^*\gamma > 1.8$ GeV chosen for the best result

Interpolation factors obtained using all three models

$b \rightarrow d\gamma$ is subtracted using $|V_{td}/V_{ts}|^2 \sim 4\%$

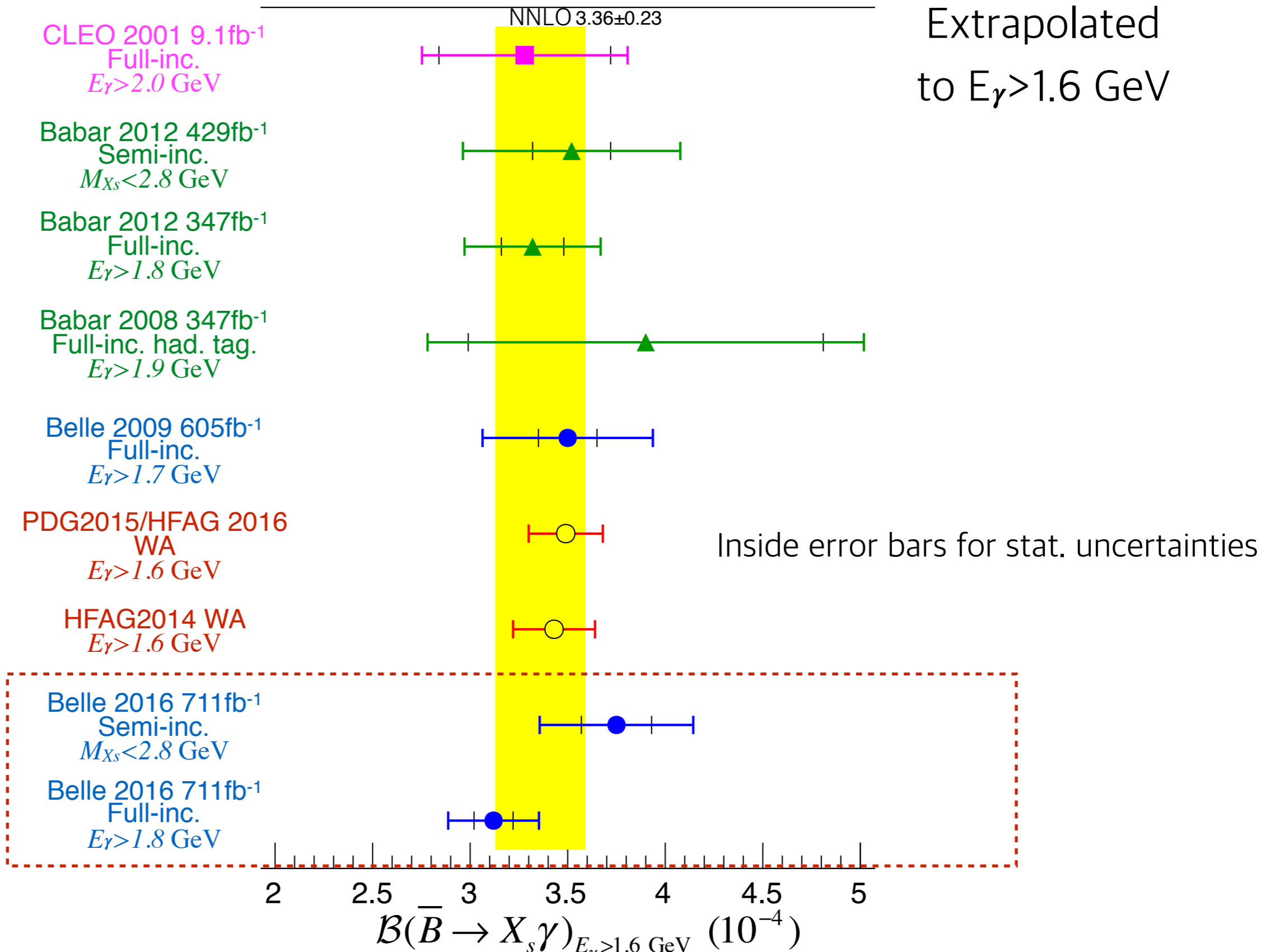


Belle preliminary **Results**

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.12 \pm 0.10_{\text{stat}} \pm 0.19_{\text{syst}} \pm 0.08_{\text{model}}) \times 10^{-4}$$

With different thresholds :	Threshold	$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) (10^{-4})$
	1.7 GeV	$3.07 \pm 0.11 \pm 0.24 \pm 0.09$
	1.8 GeV	$3.02 \pm 0.10 \pm 0.18 \pm 0.08$
	1.9 GeV	$2.95 \pm 0.09 \pm 0.14 \pm 0.07$

