Lectures on Particle Cosmology

Pre-SUSY School 2016

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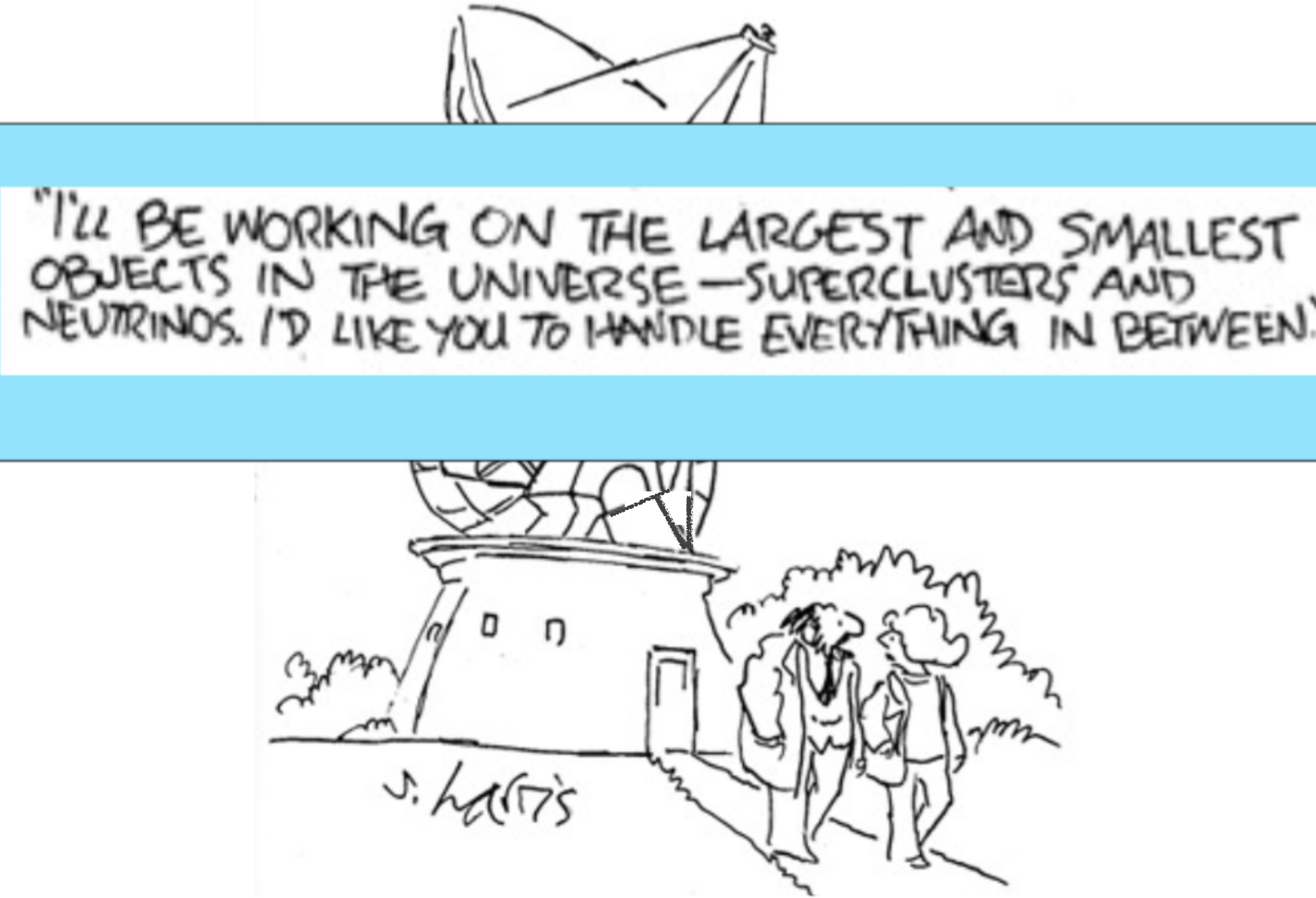
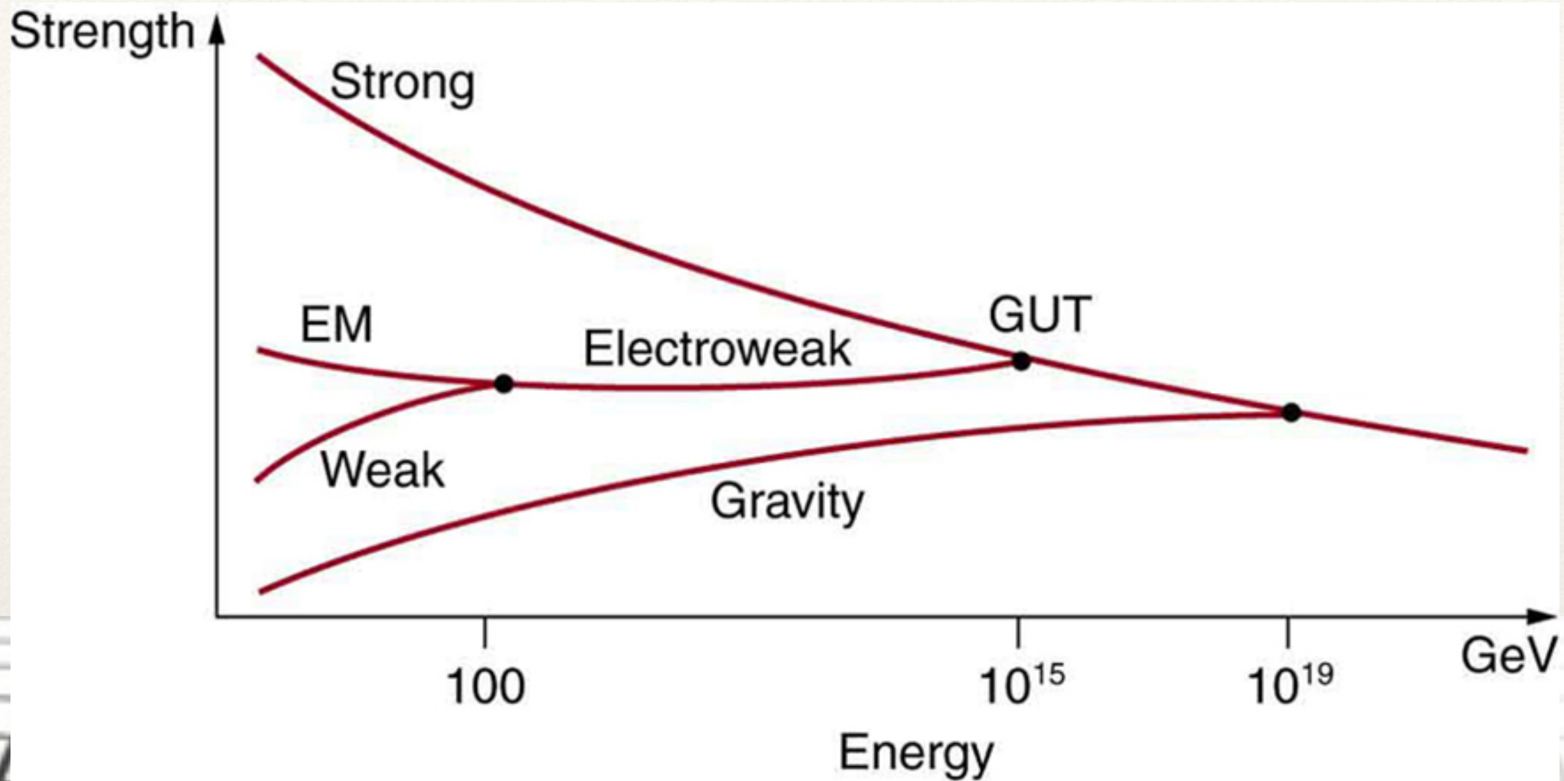
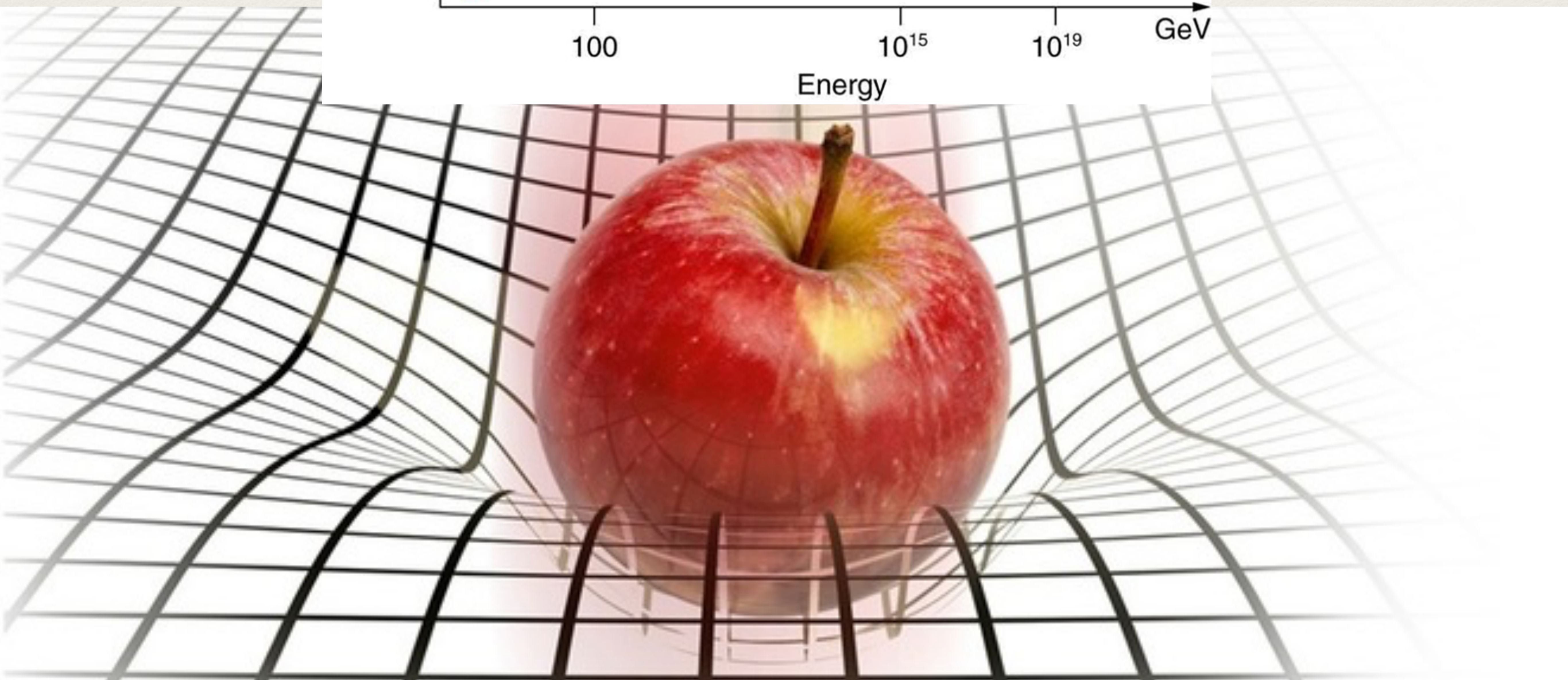

"I'LL BE WORKING ON THE LARGEST AND SMALLEST OBJECTS IN THE UNIVERSE—SUPERCLUSTERS AND NEUTRINOS. I'D LIKE YOU TO HANDLE EVERYTHING IN BETWEEN."
S. Harris

Including this diversion .....

Embrace the future ....

Unbearable Weakness of Gravity





PLAN

(1) Very Early Universe

(2) Ultra Violet (UV) aspects of Gravity

(3) Baryon Asymmetry

Dark Matter: See lectures by Stefano Profumo
Neutrino Physics: See lectures by Jenni Adams

Units

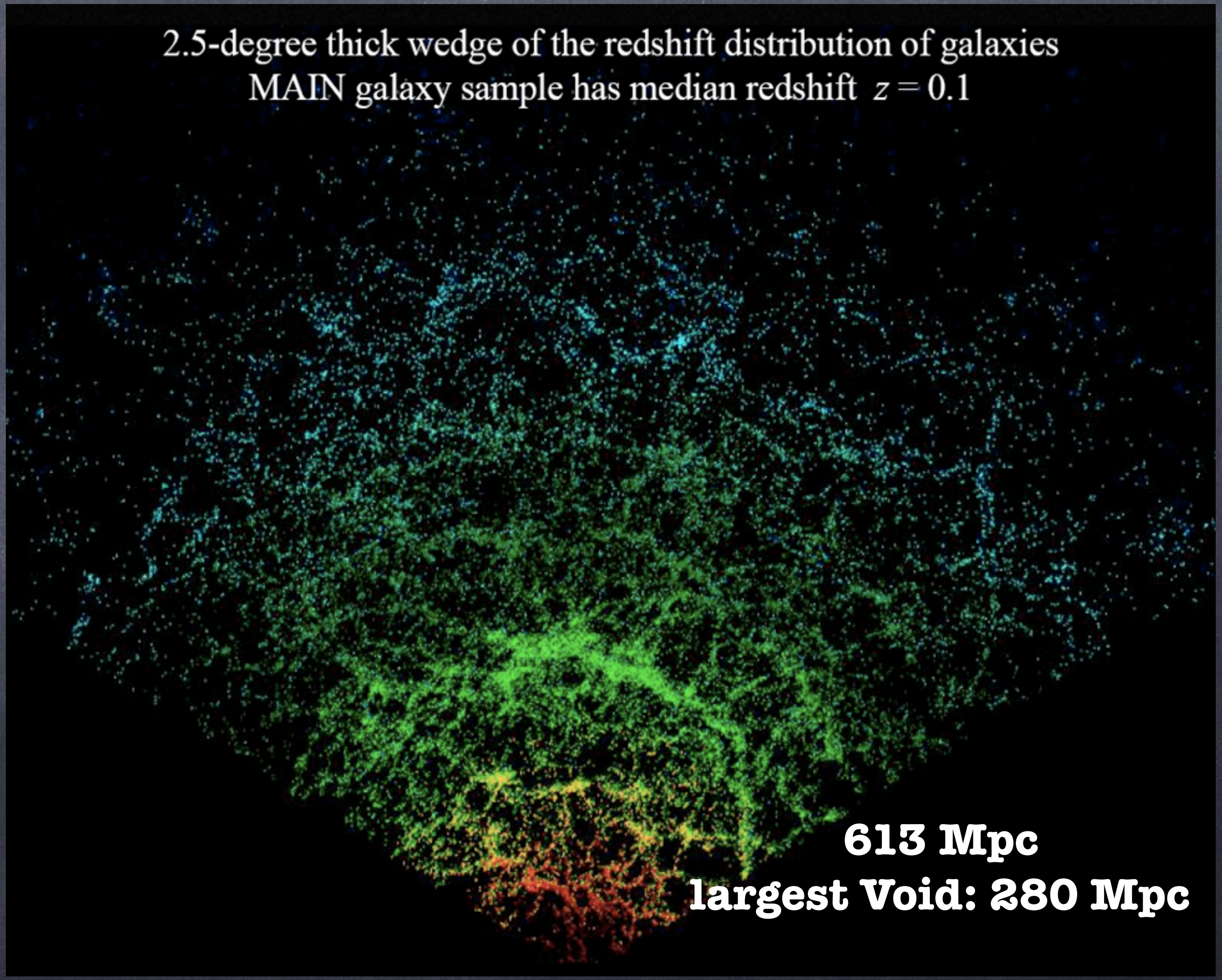
(Appendix of Kolb and Turner book)

$$\hbar = c = k_B = 1$$

$$[\text{Energy}] = [\text{Mass}] = [\text{Temperature}] = [\text{Length}]^{-1} = [\text{Time}]^{-1}$$

$$1 \text{ fermi} = 10^{-13} \text{ cm} \qquad 1 \text{ Mpc} \sim 3 \times 10^{24} \text{ cm}$$

Bohr Radius : 5.2×10^{-13}cm



$$1 \text{ GeV}^{-1} = 6.58 \times 10^{-25} \text{ sec}$$

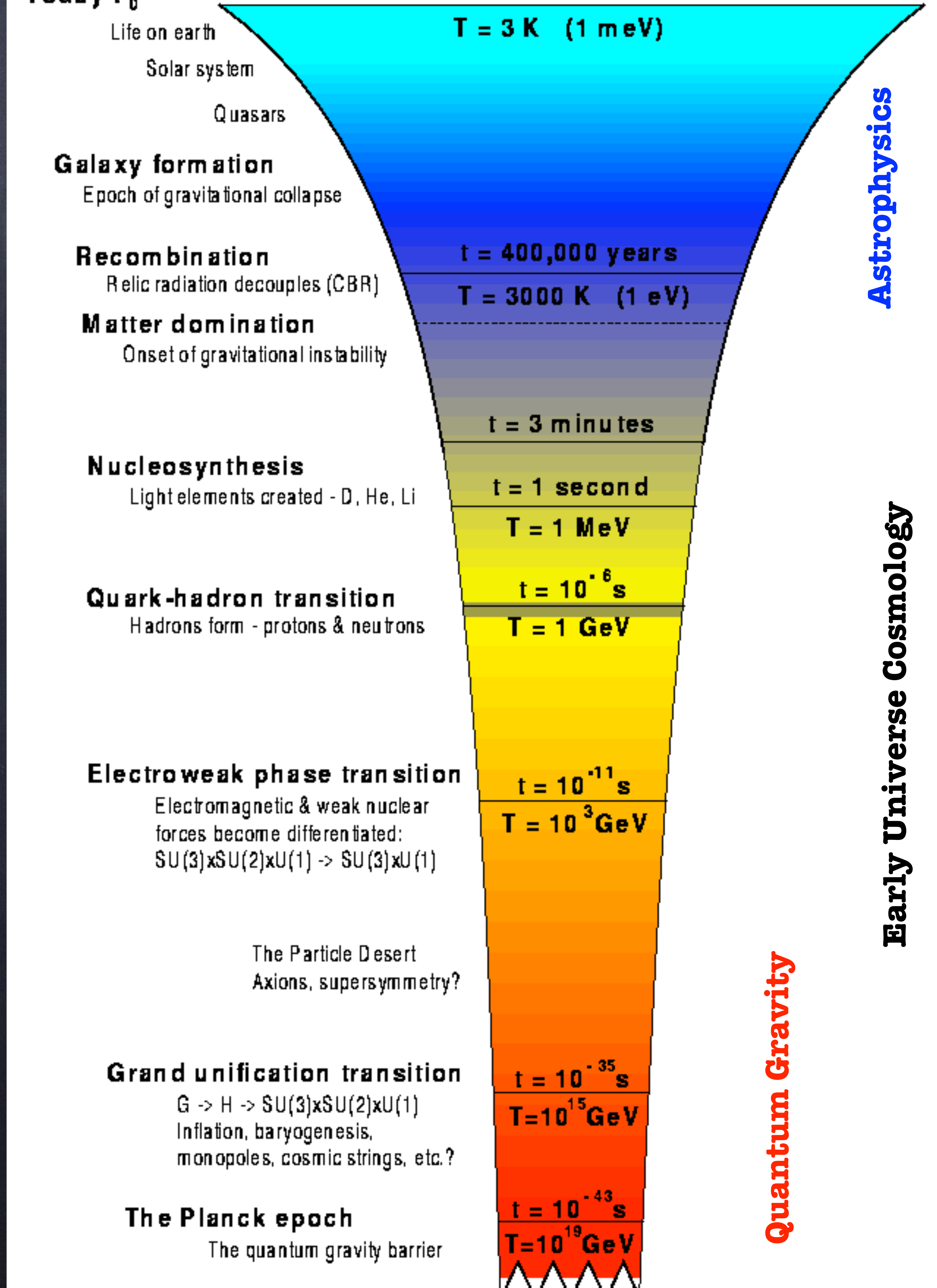
$$1 \text{ GeV}^{-1} = 1.97 \times 10^{-14} \text{ cm}$$

$$1 \text{ GeV} = 1.16 \times 10^{13} \text{ K}$$

$$1 \text{ GeV} = 1.78 \times 10^{-24} \text{ g}$$

$$M_{Pl} = \frac{1}{\sqrt{8\pi G}} = 2.4 \times 10^{18} \text{ GeV}$$

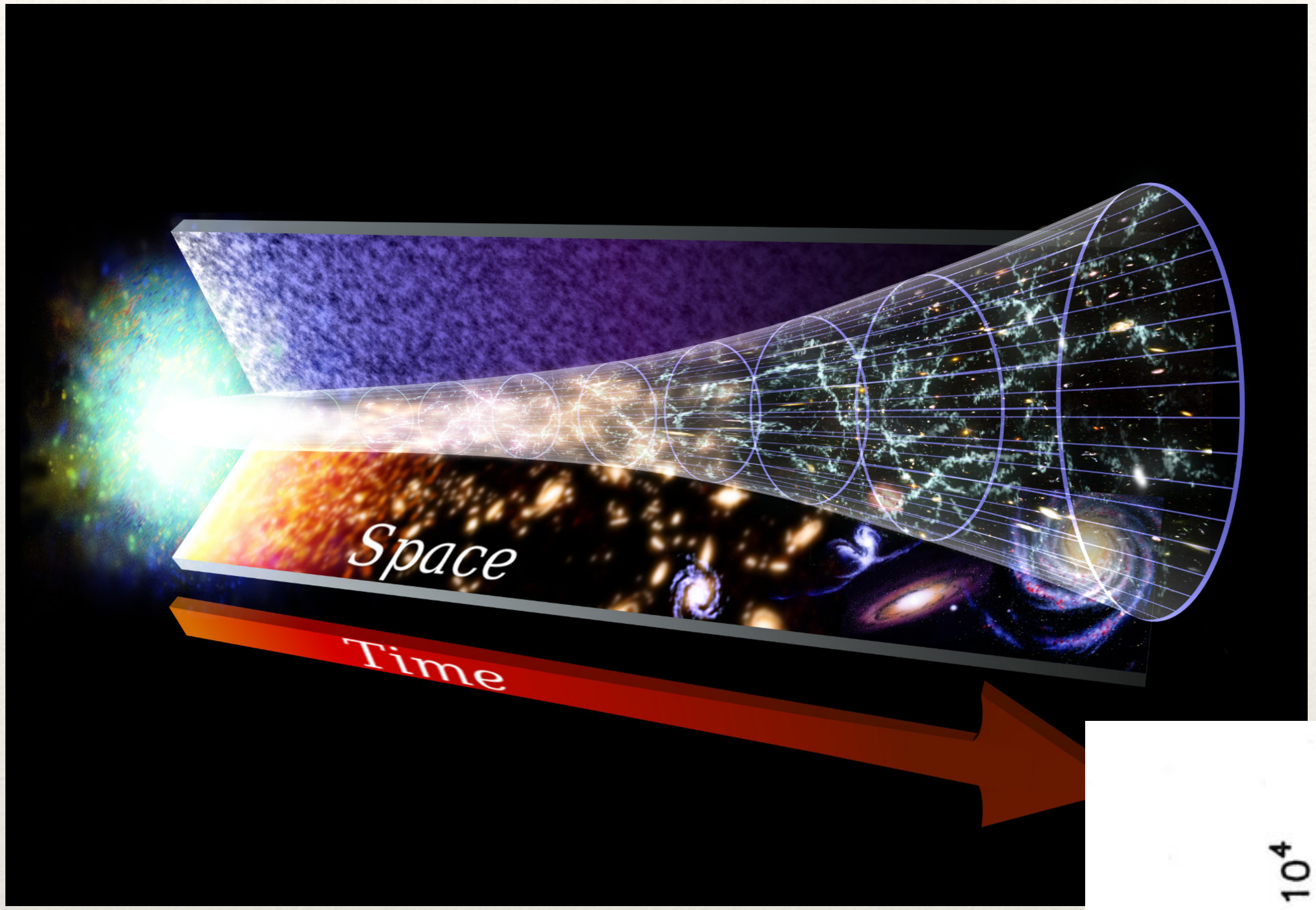
Sometimes in literature $M_p = \frac{1}{\sqrt{8\pi G}}$, or $m_{pl} = \frac{1}{\sqrt{G}} = 1.22 \times 10^{19}$ GeV

