Calorimetry Lecture 1

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Outline of Lecture 1

- Principles of Calorimetry
  - Electromagnetic and Hadronic Showers
  - Shower Profiles and Containment
- Types of Calorimeters
  - Total Absorption, Sampling
  - Scintillation, Ionization, Cherenkov
  - Signal Detection

References

- Calorimetry: Energy Measurements in Particle Physics by Richard Wigmans, 2000.
  - Big, Verbose, Expensive, but Excellent
- The Physics of Particle Detectors by Dan Green, 2000.
  - Sufficiently Detailed and Comprehensive, Not Just Calorimetry, Less Expensive
- Detectors for Particle Radiation by Konrad Kleinknecht, 1998.
  - Concise but with some detail, Not Just Calorimetry, Even Less Expensive
- Particle Data Booklet
  - Lots of Important Results but Few Explanations, Free!

Stopping Particles in Matter

- Treat each incident particle on a volume of matter as a mini-fixed target experiment and ask:
  - What interactions will have the largest cross sections, and what are the relevant length scales as determined by the particle momentum and target material?

For electromagnetic processes, the largest cross section interactions will occur at low momentum ($1/q^2$ photon propagator) in regions of the material where there is coherent non-zero net charge:

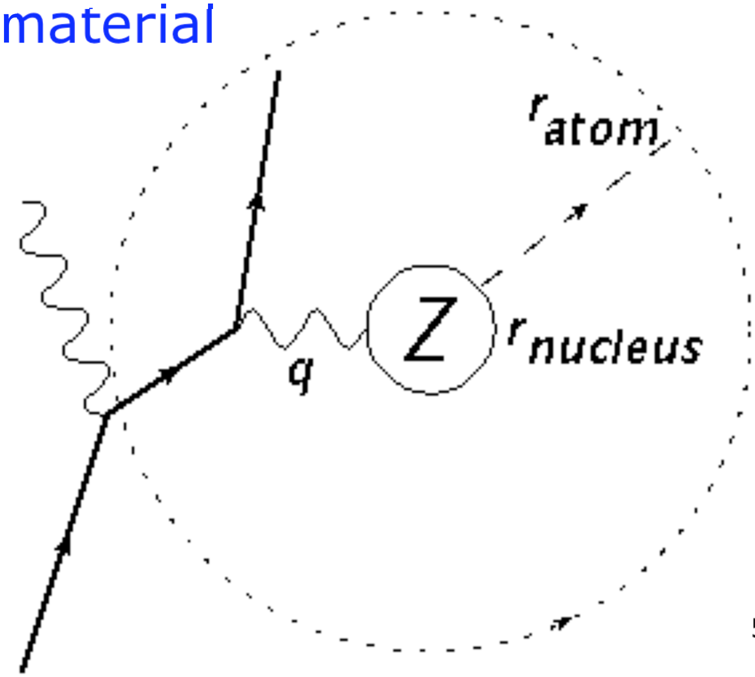
$$\frac{1}{r_{atom}} < |\vec{q}| < \frac{1}{r_{nucleus}}$$

Radiation Length

- A two-vertex electromagnetic interaction at low transverse momentum has a cross section set by the classical electron radius r_e (with πr_e^2 ~0.25 barns)
  - a radiative process (extra photon) will have an additional power of α/π and a photon propagator integral over the length scales where there is unscreened non-zero nuclear charge in the volume of the material

$$\sigma_{radiative} \approx \pi r_e^2 \left[\left(\frac{Z^2 \alpha}{\pi} \right) \int_{1/r_{atom}^2}^{1/r_{nucleus}^2} \frac{dq^2}{q^2} \right]$$

$$= 2\alpha \left(\frac{\alpha}{m_e} \right)^2 Z^2 \ln \left(\frac{r_{atom}}{r_{nucleus}} \right)$$



Radiation Length

- For a material with N atoms/cm^3 there will be n collisions for a path length L

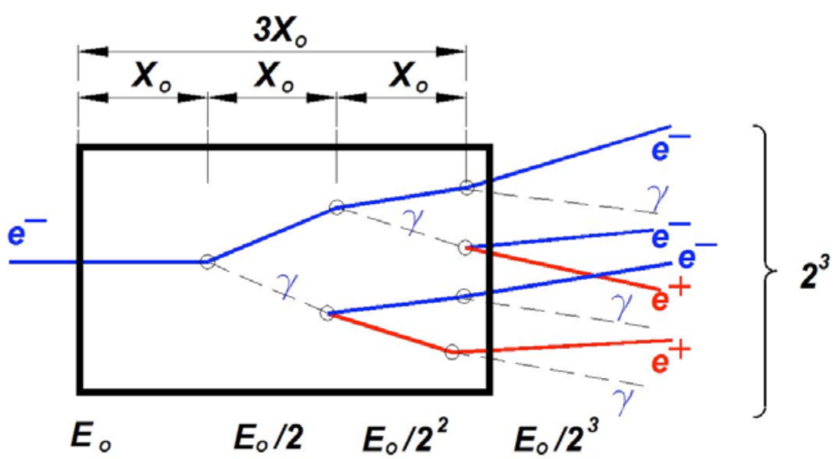
$$n = N\sigma_{radiative} L$$

- Setting n=1 and writing L=X_0, a radiation length is given by

(Inverse) Radiation Length

$$\frac{1}{X_0} = N\sigma_{radiative} \approx 4N\alpha\left(\frac{\alpha}{m_e}\right)^2 Z^2 \log\left(\frac{183}{Z^{1/3}}\right)$$

Electron Impinging on a Target



For photons

$$X_0^\gamma = \frac{9}{7} X_0$$

How about an incident proton?

- The electromagnetic interaction is no longer set by the classical electron radius: for protons (and muons) radiative processes are suppressed by $(m_e/M)^2$
  - Why is pion bremsstrahlung (seagull diagram) small?
- However, the proton-proton cross section σ(pp) is ~40 millibarns and rather constant with q^2, and similarly for σ(πp), but σ(πp) ≈ 2/3 σ(pp)
  - The equivalent nuclear interaction length is therefore (based on protons, not pions):

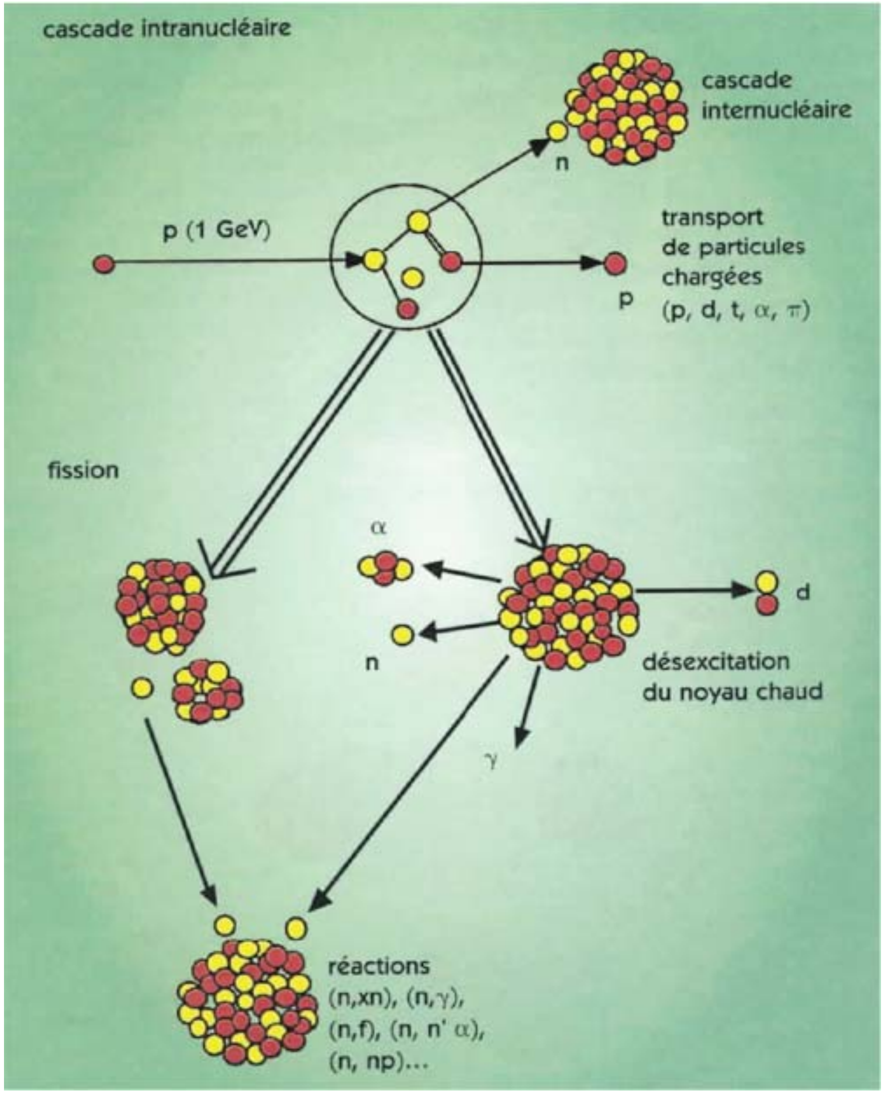
$$\frac{1}{\lambda_{\text{int}}} = NA^{2/3}\sigma(pp)$$

What is the equivalent Nλ_{int} picture?

- Completely different!

  Hadron calorimetry is not for the weak at heart

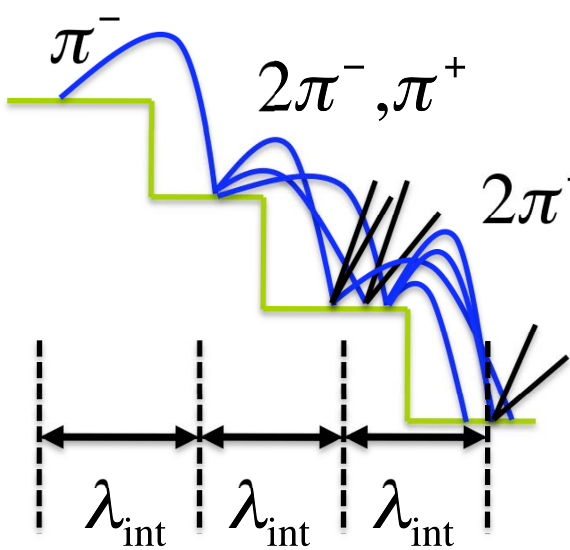
- Most notably, neutrons are abundantly produced



What is the equivalent $N\lambda_{int}$ picture?

- For the pion component, however, the behavior is a bit more clear (though erratic)
  - Pions are in a (nuclear) isospin triplet and therefore "tumble in isospin" with every nucleon they hit – once a charged pion tumbles into a neutral pion, the neutron pion rapidly decays to two photons and initiates an electromagnetic shower "one-way street"

Similarly,
$\pi^+ / \pi^0 / \pi^-$
are produced in equal amounts on average



(Only the pion component)

