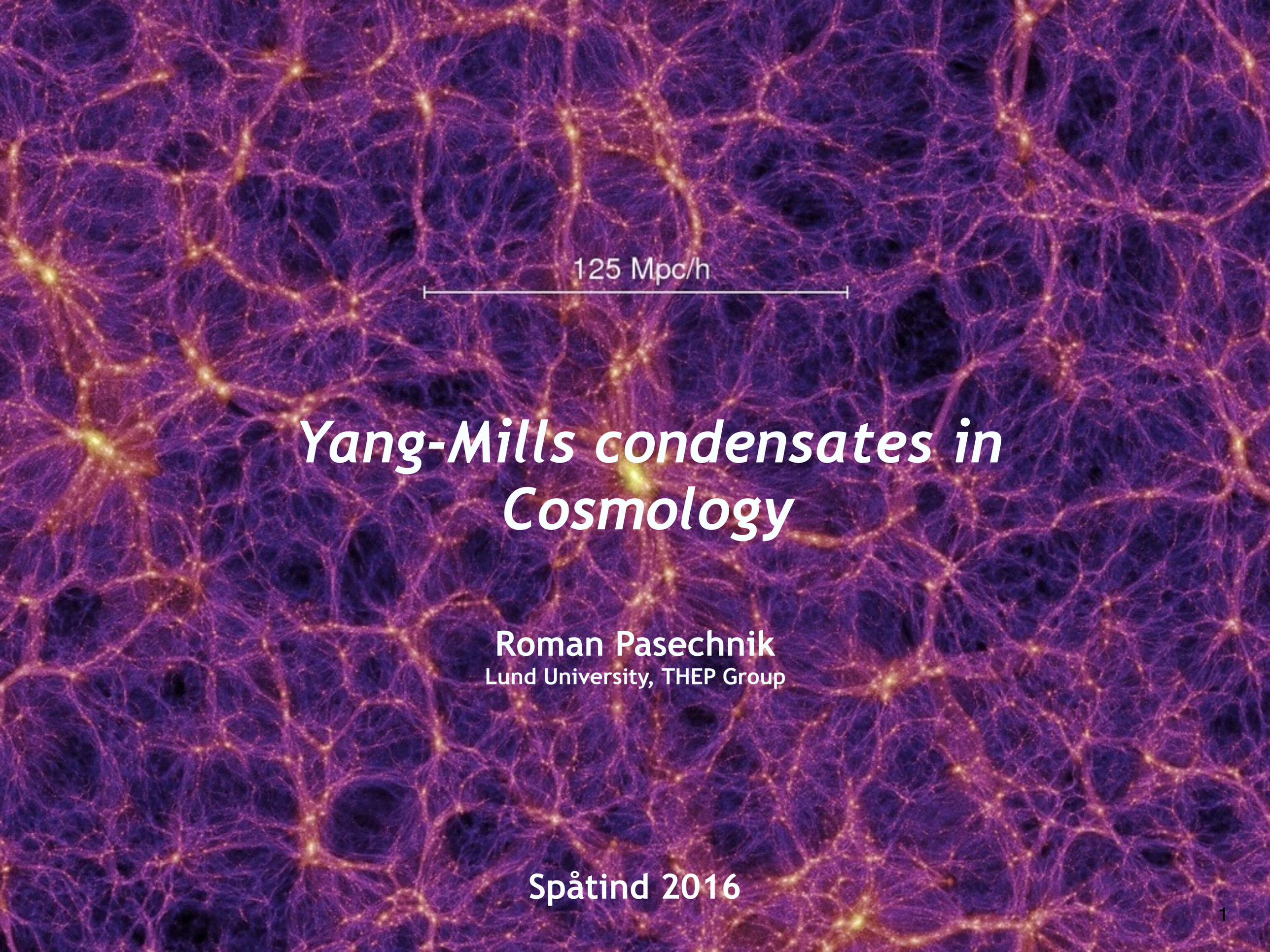
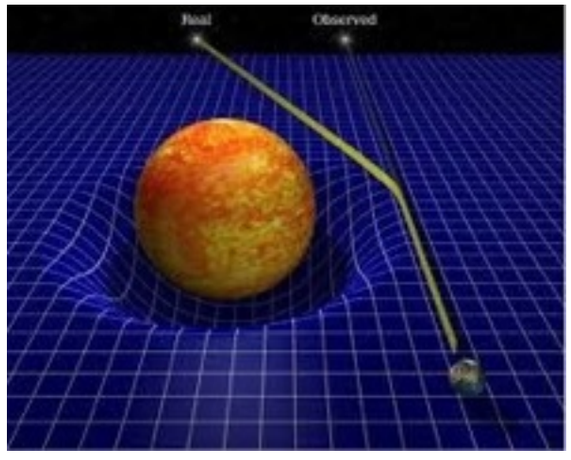