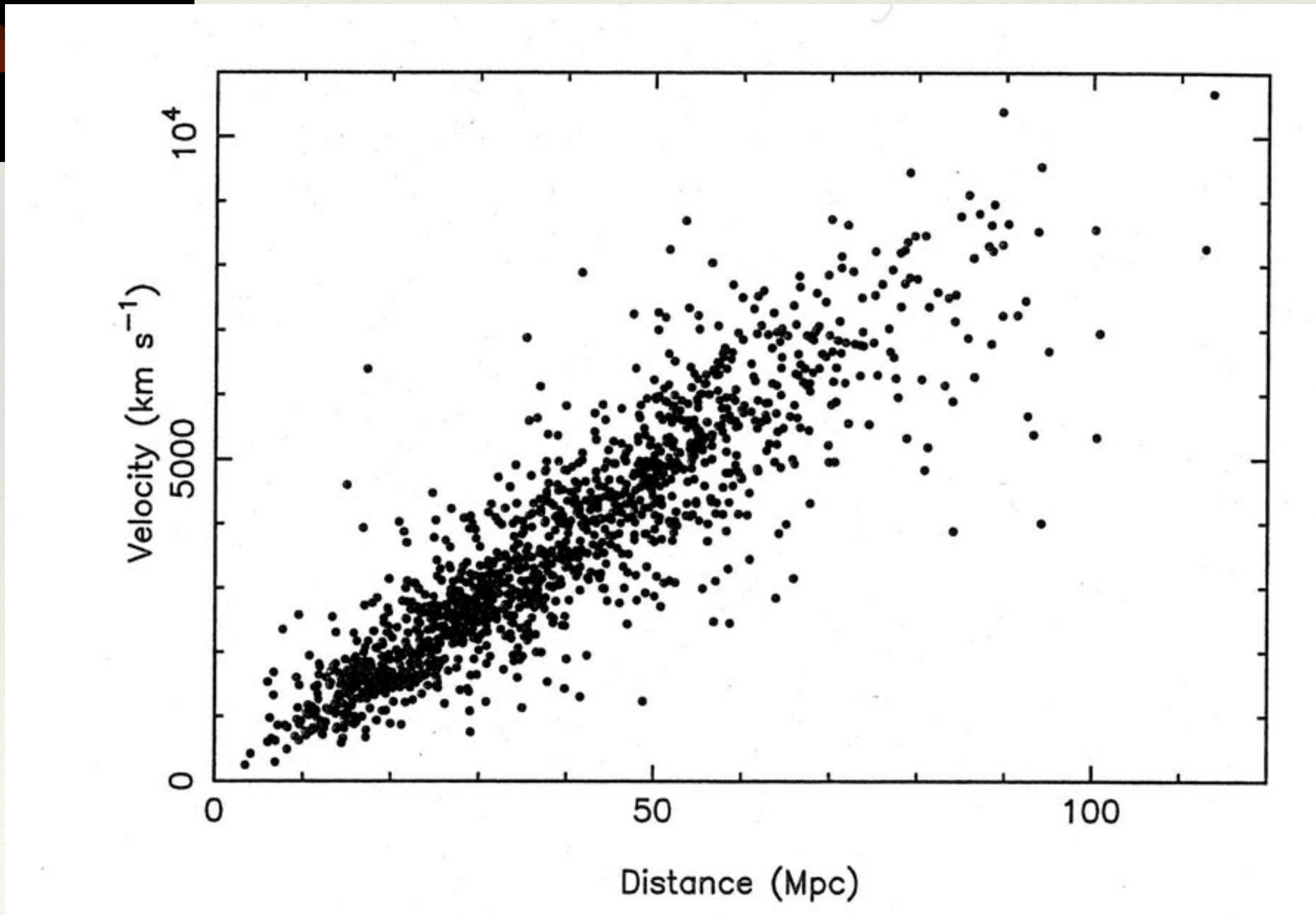


Aim:

Understanding the time line of the Universe

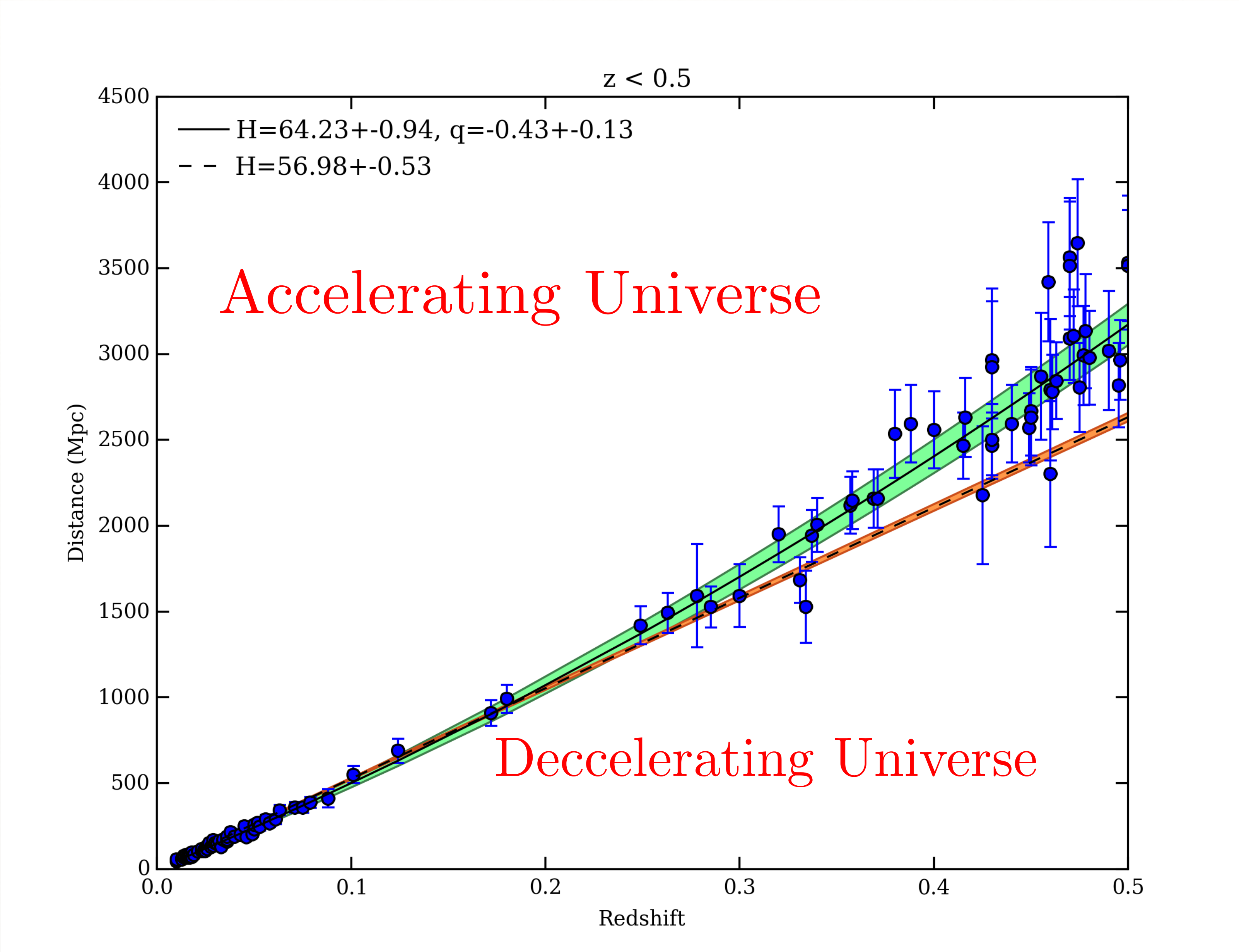
First Evidence of Expanding Universe

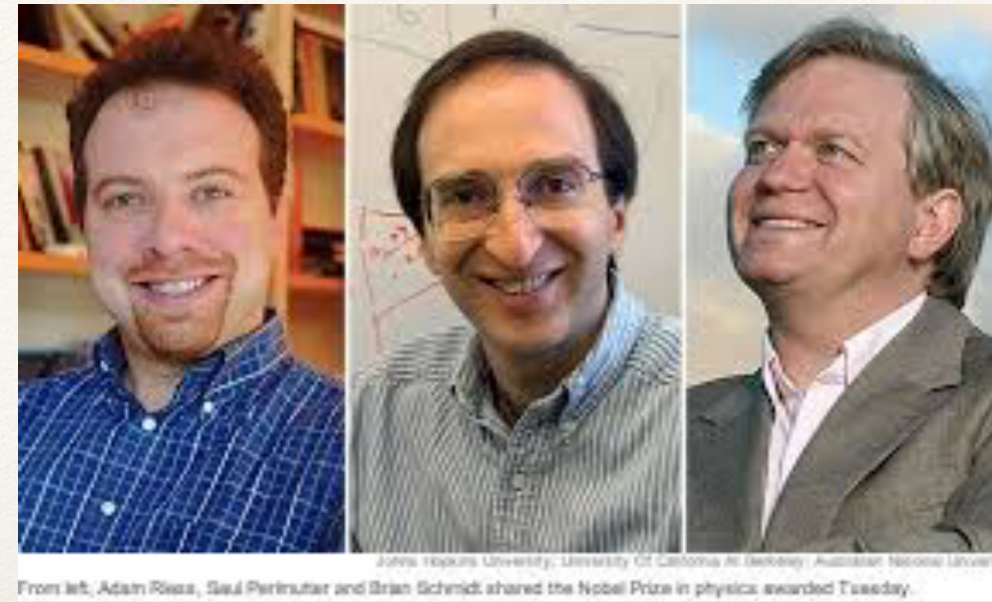




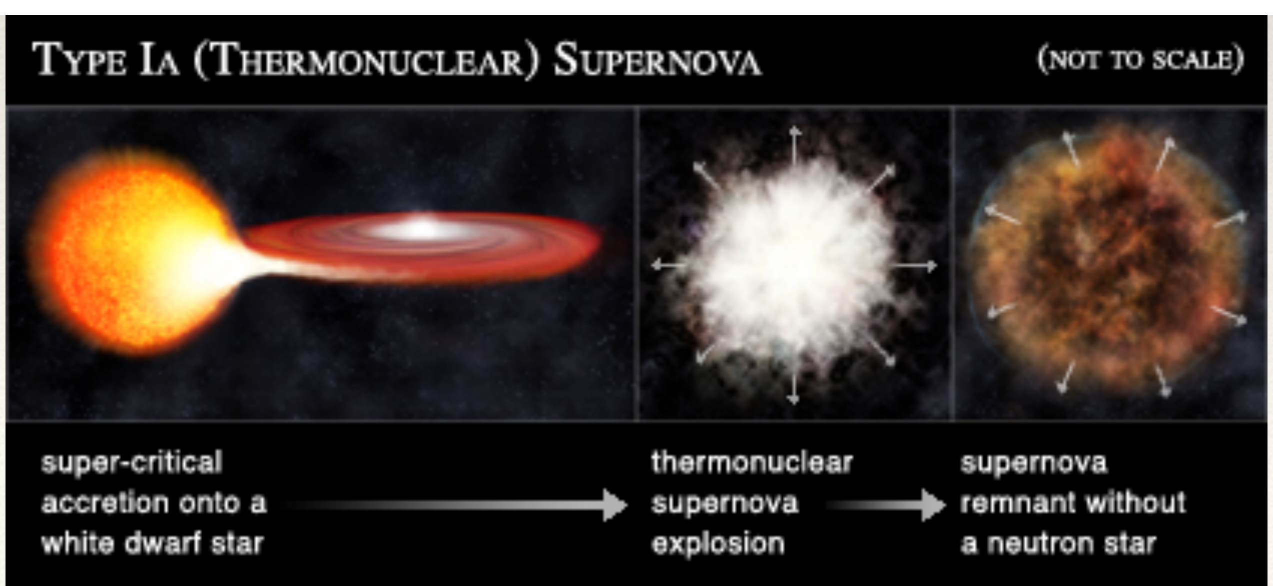
1929 Hubble discovered Universe is expanding