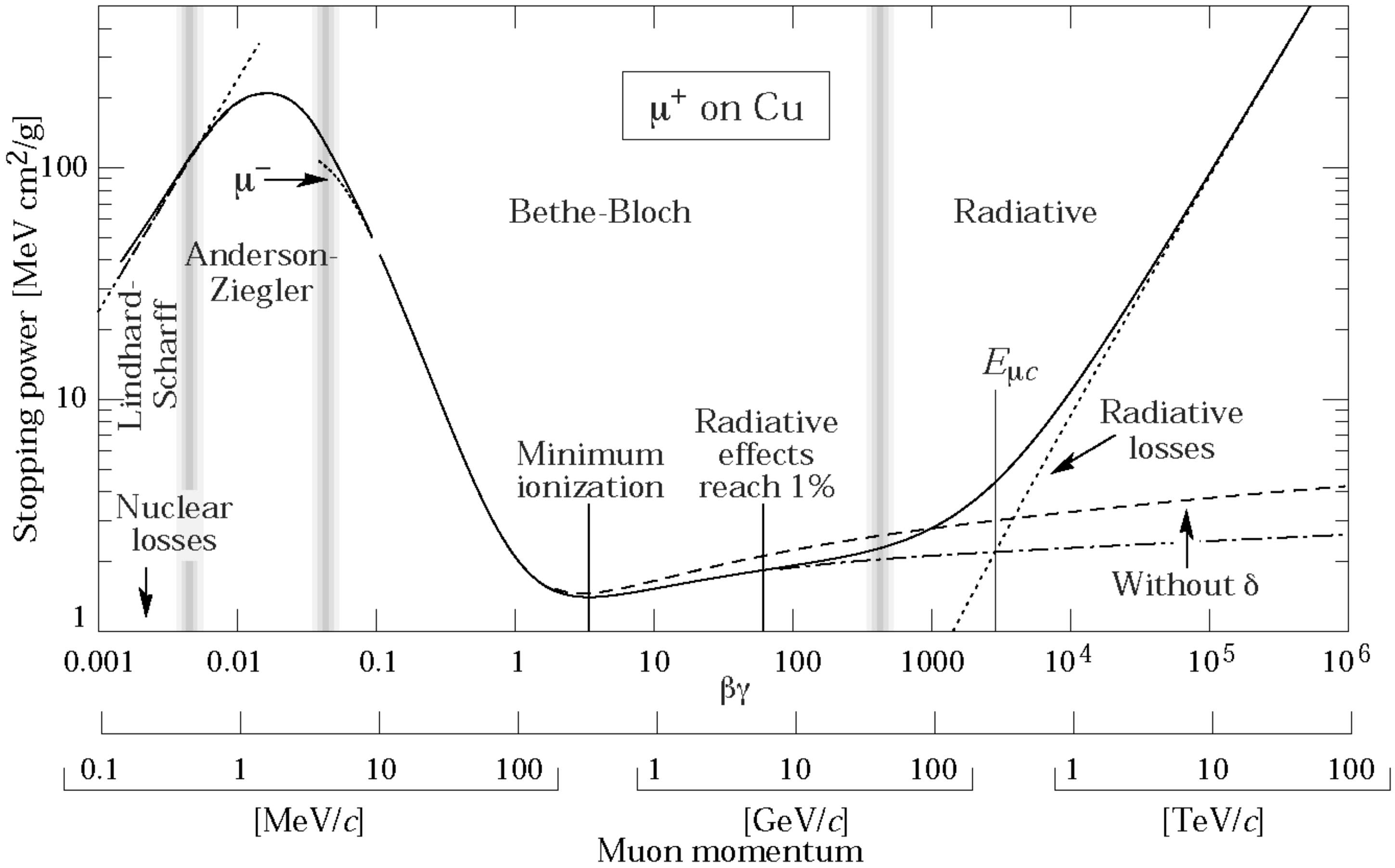
Remember this,
as some calorimeters
will only see the $\pi^0 \rightarrow \gamma\gamma$

Absorber/Active Layer/Support Properties

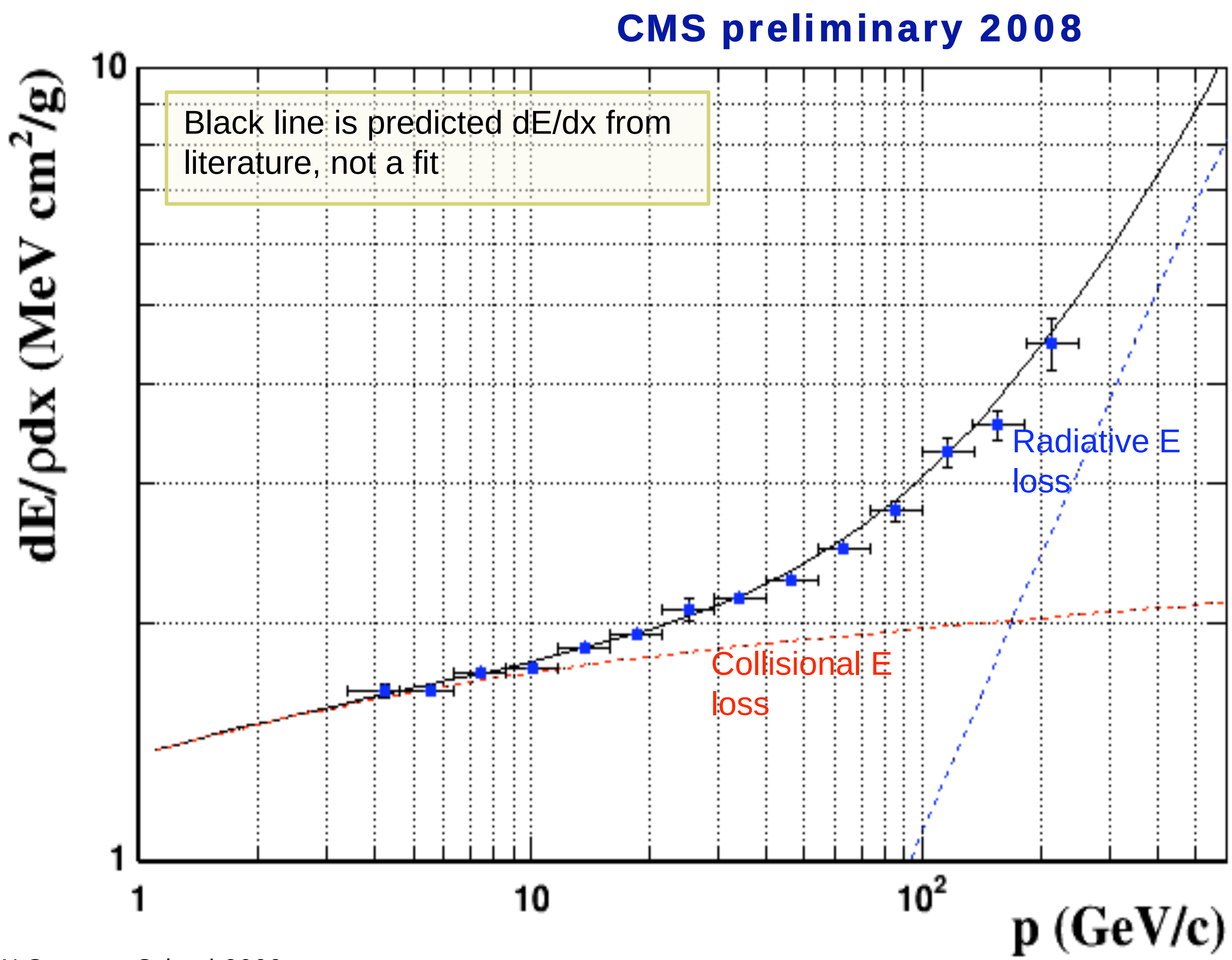
Material	Z	Density [g/cm^3]	X_0 [cm]	λ_{int} [cm]	dE/dx_{mip} [MeV/cm]
Fe	26	7.9	1.8	17	11
Cu	29	9.0	1.4	15	13
Pb	82	11	0.6	17	13
W	**74**	**19**	**0.4**	**9.6**	**22**
^{238}U	**92**	**19**	**0.3**	**11**	**21**
Plastic Scint.	**-**	**1.0**	**42**	**80**	**2.0**
LAr	**18**	**1.4**	**14**	**84**	**2.1**
Quartz	**-**	**2.3**	**12**	**43**	**3.9**
Si	14	2.3	9.4	46	3.9
Al	13	2.7	8.9	39	4.4

In between hard knocks

- When particles are not showering, the charged ones are minimum ionizing

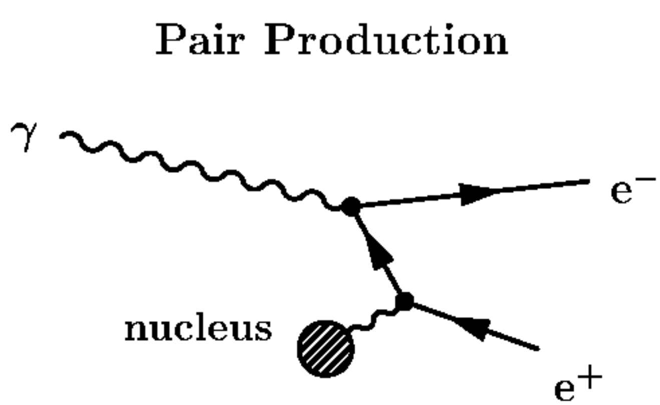


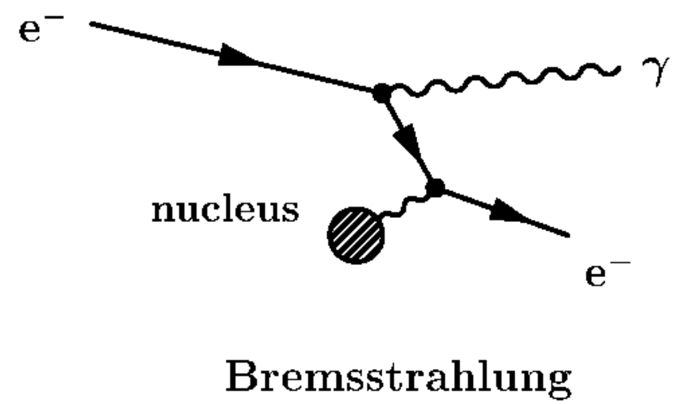
Radiative Rise from Cosmics at CMS



EM Shower Profile and Containment

- First Several Radiation Lengths of a High Energy Shower are in the particle "Amplification" stage:
  - 1→2 Processes Dominate





- As the average shower particle energy drops $E_0/2^n$, a larger fraction of 1→1 processes will occur
  - Compton Scattering, Photoelectric, Ionization
  - Only about 25% of the energy deposition is from positrons. There are two orders of magnitude more liberated atomic electrons in the shower than positrons

EM Shower Profile and Containment

- At the shower maximum, the average amplification rate in the shower goes to unity, and we have:
  - Average Particle Energy at Shower Max:

    E_c (Pb = 7.4 MeV, Fe = 22 MeV)
  - Number of Particles at Shower Max:

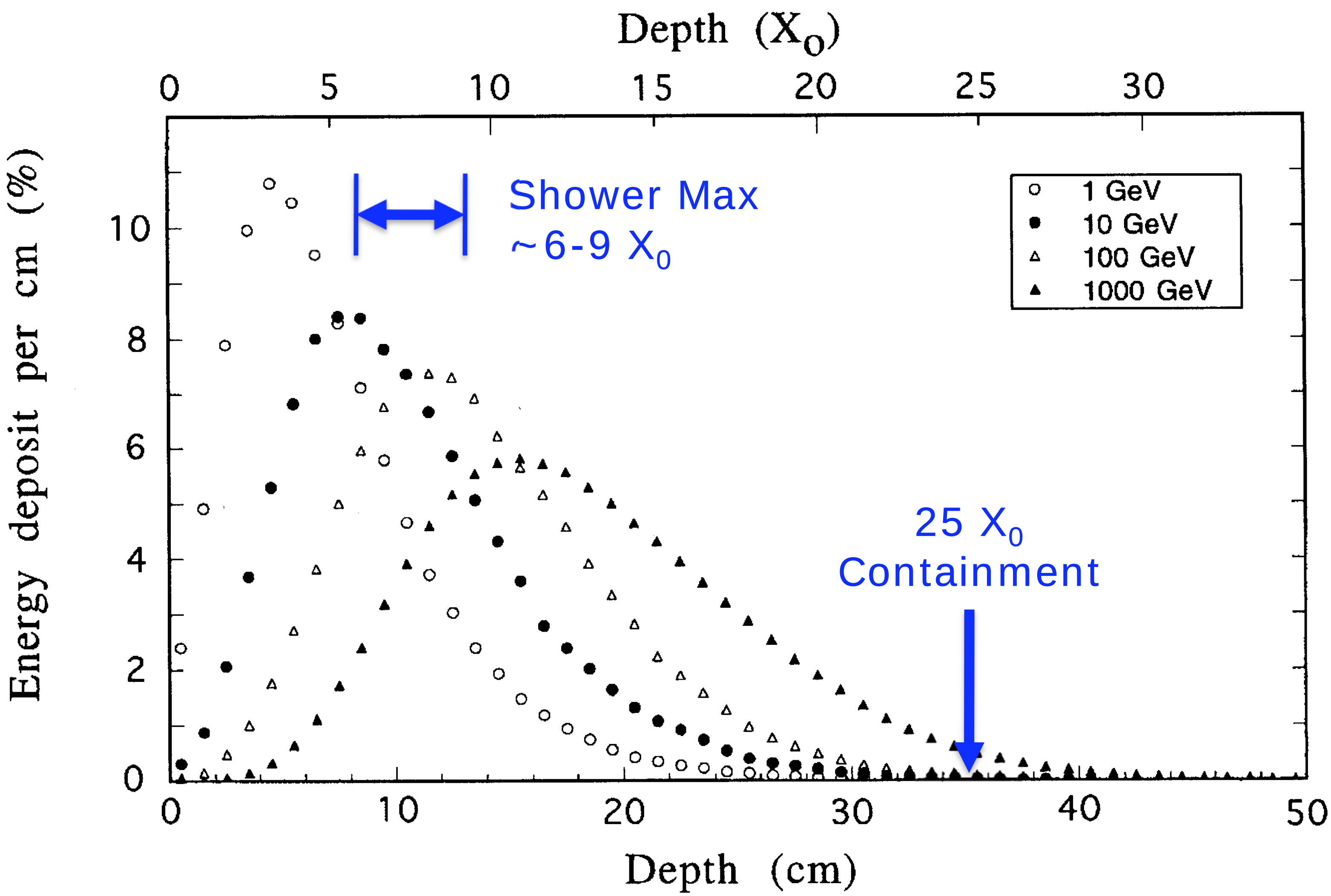
    $N_{max} = E_0/E_c \propto E_0$
  - Total "Path Length" of Showering Particles:

    $L_{tot} \sim N_{max}X_0/\ln 2 \propto E_0$
  - Depth of Shower Max:

    $L_{max} \sim \ln(E_0/E_c)\, X_0 \sim 6\text{-}9\, X_0$

- Clearly, from above, if one sampled the total path length or a known fraction of the total path length (every X_0), then there is direct proportionality between the sampled energy and the incident particle energy E_0

Longitudinal Profile of EM Shower in Cu

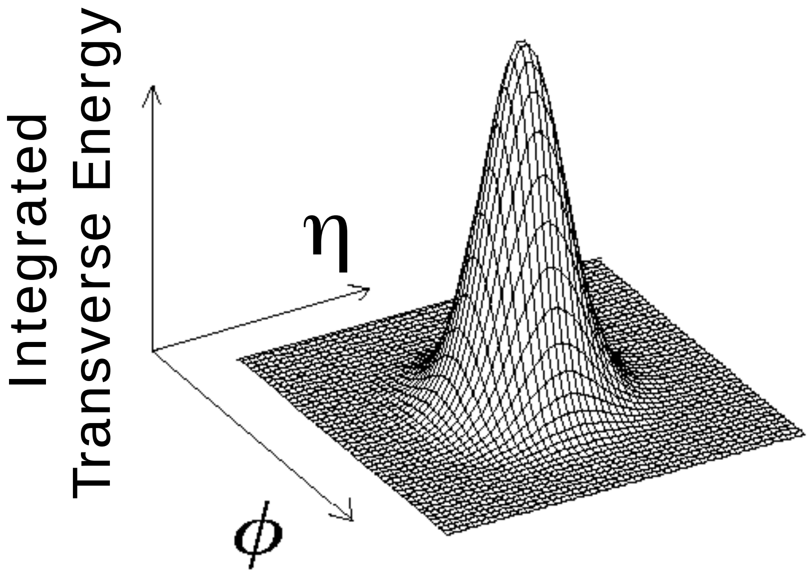


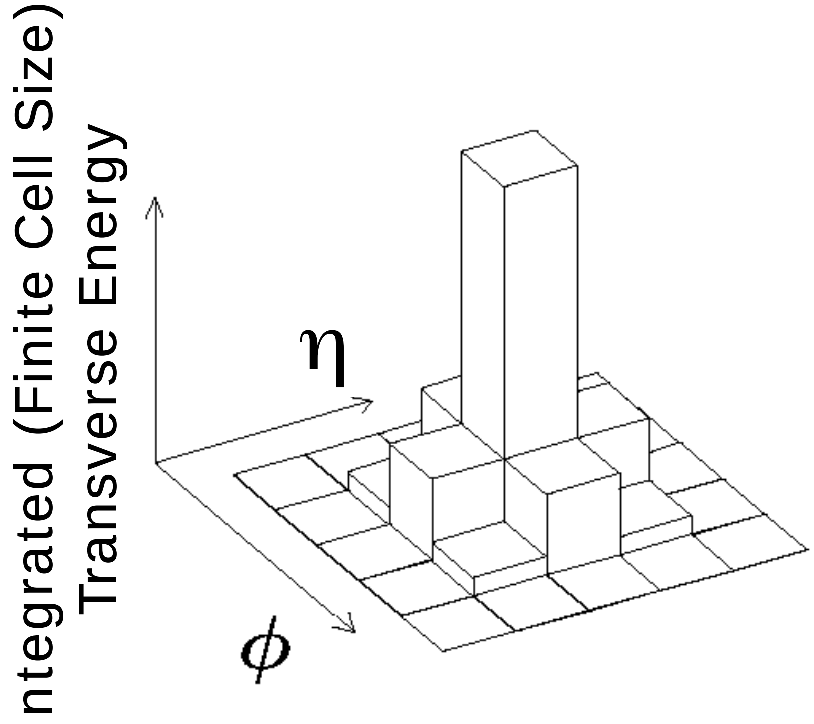
(Courtesy of R. Wigmans)

EM Transverse Profile

- The Characteristic Transverse Size of the EM Shower is given by the Moliere Radius

$\rho_M = (21\ \mathrm{MeV}/E_c)\ X_0$ (99% of shower in $3\rho_M$)





Choose Cell Granularity a bit Smaller than One Moliere Radius

Position of EM Shower in 2D

- Two common methods for computing the position of a shower within a fraction of the cell size
  - Correction to the center-of-gravity:

$$x_{cog} = \sum_i x_i E_i \Big/ \sum_i E_i$$

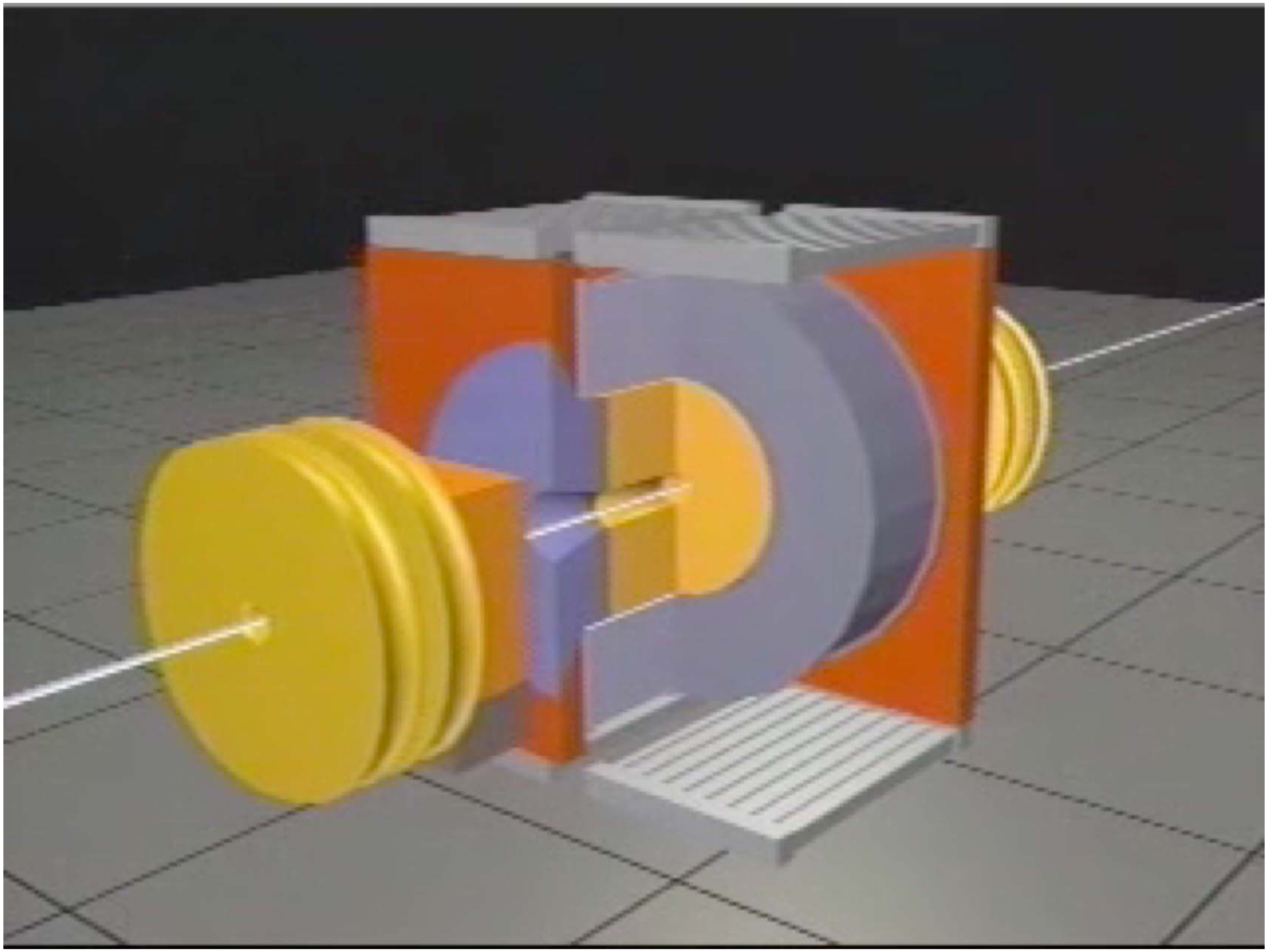
$$x_{corr} = A \tan^{-1}(B x_{cog})$$

  - Logarithmic Weighting:

$$x = \sum_i x_i w_i \Big/ \sum_i w_i \qquad w_i = w_0 + \ln\left(\frac{E_i}{\sum_i E_i}\right)$$

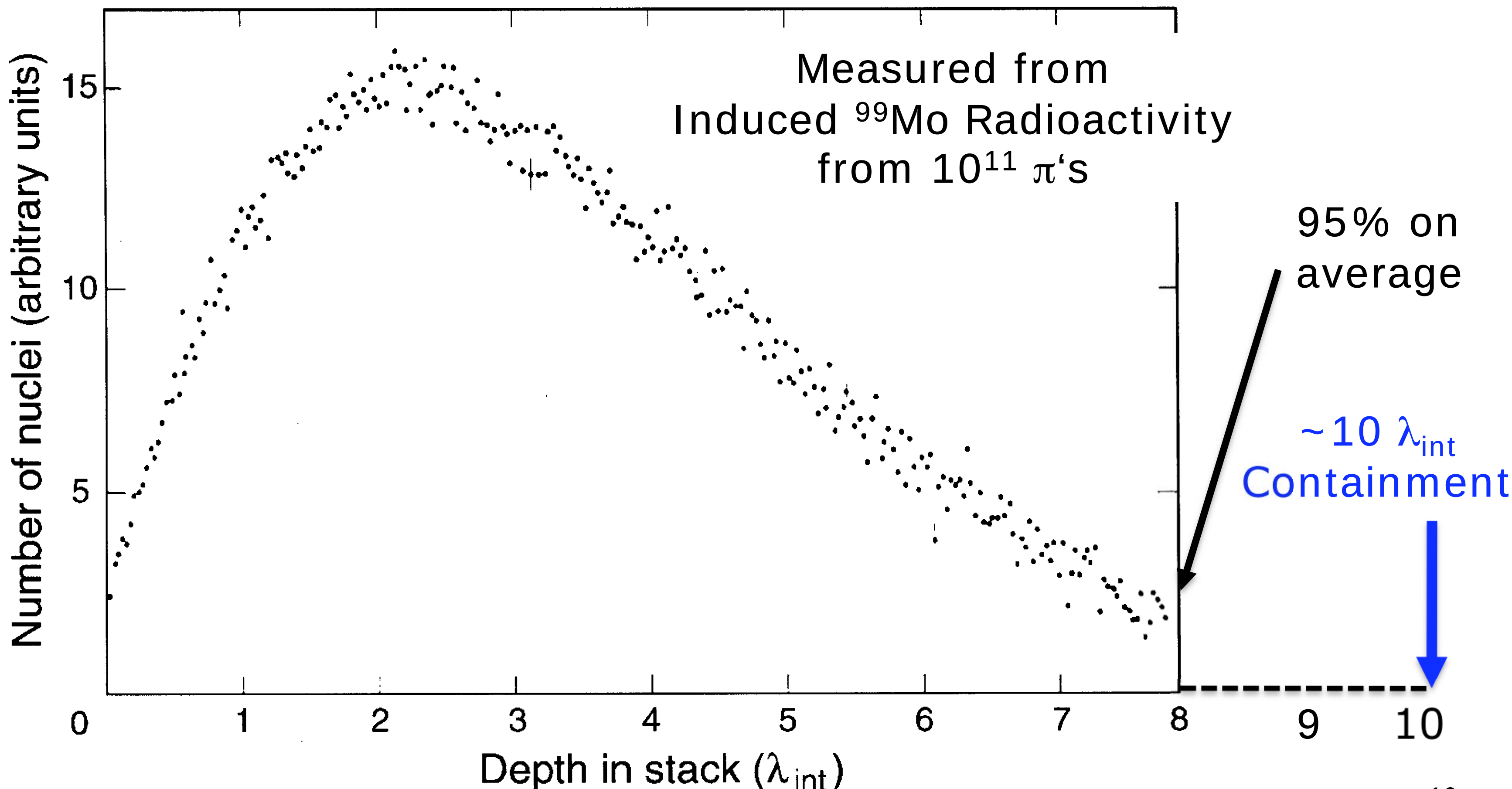
- At energies of ~45 GeV, an electron impacting on a 20mm crystal can have its impact point determined to better than 1mm in both coordinates