Yang-Mills condensates in Cosmology

Roman Pasechnik
Lund University, THEP Group

Spåtind 2016

Universe composition today: the Dark Energy



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa(\Lambda_0 g_{\mu\nu} + T^{\rm mat}_{\mu\nu}) \qquad \kappa = 8\pi G$$

Homogeneous/isotropic (FLRW) Universe:

Friedmann equation

before Planck

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G(\rho_M + \rho_{rad} + \rho_\Lambda + \rho_{curv})$$

$$\rho_c \equiv \frac{3H_0^2}{\kappa} \sim 10^{-47}\,{\rm GeV}^4 \qquad \Lambda_{\rm cosm} \sim 0.7\rho_c \qquad \rho_M \sim 0.3\rho_c$$

Dark Energy e.o.s. $\omega \equiv P/\rho = -1.13^{+0.13}_{-0.10}$

exactly minus one for pure vacuum!

- **Is the Dark Energy indeed the (non-trivial) vacuum state of the Universe?**
- **How has it been formed during evolution of the Universe?**
- **What is its time dependence and what will happen to our Universe in the future?**

Vacuum Catastrophe… or not?

Vacuum energy

in Quantum Physics

"…the worst theoretical prediction in the history of physics" (Hobson 2006)

in Cosmology

$\rho_{vac} \sim 1\,\mathrm{GeV}^4,$ **Topological QCD vacuum**

$\sim 10^8\,\mathrm{GeV}^4,$ **Higgs condensate**

$$\Lambda_{\mathrm{cosm}} \sim 10^{-47}\,\mathrm{GeV}^4$$

"Old" CC problem: Why such small and positive?
"New" CC problem: Why non-zeroth and exists at all?

Vacuum in Quantum Physics has incredibly wrong energy scale!

Possible approaches to this problem:

- **Let's forget about the "bare" vacuum (DE: "phantom", "quintessence", "ghost"… etc)**

Zero vacuum density in the Minkowski limit, by (Casimir-like) definition, then

$$\Lambda_{\mathrm{cosm}} \equiv \epsilon_{\mathrm{FLRW}} - \epsilon_{\mathrm{Mink}}$$

- **Let's look closer at the vacuum state — why does it become "invisible" to gravity?**

Dynamical cancellation of vacua at every energy scale during phase transitions???

Why may the QCD vacuum have anything to do with DE/CC?

- The ground state of the Universe has undergone many transformations during the cosmological evolution such that the DE may be a residual IR effect

- The QCD transition is the latest phase transition in the cosmological history when the properties of its current ground state might have been formed

- QCD vacuum is the only known strongly-coupled (non-perturbative) vacuum system in Nature for which the QFT principle of locality does not apply (Zhitnitsky et al)

$$\langle T^{\mu}_{\mu} \rangle \sim H\Lambda^3_{\rm QCD} \sim (10^{-3}\,{\rm eV})^4$$

Linear scaling with the Hubble parameter!

- The Yang-Mills (gluon) condensate in expanding Universe has de-Sitter solutions consistent with observations at the minimum of the effective potential (Zhang et al)

- The first-order gravitational correction to the QCD ground state provide a positive contribution to the CC consistent with observations (Pasechnik et al)

