F-theory at order α'^3

Raffaele Savelli
IPhT - CEA/Saclay

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Based on work with:
R. Minasian and T. Pugh
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Context

- Topic is quantum corrections (in g_s and α'). Need control over them to understand vacuum structure of string theory & its EFT.

- Here I focus on the Type IIB corner of the string landscape:

  - $\exists$ efficient probe into the strong g_s regime $\rightarrow$ F-theory; [Vafa `96]

  - All distinctive features of GUT models can be accommodated; [Donagi, Wijnholt; Beasley, Heckman, Vafa `08]

  - One has the Large Volume Scenario as promising paradigm of moduli stabilization. [Balasubramanian, Berglund, Conlon, Quevedo `05]

- Final target is the 4D, N=1 EFT of Type IIB on CY_3 with D7/O7:

  - Knowledge of quantum corrections is still very limited, especially in the effective Kähler potential.

  - They play a key role in the phenomenology of these vacua.

Goal

- Most notable example is a certain $\mathcal{O}(\alpha'^3)$ correction to the CY_3 volume:

$$K = -2\log\left(\mathcal{V}_3 - \frac{\zeta(3)\,\chi(\mathrm{CY}_3)}{32\pi^3\, g_s^{3/2}}\right)$$

  - Due to closed strings $\Rightarrow$ N=2 sector; [Antoniadis, Ferrara, Minasian, Narain `97]
  - Shown to survive orientifolding N=2 → N=1. [Becker, Becker, Haack, Louis `02]

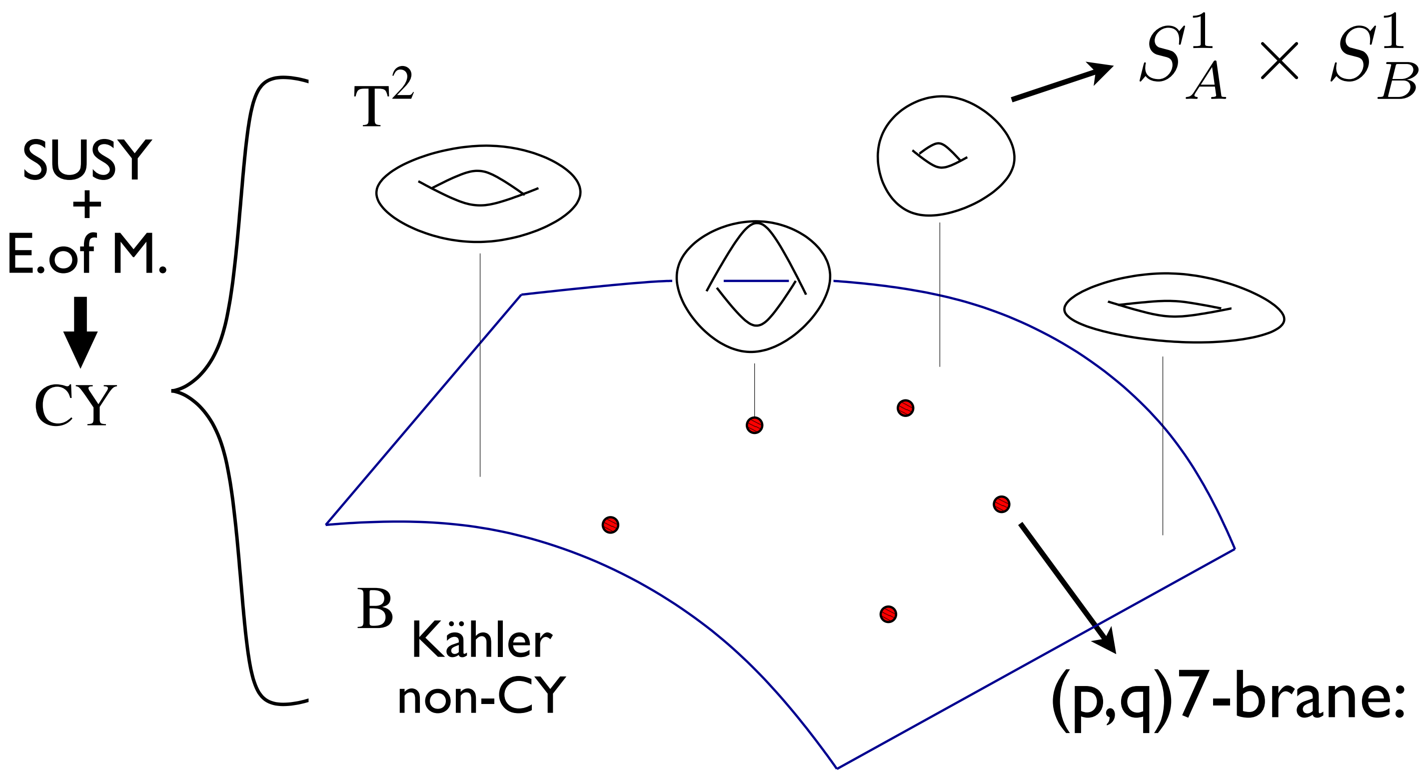
- Main aim: Study possible, genuinely N=1 modifications of CY_3 volume.