Evidence for Acceleration





Riess, Perlmutter, Schmidt
1998



Supernovae
Type-1A acts like a
Standard Candle

Chandrasekhar

Weighing the Universe

$$10^{-47}\ \mathrm{GeV}^4 \sim 10^{-29}\ \mathrm{g\ cm}^{-3}$$

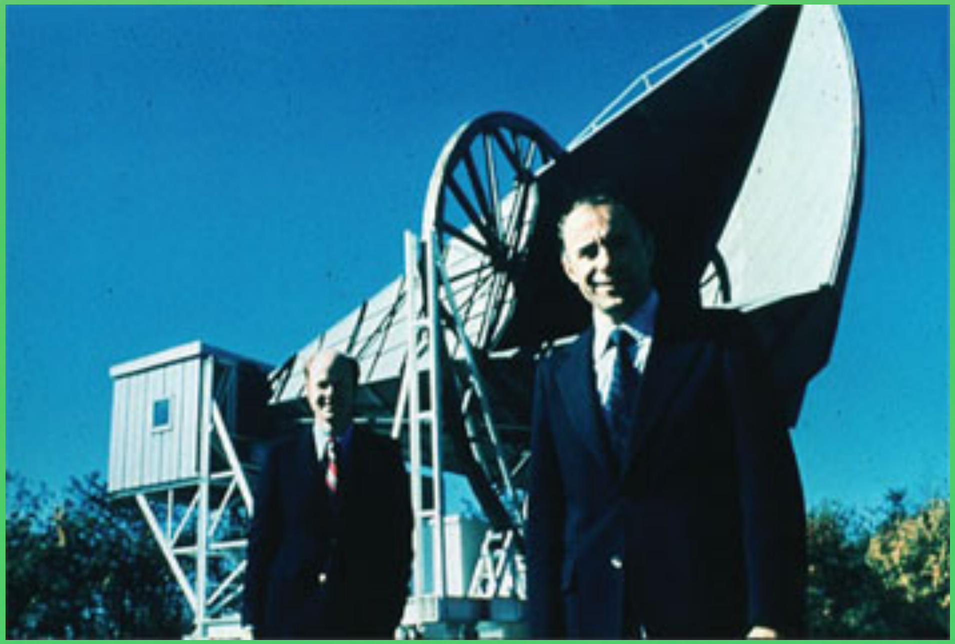
Unbearable Lightness of the Universe

Cosmic Microwave Background Radiation

Universe is Bright in
Microwave region
with 2.725 k

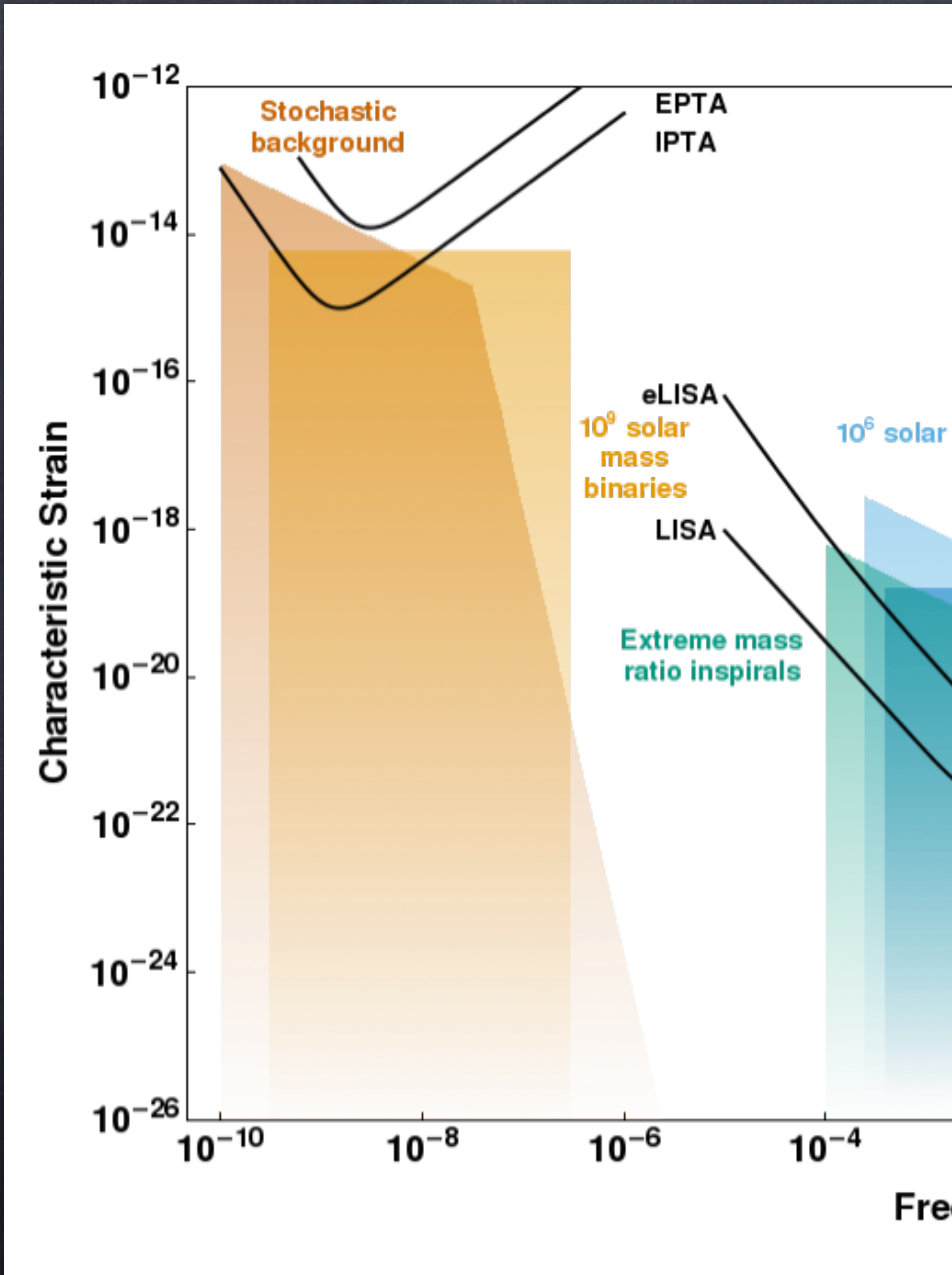
Gammow

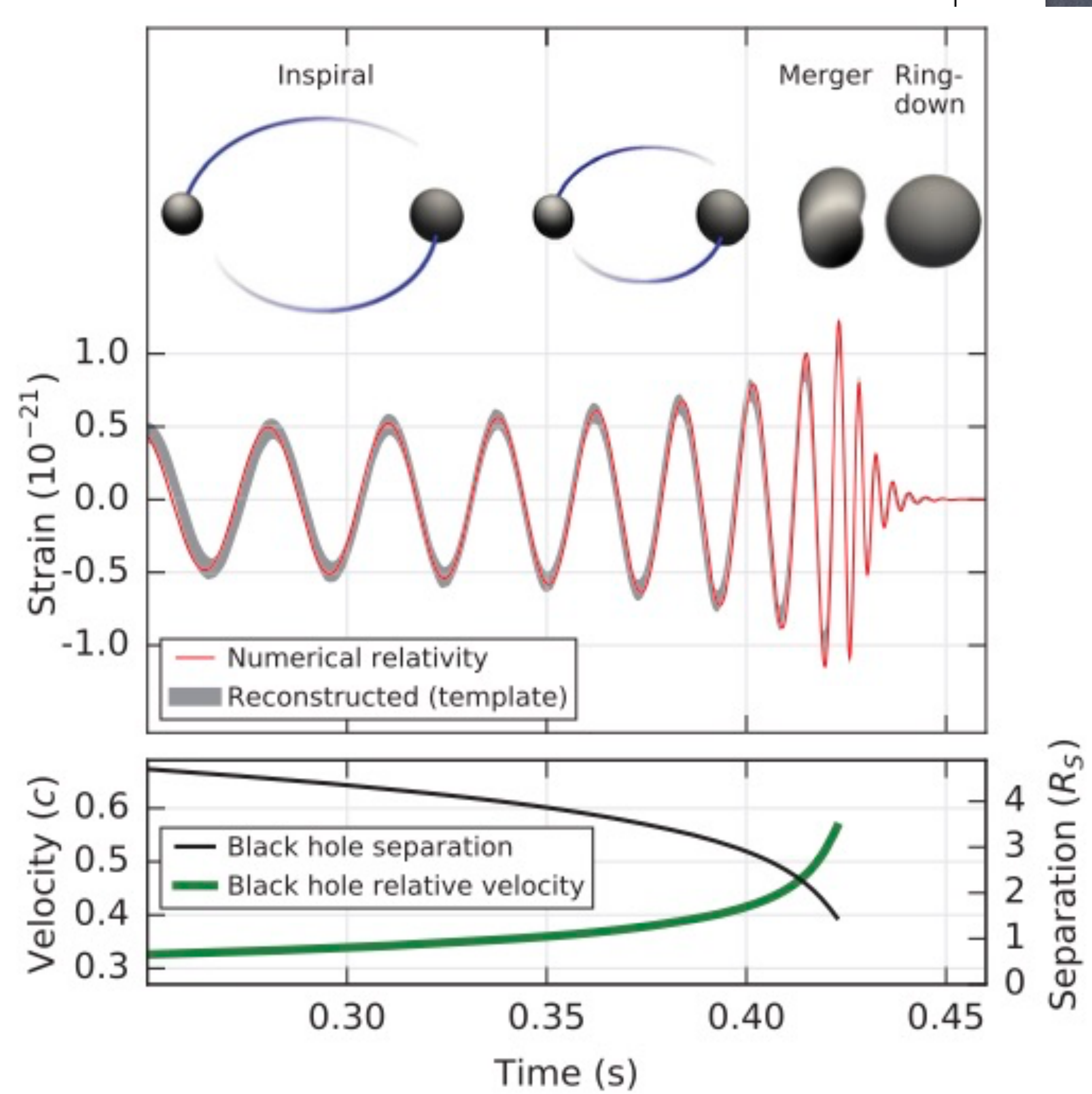
Penzias and Wilson
discovered it in 1965



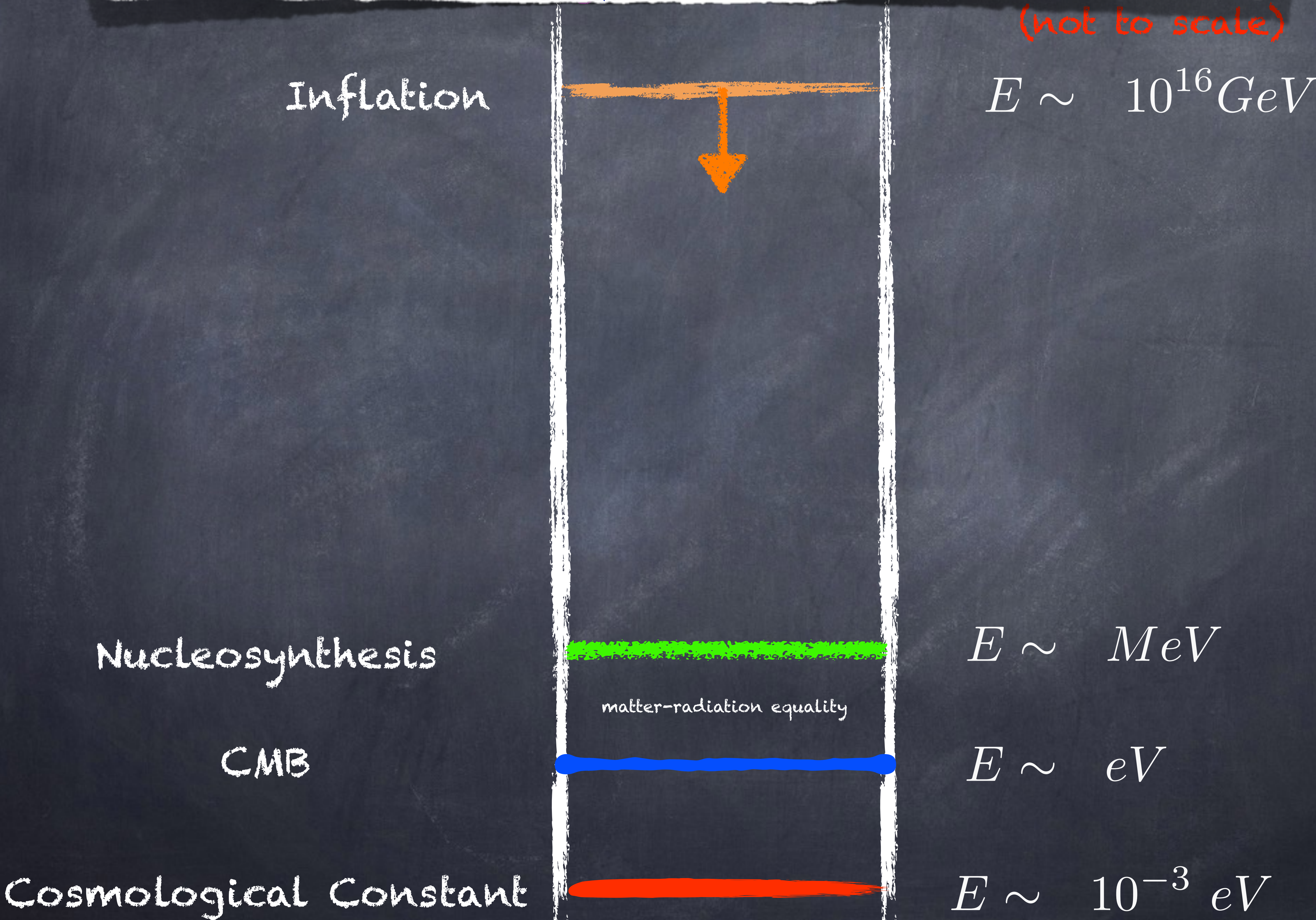
Further Confirmation of Hot Big Bang Cosmology

New Era: Gravitational Wave Astronomy





Energy Ladder



Energy Ladder

(not to scale)

Inflation — $E \sim \quad 10^{16} GeV$

We may also add 'particle physics ladder' on top if it:

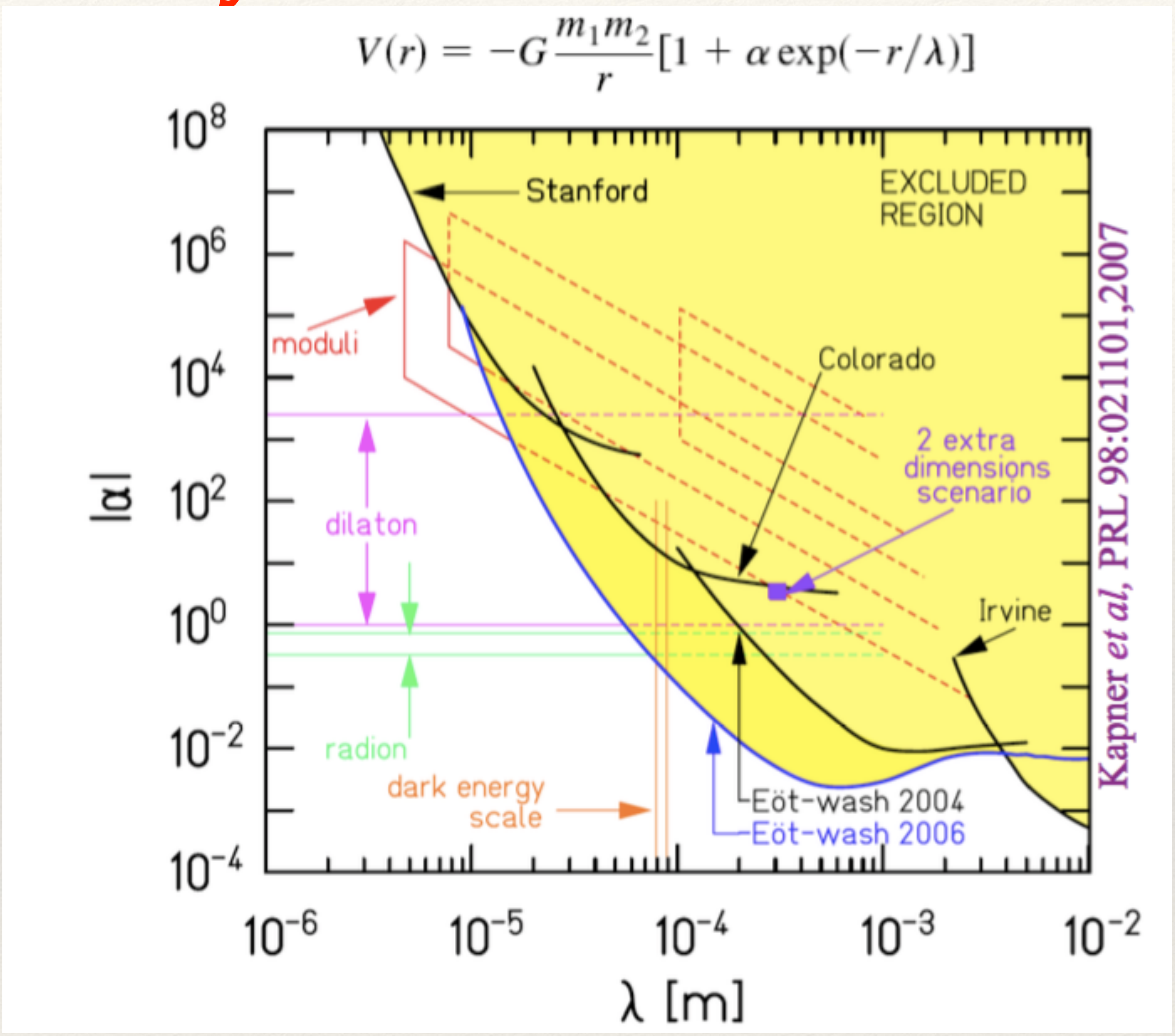
- GUT physics
- Lepto/Baryogenesis
- EW phase transition
- QCD phase transition
- SUSY
- String theory
- Dark Matter freeze out
- ....

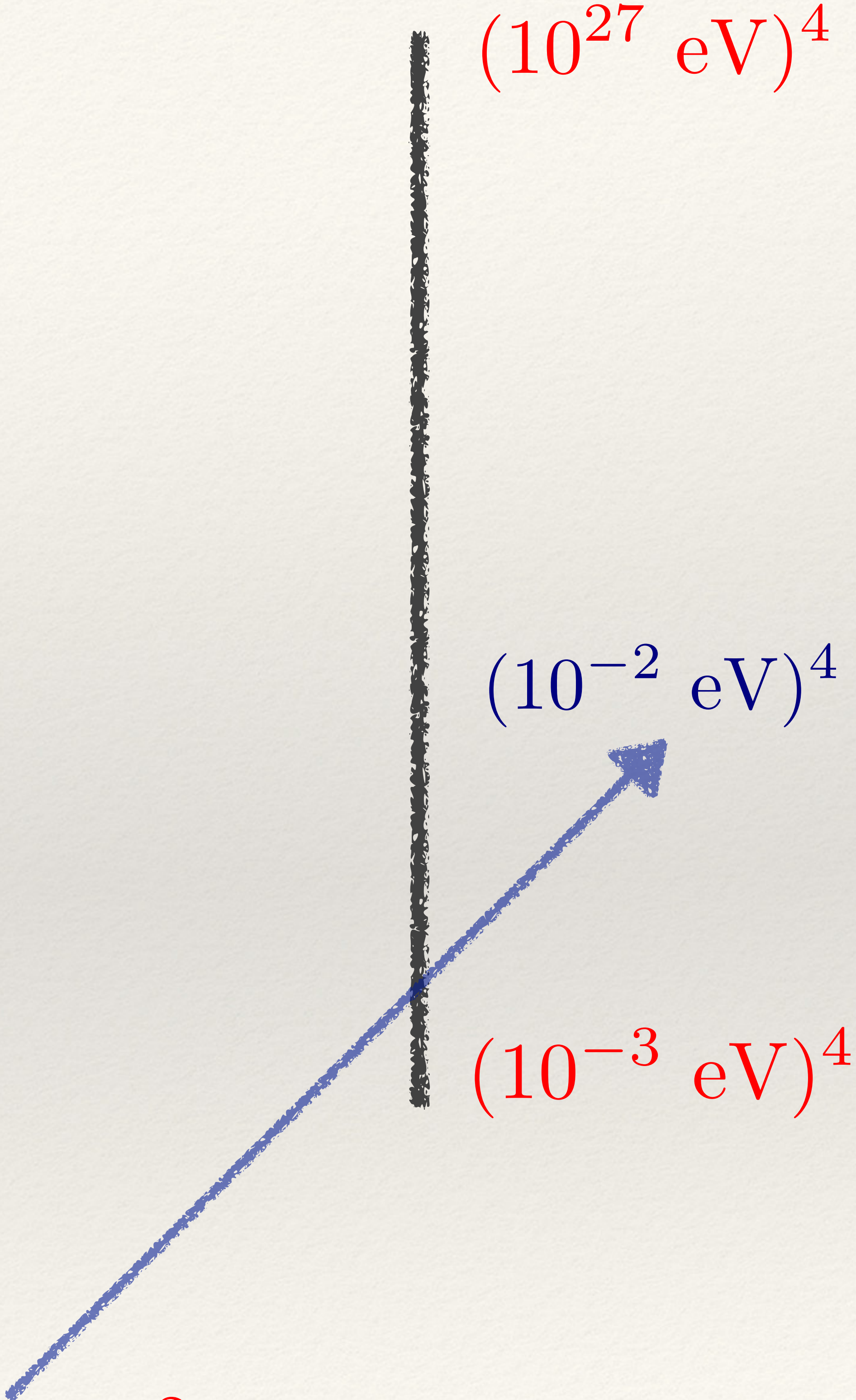
Nucleosynthesis — $E \sim \quad MeV$

CMB — $E \sim \quad eV$

Cosmological Constant — $E \sim \quad 10^{-3}\ eV$

Energy Ladder :
Very Little do we know about Gravity





No departure from Newtonian Gravity up to

$10^{-5}\ \mathrm{m} \sim 100\ (\mathrm{eV})^{-1}$ or, $M \sim 10^{-2}\ \mathrm{eV}$

Classical Aspects

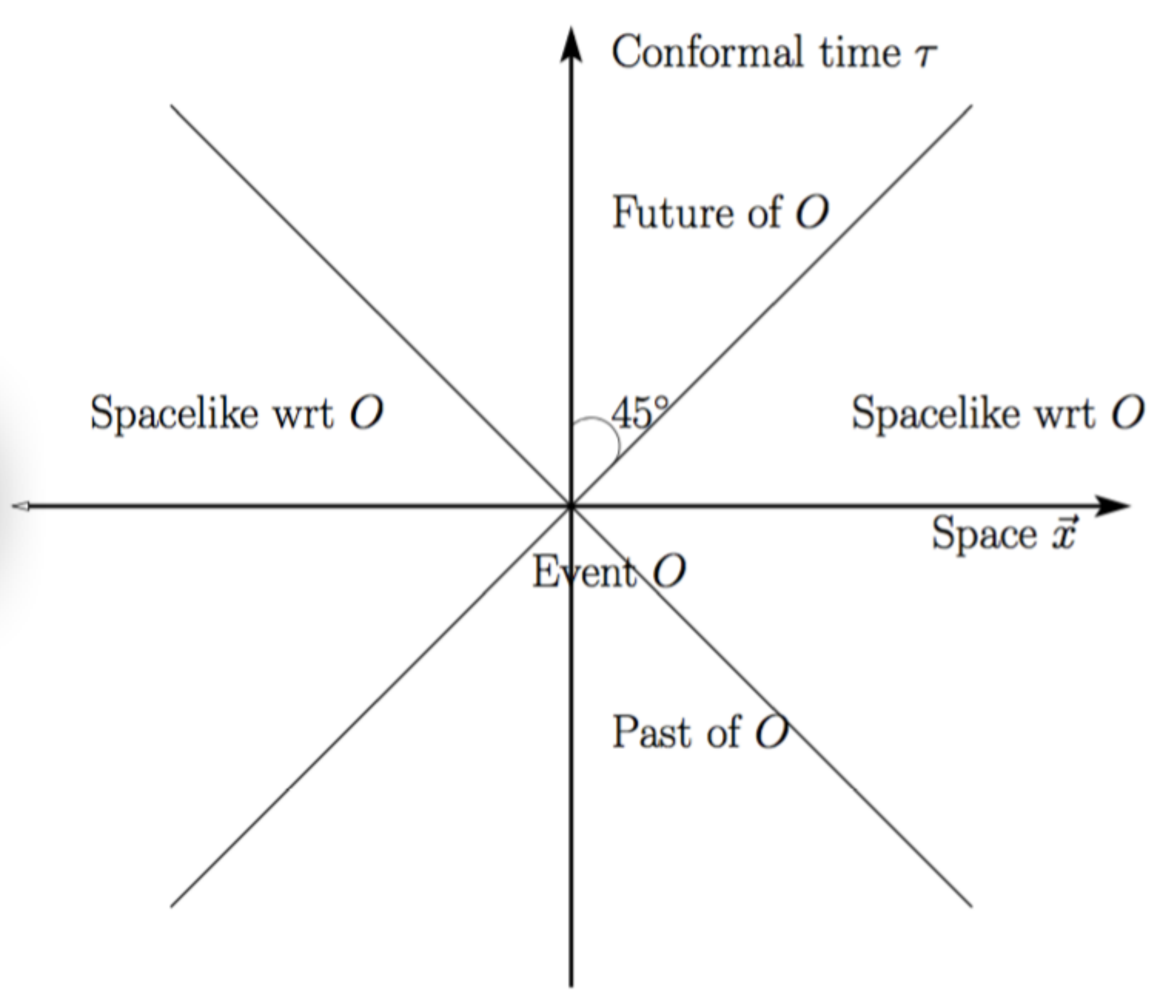
Recap of Standard Cosmology

FRW Metric: $$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right)$$ $$H = \frac{\dot{a}}{a}$$

For k = 0 $$ds^2 = a(\tau)^2 \left[-d\tau^2 + d\chi^2 + \chi^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$\chi(\tau) = \pm\tau + \text{const}$$

Conformal time: $$\tau = \int^{\tau} \frac{dt}{a(t)}$$



Geodesic distance Particle Horizon: $$d_p(t) = a(\tau)\chi_p(\tau)$$

Hubble patch/radius : $$d_p \sim a\tau \sim t \sim \frac{1}{H}$$

Recap of Standard Cosmology

$$G_{\mu\nu} = T_{\mu\nu}/M_{\rm Pl}^2$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

$$\nabla_\mu G^\mu_\nu = 8\pi G \nabla_\mu T^\mu_\nu$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm Pl}^2}\rho - \frac{k}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)$$

$$\Omega_i(a) = \frac{\rho_i(a)}{\rho_{cr}(a)}\,, \qquad \Omega_k(a) = -\frac{k}{a^2 H^2(a)}$$