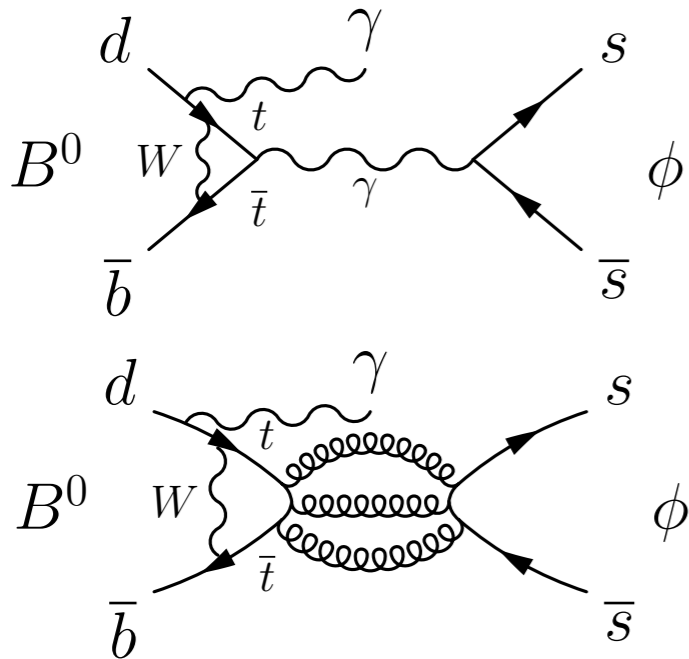
Summary of $\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma)$



$\mathcal{B}(B^0 \rightarrow \phi\gamma) / \mathcal{B}(B_s^0 \rightarrow \phi\gamma) \text{ \& } \mathcal{B}(B_s^0 \rightarrow \gamma\gamma)$

D.Dutta, B.Bhuyan, et al. (Belle Collaboration), published in [PRD 91, 011101\(R\) \(2015\)](#)

Z.King, B.Pal, A.J.Schwartz, et al. (Belle Collaboration), published in [PRD 93, 111101 \(2016\)](#)

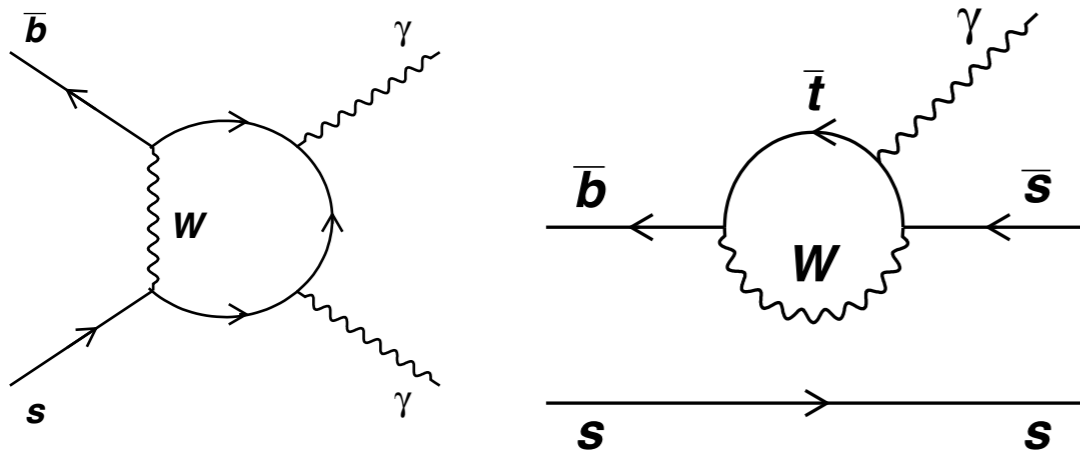


No evidence for $B^0 \rightarrow \phi\gamma$ decay, setting its upper limit for BF :

the most stringent limit on BF

$$\mathcal{B}(B^0 \rightarrow \phi\gamma) < 1.0 \times 10^{-7} \text{ at 90\% C.L.}$$

SM Prediction : $O(10^{-11} \sim -12)$



SM prediction: $\mathcal{B}(B_s^0 \rightarrow \phi\gamma) \approx 4 \times 10^{-5}$
 $\mathcal{B}(B_s^0 \rightarrow \gamma\gamma) \approx (0.5 - 1.0) \times 10^{-6}$

In R-parity violating (RPV),
 BF of $b \rightarrow s\gamma\gamma$ can be enhanced significantly

$$\mathcal{B}(B_s^0 \rightarrow \phi\gamma) = (3.6 \pm 0.5_{stat} \pm 0.3_{syst} \pm 0.6(f_s)) \times 10^{-5}$$

$$\mathcal{B}(B_s^0 \rightarrow \gamma\gamma) < 3.1 \times 10^{-6} \text{ at 90\% C.L. } \boxed{\text{the most stringent limit on BF}}$$