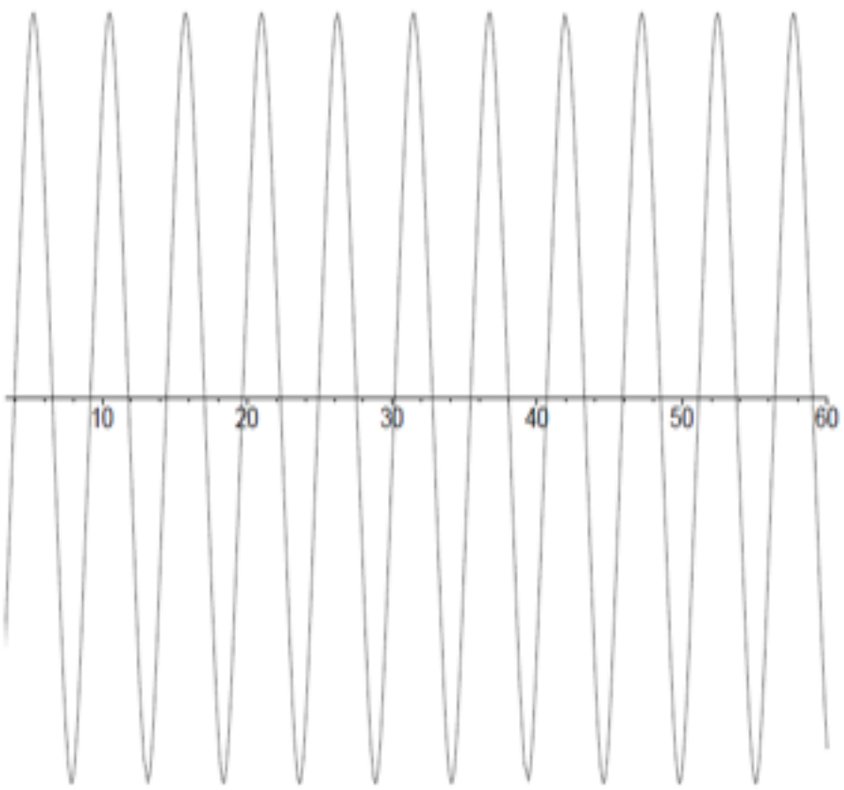
$$\varepsilon_\Lambda \sim G\Lambda^6_{\rm QCD}$$

Quantum effects on the YM ground state

temporal gauge $A_0^a = 0$ $\quad e_i^a A_k^a \equiv A_{ik}$ $\quad e_i^a e_k^a = \delta_{ik}$ $\quad e_i^a e_i^b = \delta_{ab}$

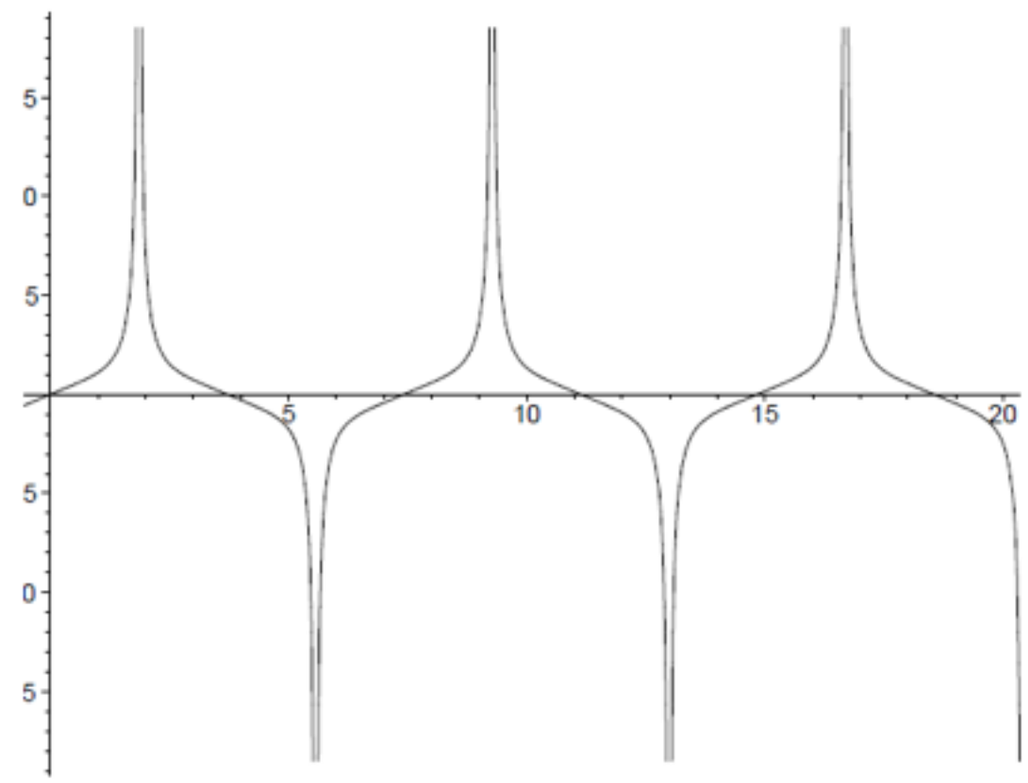
due to local SU(2) ~ SO(3) isomorphism $\quad A_{ik}(t,\vec{x}) = \delta_{ik} U(t) + \widetilde{A}_{ik}(t,\vec{x})$

Classical condensate



Quantum corrections

Savvidy vacuum



"Radiation" medium

$$\epsilon_{\rm YM} \propto 1/a^4$$

Unstable solution!