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

$$p = w\rho$$

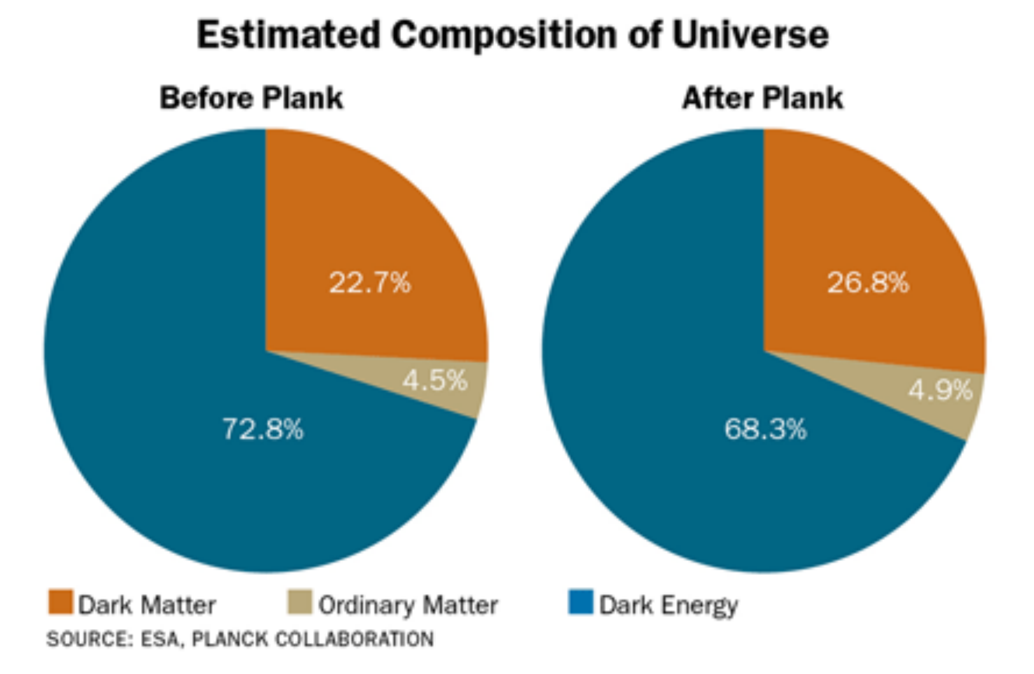
$$\rho \propto a^{-3(1+w)}$$

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+w)}} & w \neq 1 \\ e^{Ht} & w = -1 \end{cases}$$

$$\Omega_{cr} = 3M_{\rm Pl}^2 H^2$$ **Time independent**

$$\left(\frac{H^2}{H_0^2}\right) = \sum_i \Omega_{i,0} a^{-3(1+w_i)} + \Omega_{k,0} a^{-2}$$

$$\textstyle\sum_i \Omega_{i,0} + \Omega_{k,0} = 1.$$ **Flatness of the Universe**

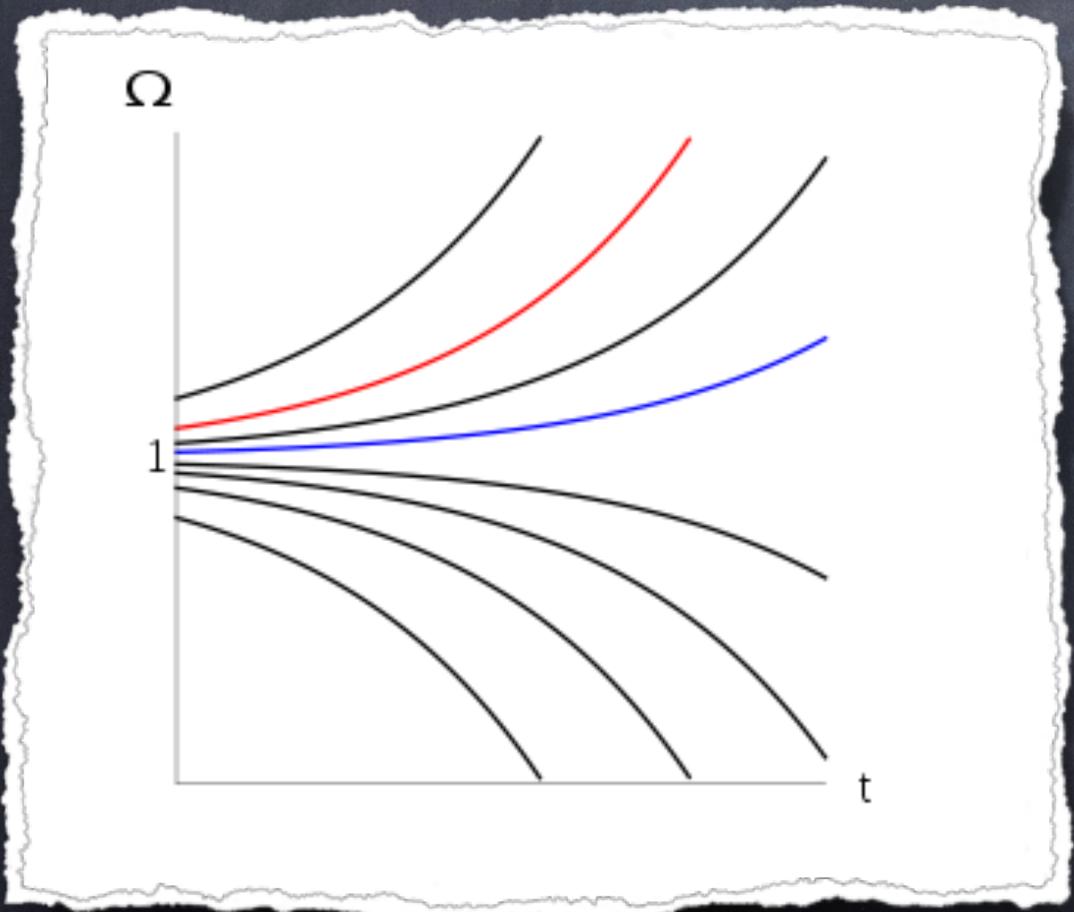


Shortcomings of the Hot Big Bang Cosmology

Flatness Problem:

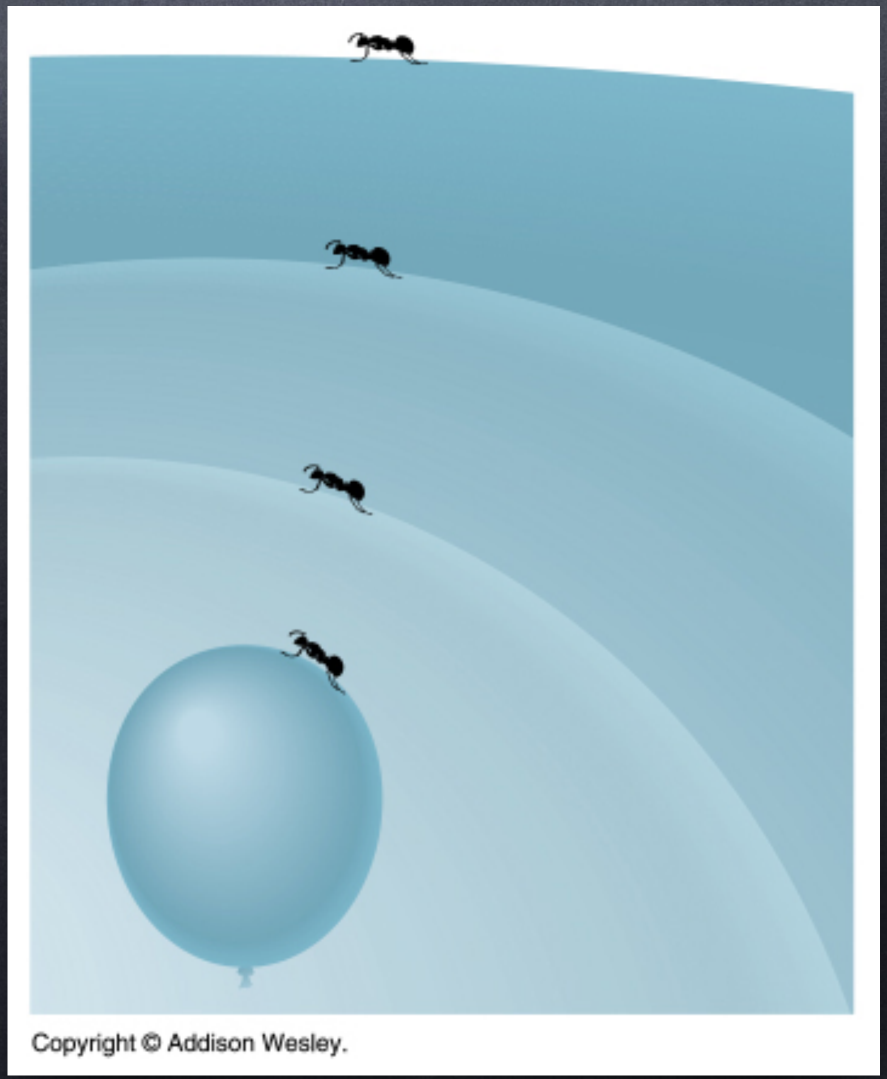
$$\dot{\Omega}_k = H\Omega_k(1+3w)\ , \qquad \frac{\partial \Omega_k}{\partial \log a} = \Omega_k(1+3w)$$