Summary

- Exclusive $b \rightarrow d, b \rightarrow s\gamma, b \rightarrow s\gamma\gamma$

$$\mathcal{B}(B^0 \rightarrow \phi\gamma) < 1.0 \times 10^{-7}$$

$$\mathcal{B}(B_s^0 \rightarrow \phi\gamma) = (3.6 \pm 0.4) \times 10^{-5}$$

$$\mathcal{B}(B_s^0 \rightarrow \gamma\gamma) < 8.7 \times 10^{-6}$$

- Semi-inclusive $b \rightarrow s\gamma$

$$\mathcal{B}(\bar{B} \rightarrow X_s\gamma) = (3.51 \pm 0.17_{stat} \pm 0.33_{syst}) \times 10^{-4}$$

- Inclusive $b \rightarrow s\gamma$ with lepton tagging

$$A_{CP}(X_{s+d}\gamma) = (2.2 \pm 3.9_{stat} \pm 0.9_{syst})\% \quad E_\gamma^* > 2.1 \text{ GeV} \quad \textit{The most precise measurement!}$$

$$\mathcal{B}(\bar{B} \rightarrow X_s\gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.12 \pm 0.10_{stat} \pm 0.19_{syst} \pm 0.08_{model}) \times 10^{-4}$$

The most precise measurements!

$$m_b(SF) = 4.626 \pm 0.028 \text{ GeV}/c^2$$

$$\mu_\pi^2(SF) = 0.301 \pm 0.063 \text{ GeV}^2 \quad (\rho = -0.701)$$

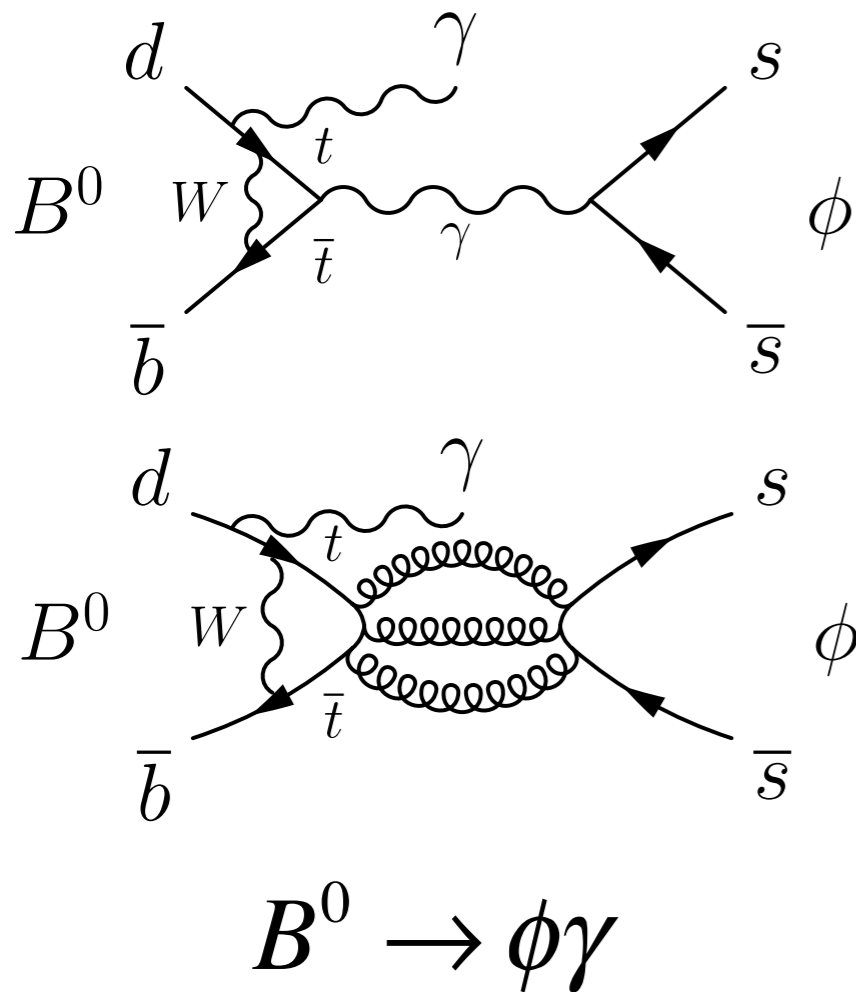
NEW Result!