Shower Development "Live Action View"



Hadronic Shower Profile and Containment

- 300 GeV π^- in Uranium



(Courtesy of R. Wigmans)

Hadronic Shower Profile and Containment

- What do we notice about hadronic showers besides the order of magnitude difference between X_0 and λ_{int} ?
  - 300 GeV π^- shower looks like an equiv. ~1 GeV electron shower in longitudinal profile with shower max at 2-3 λ_{int}
  - Multiplicity of interaction $\propto$ ln s

    1 → N ~ 9 processes in the first λ_{int},

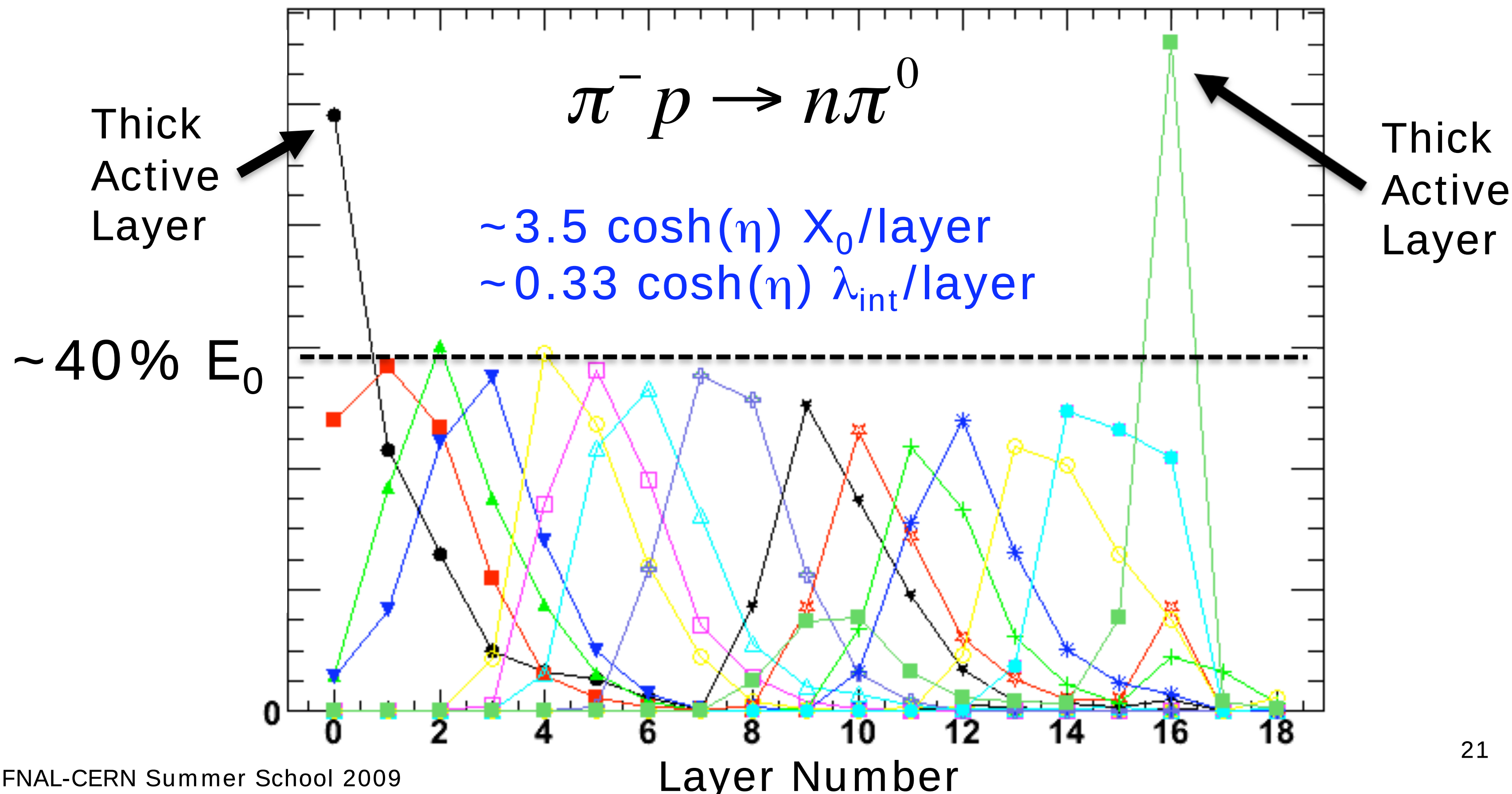
    dropping with ln s to 1 → 6 in the second step,

    1 → 3 in the 3rd step, and

    then mainly 1 → 1 processes after that (shower max)
  - Also, recall π^0's are falling out of the shower **(at that energy)** at a multiplicity fraction of 1/3 per interaction
- Another important feature of the pion is that it can remain a MIP for many λ_{int}
  - Although the probability drops off exponentially, there are many pions per event and many events to sort through

Extreme Pion Showers

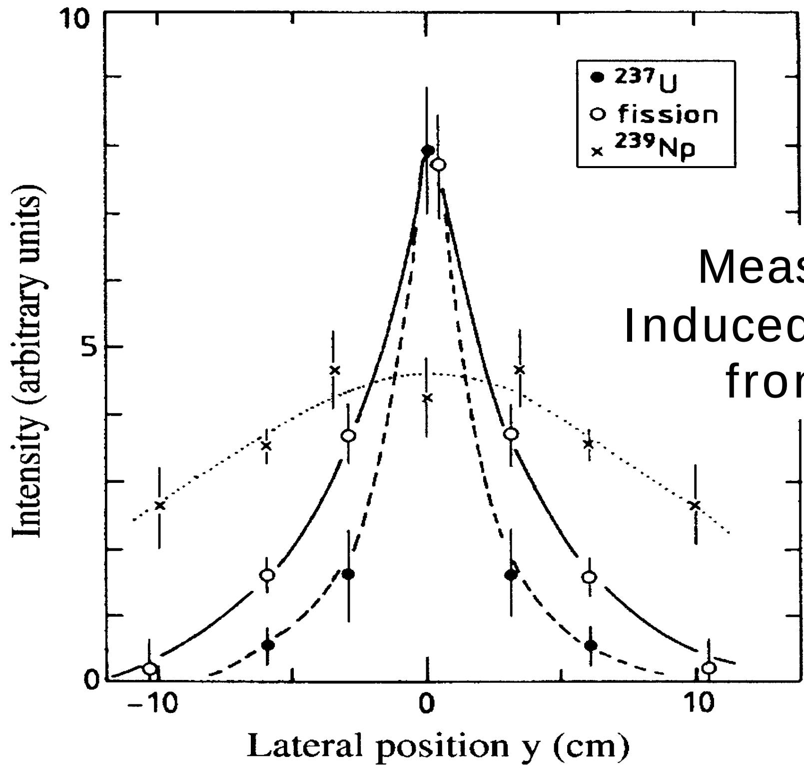
- 500 GeV π^- in Brass (protons won't do this!)
  - If you wait long enough, you'll get charged pions that MIP for the full detector and then go through charge exchange



Hadronic Shower Profile and Containment

- Transverse Profile is Shower Particle Species Dependent and Depth Dependent
  - Narrow EM core in the first few λ_{int}
  - Broad linear drop off after several λ_{int}

- More π^0's γ's in core
- Energetic neutrons and charged pions form a wider core
- Thermal neutrons generate broad tail

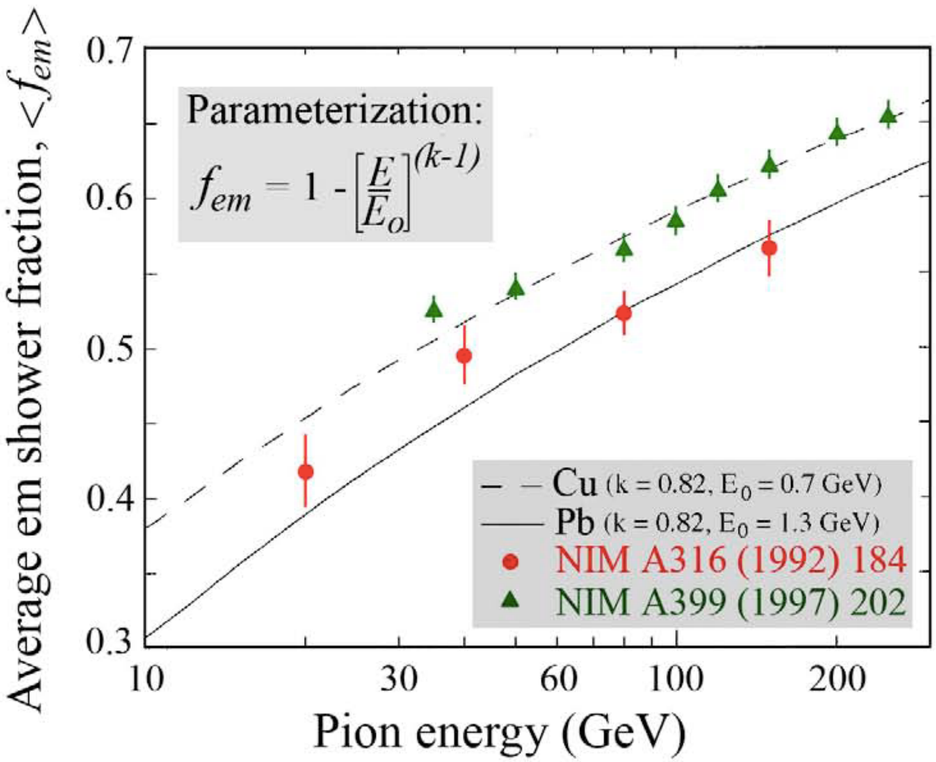


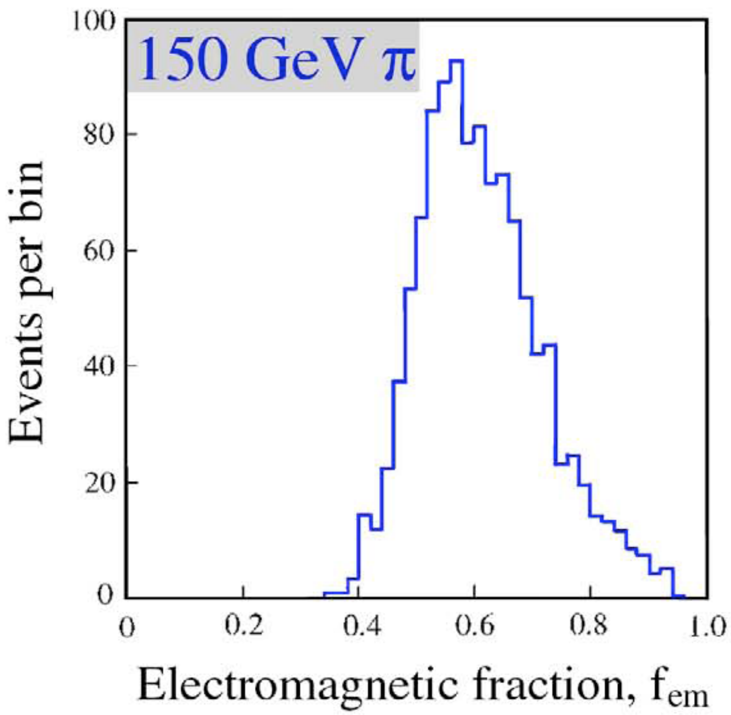
Measured from Induced Radioactivity from 10^{11} π's

(Courtesy of R. Wigmans)