$\Omega_k = 0$ Unstable point



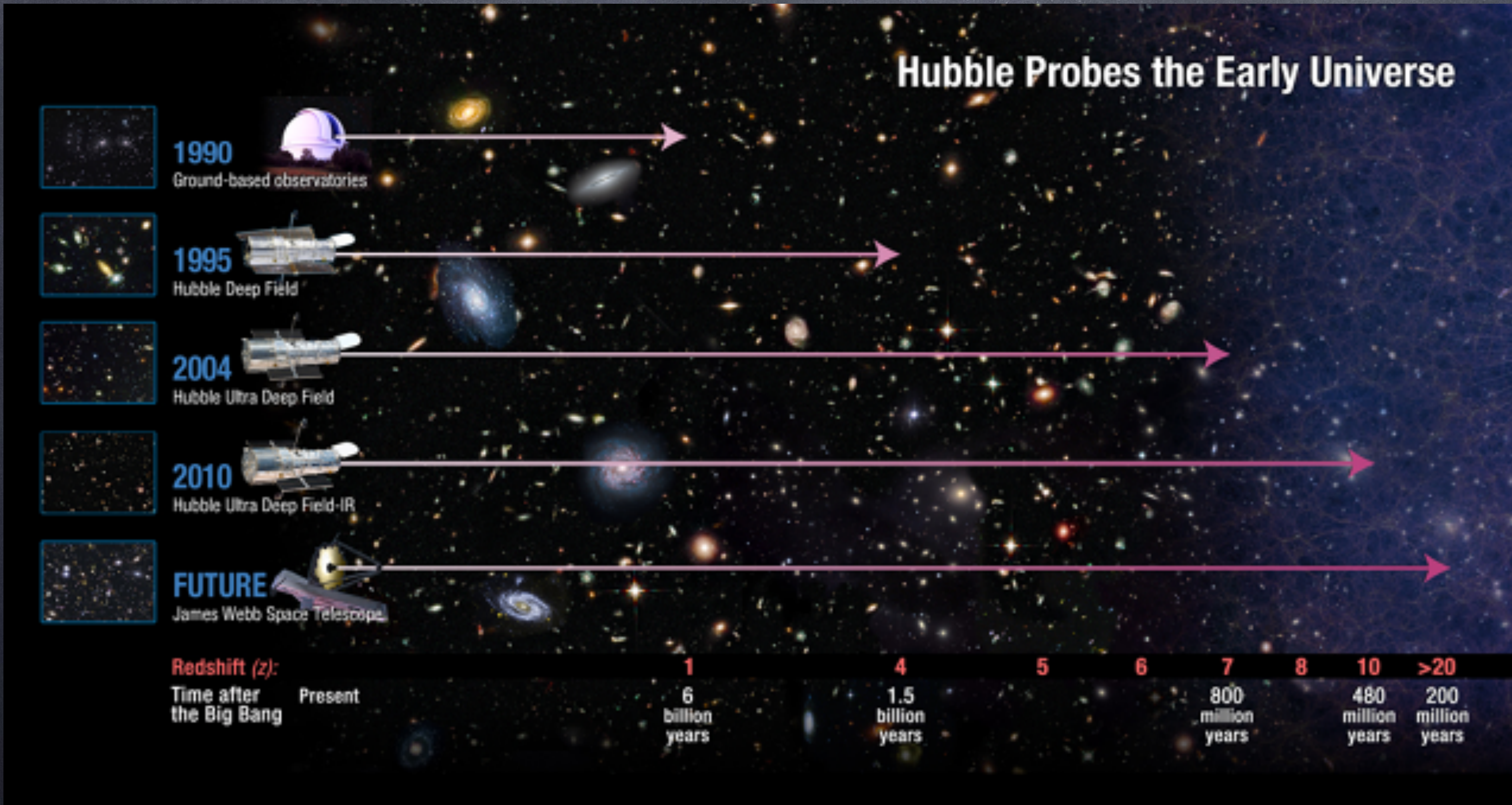
$\Omega_k < 10^{-2}$ Planck data

$|\Omega_k| < 10^{-18}$ at BBN epoch

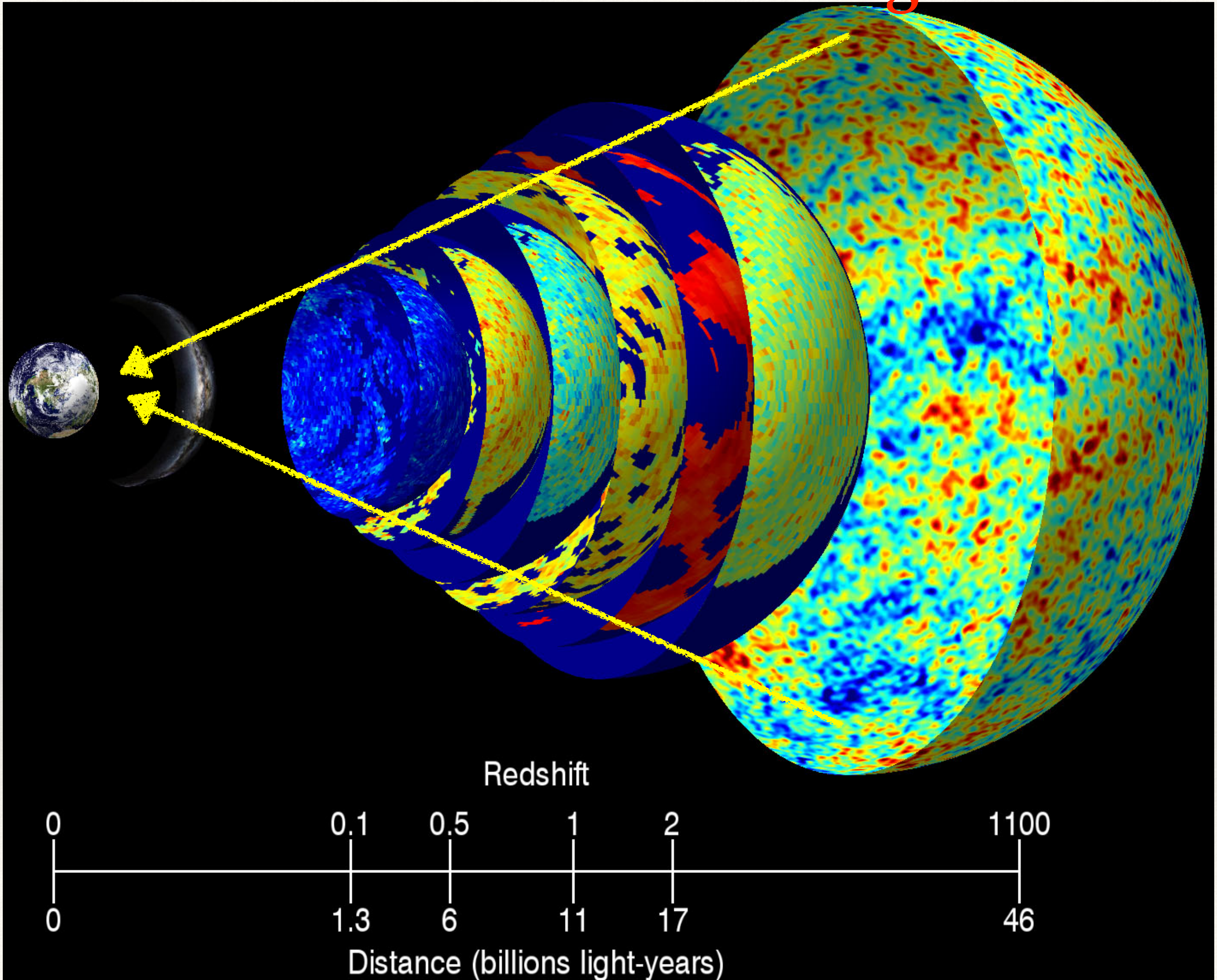


$$\Omega_k \to 0 \quad a(t) \sim e^{Ht} \text{ or } \ddot{a} > 0$$

We are constantly probing our History

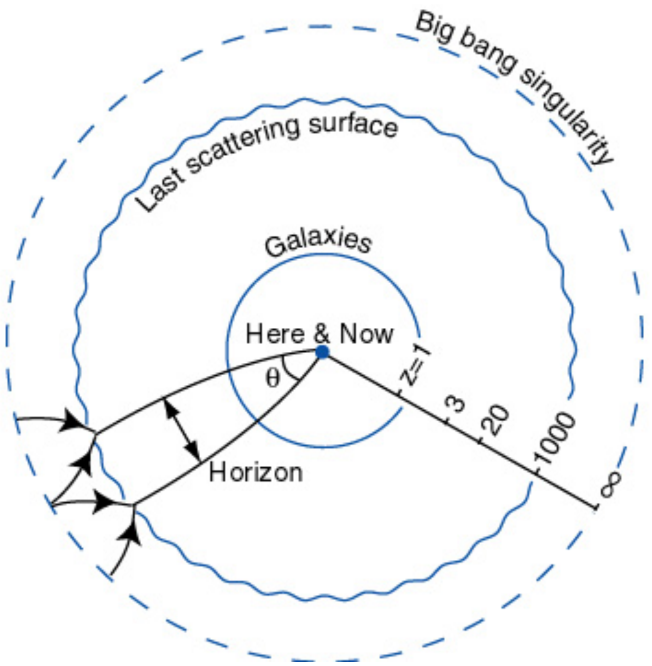


Surface of last scattering



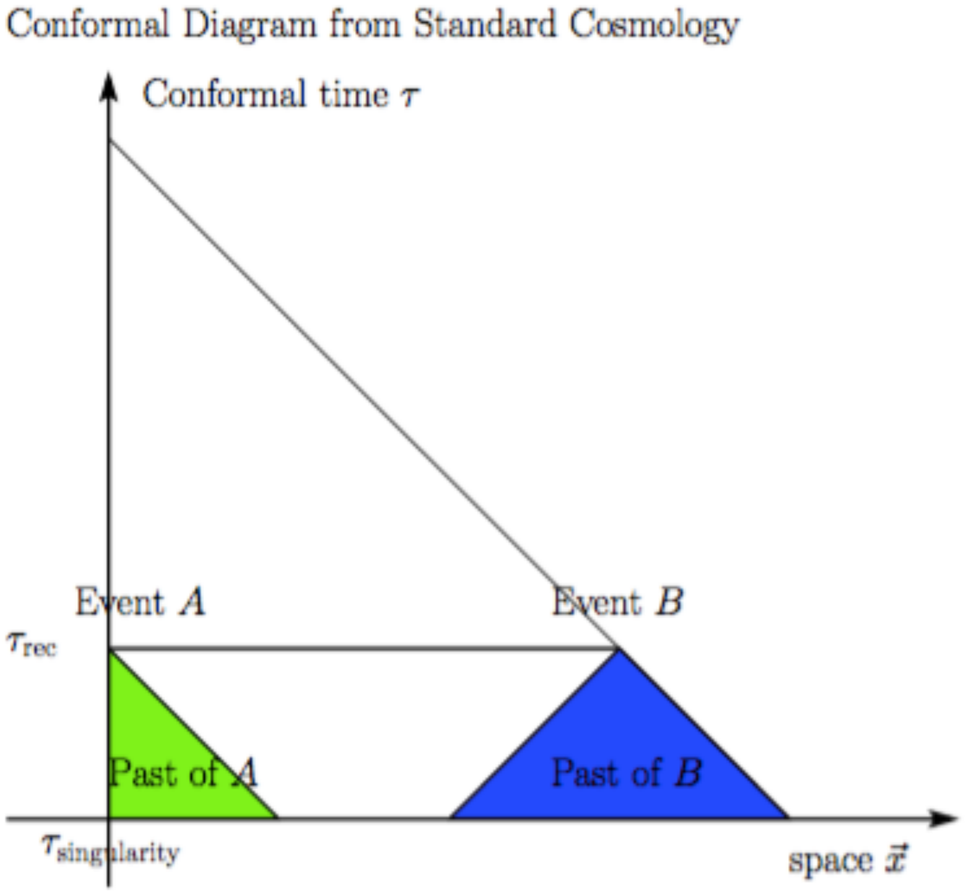
Shortcomings of Hot Big Bang

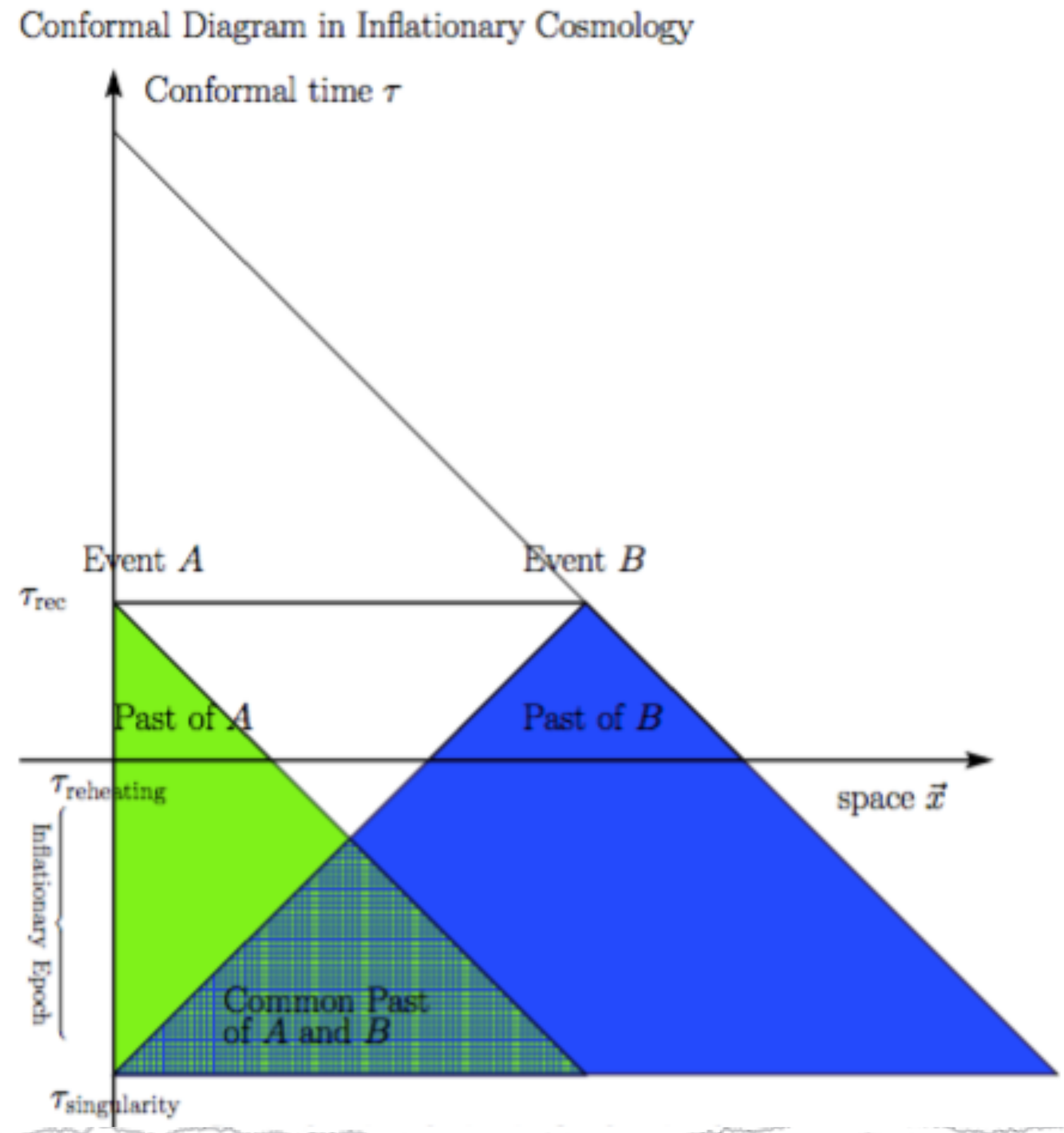
Horizon Problem:



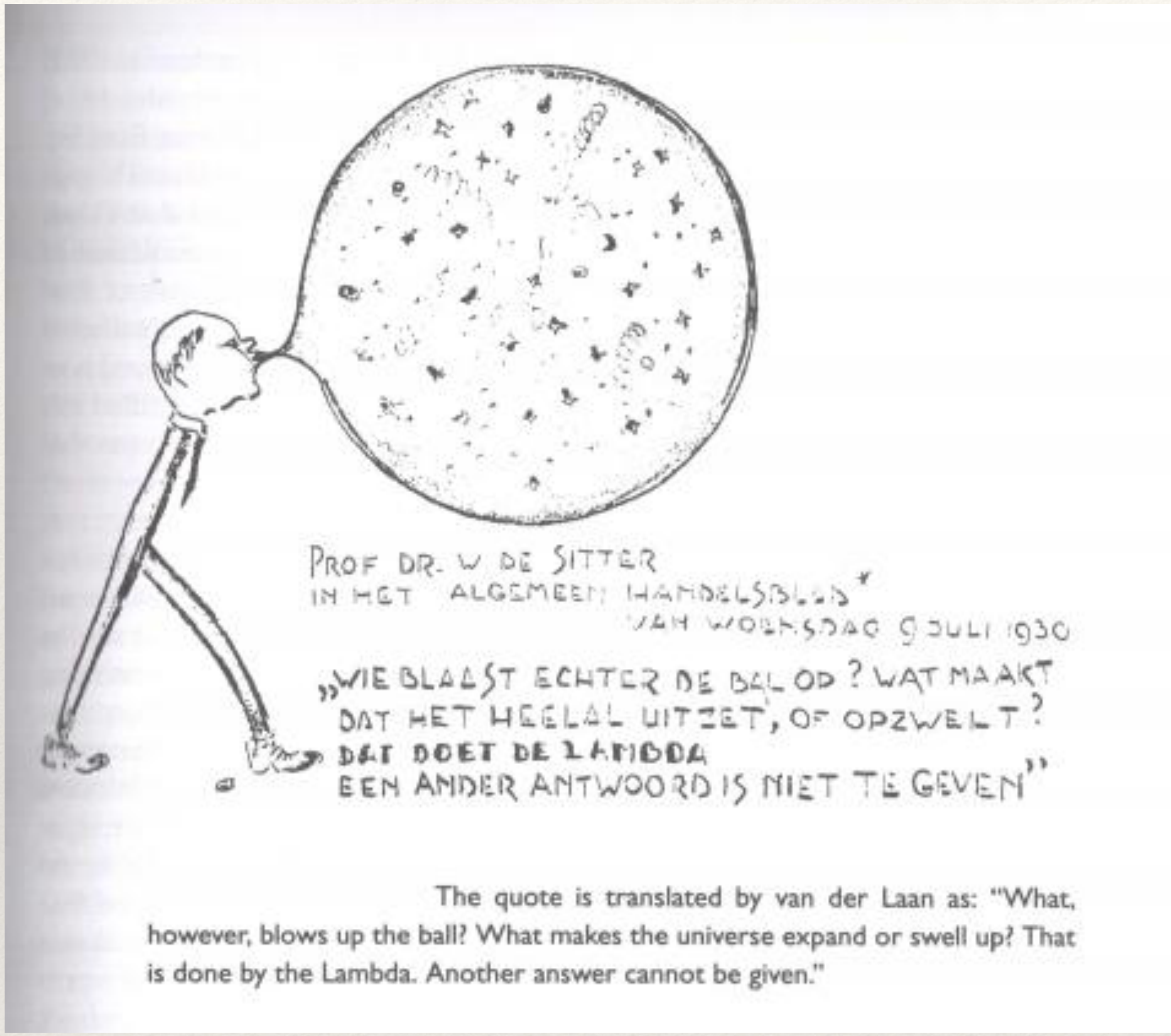
$$\chi_p(\tau) = \tau - \tau_i = \int_{\tau_i(t_i)}^{\tau(t)} \frac{dt'}{a(t')} = \int_{a_i}^{a} \frac{da}{Ha^2} \sim a^{(1+3w)/2} - a_i^{(1+3w)/2}$$

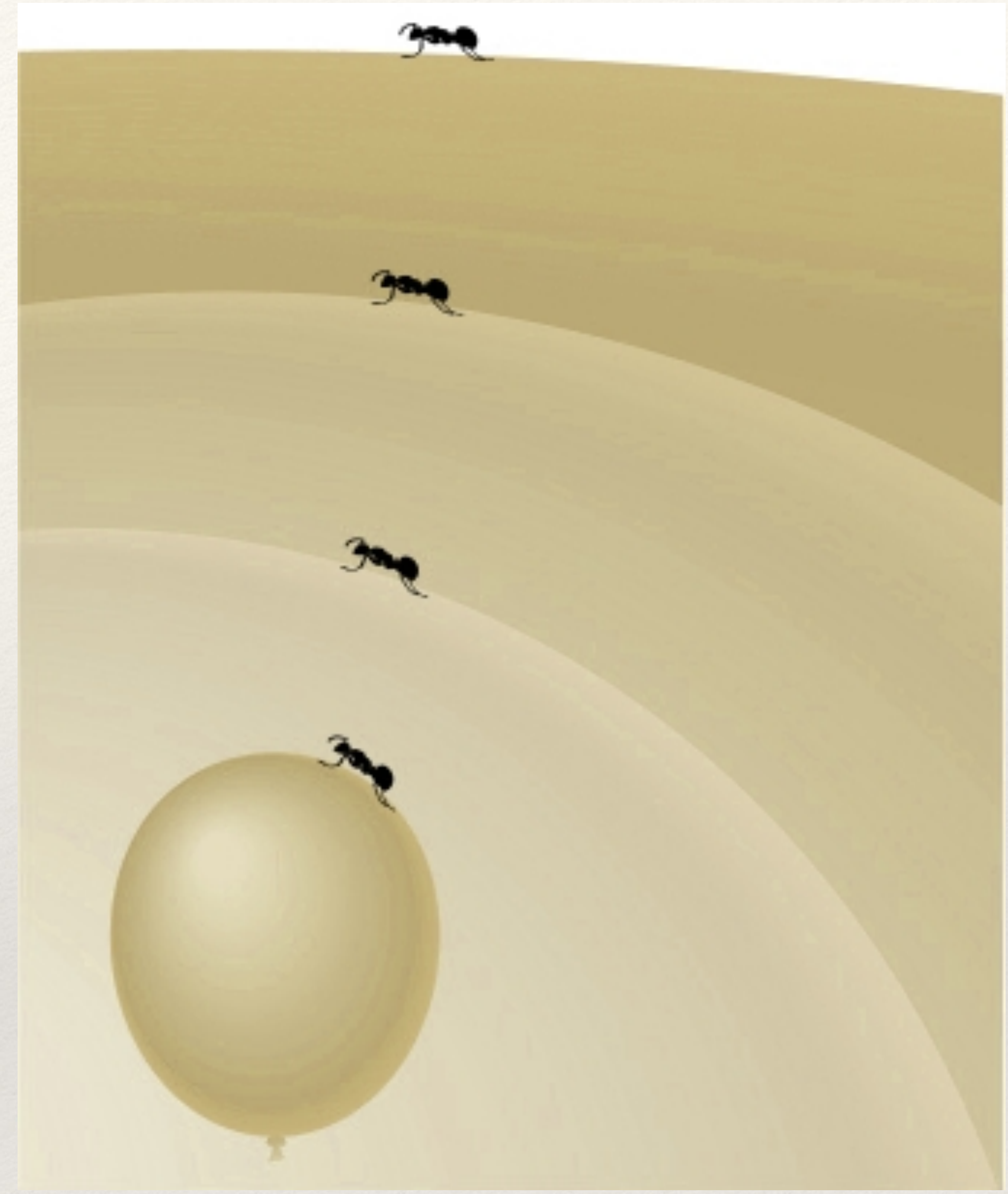
$$d_p \sim a\tau \sim t \sim \frac{1}{H}$$





Inflation Makes Universe Flat !





$$\mathbf{C.C.}:\Lambda \Longrightarrow \omega = -1$$

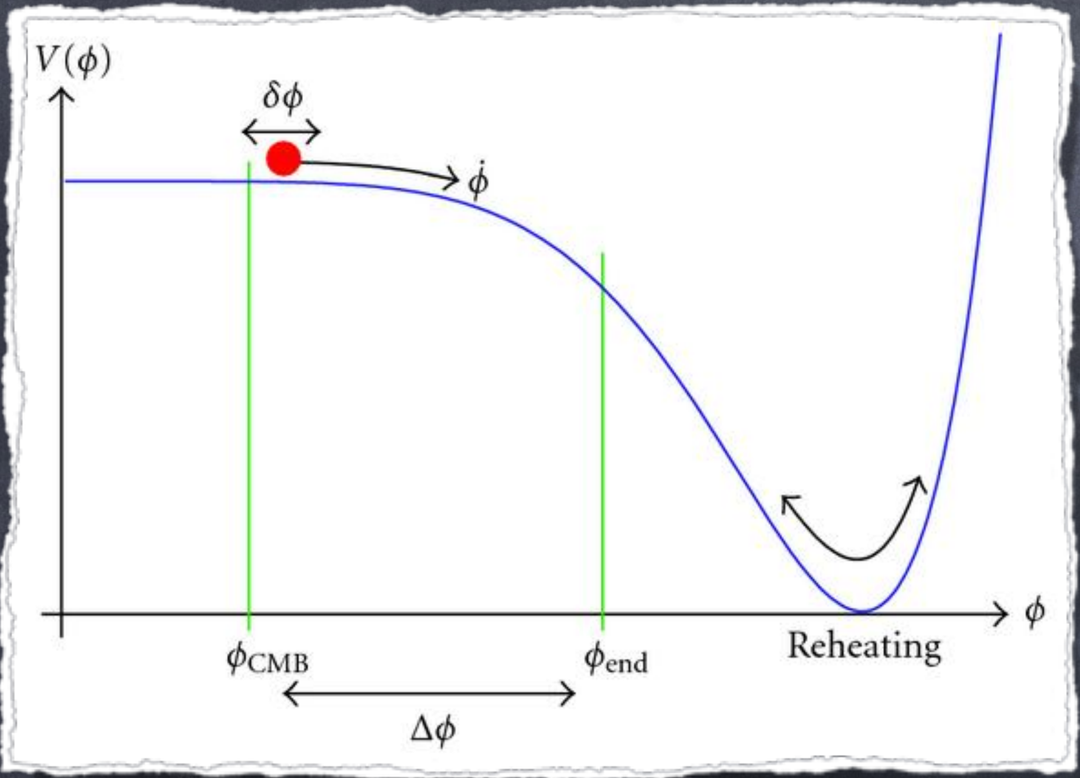
$$\mathcal{N} \sim 10^{10^{10}}$$

$$\omega = \frac{p}{\rho}$$

This idea was reinvented by Starobinsky, Guth, Linde, Steinhart, Albrecht

Dynamics of Inflation

(Liddle + Lyth Book)



$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$T_{\mu\nu}^{(\phi)} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

$$\frac{\delta S}{\delta\phi} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) + V_{,\phi} = 0 \quad \Rightarrow \quad \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \qquad \dot{\phi}^2 \ll V(\phi) \quad \Rightarrow \quad w_\phi \simeq -1 < -\frac{1}{3}$$

Slow Roll Conditions

$$\epsilon = -\frac{\dot{H}}{H^2} \sim \frac{\dot{\phi}^2}{V} \ll 1 \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

$$\varepsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 ; \; \eta_V = M_P^2\left(\frac{V''}{V}\right)$$

$$\xi_V^2 = M_P^4\left(\frac{V'V'''}{V^2}\right) ; \; \sigma_V^3 = M_P^6\left(\frac{V'^2V''''}{V^3}\right)$$

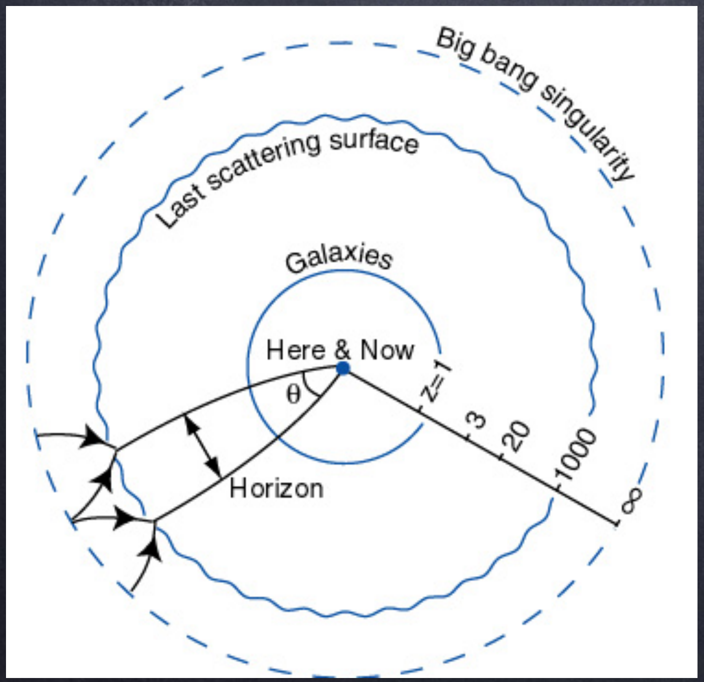
Number of e-foldings of Inflation

* **Number of e-foldings : Duration of Inflation**

$$N = \ln\left(\frac{a_{end}}{a}\right) \approx \int_t^{t_{end}} H(t)dt = \int_\phi^{\phi_{end}} \frac{H}{\dot\phi} d\phi \approx \int_{\phi_{end}}^{\phi} \frac{V}{V_{,\phi}} d\phi$$

* **Flatness & Horizon Problem can be addressed very easily:**

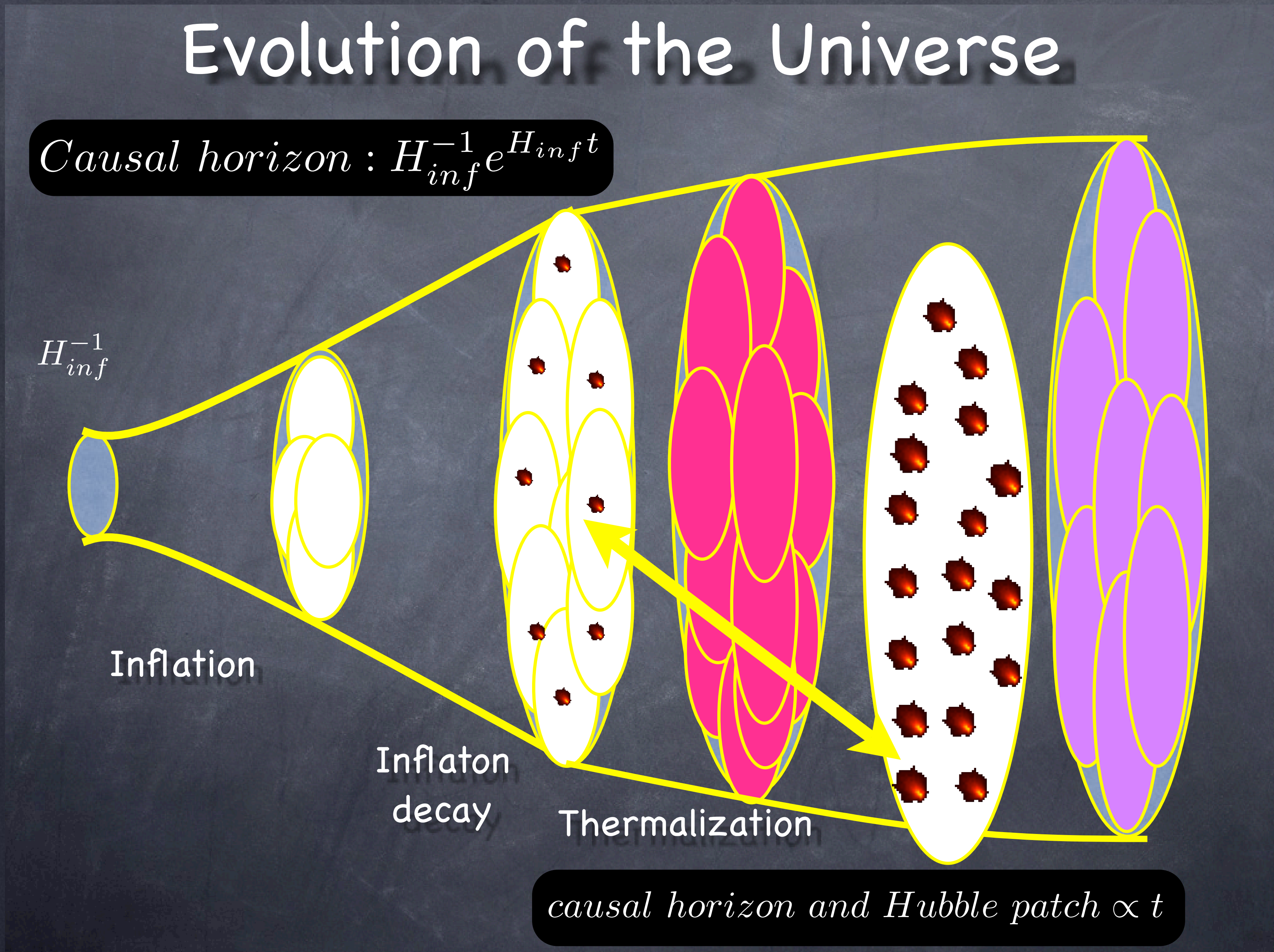
$$\Omega_k = -\frac{k}{a^2H^2} \propto \frac{1}{a^2} \to 0. \qquad \Omega_k(a_{end}) \simeq \Omega_k(a_{in})\frac{a_{in}^2}{a_{end}^2} \sim \frac{a_{in}^2}{a_{end}^2} = e^{-2N}$$



