



Stefano Profumo

**Santa Cruz Institute for Particle Physics
University of California, Santa Cruz**

An Introduction to Particle Dark Matter

**Pre-SUSY Summer School
Melbourne, June 29-July 1, 2016**

Thank you to those who came and introduced themselves,
asking lots of **great questions!**

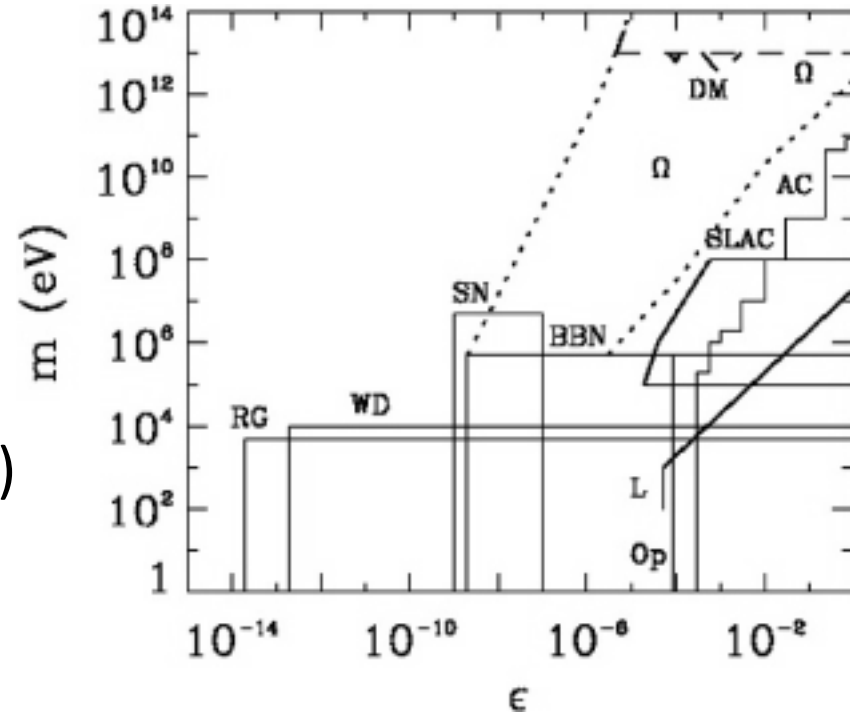
Looking forward to **more interactions!**

Quick **summary** of key concepts from Lecture 1

Dark matter key ingredient to seed **timely structure formation**
[MOND does not work: baryon acoustic oscillations...]

Dark Matter as a **particle**:

- **Dark**
- **Collisionless** ($\sigma/m < 1 \text{ cm}^2/\text{g}$, or **1 barn/GeV**)
- **Classical** (de Broglie; Pauli-blocking)
- **Fluid**
- **Right abundance**, ~ 0.3 critical



Quick **summary** of key concepts from Lecture 1

Paradigm of **thermal decoupling** $\Gamma(T_{\text{t.o.}}) \sim H(T_{\text{t.o.}})$

$$\Gamma = n \cdot \sigma \cdot v \quad H \simeq T^2 / M_P$$

Example: **hot** relic (e.g. SM neutrinos) $\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}}$

doesn't always work... **relic protons-antiprotons**

relic protons, antiprotons: $\sim 10^{-15}$ obs. baryon density
Cold relics!

Cold Relic $\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{f.o.}}{20} \left(\frac{10^{-8} \text{ GeV}^{-2}}{\sigma}\right)$

Works for **WIMPs**, but also for lighter, more weakly coupled particles

Mass Range: $m=0.1 \text{ eV}$ [1 MeV]... 120 TeV

Assuming weak interactions, $m > 10 \text{ GeV}$ [**Lee-Weinberg**]

$$\dot{n} + 3Hn = \langle \sigma v \rangle (n_{\text{eq}}^2 - n^2)$$

Proper formulation:

Boltzman equation L=C

$$\frac{dY(x)}{dx} = -\frac{x s \langle \sigma v \rangle}{H(m)} (Y(x)^2 - Y_{\text{eq}}^2(x))$$

$$\langle \sigma \cdot v_{M\phi l} \rangle = \frac{\int \sigma \cdot v_{M\phi l} e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}{\int e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}$$

There exist important "**exceptions**" to this standard story:

1. **Resonances**

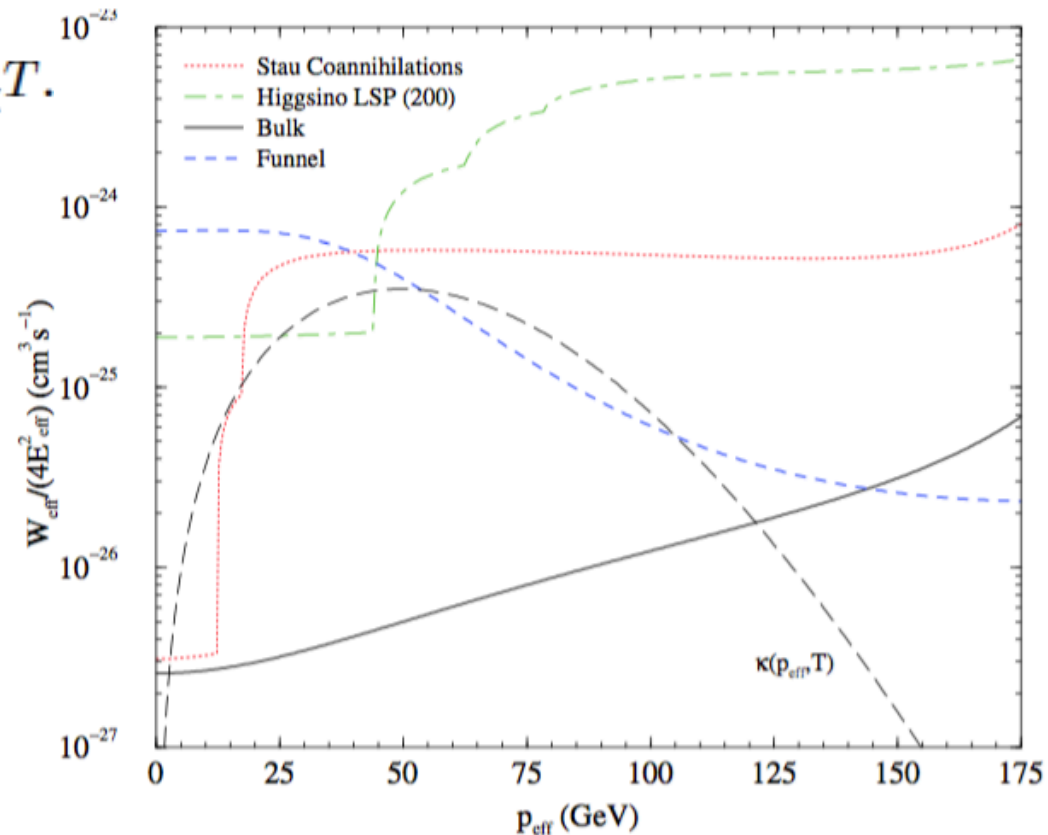
$$\langle s \rangle \simeq 4m_\chi^2 + 6m_\chi T.$$