QCD vacuum:
a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

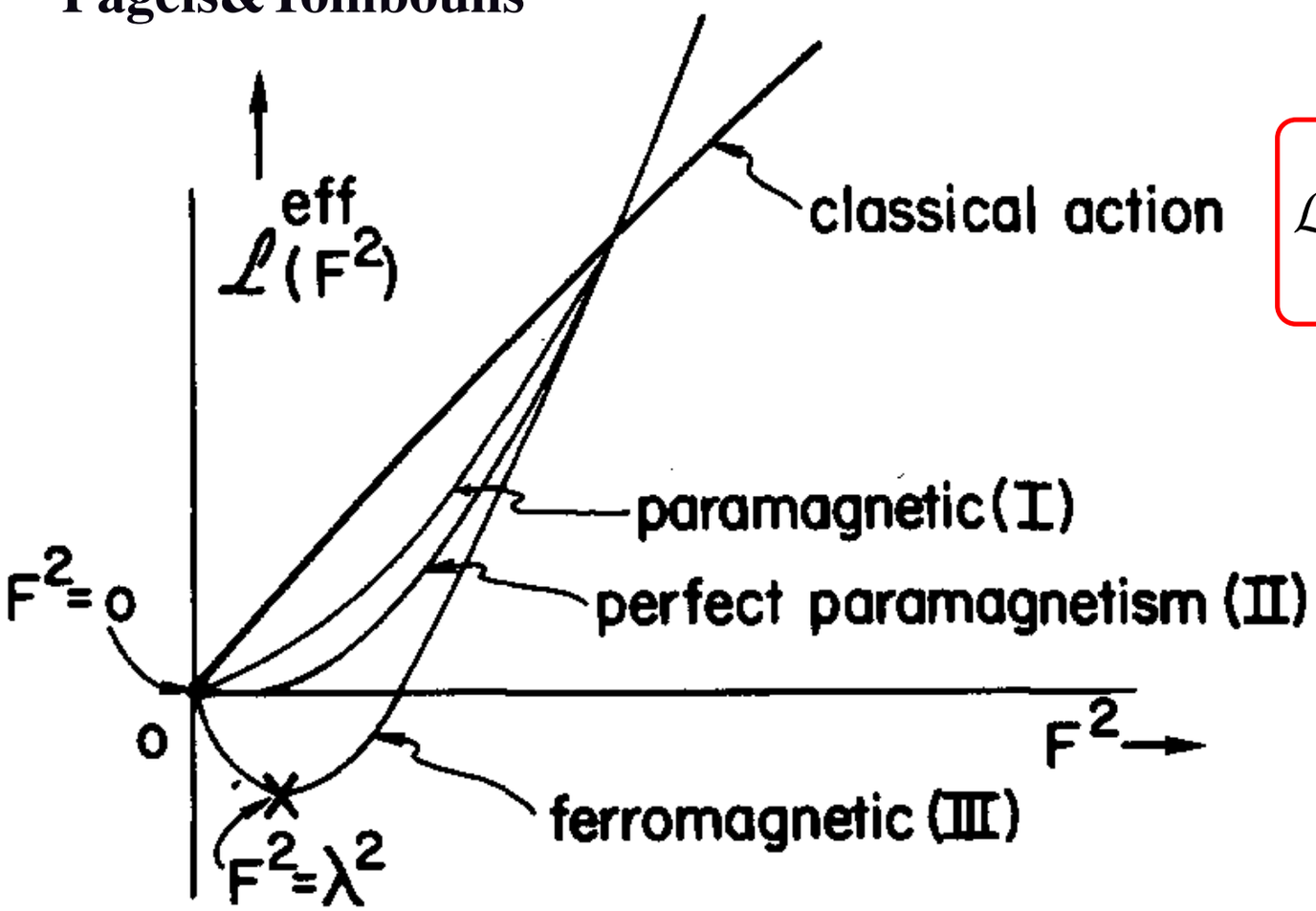
Asymptotic tracker solutions

$$\epsilon_{\rm YM} \to \pm\Lambda_{\rm YM}^4 \qquad t \to \infty$$

Stable solutions!