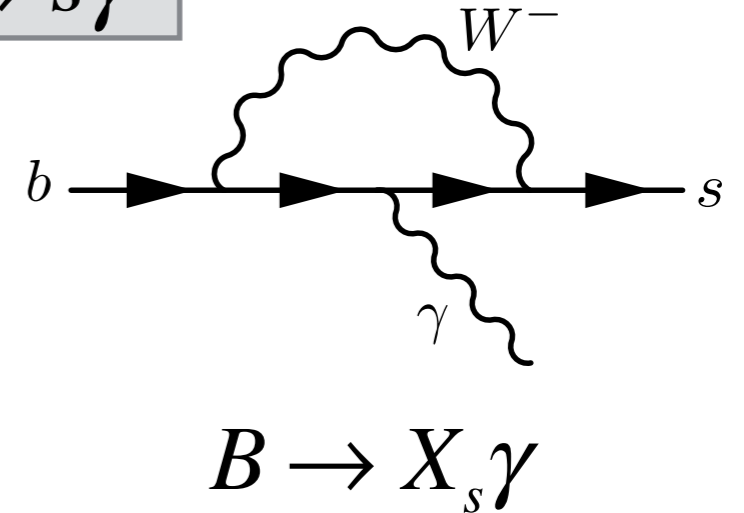
BACKUP

Feynman Diagrams

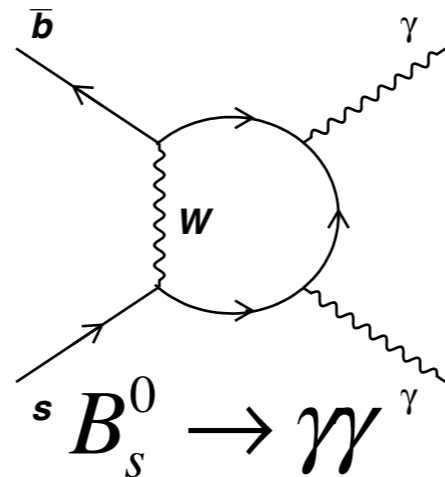
$b \rightarrow d$ annihilation



$b \rightarrow s \gamma$



$b \rightarrow s \gamma \gamma$



Observables:
 $BF, A_{CP}, \Delta_{+-} \dots$

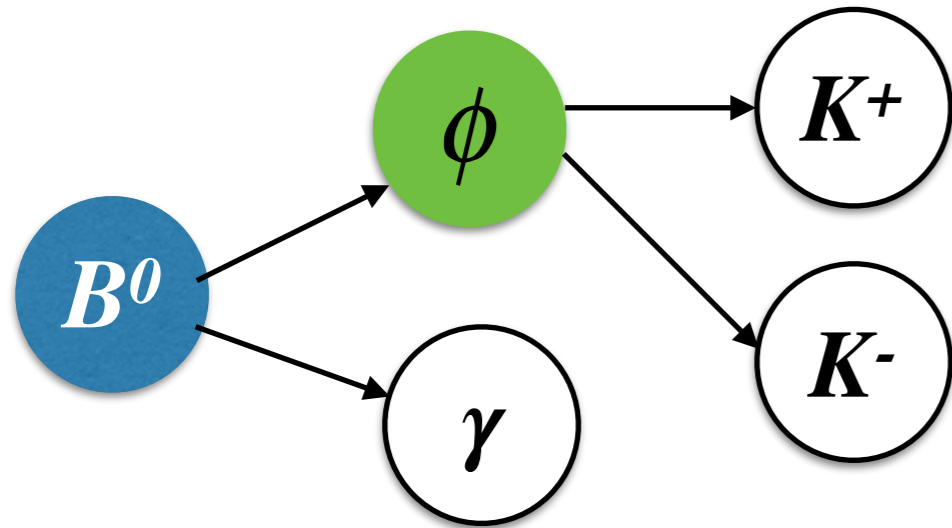
Integrated Luminosity at *Belle*
711 fb⁻¹ for Y(4S)
121 fb⁻¹ for Y(5S)

Electroweak penguin processes highly suppressed in SM corresponding to V_{td}/V_{ts} in CKM matrix
 W^- may be replaced with 2HDM H^- or SSM squarks

Probe to New Physics

Search for $B^0 \rightarrow \phi\gamma$

Z.King, B.Pal, A.J.Schwartz, et al. (Belle Collaboration), published in [PRD93, 111101 \(2016 June 20\)](#)



Event Reconstruction

- ϕ reconstructed via $\phi \rightarrow K^+ K^-$
- Candidate photon lying in [2.0, 2.8] GeV

Background suppression

$\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma \gg 50\%$ of them suppressed with likelihoods based on invariant mass

continuum suppression \gg Neural Network trained with event topology variables suppressing 89% of qq while retaining 85% of signal

Background composition

- continuum events
- rare charmless b -decays
- a negligible contribution from $b \rightarrow c$ process

Search for $B^0 \rightarrow \phi\gamma$

Z.King, B.Pal, A.J.Schwartz, et al. (Belle Collaboration), published in [PRD93, 111101 \(2016 June 20\)](#)

Systematic uncertainty

Source	Uncertainty (events)
PDF parameterization	+1.21 -1.14
Fit bias	+0.00 -0.08
C_{NN} selection efficiency	0.03
C_{NN} background sample	0.02
Tracking efficiency	0.02
PID efficiency	0.05
Photon reconstruction	0.08
MC statistics	0.01
$\mathcal{B}(\phi \rightarrow K^+ K^-)$	0.03
Number of $B\bar{B}$ events	0.05
Total	+1.22 -1.15

Largest uncertainty from fixed PDF parameters

Fitting bias estimated from MC ensembles

Result

No evidence for $B^0 \rightarrow \phi\gamma$ decay, setting its upper limit for BF :

$$\mathcal{B}(B^0 \rightarrow \phi\gamma) < 1.0 \times 10^{-7}$$

at 90% C.L.