2. **Thresholds**

3. **Co-annihilation**

$$\langle \sigma v \rangle \rightarrow \langle \sigma_{\text{eff}} v \rangle = \frac{\sum_{i < j=1}^N \sigma_{ij} \exp\left(-\frac{\Delta m_i + \Delta m_j}{T}\right)}{\sum_{i=1}^N g_i \exp\left(-\frac{\Delta m_i}{T}\right)}.$$

Affects what the **pair-annihilation** rate **today** is compared to what it was at **freeze-out**!



$$\langle \sigma_{\text{eff}} v \rangle = \int_0^\infty dp_{\text{eff}} \frac{W_{\text{eff}}(p_{\text{eff}})}{4E_{\text{eff}}^2} \kappa(p_{\text{eff}}, T) \quad E_{\text{eff}}^2 = \sqrt{p_{\text{eff}}^2 + m^2}.$$

So far we looked into what happens if we fiddle with the left hand side of

$$\Gamma = n \cdot \sigma \sim H,$$

Consider a "**Quintessence**" dark energy model – homogeneous real scalar field

$$\rho_\phi = \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 + V(\phi)$$

$$P_\phi = \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 - V(\phi)$$

$$w = P_\phi / \rho_\phi \quad \rho_\phi \sim a^{-3(1+w)} \quad \rho \sim a^{-6}$$

$$H \sim \frac{T^2}{M_P} \frac{T}{T_{\text{KRE}}} \quad (T \gtrsim T_{\text{KRE}})$$

$$n_{\text{f.o.}} \langle \sigma v \rangle \sim \frac{T^2}{M_P} \frac{T}{T_{\text{KRE}}}.$$

$$\frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^2} \sim \frac{1}{M_P} \frac{T_{\text{f.o.}}}{\langle \sigma v \rangle T_{\text{KRE}}}, \quad \Omega_{\chi}^{\text{quint}} = \frac{T_0^3}{M_P \cdot \rho_c} x_{\text{f.o.}} \left(\frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^2} \right)$$

$$\frac{\Omega_{\chi}^{\text{quint}}}{\Omega_{\chi}^{\text{standard}}} \sim \frac{T_{\text{f.o.}}}{T_{\text{KRE}}} \lesssim \frac{m_{\chi}}{20} \frac{1}{T_{\text{BBN}}} \sim 10^4 \frac{m_{\chi}}{100 \text{ GeV}},$$

After **chemical** decoupling (number density freezes out),
DM can still be in **kinetic** equilibrium
(i.e. its **velocity** distribution is in equilibrium)

generically, this is the case, since for **cold** relics

$$\begin{aligned} \chi\chi &\leftrightarrow ff && \rightarrow \Gamma = n_{\text{non-rel}} \cdot \sigma \\ \chi f &\leftrightarrow \chi f && \rightarrow \Gamma = n_{\text{rel}} \cdot \sigma \end{aligned}$$

Think of a **prototypical WIMP**:

$$\sigma_{\chi f \leftrightarrow \chi f} \sim G_F^2 T^2$$

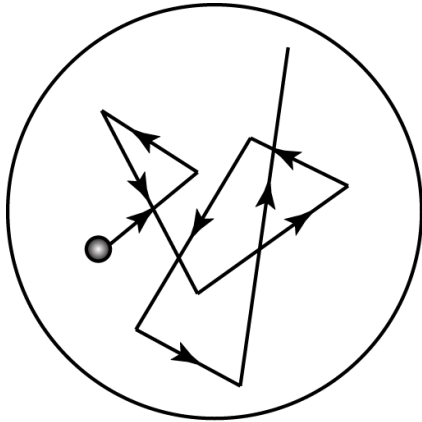
Problem: every collision has a **momentum transfer** $\delta p \sim T$,

...but we need to keep the (cold) DM momentum in equilibrium, i.e.

$$\frac{p^2}{2m_\chi} \sim T \quad ; \quad p \sim \sqrt{m_\chi T}$$

so **$\delta p \ll p$** , we need a bunch of kicks!

However, **subtlety**: kicks are in **random directions**!



$$N = \left(\frac{p}{\delta p} \right)^2 \sim \frac{m_\chi T}{T^2} = \frac{m_\chi}{T} \gg x_{\text{f.o.}} \gtrsim 20$$

Let's estimate a typical WIMP **kinetic decoupling temperature**

$$n_{\text{rel}} \cdot \sigma_{\chi f \leftrightarrow \chi f} \left(\frac{\delta p}{p} \right)^2 \sim T^3 \cdot G_F^2 T^2 \cdot \frac{T}{m_\chi} \sim H \sim \frac{T^2}{M_P}$$

$$T_{\text{kd}} \sim \left(\frac{m_\chi}{M_P \cdot G_F^2} \right)^{1/4} \sim 30 \text{ MeV} \left(\frac{m_\chi}{100 \text{ GeV}} \right)^{1/4}$$

What does this implies for **structure formation**?

$$M_{\text{ao}} \sim \frac{4\pi}{3} \left(\frac{1}{H(T_{\text{kd}})} \right)^3 \rho_{\text{DM}}(T_{\text{kd}}) \sim 30 M_{\oplus} \left(\frac{10 \text{ MeV}}{T_{\text{kd}}} \right)^3$$