Effective YM action

Pagels&Tomboulis



asymptotic chromoelectric condensate

$$\mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu\,, \qquad \mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$$

$$\boxed{\mathcal{L}_{\rm eff} = -\frac{J}{4g^2_{\rm YM}(J)}\,, \qquad J = \mathcal{F}^2 \equiv \mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a}$$

trace anomaly:

$$\hat{T}^{\mu,{\rm YM}}_\mu = \frac{\beta(g^2_{\rm YM})}{2g^2_{\rm YM}}\,J$$

$$\boxed{\begin{aligned} 2J\frac{dg^2_{\rm YM}}{dJ} &= g^2_{\rm YM}\,\beta(g^2_{\rm YM}) \\ g^2_{\rm YM} &= g^2_{\rm YM}(|J|) \end{aligned}}$$

no instantons, no propagation!

one-loop ex: $\beta(g^2_{\rm YM}) = -\frac{bg^2_{\rm YM}}{16\pi^2}\,, \qquad g^2_{\rm YM} = \frac{32\pi^2}{b\ln(|J|/\lambda^4)}$

- **In fact, both chromoelectric and chromomagnetic condensates are stable in the FLRW Universe**
- **Basic qualitative features on the non-perturbative action are noticed already at one loop**

Gluon condensate and Dark Energy

One-loop QCD effective action
(toy model)

$$\mathcal{L}_{\rm eff}^{1-{\rm loop}} = -\frac{b\,J}{128\pi^2}\,\ln\left(\frac{|J|}{(\xi\lambda)^4}\right), \qquad J = \frac{\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a}{\sqrt{-g}}$$

$$\frac{1}{\sqrt{\kappa\epsilon_0^{\rm QCD}}} \ll t \sim \frac{1}{\sqrt{\kappa\Lambda_{\rm cosm}}}$$

$$\epsilon_0^{\rm QCD} \equiv \epsilon^{\rm QCD}(t=t_0) \gg \Lambda_{\rm cosm}$$

Friedmann equation in conformal time
with evolving QCD-induced DE

$$\begin{cases} \dfrac{3}{\kappa}\dfrac{(a')^2}{a^4} = \epsilon_{\rm mat} + \Lambda_{\rm cosm} \\[2ex] \Lambda_{\rm cosm}(t) \equiv \epsilon^{\rm QCD}_{(+)}(t) + \epsilon^{\rm QCD}_{(-)}(t) > 0 \end{cases}$$

Asymptotic Einstein-YM solution

$$a(t) \simeq a^*\left(\frac{3\kappa\epsilon_0^{\rm QCD}}{4}\right)^{1/3} t^{2/3}, \qquad \Lambda_{\rm cosm}(t) \simeq \frac{4}{3\kappa t^2}, \qquad p^{\rm QCD}(t) = -\Lambda_{\rm cosm}(t)\left(1+\frac{4\zeta(t)}{3}\right)$$

No dependence on initial conditions!

$$\epsilon^{\rm QCD}(t_U) \simeq \frac{4}{3\kappa t_U^2} \simeq 1.8\times 10^{-47}\,{\rm GeV}^4$$

Extended variational principle

D. J. Shaw and J. D. Barrow, Phys. Rev. D **83**, 043518 (2011).
J. D. Barrow and D. J. Shaw, Phys. Rev. Lett. **106**, 101302 (2011).

Proposal:

- **Promote the CC into a "field"**
- **Restrict the variations of the action w.r.t. this "field" by causality**
- **Require that the cosmological wave function possesses a classical limit**

Predictions:

- **The density of such a "field" is related to other measured quantities:**

$$\Lambda_{\rm cosm} \sim 1/t_U^2$$

- **The spatial curvature in non-trivial, small but testable:**

$$\Omega_{curv} = \rho_{curv}/\rho_c \simeq -0.0055$$

Outlook:

The gluon condensate appears to be the most natural Particle Physics candidate for such a "field"!