EM Fraction of Charged Pion Showers

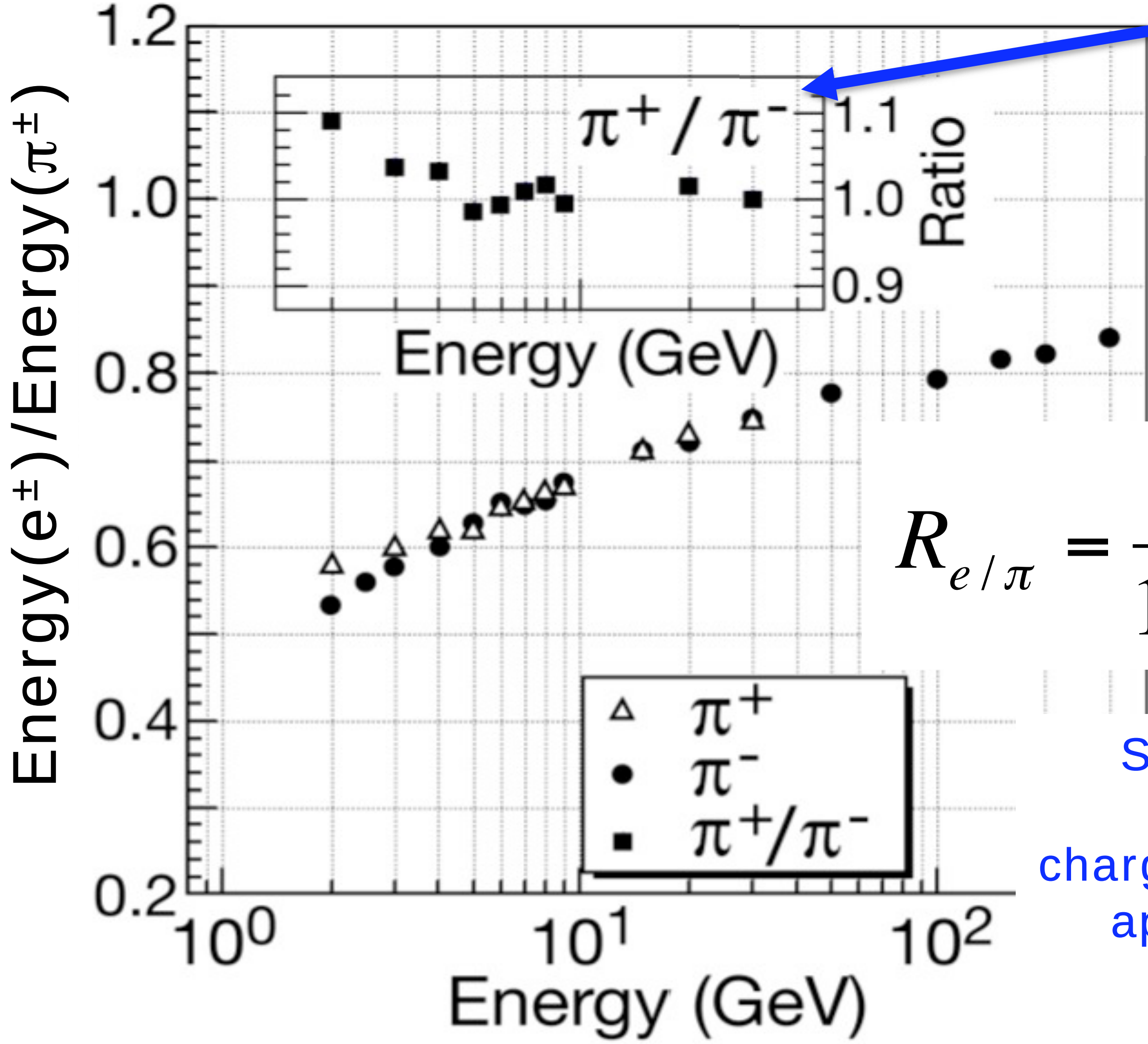
- Growth of EM Fraction with Energy
  - Non-trivial fluctuations of EM fraction





(Courtesy of R. Wigmans)

Response of Electrons/Charged Pions



Compare:

$$\pi^- p \rightarrow n\pi^0$$
$$\pi^+ n \rightarrow p\pi^0$$

More neutrons than protons in heavy nuclei

$$R_{e/\pi} = \frac{e/h}{1 - f_{em}(1 - e/h)}$$

Shower deposition of a high momentum charged pion asymptotically approaches purely EM

Compensation

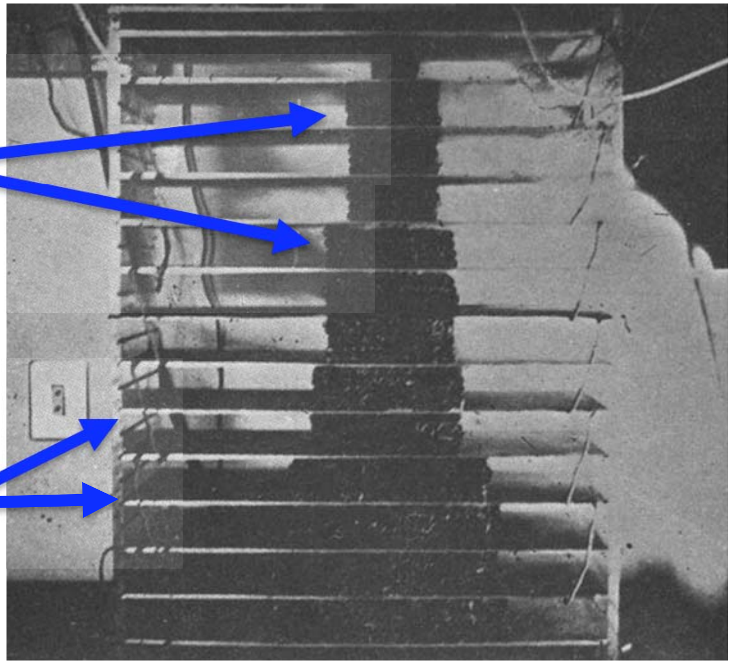
- Calorimeters can have greatly different efficiency for detecting electromagnetic as opposed to hadronic (mainly sensitive to MIP part) energy depositions
  - The ratio of the efficiencies is known as e/h
  - The "h" means the hadronic energy in a shower, not the energy from the shower of a hadron!
  - An incident charged pion will generate both electromagnetic and hadronic energy depositions in the calorimeter (EM fraction is energy dependent and fluctuates from shower-to-shower) (see previous slide)
- Compensation means to design a calorimeter that has equal efficiency for both types of deposition e/h=1
  - We don't have this luxury anymore
  - Bunch crossings are too frequent to "wait" for neutrons
  - Can the EM and hadronic energies be measured separately for each shower? (wait for 3rd lecture)

Seeing the particle showers

- The days of optical spark chambers are gone, but fondly enshrined in display cases (some still working)

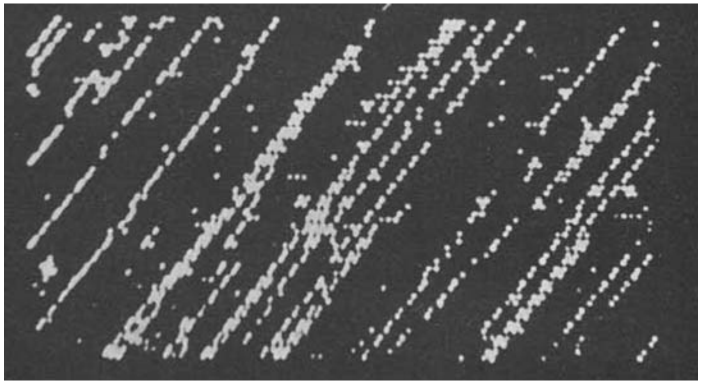
Neon Flash Tubes

Metal Plates

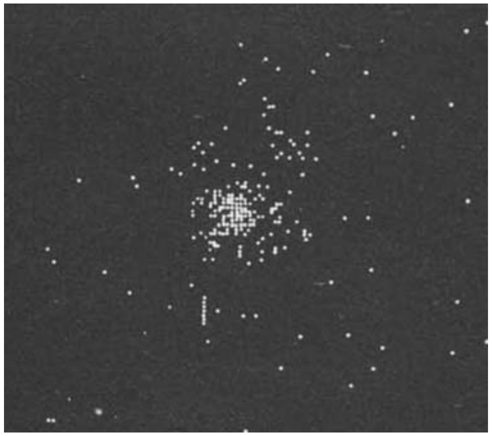


Optical Spark Chamber/Hodoscope
Partially Filled with Neon Flash Tubes
Circa. 1955

Longitudinal Profile



Transverse Profile



Types of Calorimeters

- Particle showers are measured with (modern) fast, low-power instrumentation, but the adage is still true, what you see is what you get
  - One quickly finds that there are two major movements in the calorimeter world:
    the total absorptionists and the samplers
- A **total absorption calorimeter** is a (usually homogeneous) material in which the entire volume is sensitive to energy depositions
- A **sampling calorimeter** in an interleaving of absorber/dead material and active layers used to periodically sample the particle flow in the shower

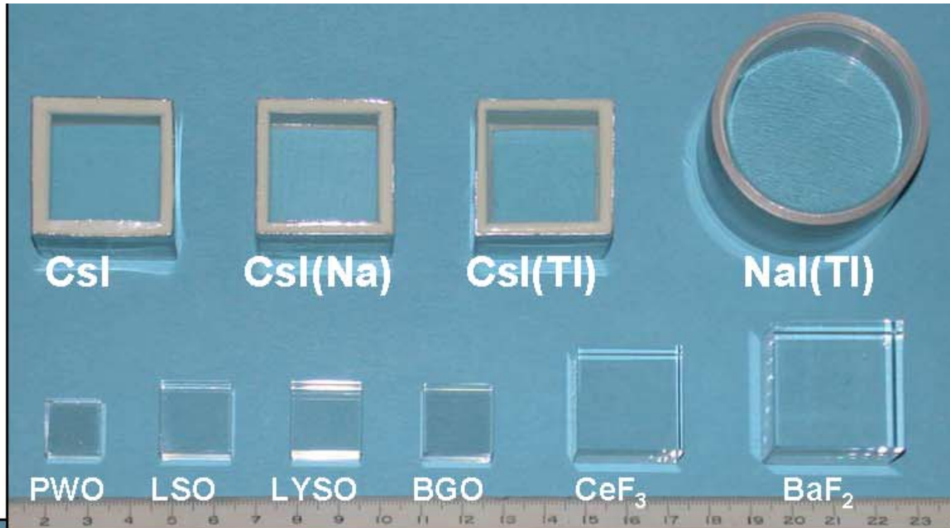
Total Absorption Calorimeters

- The resolution of an energy measurement will depend on counting statistics of the quanta released by the active material and collected by a counting device
  - In a depleted solid-state detector (Si, Ge(Li)), an electron-hole pair can be liberated with ~3.8 eV on average (band gap of Si is 1.1 eV) with most of the deposited energy going into electron-hole pair creation
  - In a scintillator, visible light with energies 2-3 eV can be emitted for a given amount of energy deposition in the crystal (not all energy goes into light)

$$E(\mathrm{eV}) = \frac{1240}{\lambda[\mathrm{nm}]}$$

  - A Cherenkov radiator, such as Lead-Glass or Quartz, will emit in the UV (~3-6 eV) for relativistic charged particles

Inorganic Scintillating Crystals



1.5 X_0 Samples:

Hygroscopic Halides

Non-hygroscopic

Full Size Crystals:

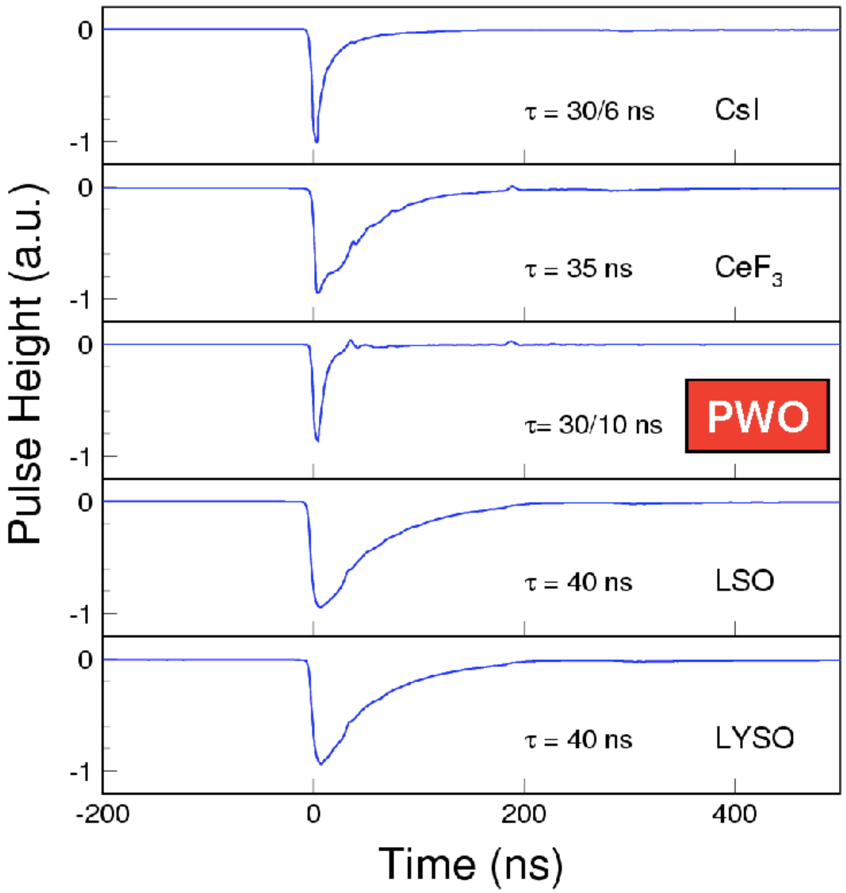
BaBar CsI(Tl): 16 X_0

L3 BGO: 22 X_0

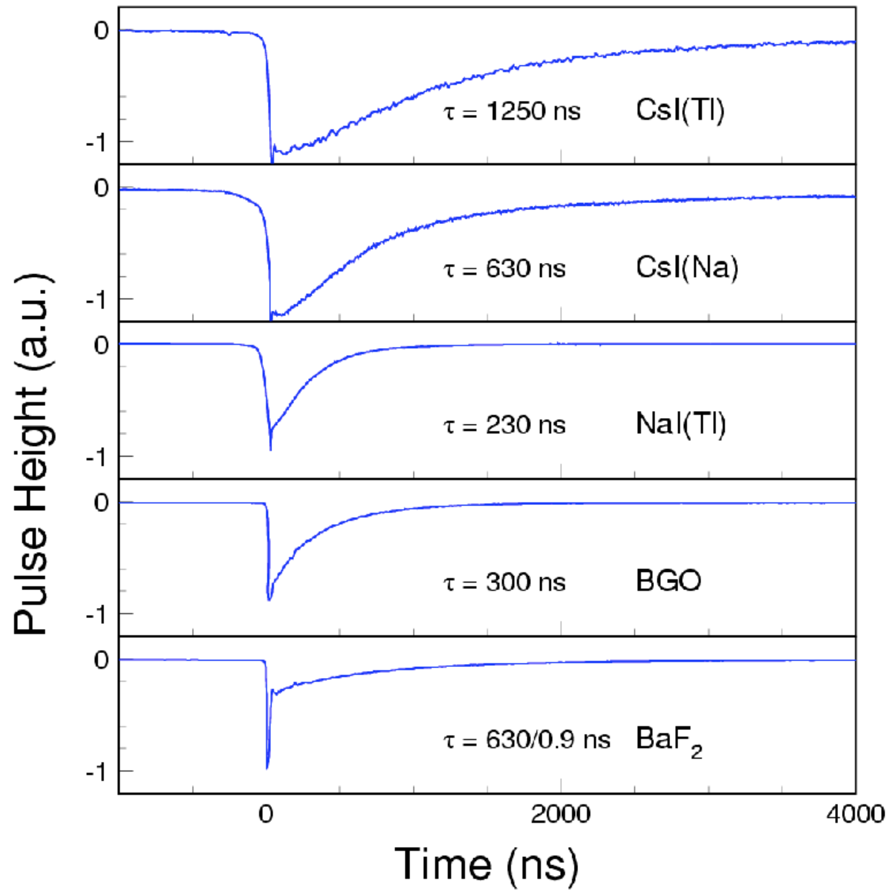
CMS PWO(Y): 26 X_0

Decay Time Constant for Crystals

Fast Scintillators



Slow Scintillators



Properties of Crystals

Crystal	NaI(Tl)	CsI(Tl)	CsI	BaF_2	BGO	PWO(Y)	LSO(Ce)	GSO(Ce)
Density (g/cm^3)	3.67	4.51	4.51	4.89	7.13	8.3	7.40	6.71
Melting Point (°C)	651	621	621	1280	1050	1123	2050	1950
Radiation Length (cm)	2.59	1.86	1.86	2.03	1.12	0.89	1.14	1.38
Molière Radius (cm)	4.13	3.57	3.57	3.10	2.23	2.00	2.07	2.23
Interaction Length (cm)	42.9	39.3	39.3	30.7	22.8	20.7	20.9	22.2
Refractive Index [a]	1.85	1.79	1.95	1.50	2.15	2.20	1.82	1.85
Hygroscopicity	Yes	Slight	Slight	No	No	No	No	No
Luminescence [b] (nm) (at peak)	410	550	420 310	300 220	480	425 420	402	440
Decay Time [b] (ns)	230	1250	30 6	630 0.9	300	30 10	40	60
Light Yield [b,c] (%)	100	165	3.6 1.1	36 3.4	21	0.29 .083	83	30
d(LY)/dT [b] (%/ °C)	-0.2	0.3	-1.3	-1.3	-0.9	-2.7	-0.2	-0.1
Experiment	Crystal Ball	CLEO BaBar BELLE BES III	KTeV	TAPS (L*) (GEM)	L3 BELLE PANDA?	CMS ALICE PrimEx PANDA?	-	-

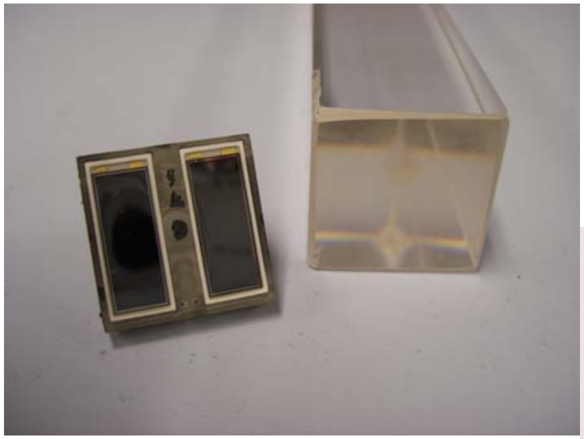
a. at peak of emission; b. up/low row: slow/fast component; c. PMT QE taken out.

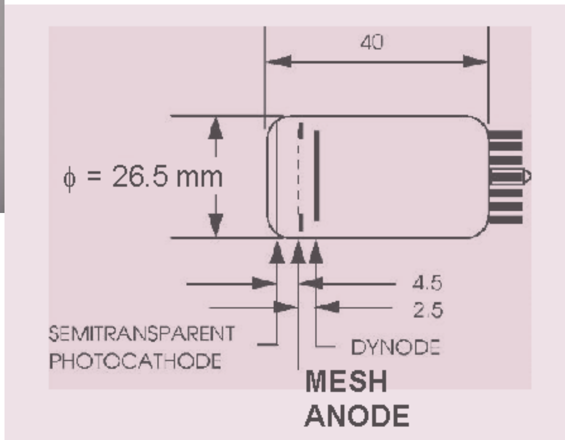
37,700 photons/MeV for NaI(Tl) (0.1 MeV of light!)

Photodetectors that Operate in Magnetic Field

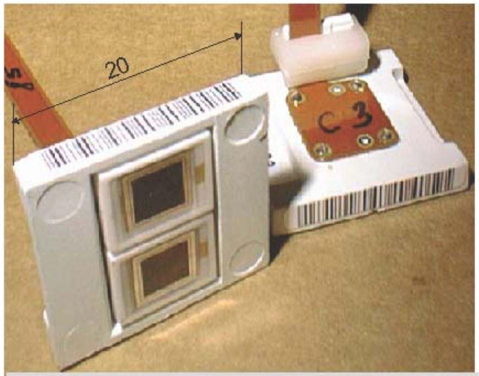
Vacuum PhotoTriode (VPT)
Gain ~x10

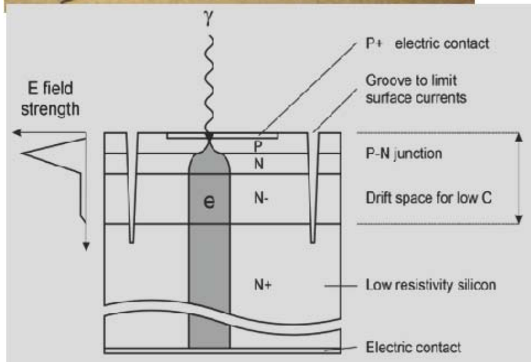
PIN Diodes
Unity Gain





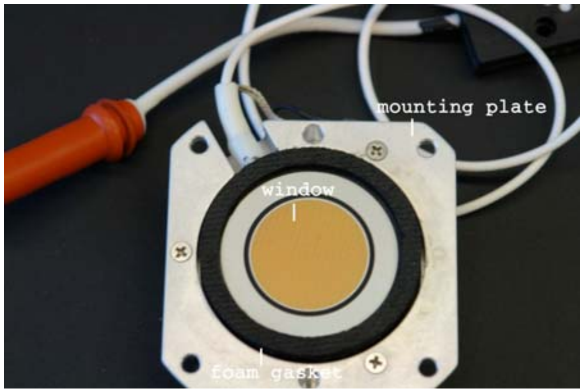
Avalanche PhotoDiode (APD)
Gain ~x50

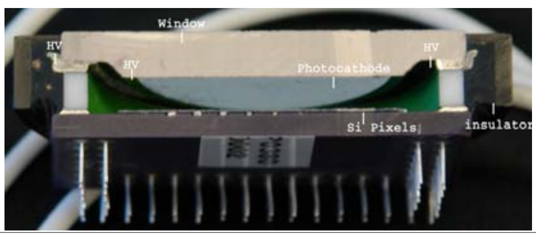


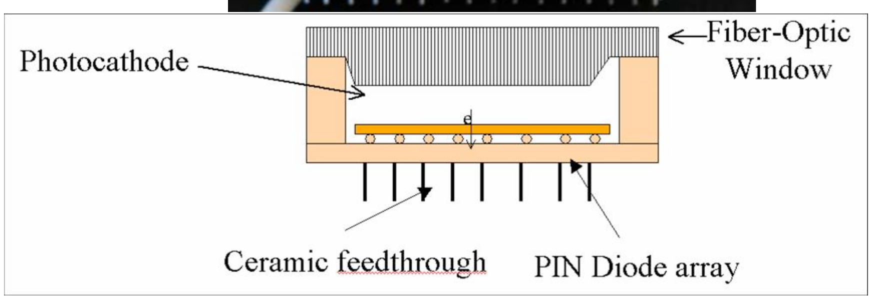


Photodetectors that Operate in Magnetic Field

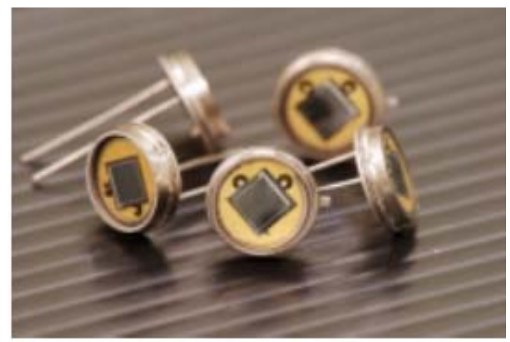
Hybrid PhotoDiode (HPD)
Gain ~x2000



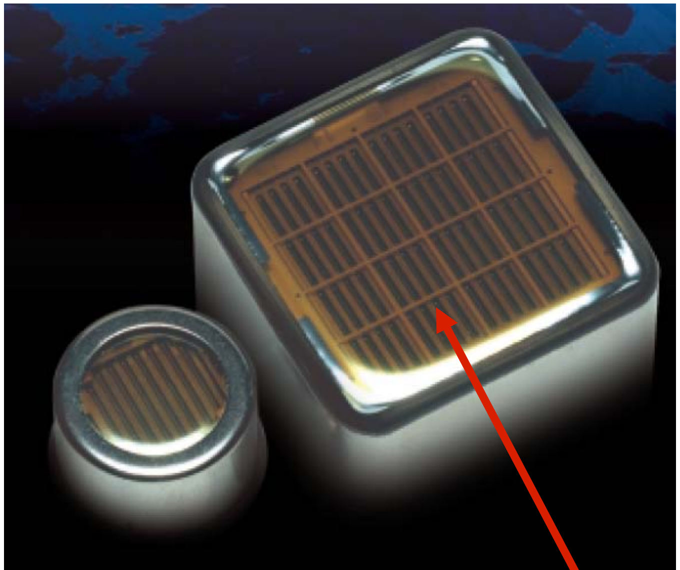




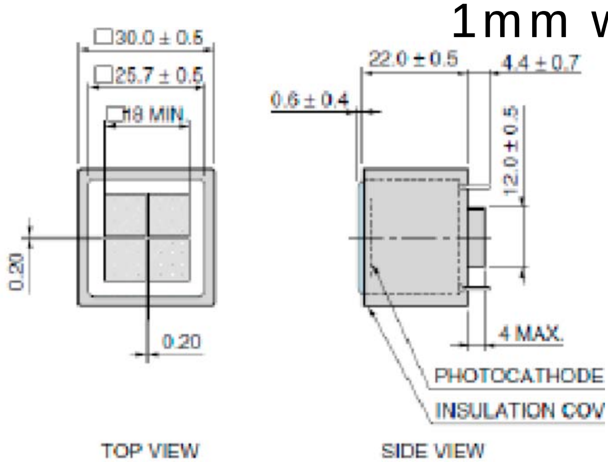
Silicon PhotoMultipliers (SiPM)
Micro-pixel Avalanche PhotoDiodes (MAPD)
Multi-Pixel Photon Counters (MPPC)
Gain ~x60,000-1,000,000