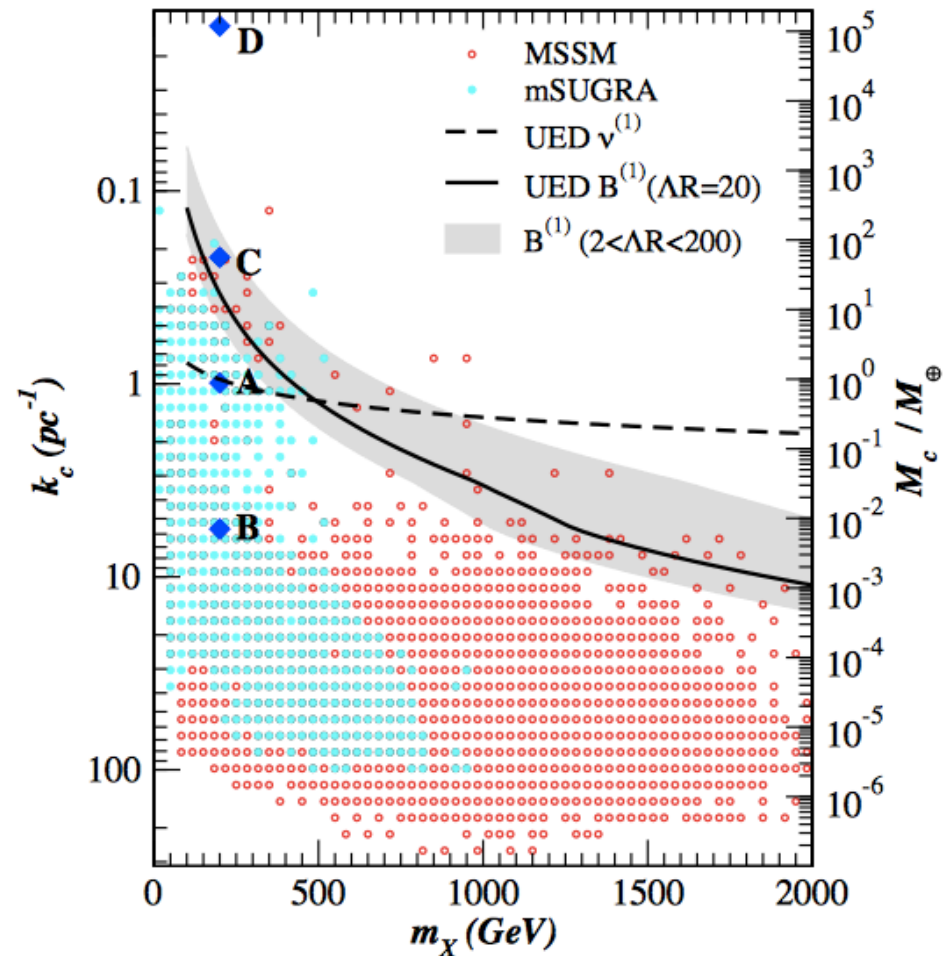
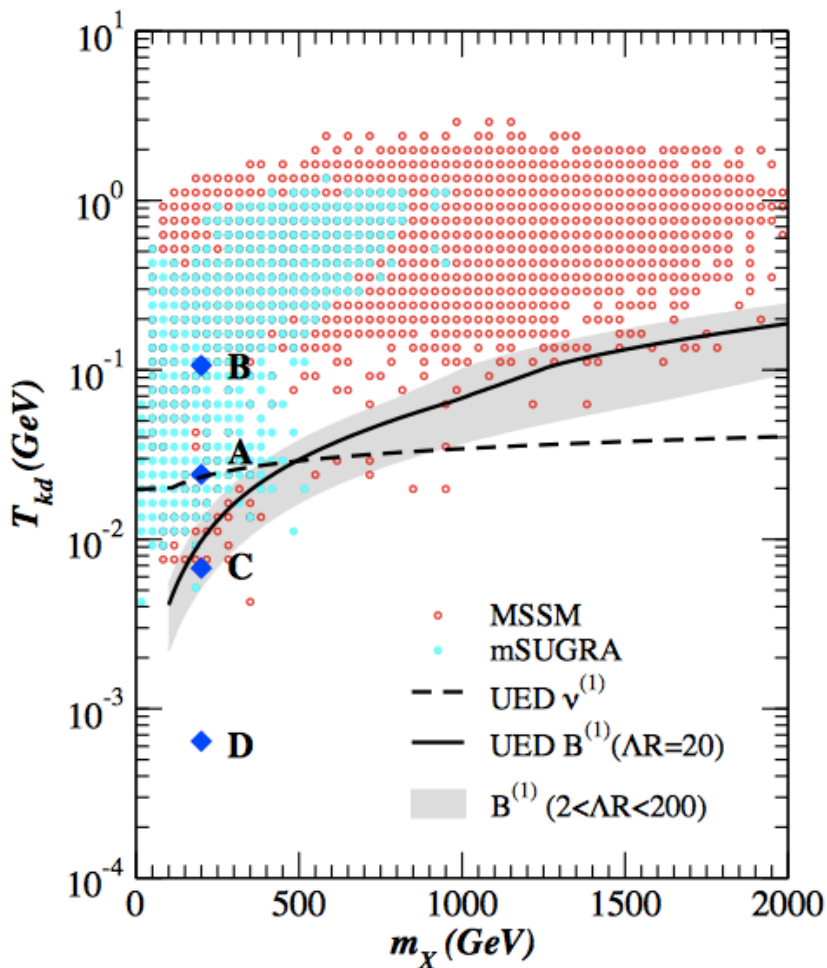
$$M_{\oplus} \simeq 3 \times 10^{-6} M_{\odot}$$

First structures that collapse are these tiny **minihalos**
(maybe some survive today?)

Structures then **merge** into bigger and bigger halos
(**bottom-up** structure formation)

Notice that the kinetic decoupling/cutoff scale **varies** significantly even for a selected particle dark matter scenario!

e.g. for **SUSY, UED**



M_c / M_\oplus

What happens instead for **hot relics**?

They decouple when **$T \gg m_\nu$**

Structures can only collapse when **$T \sim m_\nu$**

(i.e. when things slow down enough for gravitational collapse!)

Structures are cutoff to the **horizon size** at that temperature

$$d_\nu \sim H^{-1}(T \sim m_\nu) \qquad d_\nu \sim \frac{M_P}{m_\nu^2}$$

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$$M_{\text{cutoff, hot}} \sim \left(\frac{1}{H(T = m_\nu)} \right)^3 \rho_\nu(T = m_\nu) \sim \left(\frac{M_P}{m_\nu^2} \right)^3 m_\nu \cdot m_\nu^3 = \frac{M_P^3}{m_\nu^2}$$

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15} M_\odot \left(\frac{m_\nu}{30 \text{ eV}} \right)^{-2} \sim 10^{12} M_\odot \left(\frac{m_\nu}{1 \text{ keV}} \right)^{-2}$$

How does this compare with **observations**?

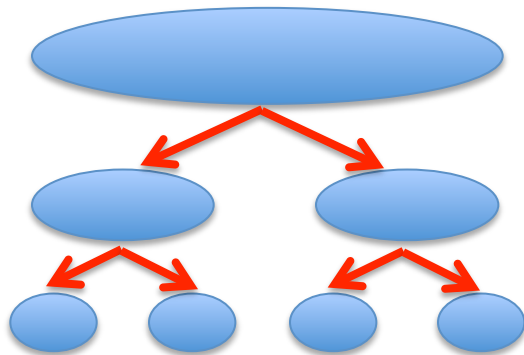
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Observational **constraints** give

$$M_{\text{cutoff}} \ll M_{\text{Ly}-\alpha} \simeq 10^{10} M_\odot$$

So at best dark matter can be **keV** scale, if produced thermally

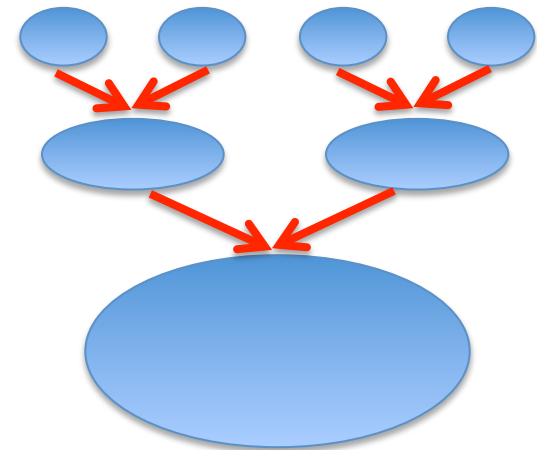
Structure formation looks strikingly different
for hot and cold dark matter