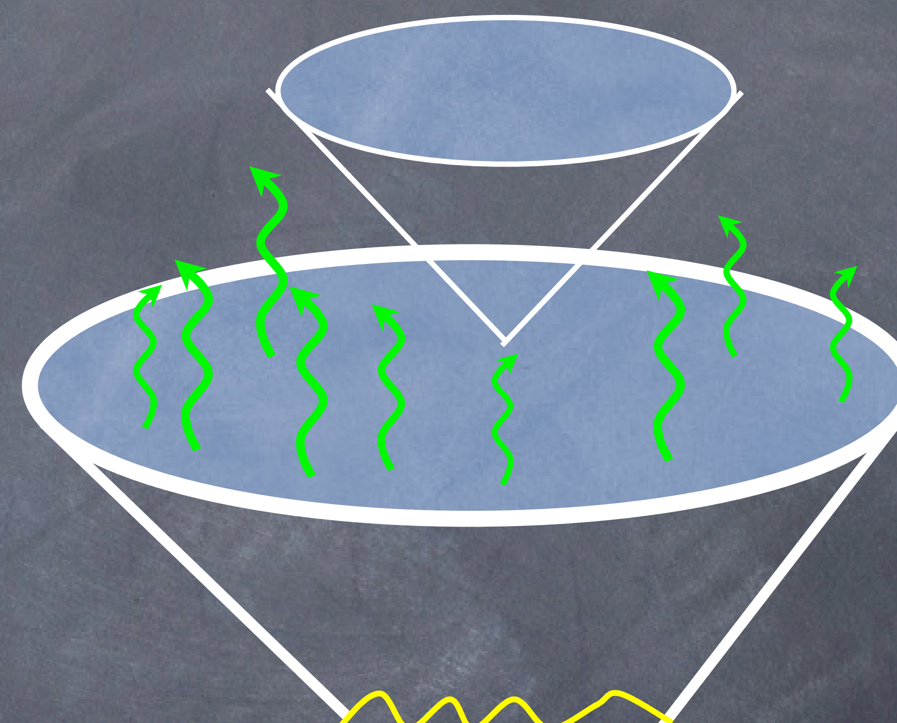
$$d_p = a(t_L)\int_{t_{in}}^{t_{end}} \frac{dt}{a(t)} \simeq \frac{a(t_L)}{a_{end}H_I}e^N \qquad a(t) = a(t_{end})e^{H_I(t-t_{end})}$$

$$d_p \geq d_L = \frac{1}{H_0}\frac{a(t_L)}{a_0} \Rightarrow N \geq \ln\left(\frac{a_{end}H_I}{a_0H_0}\right)$$

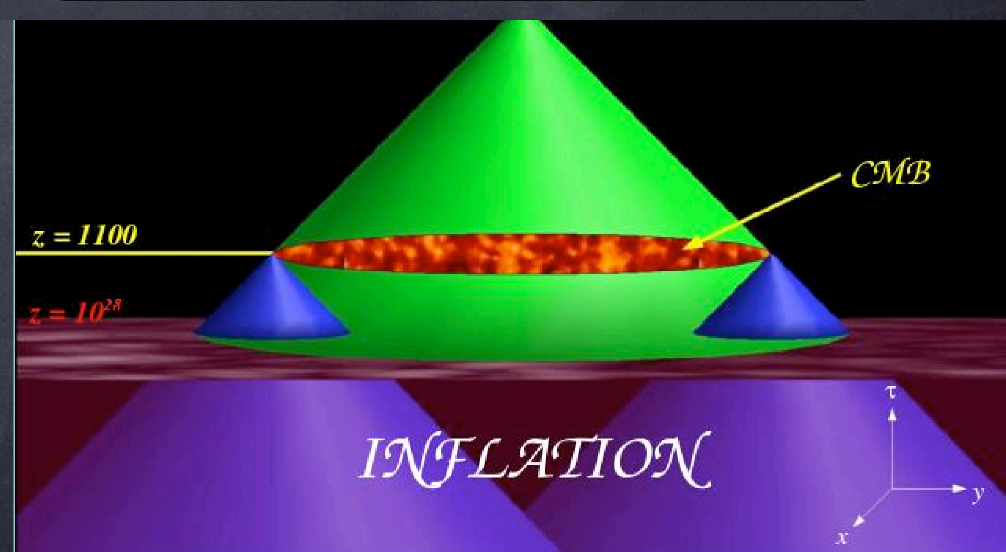
Inflationary/Post-Inflationary Evolution



Inflation is great !!



Particle Horizon

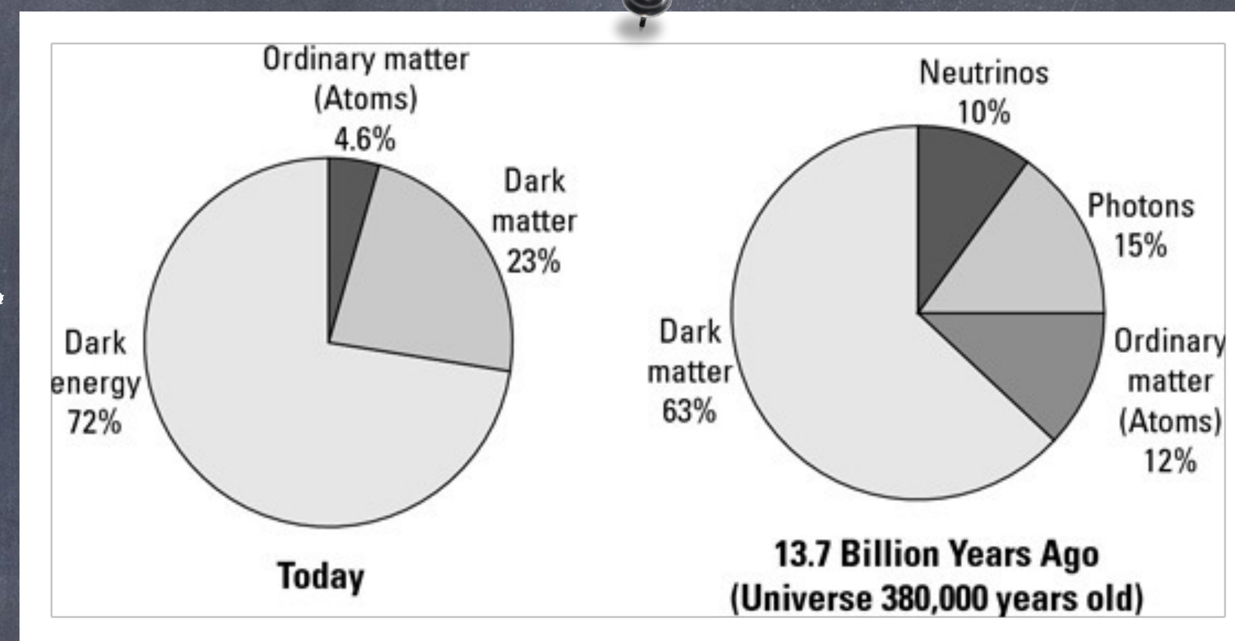


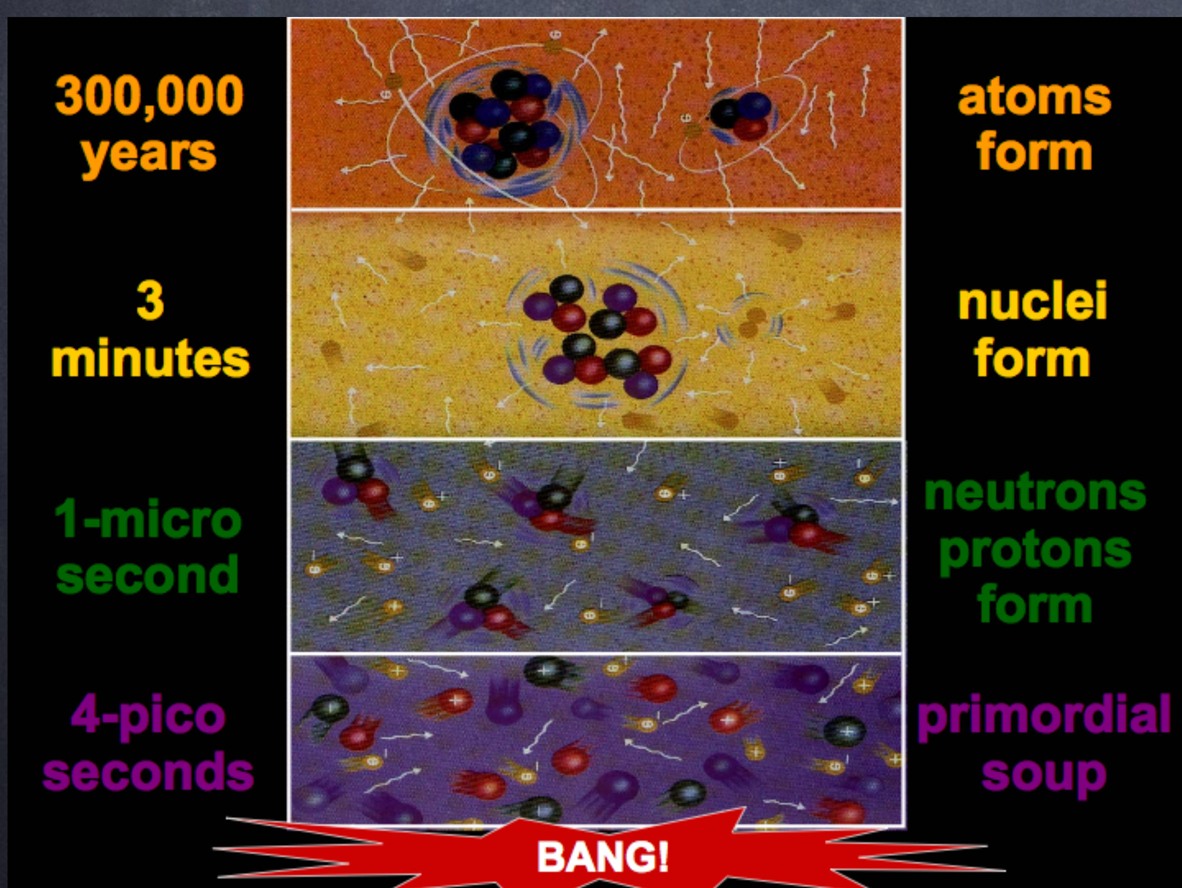
What is an Inflaton?

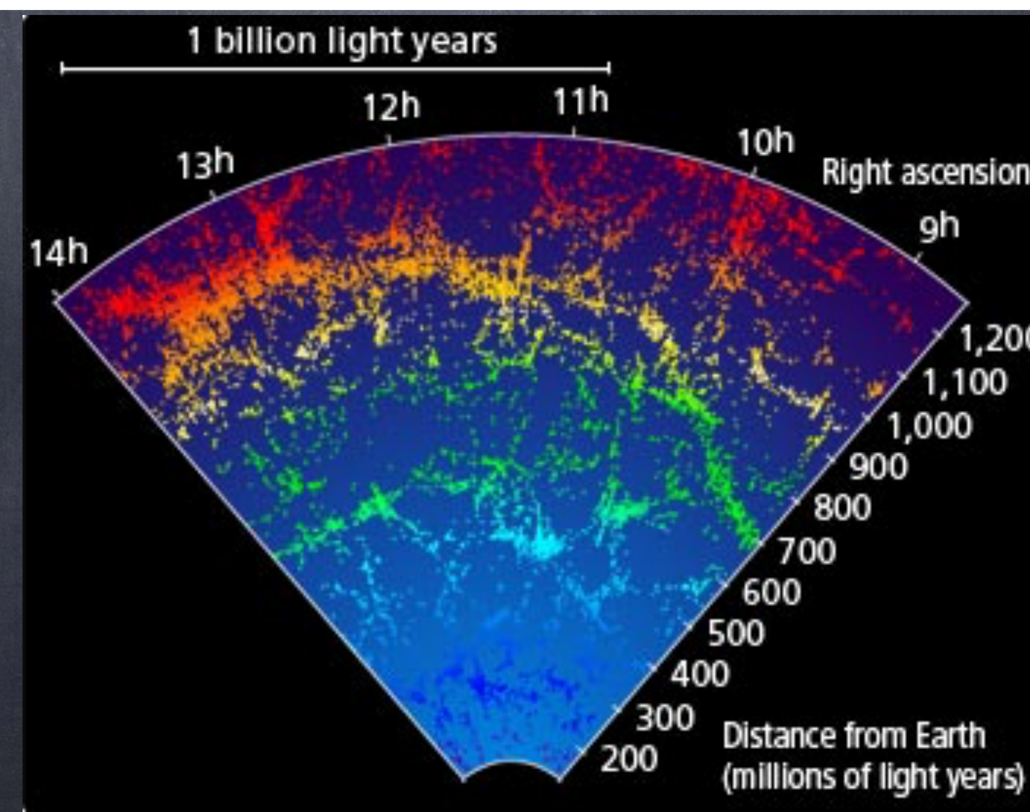
ϕ

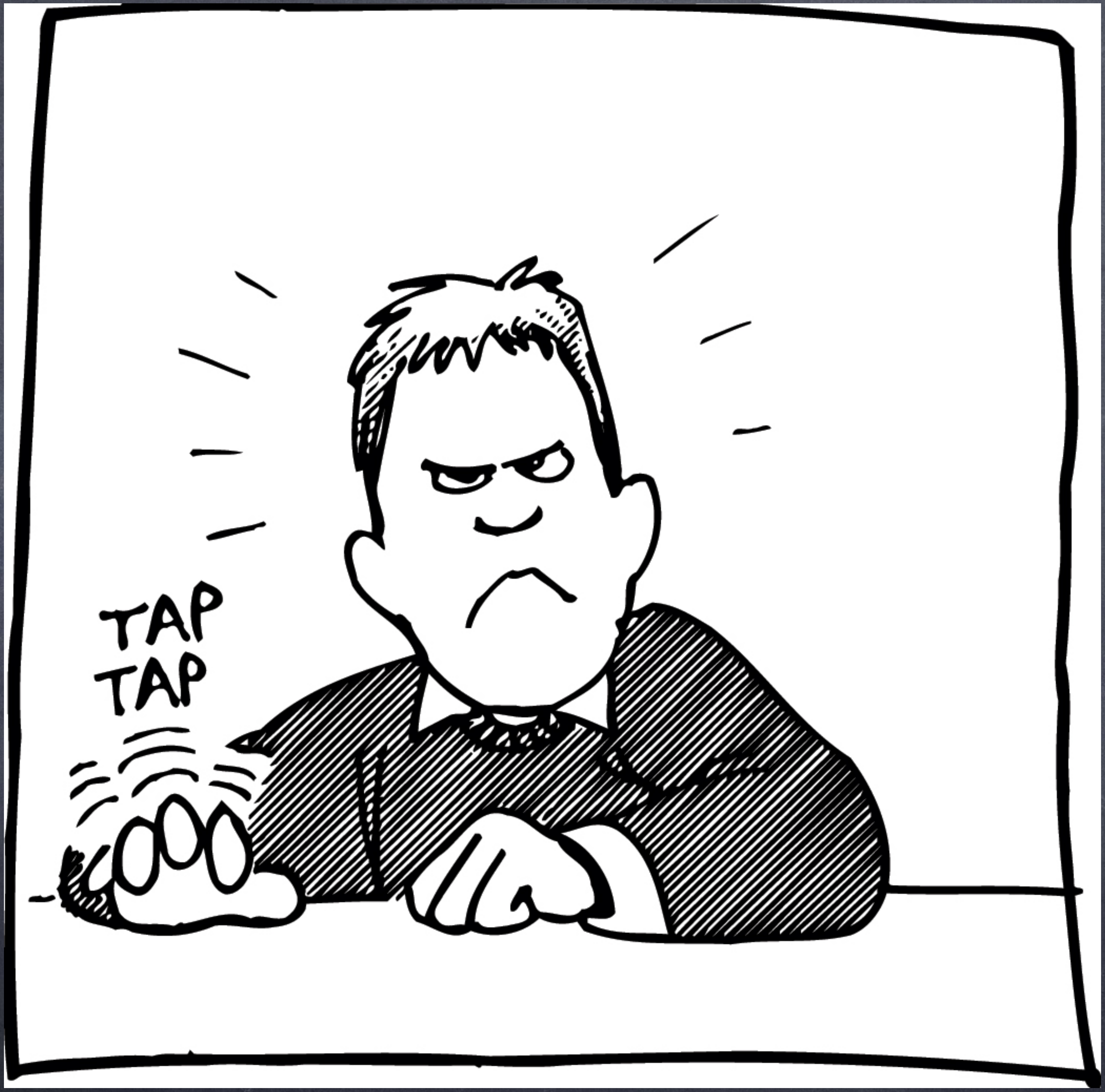
We don't know the identity,

we will discuss some possibilities later





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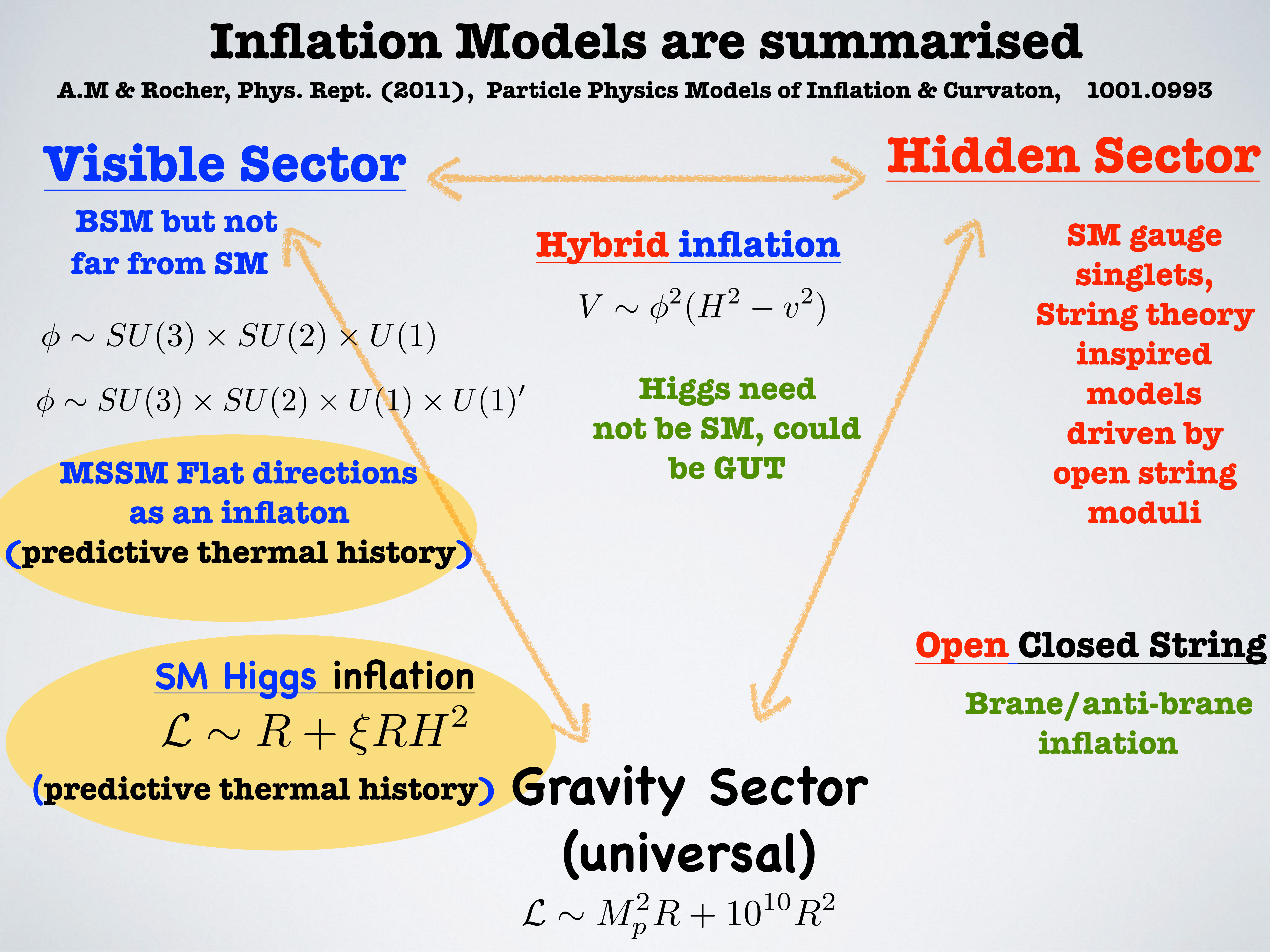