- Using F-theory is convenient, as:
  - It "geometrizes" all g_s effects of Type IIB, and it only needs to be corrected in α';
  - It includes open string effects, through 7-brane backreaction.

F-theory

12D theory: Auxiliary T^2 fibered over the 10D string space

T^2 cplx-str = axio-dilaton $\tau = C_0 + ie^{-\phi}$ varies holomorphically with $SL(2,\mathbb{Z})$ transitions



(p,q)7-brane: divisor on which $pS^1_A + qS^1_B$ collapses

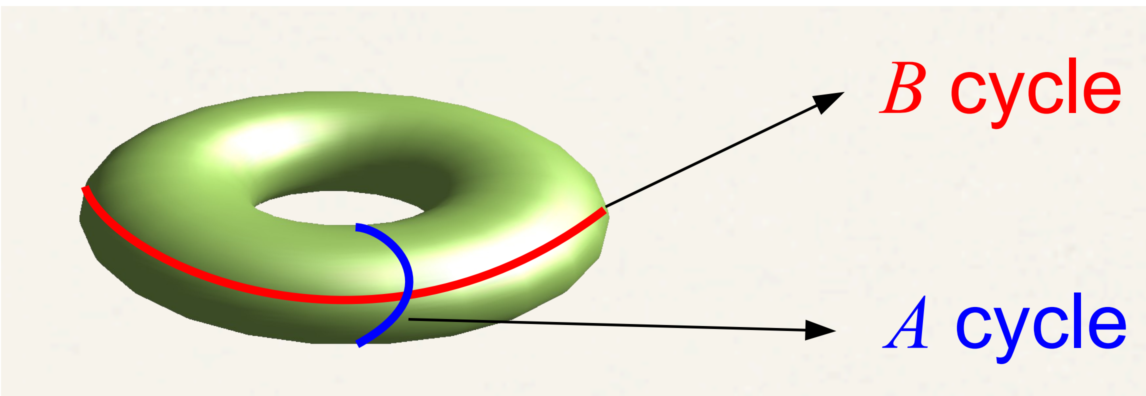
Torus size is not physical!
Bookkeeping device of
7-brane backreaction

∄ fundamental description (yet): ∄ (1,11) sugra ⇒ beyond metric spaces (?)

So far, F-theory is accessed only through limits and dualities ...

F/M-theory duality

- Start with M-theory on a T^2 :



- Reduce M to IIA along A.
- T-dualize IIA to IIB along B.
- Take the $v \equiv \mathrm{Vol}(T^2) \to 0$ limit.

➡ Type IIB string theory with varying axio-dilaton.

- IIB metric in Einstein frame and 10D Poincaré: $r \sim v^{-3/4}$.
- M-theory scale is <u>small</u> compared to string length: $l_M / l_s \sim v^{1/4}$.