Hot Dark Matter

Top-Down

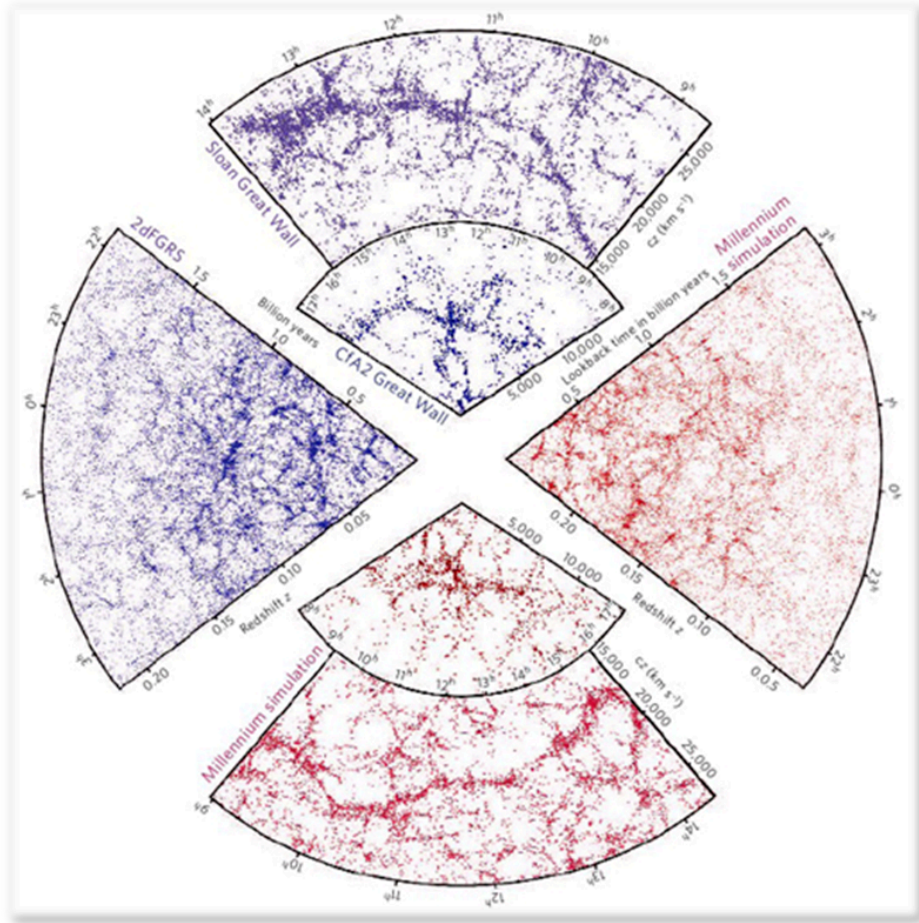
[doesn't work!]



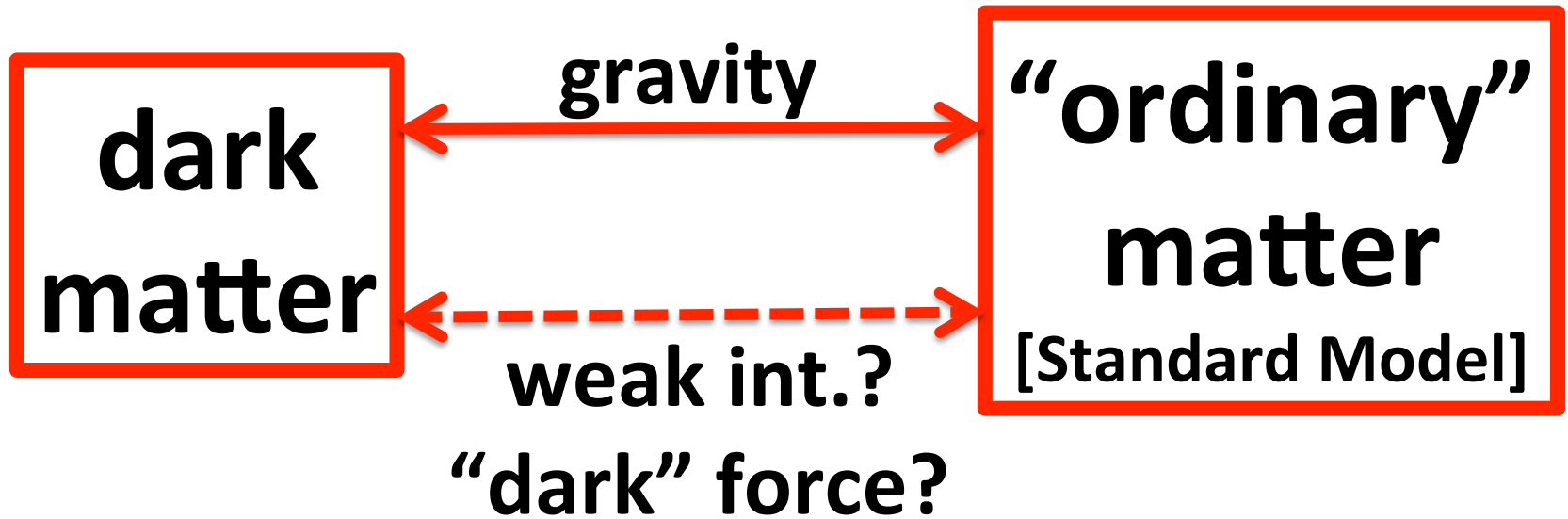
Cold Dark Matter

Bottom-Up

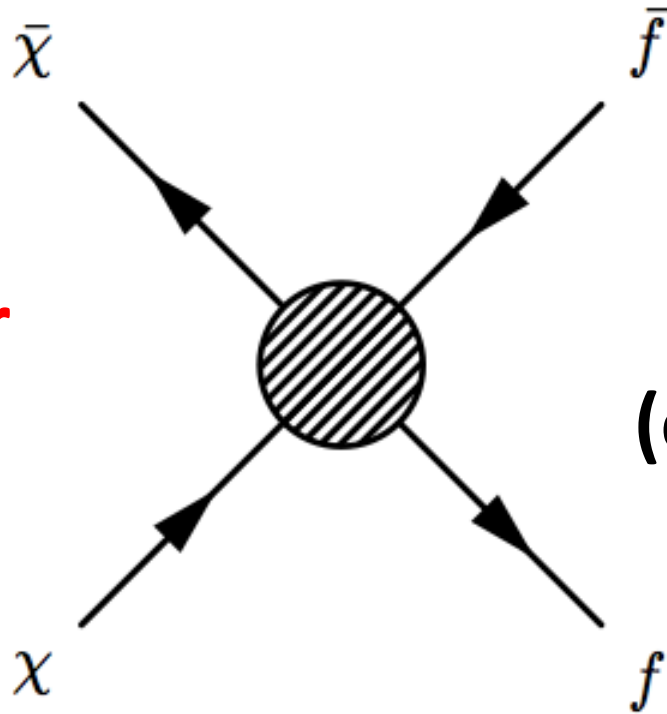
[Yeah!]



1980's: Davis, Efstathiou, Frenk and White show that simulations of structure formation in a universe with **cold dark matter** match observed structure incredibly well!!