Inflation Models are summarised
A.M & Rocher, Phys. Rept. (2011), Particle Physics Models of Inflation & Curvaton, 1001.0993
Visible Sector
Hidden Sector
BSM but not far from SM
$\phi \sim SU(3)\times SU(2)\times U(1)$
$\phi \sim SU(3)\times SU(2)\times U(1)\times U(1)'$
MSSM Flat directions as an inflaton (predictive thermal history)
SM Higgs inflation
$\mathcal{L} \sim R + \xi R H^2$
(predictive thermal history)
Hybrid inflation
$V \sim \phi^2 (H^2 - v^2)$
Higgs need not be SM, could be GUT
SM gauge singlets, String theory inspired models driven by open string moduli
Open Closed String
Brane/anti-brane inflation
Gravity Sector (universal)
$\mathcal{L} \sim M_p^2 R + 10^{10} R^2$

Thermal Universe

Inflation dilutes everything, we nee to regenerate matter

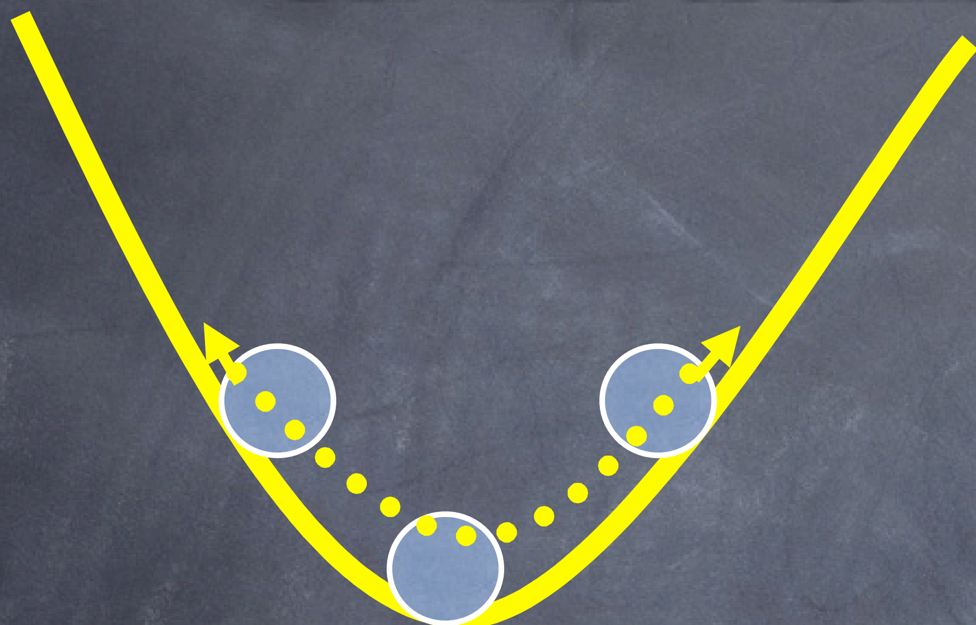
How to obtain Hot Universe ?

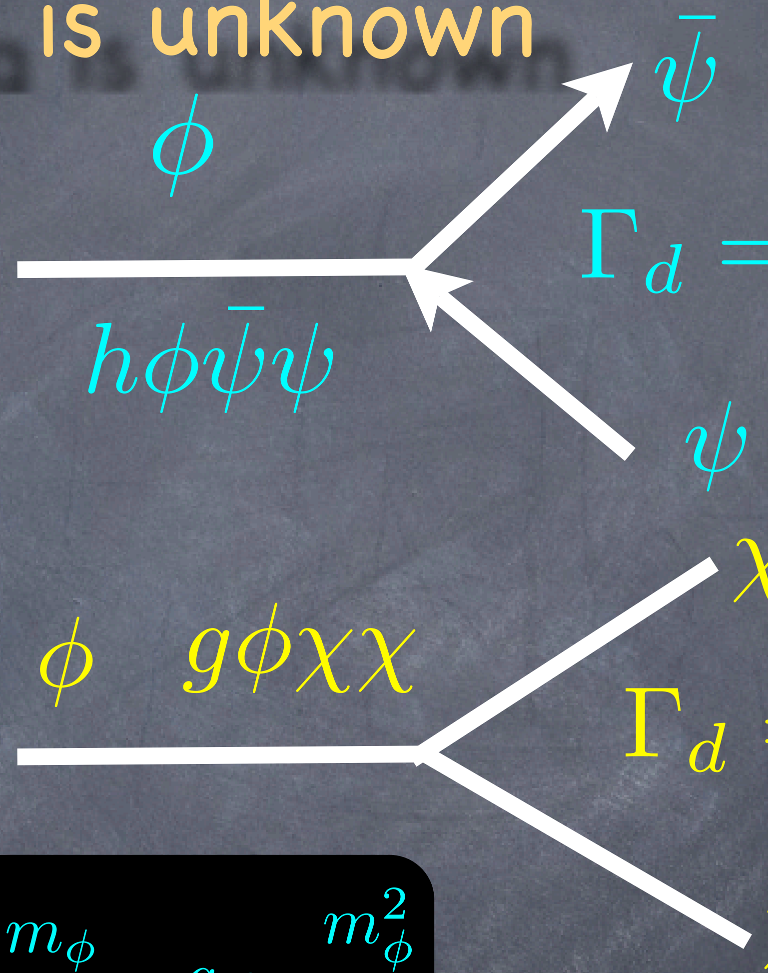
$$\frac{d\rho_\phi}{dt} + (3H + \Gamma_\phi)\rho_\phi = 0$$

Perturbative Decay
Kolb+Turner Book

Inflaton decays

but the coupling is unknown





$$\Gamma_d = h^2 \frac{m_\phi}{8\pi}$$

$$\Gamma_d = \frac{g^2}{8\pi m_\phi}$$

$$\mathcal{L}_{\phi gg} = \frac{\phi}{M_{\rm P}} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_{\phi qq} = \frac{\phi}{M_{\rm P}} (H \bar{q}_L) q_R$$

Jaikumar, Mazumdar. (2003)

$$h \sim \frac{m_\phi}{M_P} \qquad g \sim \frac{m_\phi^2}{M_P}$$

$$\Gamma_d \sim \frac{m_\phi^3}{M_P^2}$$

Last stage of inflaton decay is always perturbative

Non Perturbative Creation of particles

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} g^2 \chi^2 \phi^2$$

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k (\chi_k^*(t) \hat{a}_k e^{i\mathbf{kx}} + \chi_k(t) \hat{a}_k^\dagger e^{-i\mathbf{kx}})$$

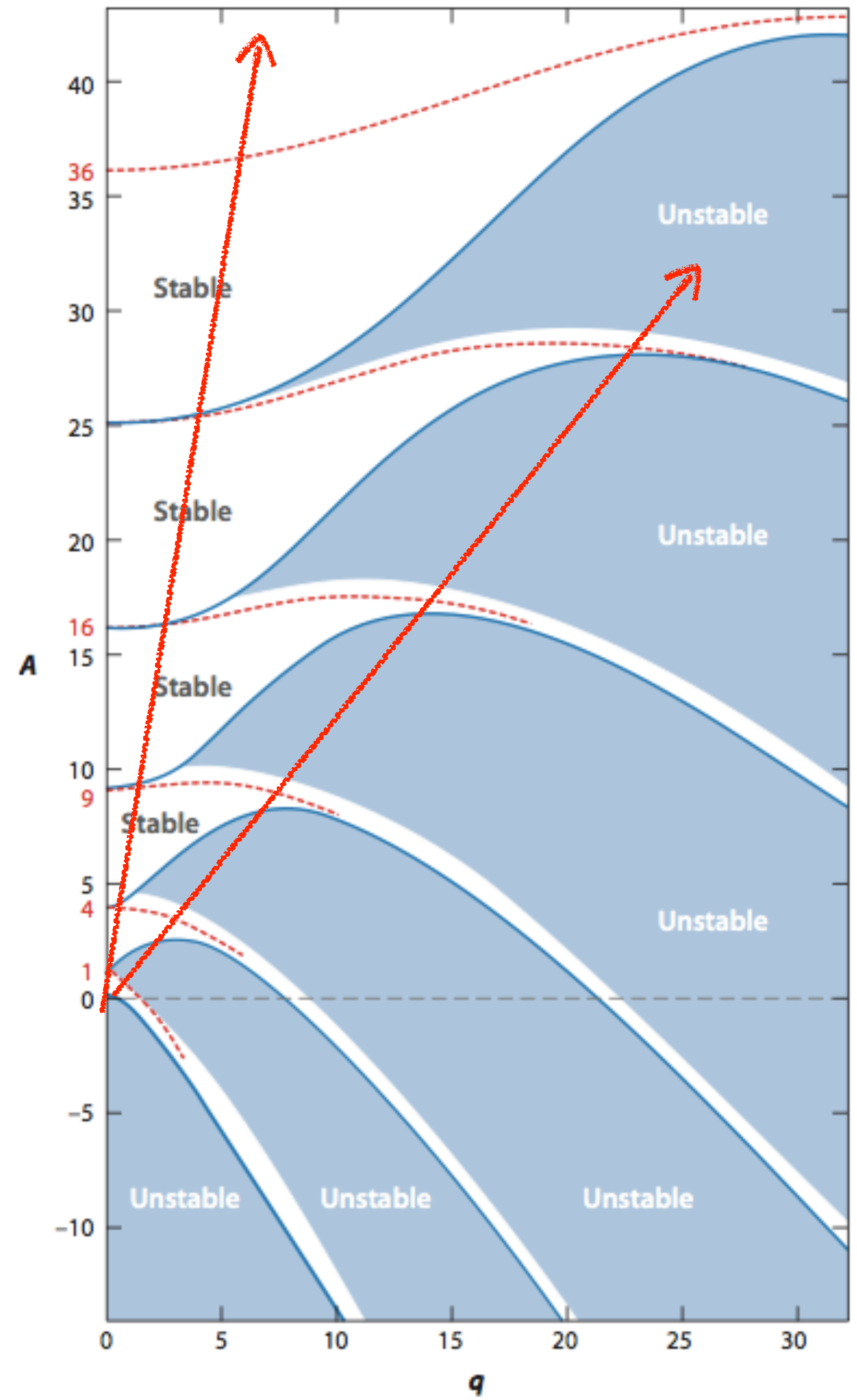
$$\ddot{\chi}_k + (k^2 + m_\chi^2 + g^2 \Phi^2 \sin^2(mt)) \chi_k = 0$$

$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0 \qquad z = mt$$

$$A_k = \frac{k^2 + m_\chi^2}{m^2} + 2q, \text{ where } q = \frac{g^2 \Phi^2}{4m^2}$$

$\chi_k \propto \exp(\mu_k z)$ Resonant preheating : $q \gg 1$

Narrow resonance : $q \leq 1, \ \mu_k \sim m$

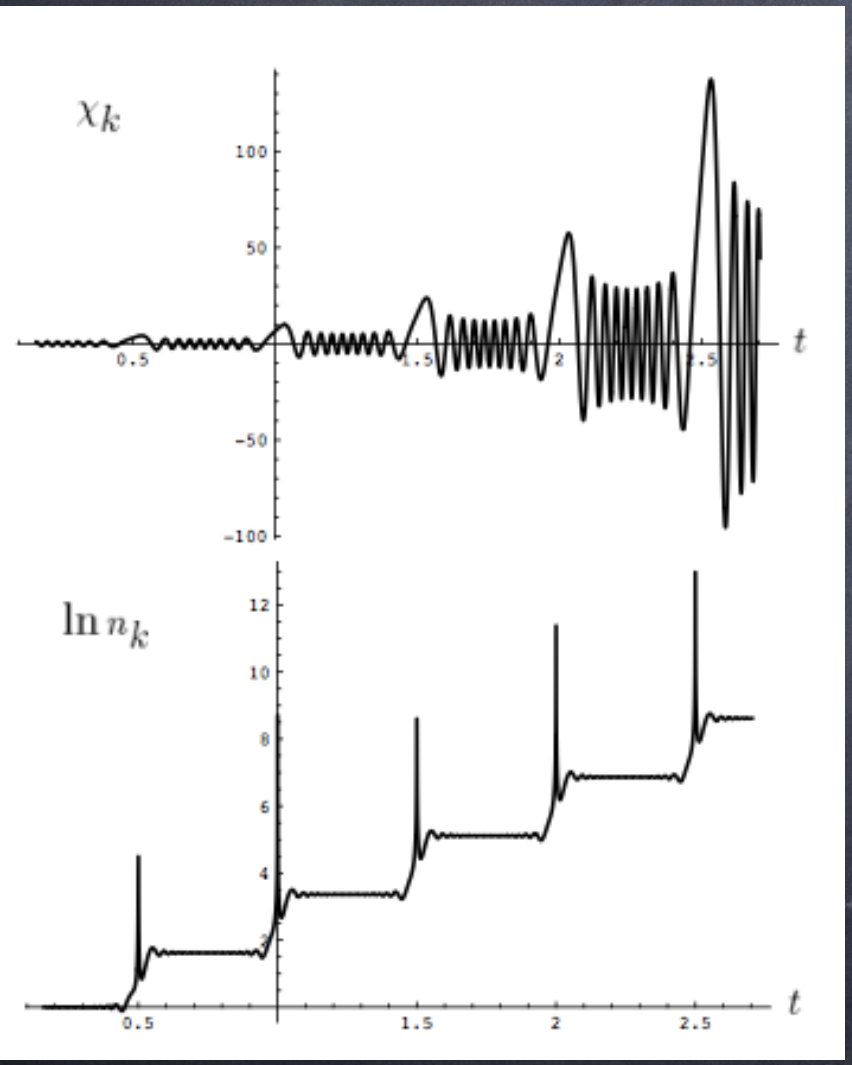


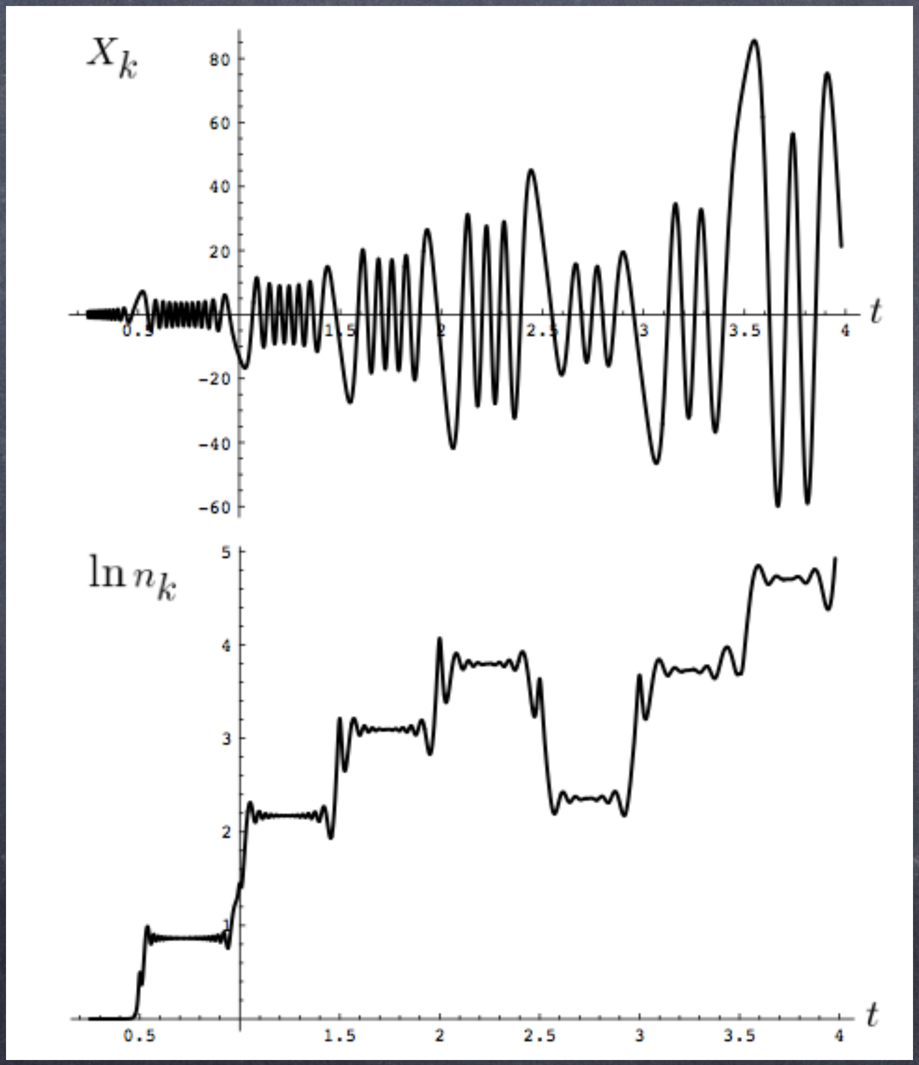
Brandenberger (1994, 1996), Koffman, Linde, Starobinsky (1997, 1998, 1999)

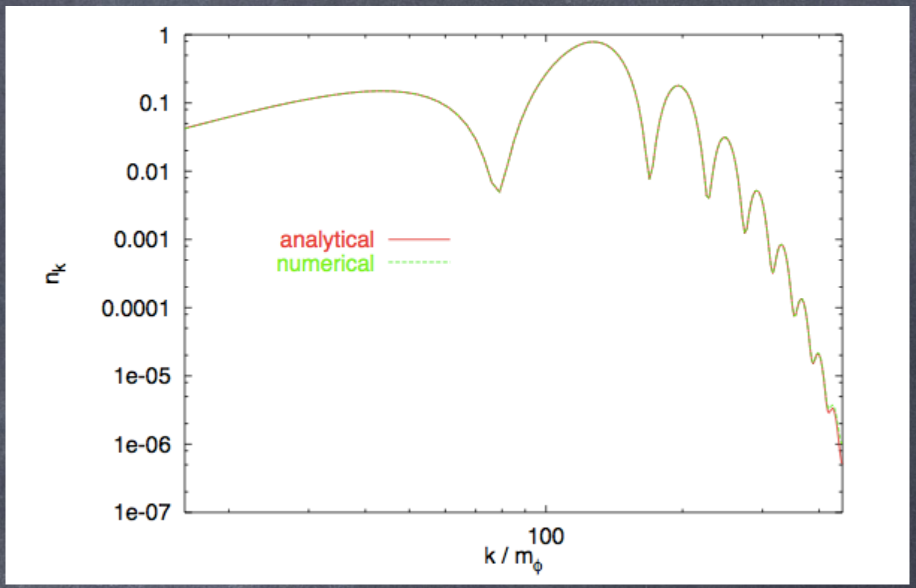
Non Perturbative Creation of Particles

$$\text{Adiabatic limit} : \chi_k \propto e^{\pm i \int \omega_k dt} \qquad \frac{d\omega_k^2}{dt} \leq 2\omega_k^3$$

Violation of adiabaticity leads to explosive particle creation







Fermion production

Koffman (2001),

Peloso, Riotto, Tkachev (2001)