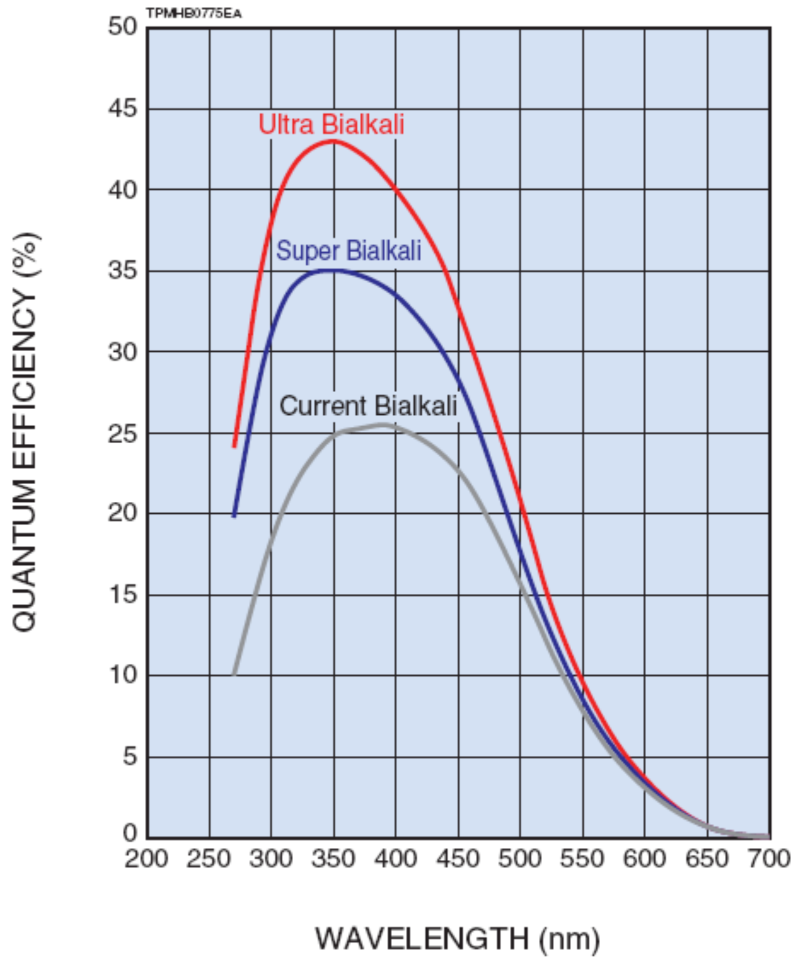
Multi-Anode High-QE PMTs



1mm window



PhotoMultiplier Tube (PMT)
Gain ~few x1,000,000

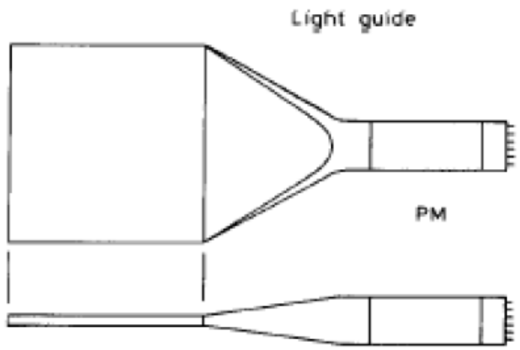


Light-Gathering Power

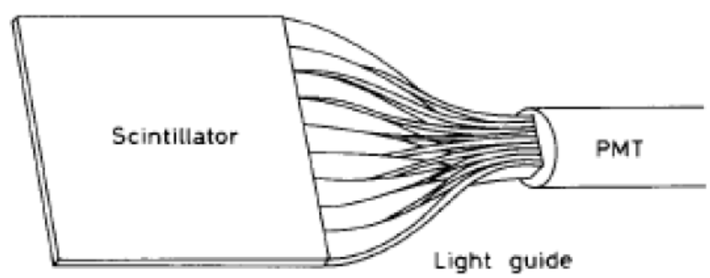
- There are limits as to how much direct light can be guided into a given photodetection area
  - Optical Invariant/Lagrange Invariant limits the Geometrical/Optical Extent of the light collection aperature

$$A_S\Omega_S = A_I\Omega_I$$

  - The art of maximizing the optical extent between the source (scintillator) and image (photomultiplier) is known as the light guide:



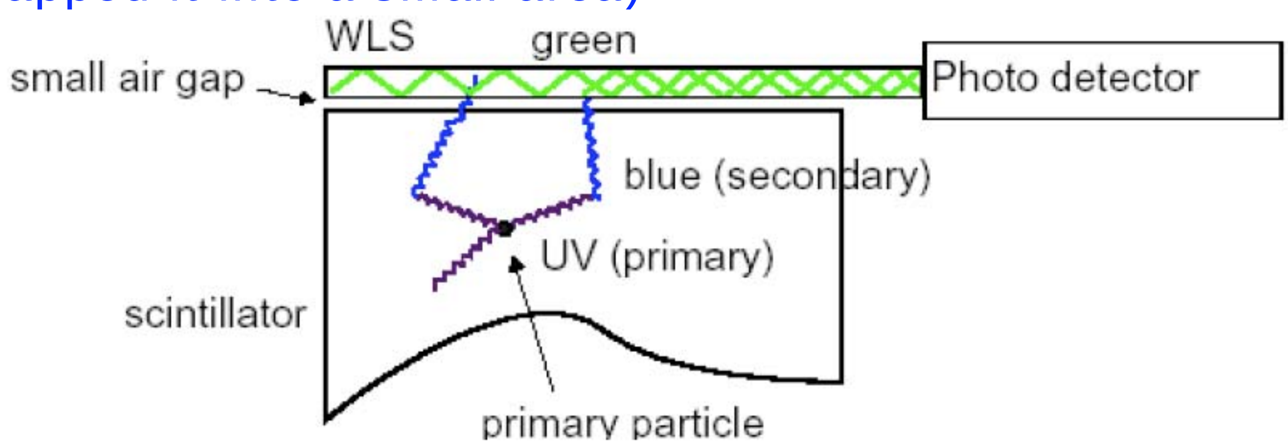
"fish tail"



adiabatic

Light-Gathering Power

- Not everyone plays by the rules:
  - If there is plenty of light, but difficulty in collecting the light uniformly over the volume of the scintillator, a new device can be optically coupled into the scintillator called a wavelength shifter
  - If the scintillator light is absorbed (the Lagrange invariant collapses) and re-emitted into a new optically constrained geometry (a fiber with internal reflection), then the effective photodetector area can be dramatically reduced (there is no extra light, but we have uniformly sampled the whole volume and mapped it into a small area)



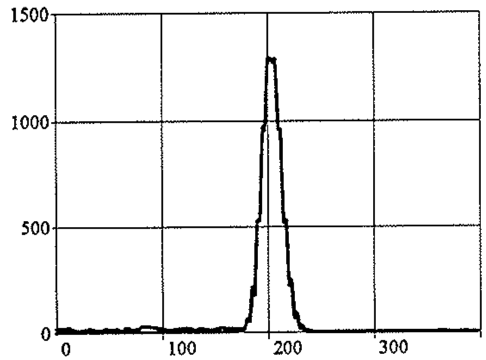
Light Emission Effects

- Birk's Law
  - At very high ionization densities, the light yield from a crystal is diminished from local saturation of excited levels. This means that heavily ionizing nuclei and slow protons will generate less light per MeV of deposited energy $\frac{dL}{dx} = L_0 \frac{dE/dx}{1 + k_B dE/dx}$ with dE/dx [MeV cm^2/g]
- Cherenkov light fraction
  - Ultra-relativistic charged particles (electrons and positrons) will radiate a spectrum of photons peaked in the UV known as Cherenkov radiation
  - For very low light yield crystals (~100 photons/MeV), the contribution of Cherenkov light to the total light collected can be of order a few percent or more
  - Therefore, EM showers will have a slightly higher relative light yield than MIP depositions

Measuring Birk's Coefficient

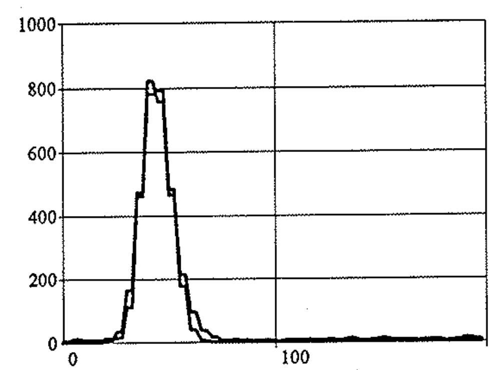
- Measurement at PSI
  - Calibrate light yield with stopping protons

405 MeV/c *p*



$\mu = 202.4$
$\sigma/\mu = 4.6\%$
(no uniformity correction)

405 MeV/c π



$\mu = 45.2$
$\sigma/\mu = 16.4\ \%$
(no uniformity correction)

$$\frac{\mathbf{dE}}{\mathbf{dx}} = \frac{45.2}{202.4} \times \frac{80\ \mathrm{MeV}}{2\ \mathrm{cm}} = 8.9\ \mathrm{MeV/cm}$$

$$k_B \approx 0.046$$

However, one finds dE/dx = 10.2 MeV/cm
when calibrated with electrons

Energy Resolution

- The energy resolution for a calorimeter can be written in the general form:

$$\frac{\sigma_E}{E} = \frac{N}{E} \oplus \frac{S}{\sqrt{E}} \oplus C$$

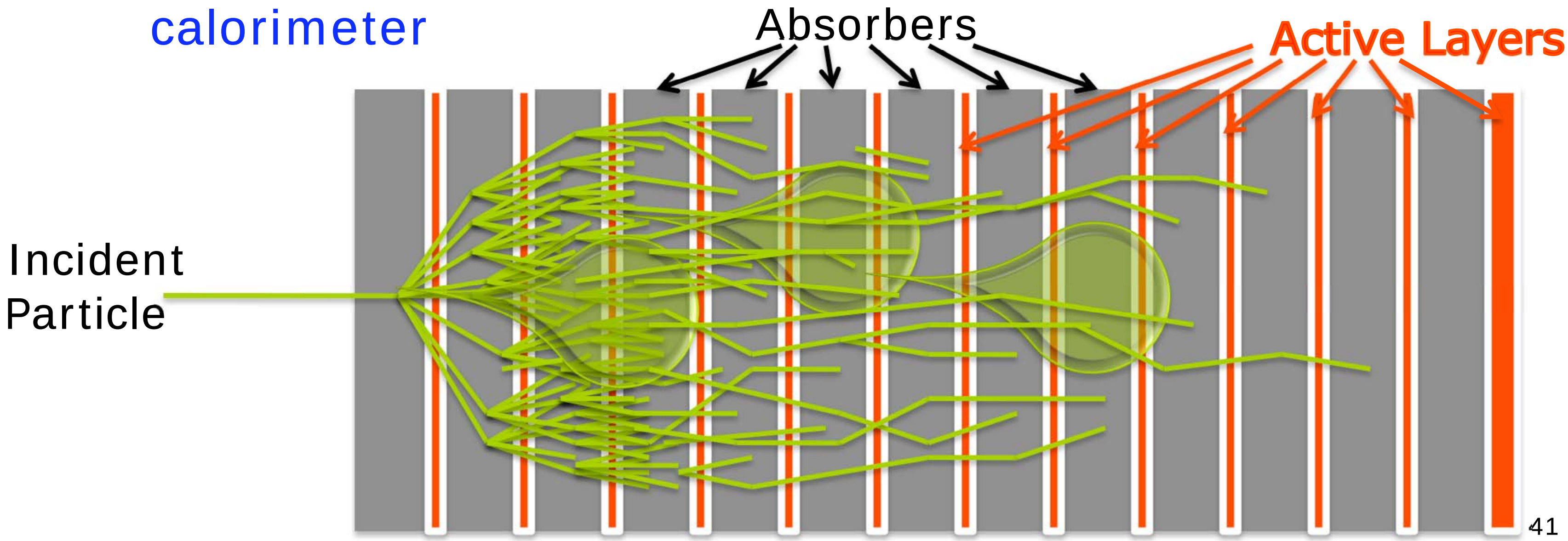
  - "N" is the electronic noise term and is set by the square root of the number of detector cells being summed (for incoherent noise) and the RMS electronic noise per channel **in units of energy**, i.e. ~6000e^- of preamp electronic noise is to be compared with the number of photoelectrons times the photodetector gain collected per MeV of deposited energy
  - "S" is the stochastic term and is a form of $1/\sqrt{N}$ counting statistics (signal quanta counting within a fixed volume)
  - "C" is the constant term and comes from intrinsic non-uniformities in how one computes the incident particle energy, i.e. variations in the mean response that depend on parameters that are not tracked (temperature, etc.)