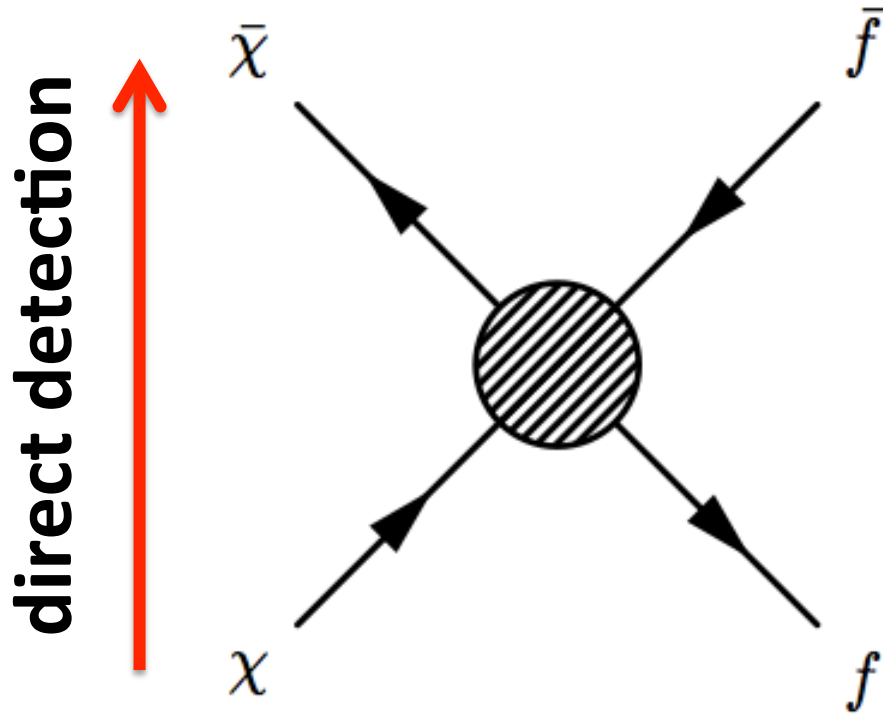


**Dark Matter
Particles**



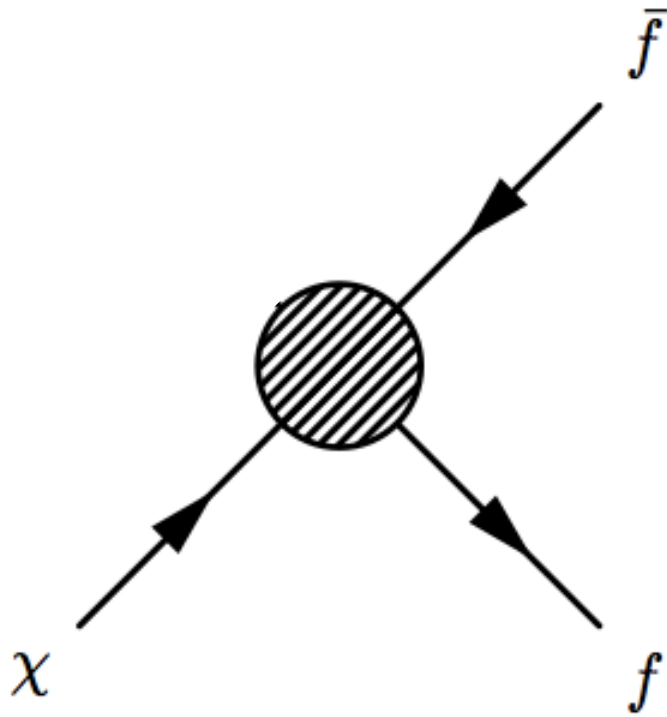
**Standard Model
(ordinary) Particles**

collider production



thermal equilibrium ?
[pair annihilation]

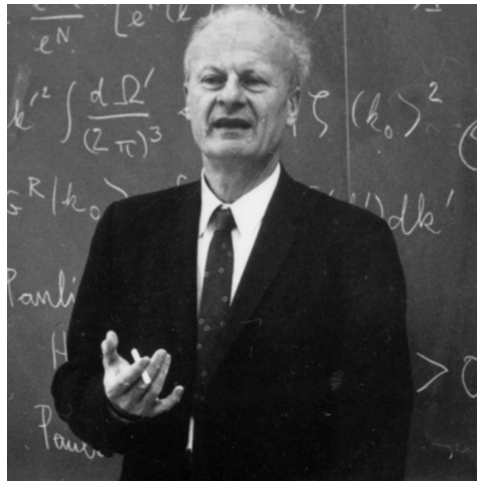




long-lived, but **metastable**

Consider **direct** detection

Detecting particles that interact **weakly** has always been known to be a **tough job**



After **estimating** in 1934 the **cross section** for $\bar{\nu}_e + p \rightarrow e^+ + n$

$$\sigma_{\bar{\nu}_e + p \rightarrow e^+ + n} \approx 10^{-43} (E_\nu / \text{MeV})^2 \text{ cm}^2.$$

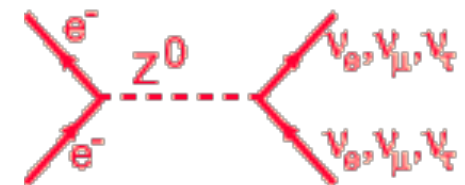
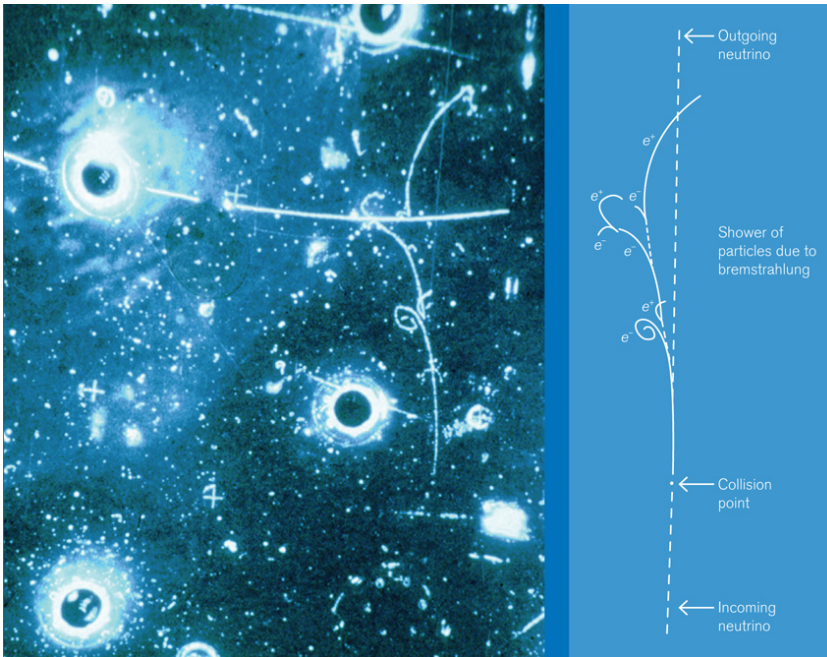
“It is therefore **absolutely impossible** to observe processes of this kind”

Bethe and Peierls were too **pessimistic/conservative**:
neutrinos were detected in 1953, abundantly in 1956

Inelastic process (maybe relevant for DM?)



Elastic neutrino scattering took **much longer** (Gargamelle 1973)



Let's use **WIMPs** again as **prototypical** DM particles

First, which **energies** and what **masses** are we talking about?

maximal recoil momentum for a DM particle
with velocity v is $2m_\chi v$, so maximal energy

$$E_{\max} = (2m_\chi v)^2 / (2m_N)$$

Now, the **maximal velocity** a DM particle can have in the
Galaxy is the **escape velocity** $v_{\max} \sim 500-700 \text{ km/s}$
 $\rightarrow E \sim \text{keV}$ for GeV particles!