Spin-3/2, Gravitino preheating

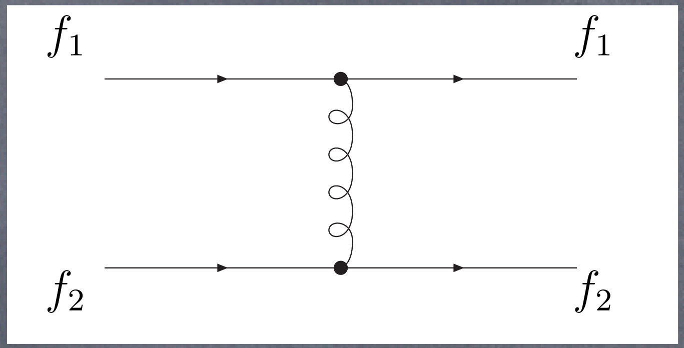
Maroto, Mazumdar (1999)

without expansion bosonic preheating

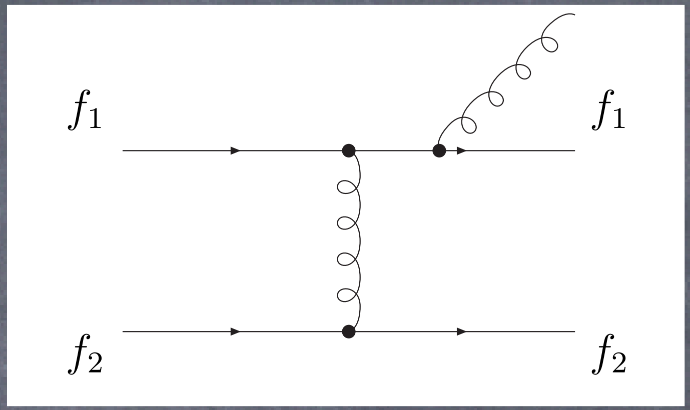
with expansion bosonic preheating

Towards full equilibrium

- Number conserving process --> Kinetic equilibrium Γ_{kin}



- Number violating process --> Chemical equilibrium Γ_{th}



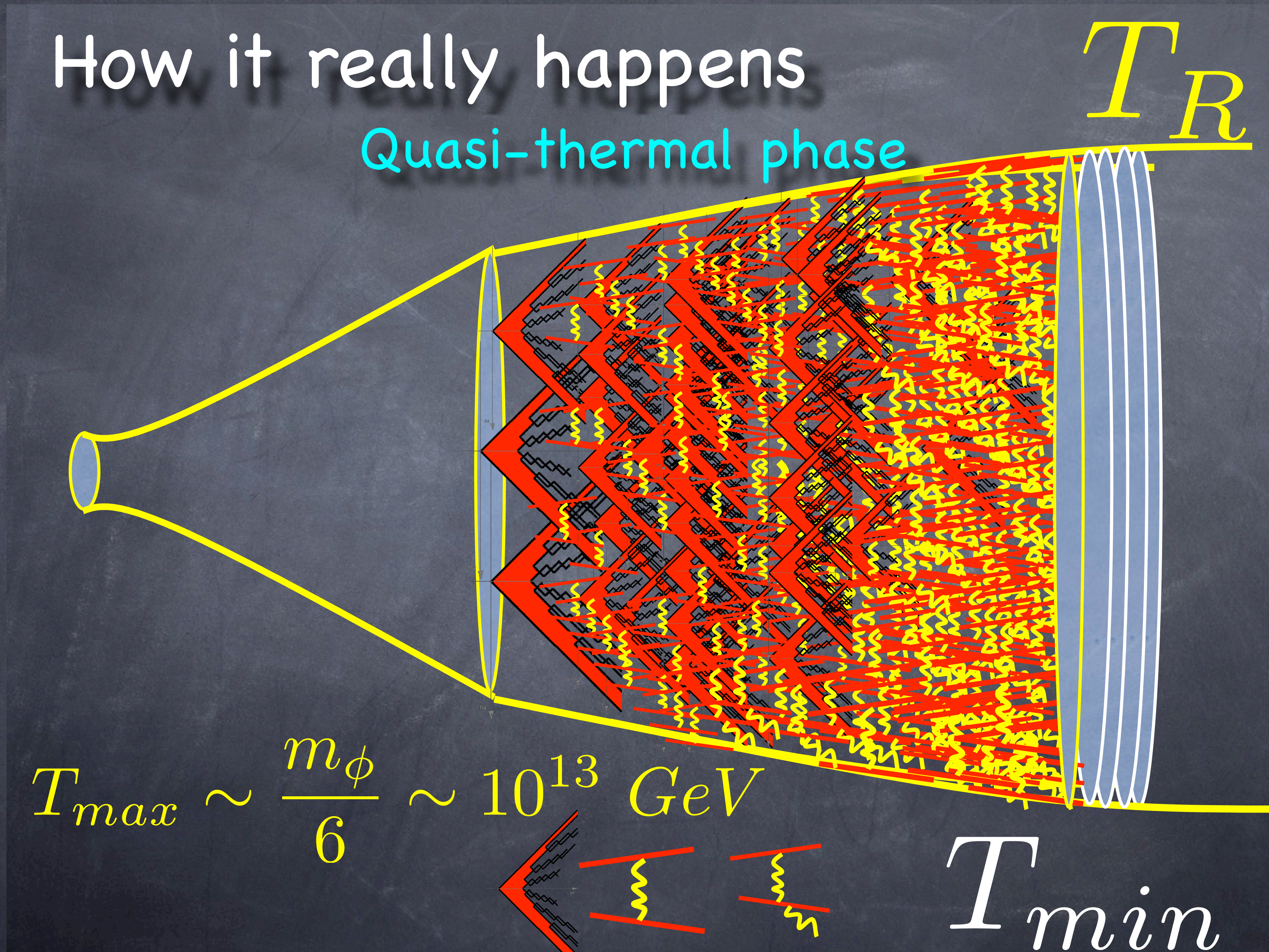
Three time scales involved: $\Gamma_d^{-1},\ \Gamma_{kin}^{-1},\ \Gamma_{th}^{-1}$

Particles with gauge interactions: $\Gamma_{th} \sim \alpha^3 \left(\frac{M_P}{m_\phi}\right) \Gamma_d$

$$\alpha \sim 10^{-2}, \quad m_\phi \leq 10^{13}\ GeV \Rightarrow \Gamma_{th} \geq \Gamma_d$$

Davidson, sarkar(2000), Allahverdi (2001), Allahverdi, Drees(2003), Jaikumar, Mazumdar (2003)

Reheating & Thermalisation



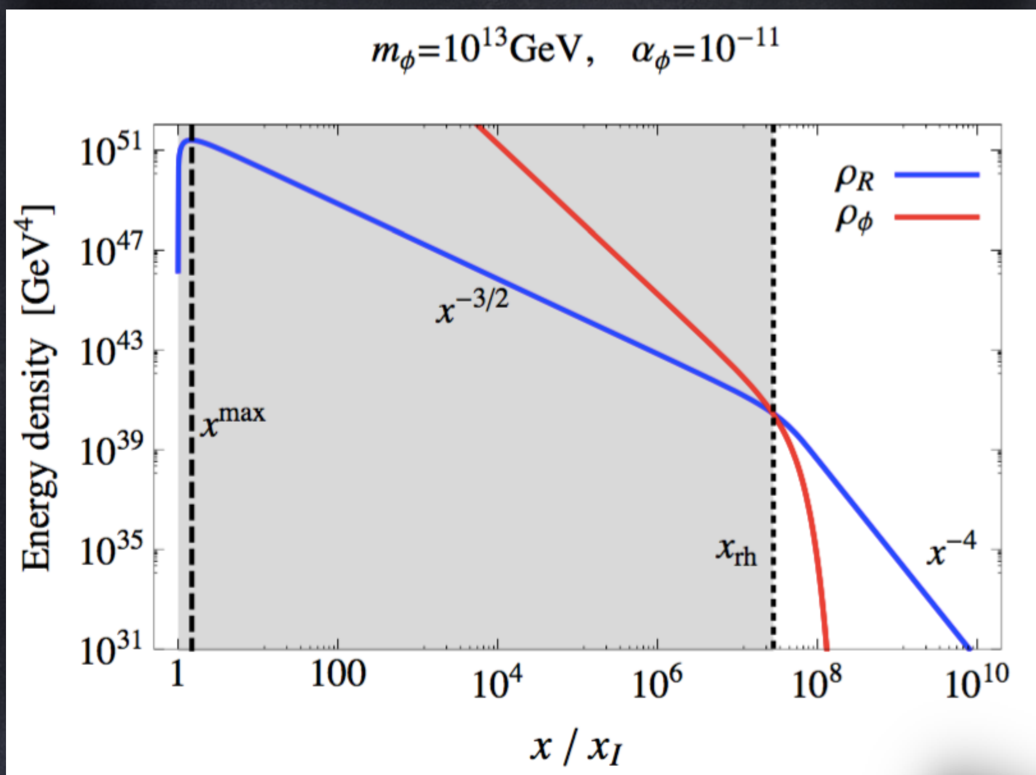
Instant Reheating Temp.

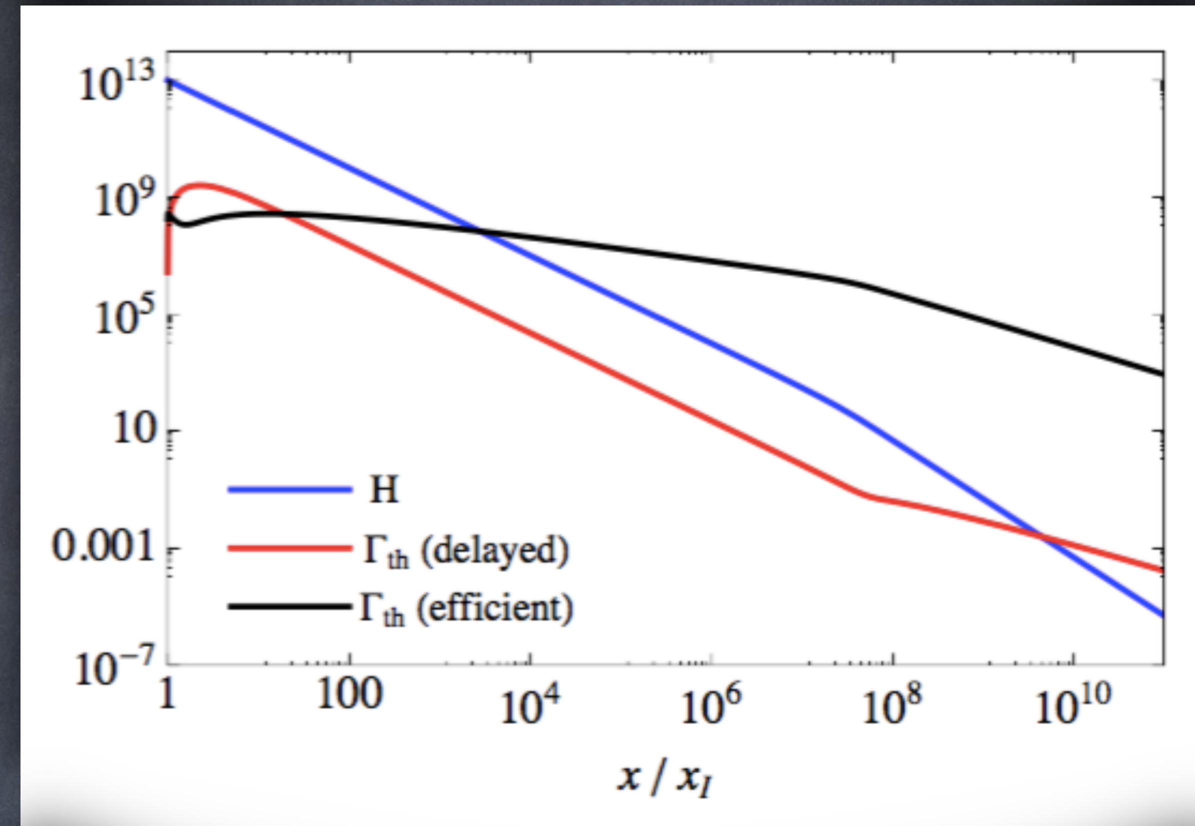
$$T_{rh} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4} \sqrt{\Gamma_\phi M_P}$$

Warning : More subtle, see next:

Boltzmann Equation

$$\begin{cases} \dot{\rho}_\phi + 3H(t)\rho_\phi = -\Gamma_\phi \rho_\phi \\ \dot{\rho}_R + 4H(t)\rho_R = \Gamma_\phi \rho_\phi + \Gamma_{\rm th}(\rho_R - \rho_R^{\rm eq}) \end{cases}$$





$$T_{rh} = T(x), \qquad x = \max(x_{th}, x_{rh})$$

Efficient Thermalisation:

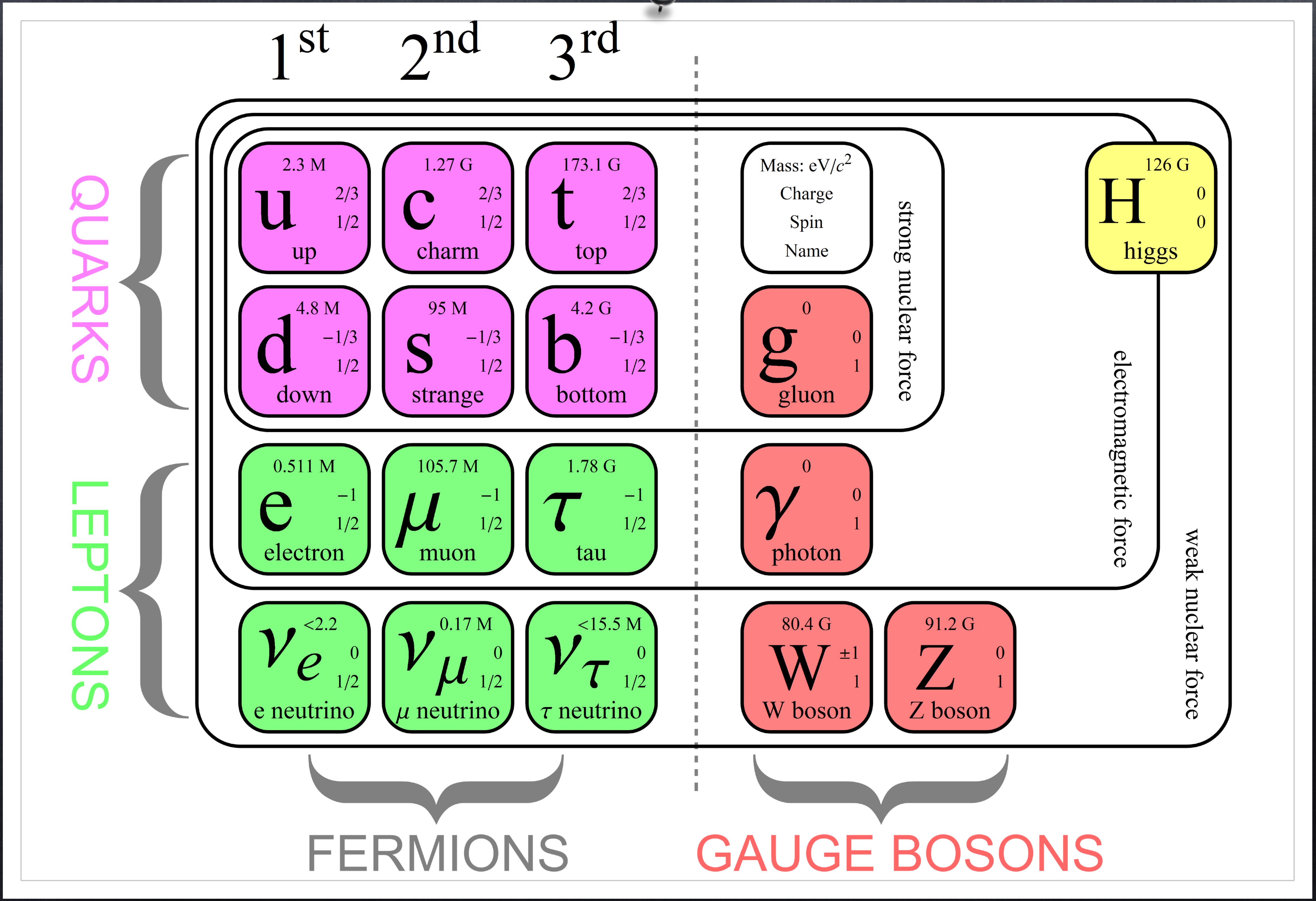
$$T_{rh}(x_{th} \ll x_{rd}) \approx \frac{0.6}{g_*^{1/4}} \sqrt{\alpha_\phi \, m_\phi \, M_P}$$

Delayed Thermalisation:

$$T_{rh}(x_{th} \gg x_{rd}) \approx (7 - 70) \times \frac{\alpha_s^3 \, \alpha_\phi^{3/2} M_P^{5/2}}{g_*^{1/4} m_\phi^{3/2}}$$

Mazumdar, Zaldivar 1310.5143

Visible matter : Thermal Sector



Thermal History of the visible Universe

Quarks	t	174.2 ± 3.3GeV	$\bar{t}$	spin=$\frac{1}{2}$	$g = 2 \cdot 2 \cdot 3 = 12$	
	b	4.20 ± 0.07GeV	$\bar{b}$	3 colors		
	c	1.25 ± 0.09GeV	$\bar{c}$			
	s	95 ± 25MeV	$\bar{s}$			
	d	3–7MeV	$\bar{d}$			
	u	1.5–3.0MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin=1	$g = 2$	16
Leptons	τ^-	1777.0 ± 0.3MeV	τ^+	spin=$\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	μ^-	105.658MeV	μ^+			
	e^-	510.999keV	e^+			
						12
	ν_τ	< 18.2MeV	$\bar{\nu}_\tau$	spin=$\frac{1}{2}$	$g = 2$	
	ν_μ	< 190keV	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	W^+	80.403 ± 0.029GeV		spin=1	$g = 3$	
	W^-	80.403 ± 0.029GeV				
	Z^0	91.1876 ± 0.0021GeV				
	γ	0 $(< 6 \times 10^{-17}$eV)			$g = 2$	
						11
Higgs boson (SM)	$H^0 = 126 GeV$			spin=0	$g = 1$	1
						$g_f = 72 + 12 + 6 = 90$
						$g_b = 16 + 11 + 1 = 28$

$$\rho(T) = \frac{\pi^2}{30} g_*(T) T^4 \quad g_*(T) = g_b(T) + \frac{7}{8} g_f(T)$$

$$s(T) = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

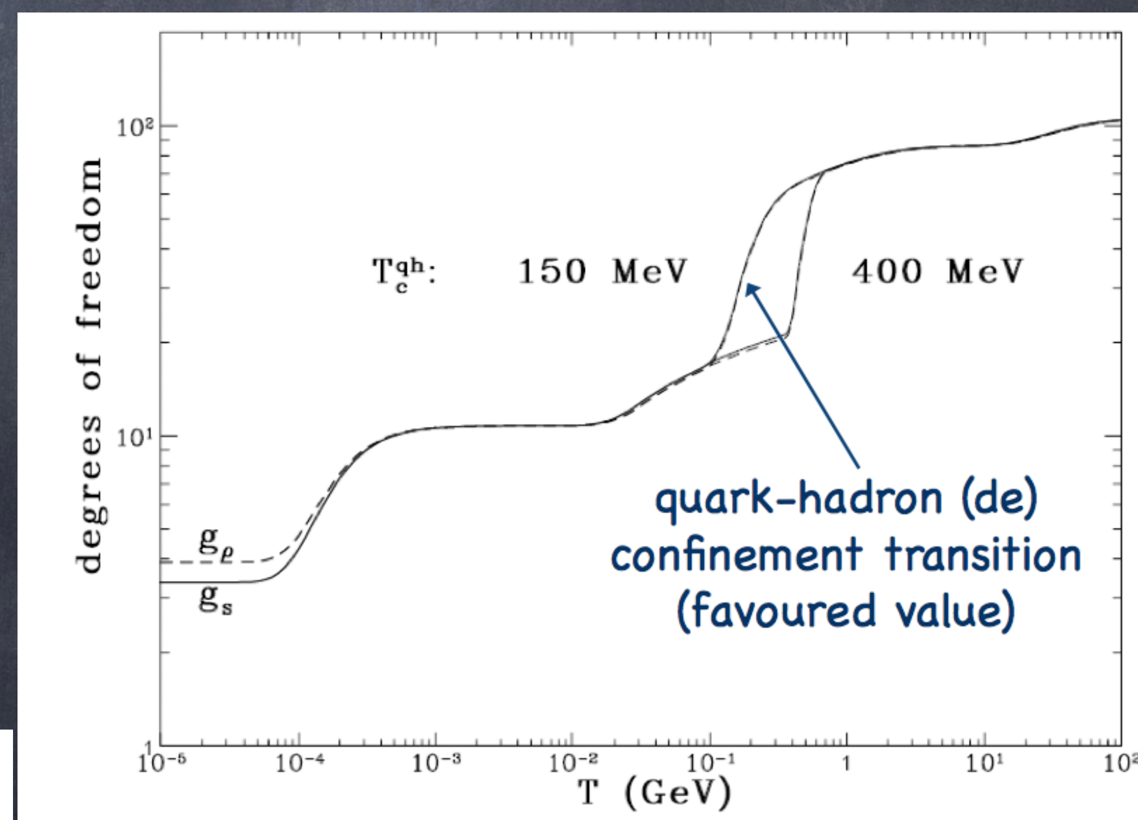
$$g_*(T) = \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^4$$

$$g_{*s}(T) = \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^3$$

$$g_b = 28 \qquad \text{gluons } 8\times2\text{, photons } 2\text{, } W^\pm \text{ and } Z^0\ 3\times3\text{, and Higgs } 1$$

$$g_f = 90 \qquad \text{quarks } 12\times6\text{, charged leptons } 6\times2\text{, neutrinos } 3\times2$$

$$g_* = 106.75.$$



$$t = \frac{1}{2} H^{-1} = \sqrt{\frac{45}{16\pi^3 G}} \frac{T^{-2}}{\sqrt{g_*}} = 0.301 g_*^{-1/2} \frac{m_{\rm Pl}}{T^2} = \frac{2.4}{\sqrt{g_*}} \left(\frac{\rm T}{\rm MeV}\right)^{-2} \text{s}$$