Total Absorption Calorimeter

- For a total absorption calorimeter, the stochastic term of the energy resolution is largely due to the counting statistics of light collection, but it's a little more involved:
  - Take $PbW0_4$ with ~100,000 photons/GeV, gives 0.3%
  - Apply the photodetection area and quantum efficiency of photodetector, now one has 4000 photoelectrons/GeV and 1.6%
  - Include the excess noise factor F=2 for amplification shot noise from the APD, 1.6%*√2=2.2%
  - Now limit the lateral sum to keep the number of channels contributing to the electronic noise to 25, the containment fluctuations add 1.5% in quadrature, giving a total stochastic term of S=2.7%

Sampling Calorimeters

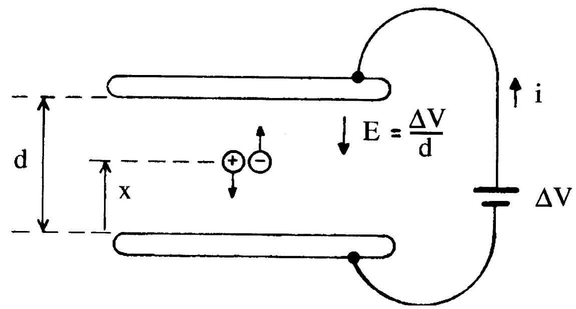
- There are no known total absorption detectors that can stop hadronic showers ($10\lambda_{int}$) in a finite thickness (~few meters or less) and for finite cost
  - There are materials (Fe, Cu, Pb, W, ^{238}U) than can do this in 1-2 meters, but they are passive materials
  - One can therefore consider interleaving active and passive (absorber) materials to form a "sampling" calorimeter

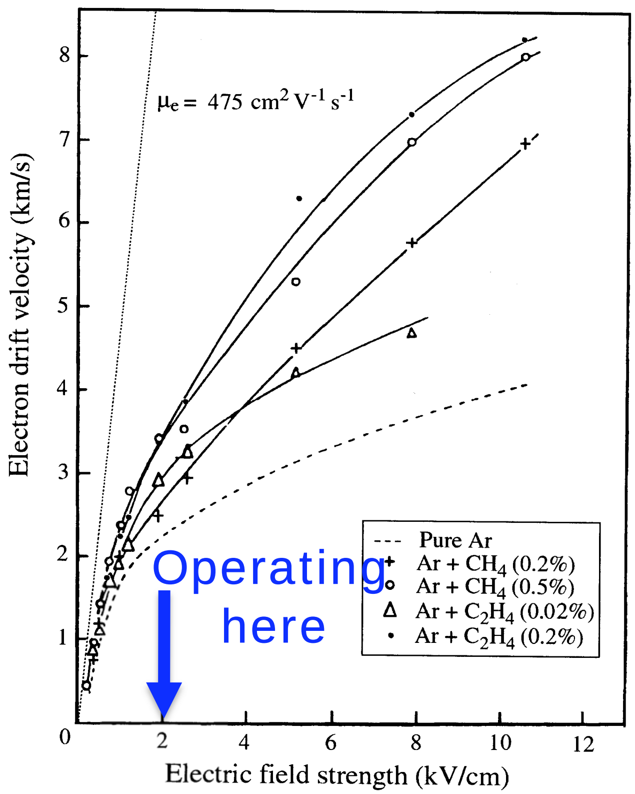


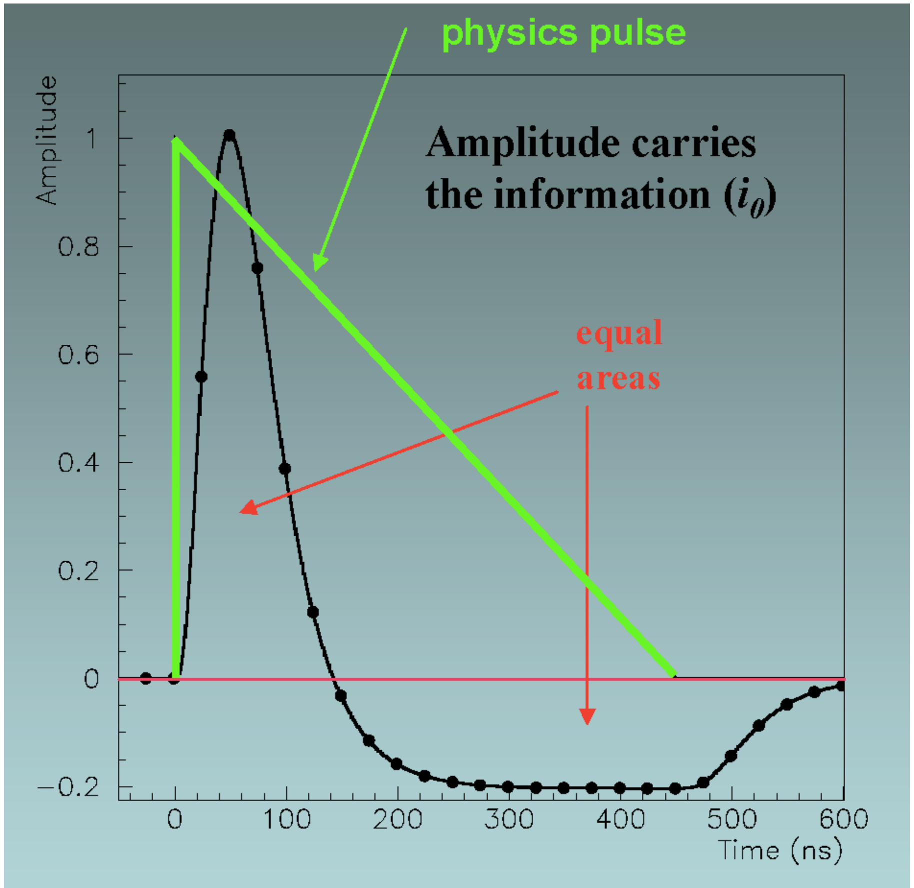
Ionization

- There are examples of sampling calorimeters with gas active layers (such as the L3 Experiment uranium/gas hadron calorimeter)
  - Center-wire anode proportional tubes, however, are slow as they require the ions (not the electrons) to climb out of the potential to generate the electrical signal
- A cryogenic noble liquid with a planar gap geometry can generate an electric signal from the collection of electrons (and not the slower ions)
  - The higher density of the liquid provides enough ionization that no "gas" amplification is needed (not possible anyhow due to the lower mean free path in the liquid)
  - However, the electron drift is still not that fast (hundreds of nanoseconds). Also, the drift velocity saturates at very high E field, so there are intrinsic limits to the charge collection time

Ionization







This isn't an accident – it's the genius that allows LAr to be used at the LHC

(Ref. V. Radeka)

Light-Emitting Active Layers

- Plastic Scintillator is a common active layer
  - Light signals can be brought out for photodetector readout using wavelength shifting (WLS) fibers
  - Absorbers can be oriented “orthogonal” or “parallel” or “zig-zag” to the beam axis (we will see explicit examples of several orientation options in LHC detectors)
- Longitudinal densely packed fibers (scintillating or cherenkov radiators) can also be used as the active elements of the sampling calorimeter (we will see examples of these in lecture 2 as well as mix and match combos of fibers and layers)

Sampling Calorimeters

- There are two leading parameters in the performance of a sampling calorimeter:
  - *Sampling fraction per layer* – is the fraction of the MIP energy deposition in an active layer compared to sum of the corresponding MIP energy deposition in the absorber preceding the active layer and the active layer

$$f_{samp} = \int_{active} dE/dx_{MIP}\,dx \Big/ \int_{absorber+active} dE/dx_{MIP}\,dx$$

  - *Sampling frequency per unit length* – for a fixed sampling fraction, the number density of active layers is inversely proportional to a linear dimension of the active layer (thickness of plate, diameter of fiber, etc.)

$$\nu_{samp} \propto 1/d_{active}$$

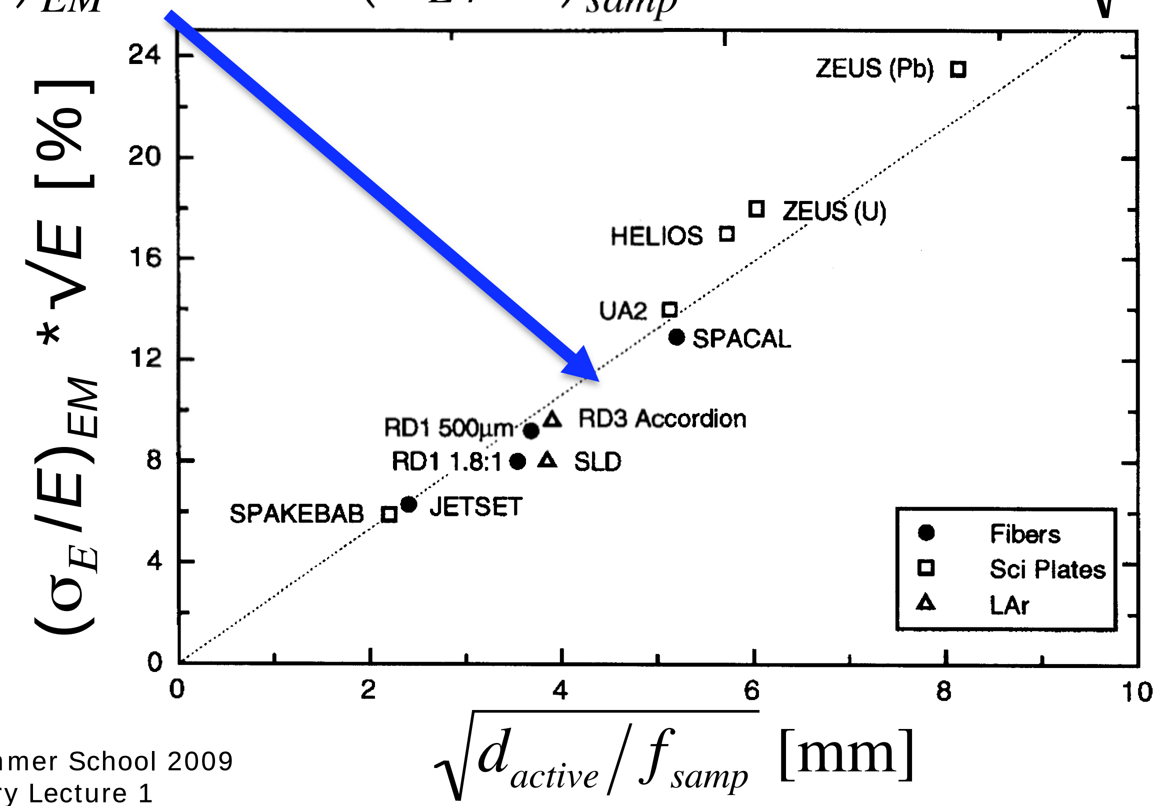
- The stochastic part of the energy resolution of a sampling fluctuations can be parameterized by:

$$\left(\sigma_E/E\right)_{samp} \propto \sqrt{d_{active}/f_{samp}} \Big/ \sqrt{E}$$

Sampling Calorimeters

- For EM showers in a sampling calorimeter, the energy resolution is dominated by the sampling fluctuations:

$$(\sigma_E/E)_{EM} \cdot \sqrt{E} \approx (\sigma_E/E)_{samp} \cdot \sqrt{E} = 2.7\% \sqrt{d_{active}/f_{samp}}$$



Sampling Calorimeter

- The relative size of the sampling fluctuation energy resolution term for hadronic showers can be estimated for similar calorimeter geometries from the relative thickness of the absorber layers in units of λ_{int}
  - For example, a calorimeter with a 45%/√E stochastic term from hadronic shower sampling fluctuations would compare with a 80%/√E resolution calorimeter with 3 times the absorber thickness t_{had} in units of λ_{int}

$$\sigma_E^{had,samp}(t_{had}=0.33)/\sigma_E^{had,samp}(t_{had}=0.11) \approx \sqrt{t_{had}^{rel}} = \sqrt{3}$$

	X_0 (cm)	λ_{int}(cm)
Pb	0.56	17.0
$PbWO_4$	0.89	18.0
Fe	1.76	16.8
Cu	1.43	15.1

	t_{em}	t_{had}
ATLAS, Tilecal (Fe)	1.0	0.11
CMS HCAL (Cu)	3.5	0.33

Sampling Calorimeters

- However, sampling fluctuations are not the whole story, and several other terms contribute to the energy resolution. Other contributions include:
  - Visible Energy Fluctuations (Nuclear Binding Energy and Escaping Neutrons)
  - Electromagnetic Fraction Fluctuations (for non-compensating calorimeters)
  - Containment (Longitudinal and Lateral) Fluctuations
  - Electronic Noise (at low Energy)
  - Counting Statistics of Signal Quanta (for very low yield active layers)
  - Excess Noise Factor in the Gain Mechanism of the Signal Quanta Detector
- Tevatron/LHC calorimeter examples will be covered in Lecture 2