Plug in numbers for a detector with an energy threshold $\sim \text{keV}$...

minimal detectable DM mass $\sim \text{GeV}$

OK, now what about the **event rate**? $\bar{R} = K\phi\sigma.$

$$K \simeq 6.0 \times 10^{26} / A \quad \phi = v\rho_{\text{DM}}/m_\chi$$

Plug in sensible **benchmark** values...

$$R = \frac{0.06 \text{ events}}{\text{kg day}} \left(\frac{100}{A} \right) \left(\frac{\sigma}{10^{-38} \text{ cm}^2} \right) \left(\frac{\rho_{\text{DM}}}{0.3 \text{ GeV/cm}^3} \right) \left(\frac{v}{200 \text{ km/s}} \right)$$

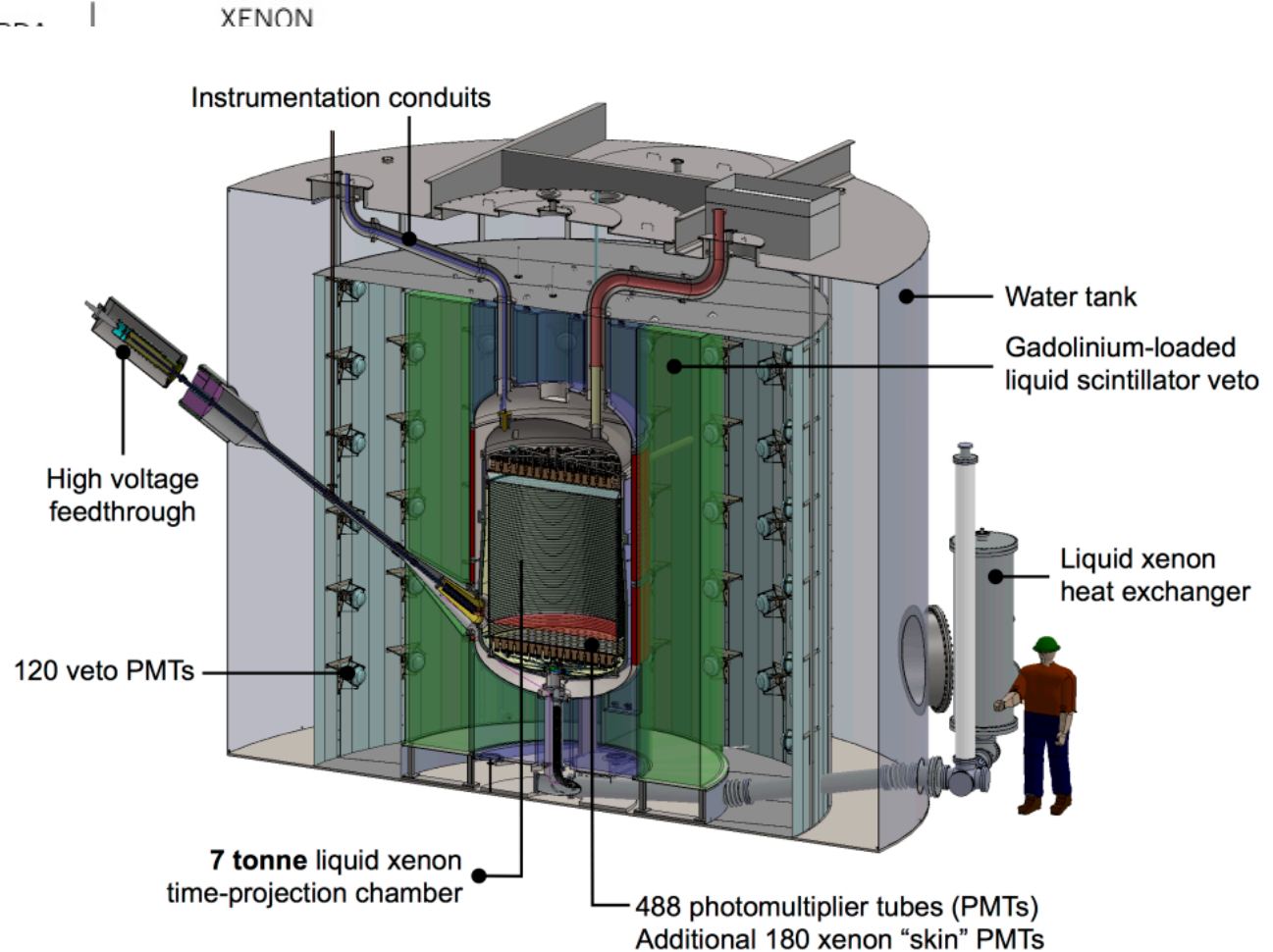
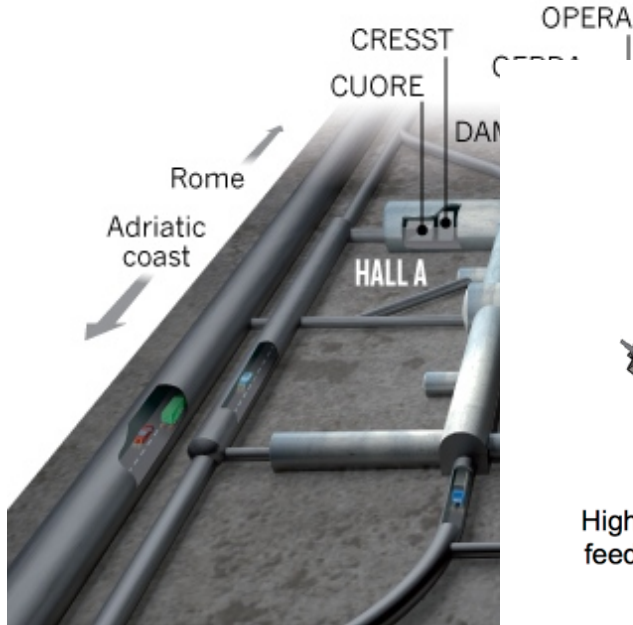
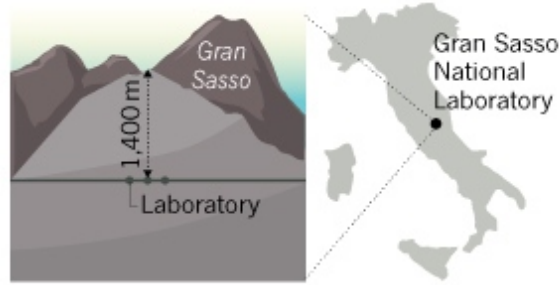
To have a detection need both enough **signal** events,
and enough **background** suppression

1. slowly decaying "primeval" nuclides (U, Th, ^{40}K),
ab. 10^{-4} , half lives $\sim 10^9$ yr
2. rare, fast decaying trace elements like tritium, ^{14}C :
ab 10^{-18} , half lives 10 yr

Big detectors, in **underground**, actively **shielded** environments...