➡ F-theory α' corrections ⇔ 11D sugra l_M corrections surviving $v \to 0$.

Two derivatives

- Let's warm-up with $O\,(\alpha'^0)$.

  - Take M-theory $O\,(l_M{}^0)$ action and reduce it (to 0-modes) on T^2 :

$$S_0^{(11)} \sim \frac{1}{l_M^9} \int R_{\rm sc}^{(11)} *_{11} 1 \quad \xrightarrow[\text{red.}]{T^2} \quad S_0^{(9)} \sim \frac{\nu\, l_M^2}{l_M^9} \int \left(R_{\rm sc}^{(9)} - 2\,P\cdot\bar{P}\right) *_9 1$$

$$\xrightarrow{\nu\to 0} \quad S_0^{(10)} \sim \frac{1}{l_s^8} \int \left(R_{\rm sc}^{(10)} - 2\,P\cdot\bar{P}\right) *_{10} 1$$

  where we used the T^2 metric $\frac{\nu}{\mathrm{Im}\,\tau}\begin{pmatrix} 1 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & |\tau|^2 \end{pmatrix}$

  and P is the $U(1)_R$ -covariant quantity $P = \frac{i}{2\,\mathrm{Im}\,\tau}\nabla\tau$ of charge 2.

  - We reach same result by U(1)$_R$-invariance ⇔ "12D diff-invariance" ,

  i.e. reduce on T^2 $S_0^{(12)} \sim \frac{1}{l_s^8}\int R_{\rm sc}^{(12)} *_{10} 1$ → integral on 10D slice.

  Warning : No 12D lift of the 10D measure !

Eight derivatives

- At $\mathcal{O}(\alpha'^3)$ we have non-trivial dynamics.

  - 0-mode reduction of $\mathcal{O}(l_M^6)$ action is now killed by $\nu \to 0$:

$$S_3^{(9)} \sim \frac{1}{r} \longrightarrow 0$$

  - We must take into account KK modes on T^2 ! How?

- Make them run in the loop of scattering amplitude of 4 9D gravitons:
  [Green, Gutperle, Vanhove `97]

  - The loop generates an extra factor of $f_0(\tau,\bar{\tau}) \cdot \nu^{-3/2}$, so that

$$S_3^{(11)} \sim \frac{1}{l_M^3} \int t_8 t_8 (R^{(11)})^4 *_{11} 1 \quad \xrightarrow[\text{loop}]{\text{KK}} \quad S_3^{(10)} \sim \frac{1}{l_s^2} \int f_0(\tau,\bar{\tau})\, t_8 t_8 (R^{(10)})^4 *_{10} 1$$

  - f_0 holds all g_s corrections compatible with SUSY. For $(\mathrm{Im}\,\tau)^{-1} = g_s << 1$

$$f_0(\tau,\bar{\tau}) \approx \underbrace{\frac{2\zeta(3)}{g_s^{3/2}}}_{\text{tree-level}} + \underbrace{\frac{2\pi}{3} g_s^{1/2}}_{1-\text{loop}} + \underbrace{\mathcal{O}(e^{-1/g_s})}_{\mathrm{D}(-1)}$$

From 12D to 10D

- A richer kinematics is missing here: Can get it ALL from the 12D trick!

  0-mode reduction on T^2 of $S_3^{(12)} \sim \frac{1}{l_s^2} \int t_8 t_8 (R^{(12)})^4 *_{10} 1$.

- The full 10D action is a complicated sum of $U(1)_R$-invariant couplings, written in terms of $R^{(10)}$, P and DP ($D \to U(1)_R$-covariant derivative).

- Check ? Possible at 4pt level. Result known from tree-level string amplitudes: [Policastro, Tsimpis `06 -`08]

$$S_3^{(10)}|_{4\mathrm{pt}} \sim \frac{1}{l_s^2} \int \left\{ t_8 t_8 \left[(R^{(10)})^4 + 24\,(R^{(10)})^2\,|DP|^2 \right] + \widehat{\mathcal{O}}_1 \left[(|DP|^2)^2 \right] \right\} *_{10} 1$$

✓ perfect match!