previous upper limit^(*) : $\mathcal{B}(B^0 \rightarrow \phi\gamma) < 8.5 \times 10^{-7}$

(*) B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 72, 091103 (2005)

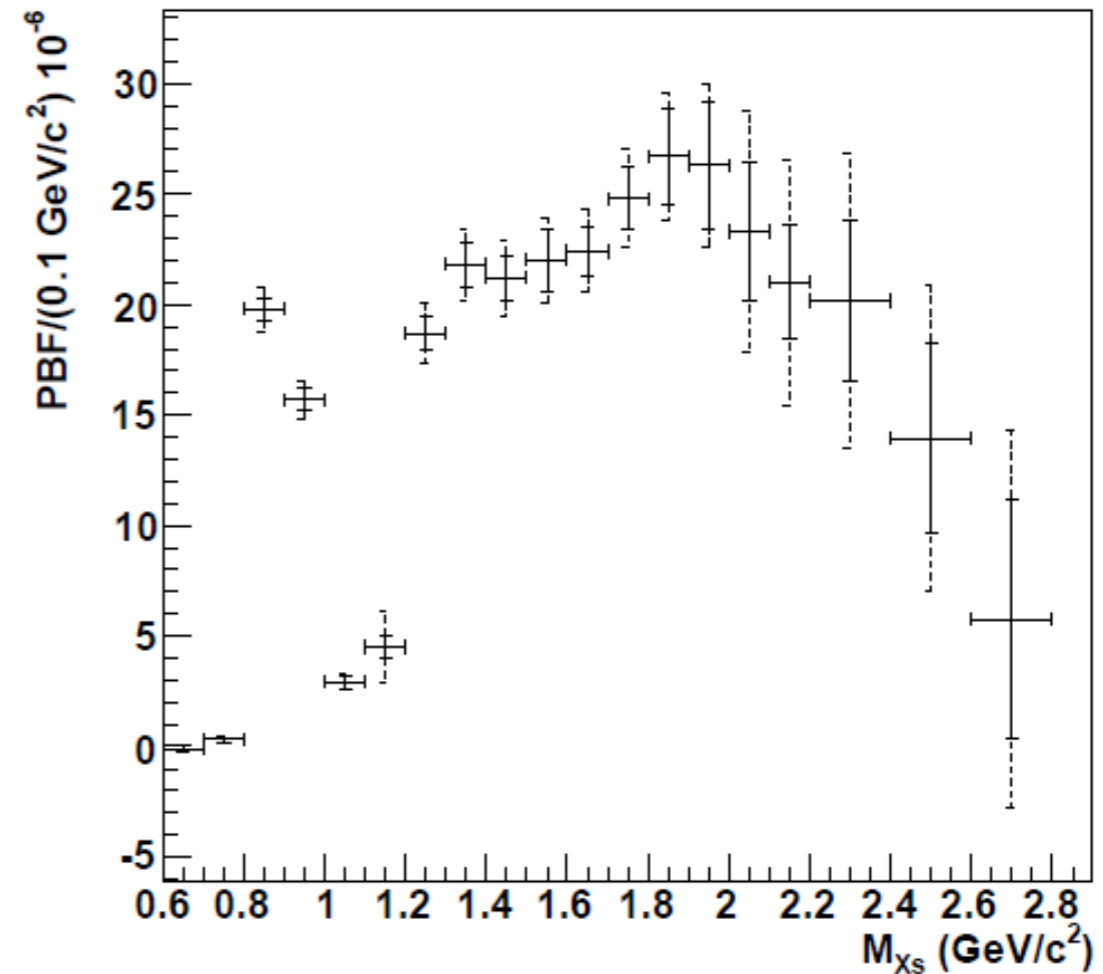
Reconstructed modes

Mode ID	Final state	Mode ID	Final state	Mode ID	Final state
1	$K^+\pi^-$	16	$K_s\pi^+\pi^+\pi^-\pi^0$	31	$K^+\eta\pi^-\pi^0$
2	$K_s\pi^+$	17	$K^+\pi^0\pi^0$	32	$K_s\eta\pi^+\pi^0$
3	$K^+\pi^0$	18	$K_s\pi^0\pi^0$	33	KKK
4	$K_s\pi^0$	19	$K^+\pi^-\pi^0\pi^0$	34	KKK_s
5	$K^+\pi^+\pi^-$	20	$K_s\pi^+\pi^0\pi^0$	35	KK_sK_s
6	$K_s\pi^+\pi^-$	21	$K^+\pi^+\pi^-\pi^0\pi^0$	36	$K^+K^+K^-\pi^-$
7	$K^+\pi^+\pi^0$	22	$K_s\pi^+\pi^-\pi^0\pi^0$	37	$K^+K^-K_s\pi^+$
8	$K_s\pi^+\pi^0$	23	$K^+\eta$	38	$K^+K^+K^-\pi^0$
9	$K^+\pi^+\pi^-\pi^-$	24	$K_s\eta$		
10	$K_s\pi^+\pi^+\pi^-$	25	$K^+\eta\pi^-$		
11	$K_s\pi^+\pi^0$	26	$K_s\eta\pi^+$		
12	$K_s\pi^+\pi^0$	27	$K^+\eta\pi^0$		
13	$K^+\pi^+\pi^+\pi^-\pi^-$	28	$K_s\eta\pi^0$		
14	$K_s\pi^+\pi^+\pi^-\pi^-$	29	$K^+\eta\pi^+\pi^-$		
15	$K_s\pi^+\pi^+\pi^-\pi^0$	30	$K_s\eta\pi^+\pi^-$		