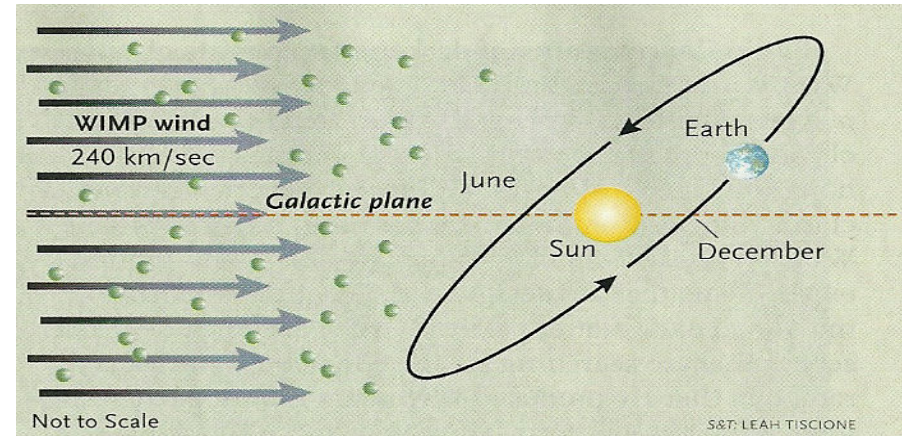
THE A, B AND C OF GRAN SASSO

Experiments at the Gran Sasso National Laboratory are housed in and around three huge halls carved deep inside the mountain, where they are shielded from cosmic rays by 1,400 metres of rock.

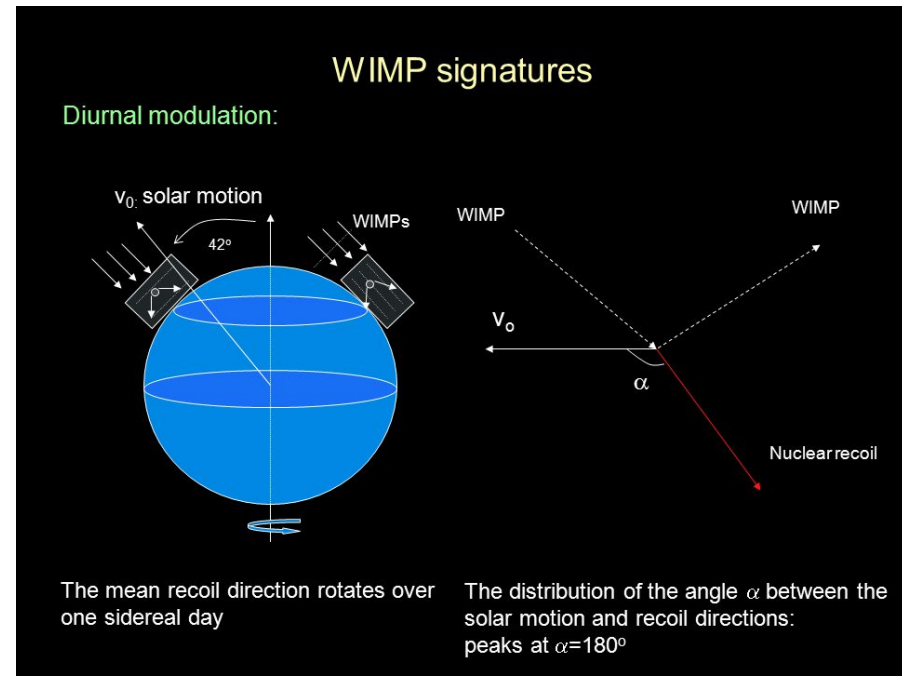


Other handles on a DM **signal** versus radioactive **background**:

1. **Seasonal** modulation



2. **Diurnal** modulation



3. **Directional** information

Now: direct detection **event rates**, for real!

$$\frac{dR}{dE_R} = N_T n_\chi \left\langle v_\chi \frac{d\sigma}{dE_R} \right\rangle$$

$$E_R = \frac{q^2}{2m_T} = \frac{\mu_T^2}{m_T} v_\chi^2 (1 - \cos \theta)$$

$$dE_R = (d \cos \theta) (\mu_T^2 / m_T) v^2$$

$$\frac{dR}{dE_R} = N_T \frac{\rho_{DM} m_T}{m_\chi \mu_T^2} \int_{v_{\min}}^{v_{\text{esc}}} d^3 v \frac{f(v)}{v} \frac{d\sigma}{d \cos \theta}$$

How do we calculate the scattering **cross section**?

Non-relativistic limit, the scattering **matrix element** is the Fourier transform of WIMP-nucleus potential

$$\mathcal{M}(q^2) \sim \int \langle f | V(\vec{r}) | i \rangle e^{i\vec{q} \cdot \vec{r}} d\vec{r},$$

to the lowest order in velocity, the potential is just a **contact interaction** of spin-independent and axial

$$V(\vec{r}) = \sum_{\text{nucleons } n} (G_s^n + G_a^n \vec{\sigma}_\chi \cdot \vec{\sigma}_n) \delta(\vec{r} - \vec{r}_n).$$

where the G's are the effective DM-nucleon interactions for **scalar** and **axial** interactions

Coherence requires the nucleus size to be much smaller than the momentum transfer wavelength ($1/q$)

$$qR_{\text{nucleus}} \ll 1$$

Loss of coherence is phenomenologically accounted for by introducing **form factors** describing the nucleus response

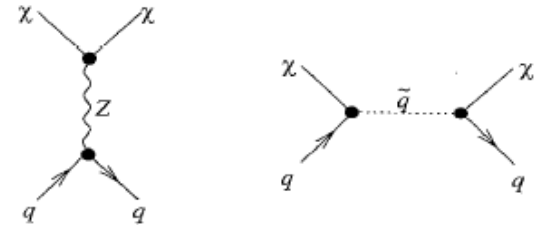
$$\mathcal{M}(q^2) = T(0)F(q^2)$$

Given a **microscopic** theory of dark matter,
how does one get to the **DM-nucleus cross section**?

An interesting **multi-layered** problem in **effective field theory**!