Partial BF (semi-inclusive)

Table 9.12: The partial branching ratio on M_{X_s}

M_{X_s} bin(GeV/c^2)	$\mathcal{BR}(10^{-6})$
0.6-0.7	$-0.1 \pm 0.1 \pm 0.0$
0.7-0.8	$0.3 \pm 0.1 \pm 0.1$
0.8-0.9	$19.8 \pm 0.5 \pm 0.9$
0.9-1.0	$15.7 \pm 0.5 \pm 0.7$
1.0-1.1	$2.9 \pm 0.3 \pm 0.2$
1.1-1.2	$4.8 \pm 0.5 \pm 1.5$
1.2-1.3	$18.7 \pm 0.8 \pm 1.1$
1.3-1.4	$21.8 \pm 1.0 \pm 1.3$
1.4-1.5	$21.2 \pm 1.0 \pm 1.4$
1.5-1.6	$22.0 \pm 1.4 \pm 1.3$
1.6-1.7	$22.4 \pm 1.1 \pm 1.5$
1.7-1.8	$24.8 \pm 1.4 \pm 1.7$
1.8-1.9	$26.7 \pm 2.2 \pm 1.9$
1.9-2.0	$26.3 \pm 2.9 \pm 2.3$
2.0-2.1	$23.3 \pm 3.1 \pm 4.5$
2.1-2.2	$21.0 \pm 2.6 \pm 4.9$
2.2-2.4	$40.3 \pm 7.2 \pm 11$
2.4-2.6	$27.9 \pm 8.6 \pm 11$
2.6-2.8	$11.5 \pm 11 \pm 13$



- (a) Partial branching ratio. The first solid error is the statistical one and the second dashed error is a quadratic sum of the statistical and systematic errors.

Partial BF (full-inclusive)

E_γ^* bin	PBF	Stat	Syst	Model	
1.6-1.8	12.3	735.4	243.0	693.9	18.5
1.8-1.9	11.6	441.2	135.0	419.9	10.9
1.9-2.0	16.7	166.7	64.5	153.6	6.1
2.0-2.1	24.2	57.2	39.5	41.1	4.5
2.1-2.2	34.7	31.1	23.4	20.0	4.7
2.2-2.3	47.6	18.6	14.8	10.6	3.9
2.3-2.4	61.1	12.3	10.5	5.6	3.0
2.4-2.5	63.1	10.8	8.8	5.0	3.7
2.5-2.6	43.7	12.4	10.9	5.4	2.6
2.6-2.7	20.1	22.3	20.2	9.3	2.6

TABLE III. Partial branching fractions of the $\bar{B} \rightarrow X_{s+d}\gamma$ spectrum and uncertainties, in units of 10^{-6} .

Inclusive $\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma)$ with lepton tagging

NEW Result!

REFERENCE

>> *BF measurement*

Before extrapolation

Belle preliminary

Threshold	$\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma) (10^{-4})$	$\mathcal{B}(\bar{B} \rightarrow X_s\gamma) (10^{-4})$
1.7 GeV	$3.21 \pm 0.11 \pm 0.25 \pm 0.10$	$3.07 \pm 0.11 \pm 0.24 \pm 0.09$
1.8 GeV	$3.16 \pm 0.10 \pm 0.19 \pm 0.08$	$3.02 \pm 0.10 \pm 0.18 \pm 0.08$
1.9 GeV	$3.08 \pm 0.09 \pm 0.15 \pm 0.07$	$2.95 \pm 0.09 \pm 0.14 \pm 0.07$
2.0 GeV	$2.92 \pm 0.08 \pm 0.12 \pm 0.05$	$2.79 \pm 0.08 \pm 0.11 \pm 0.05$

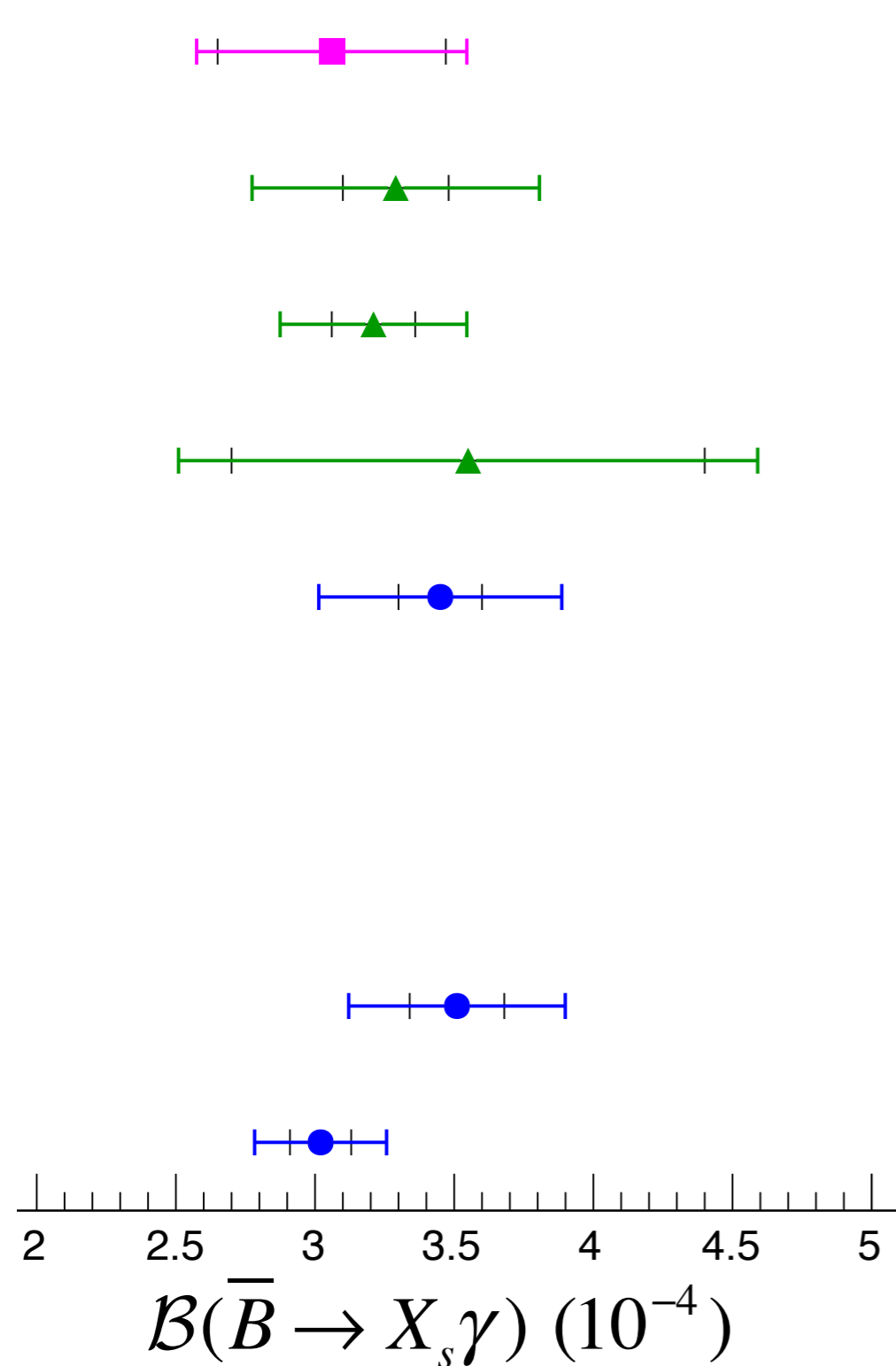
BF thresholds (full-inclusive)

Threshold	Selection eff. (%)	Conversion factor	$\mathcal{B}_{s+d\gamma}$	$\mathcal{B}_{s\gamma}$
1.7 GeV	2.392 ± 0.070	1.0135 ± 0.0024	$3.21 \pm 0.11 \pm 0.25 \pm 0.10$	$3.07 \pm 0.11 \pm 0.24 \pm 0.09$
1.8 GeV	2.442 ± 0.059	1.0216 ± 0.0031	$3.16 \pm 0.10 \pm 0.19 \pm 0.08$	$3.02 \pm 0.10 \pm 0.18 \pm 0.08$
1.9 GeV	2.508 ± 0.055	1.0334 ± 0.0039	$3.08 \pm 0.09 \pm 0.15 \pm 0.07$	$2.95 \pm 0.09 \pm 0.14 \pm 0.07$
2.0 GeV	2.595 ± 0.045	1.0526 ± 0.0046	$2.92 \pm 0.08 \pm 0.12 \pm 0.05$	$2.79 \pm 0.08 \pm 0.11 \pm 0.05$