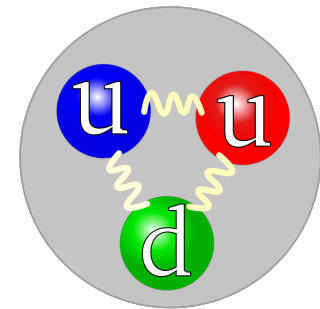
Low-energy EFT

Dark Matter-quark



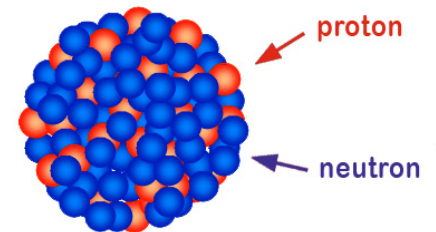
Nucleon matrix elements

Dark Matter-nucleon



Form factors

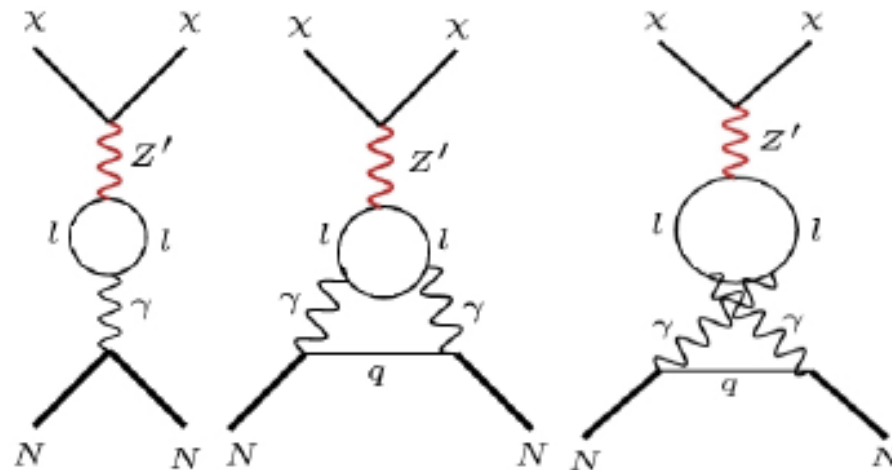
Dark Matter-nucleus

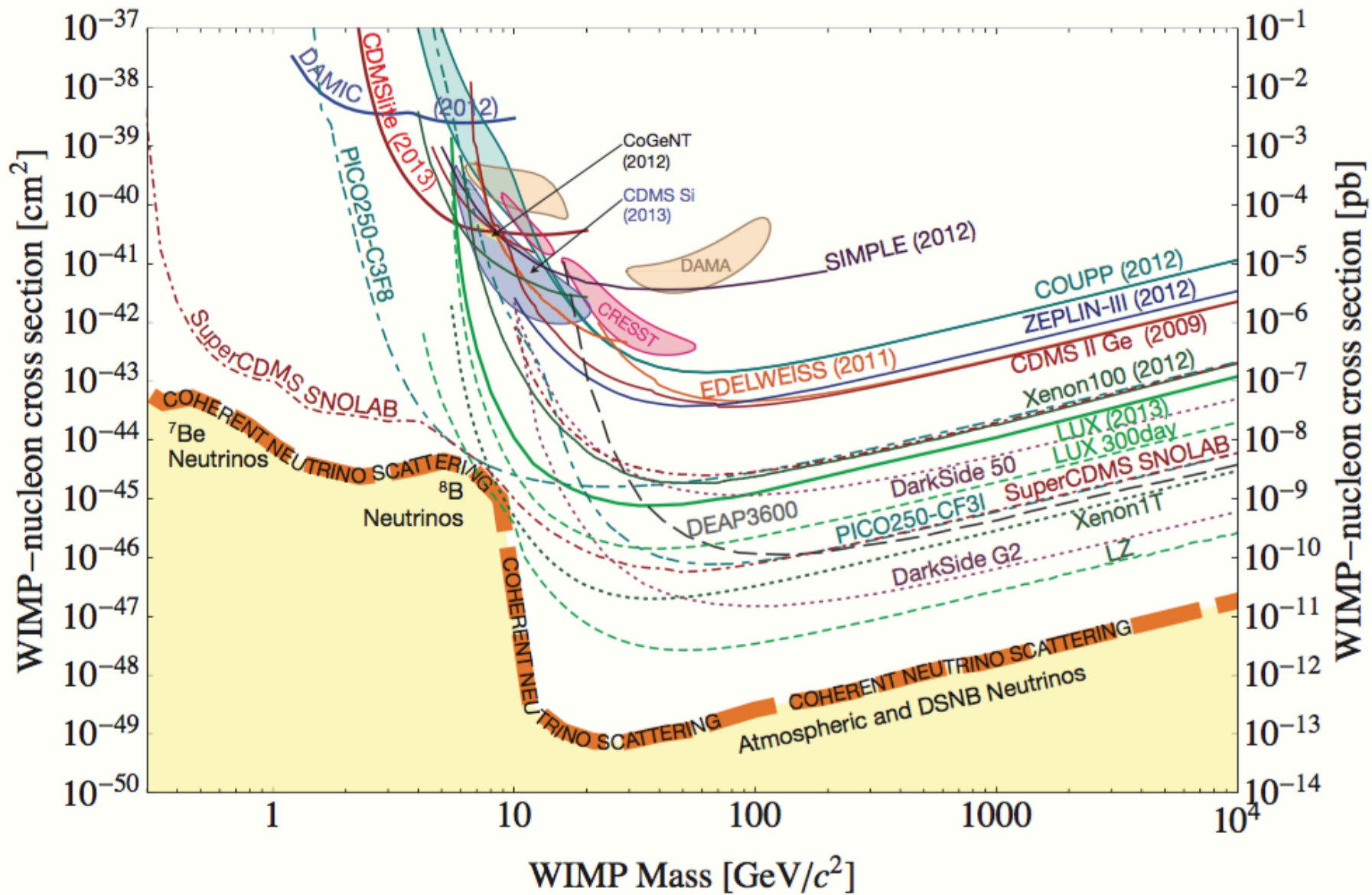


Sometimes life is simpler, e.g. if DM is (**milli-electric-**)**charged**

$$\sigma_N = \frac{16\pi\alpha^2\epsilon^2 Z^2 \mu_N^2}{q^4}$$

Sometimes life is nastier, e.g. if DM is **lepto-philic**





Now off to **indirect** dark matter detection

Idea: use the **debris** of DM **pair-annihilation**
(likely large if thermal relic) or **decay**

$$\Gamma_{\text{SM, ann}} \sim \left(\int_V \frac{\rho_{\text{DM}}^2}{m_\chi^2} dV \right) \times (\sigma v) \times (N_{\text{SM, ann}}),$$
$$\Gamma_{\text{SM, dec}} \sim \left(\int_V \frac{\rho_{\text{DM}}}{m_\chi} dV \right) \times \left(\frac{1}{\tau_{\text{dec}}} \right) \times (N_{\text{SM, dec}})$$

What do we know about these **rates**?
 σv from **thermal production** (with caveats!)

How about **decay rate**?

Suppose DM decay mediated by **high-scale** physics at scale **M**

$$\Gamma_5 \sim \frac{1}{M^2} m_\chi^3$$

$$\tau_5 \sim 1 \text{ s} \left(\frac{1 \text{ TeV}}{m_\chi} \right)^3 \left(\frac{M}{10^{16} \text{ GeV}} \right)^2$$

Dimension-5 operator doesn't work – would be too **short lived!**

$$\Gamma_6 \sim \frac{1}{M^4} m_\chi^5,$$

Interesting, well motivated!

$$\tau_6 \sim 10^{27} \text{ s} \left(\frac{1 \text{ TeV}}{m_\chi} \right)^5 \left(\frac{M}{10^{16} \text{ GeV}} \right)^4$$

What about annihilation **final state**?

Very **model-dependent**

1. if DM belongs to an SU(2) **multiplet**, then well-defined combination of ZZ, WW final states...

2. In UED, DM is KK-1 mode of **hypercharge gauge boson**, thus

$$|M|^2 \propto |Y_f|^4 \quad [Y_{u_L} = 4/3, \quad Y_{e_R} = 2]$$

3. Special "**selection rule**", e.g. helicity suppression for Majorana fermion (analogous to charged pion decay)

$$|M|^2 \propto m_f^2$$