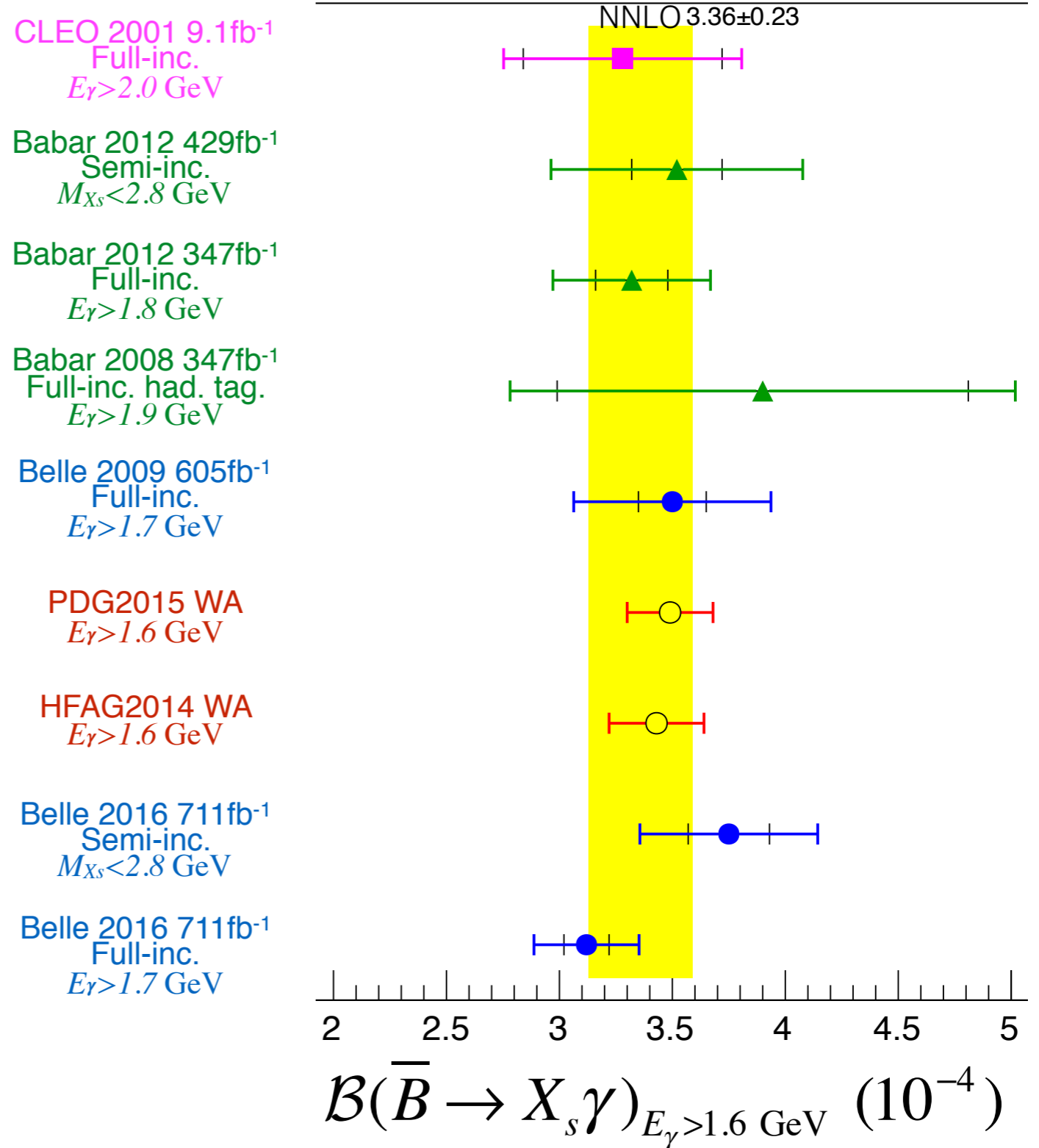
TABLE I. Inclusive $\bar{B} \rightarrow X_{s+d}\gamma$ and $\bar{B} \rightarrow X_s\gamma$ branching fractions for different energy thresholds up to 2.8 GeV, in units of 10^{-4} . The uncertainties are statistical, systematic and from the modeling.

Summary of $\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma)$

With thresholds



With Extrapolation



CLEO 2001 9.1fb⁻¹
Full-inc.
 $E_\gamma > 2.0 \text{ GeV}$

Babar 2012 429fb⁻¹
Semi-inc.
 $M_{X_s} < 2.8 \text{ GeV}$

Babar 2012 347fb⁻¹
Full-inc.
 $E_\gamma > 1.8 \text{ GeV}$

Babar 2008 347fb⁻¹
Full-inc. had. tag.
 $E_\gamma > 1.9 \text{ GeV}$

Belle 2009 605fb⁻¹
Full-inc.
 $E_\gamma > 1.7 \text{ GeV}$

PDG2015 WA
 $E_\gamma > 1.6 \text{ GeV}$

HFAG2014 WA
 $E_\gamma > 1.6 \text{ GeV}$

Belle 2016 711fb⁻¹
Semi-inc.
 $M_{X_s} < 2.8 \text{ GeV}$

Belle 2016 711fb⁻¹
Full-inc.
 $E_\gamma > 1.7 \text{ GeV}$

Summary of $\mathcal{B}(\bar{B} \rightarrow X_{(s+d)}\gamma)$