- Things work also for the parity odd/odd sector:

  0-mode reduction on T^2 of $S_3^{(12)} \sim \frac{1}{l_s^2} \int \epsilon_{12} \epsilon_{12} (R^{(12)})^4 *_{10} 1$.

- Restricting to 4pt $\Rightarrow$ $S_3^{(10)}|_{4\mathrm{pt}} = 0$ ✓ as expected!

Beyond tree-level

- Price we pay: Miss τ - dynamics and hence all g_s corrections.

  - Use SUSY + SL(2)-invariance to restore them.

  - At perturbative level: Just 1-loop for all terms → $2\pi/3$.

  - "Cusp forms" are ruled out: D(-1) corrections are universally given by f_0 . [Pioline `98]

- Our final result :

$$S_3^{(12)} = \frac{1}{(2\pi)^7 \cdot 3 \cdot 2^{11} \cdot l_s^2} \int f_0(\tau,\bar\tau)\, [t_8 t_8 + \tfrac{1}{96}\epsilon_{12}\epsilon_{12}]\, (R^{(12)})^4 *_{10} 1$$

It reproduces the g_s-exact Type IIB action at $O(\alpha'^3)$ with no flux.

It contains couplings of R & τ at all-orders $\Rightarrow$ prediction beyond 4pt.

4D N=1 compactifications

- Reduce F-theory on smooth CY_4, elliptically fibered over B_3, with zero-section: $B_3 \to CY_4$.

  - Two-derivative action yields classical volume of base:

$$S_0^{(4)} = \frac{1}{2\pi\alpha'} \int \mathcal{V}_b \, R_{\rm sc}^{(4)} \; *_4 1$$

  - Eight-derivative action leads to the correction:

$$\boxed{S_{0+3}^{(4)} = \frac{1}{2\pi\alpha'} \int \left(\mathcal{V}_b - \frac{1}{64\pi^3} \int_{\rm B_3} f_0(\tau,\bar{\tau}) \, c_3({\rm CY}_4)|_{\rm B_3} \right) R_{\rm sc}^{(4)} \; *_4 1}$$

  - For constant τ $\Rightarrow$ old N=2 correction ($B_3 = CY_3$);
  - N=1 vacua: Correction is non-topological (τ varies over B_3);
  - Weyl rescaling induces correction to 4D, N=1 Kähler potential.

Weak coupling

- Correction simplifies when going to weak string coupling.

  - Sen (`97): Restrict CY_4 -complex structure s.t. $g_s \to 0$ is well-defined.

➡ Type IIB on $CY_3 \to B_3$ branched double cover.

➡ O7-plane wrapping branch locus: In cohomology $D_{O7} = c_1(B_3)$.

$$\tilde{\mathcal{V}}_3 = \mathcal{V}_3 - \frac{\zeta(3)}{32\pi^3 g_s^{3/2}} \left(\chi(\mathrm{CY}_3) + 2 \int_{\mathrm{CY}_3} D_{\mathrm{O7}}^3 \right) + \mathcal{O}(g_s^{-1/2})$$

- Topological correction at closed-string tree-level.
- Next-to-leading: tree-level of open + unorientable strings.
- Arises from graviton two-point function in orientifold backgrounds.
- Absent in toroidal models!

Caveats / Open questions

- Our 12D $\mathcal{O}(\alpha'^3)$ action is only checked at 4pt!
  - Test it beyond 4pt : 5pt amplitude computation or M/F duality.
- Employ 12D logic to study the F_3 / H_3 -flux sector in 10D at $\mathcal{O}(\alpha'^3)$.
- Correction to Weyl rescaling does not fully determine Kähler potential!
  - Derive corrections to kinetic terms of moduli. [Berg, Haack, Kang, Sjörs `14]
- Include corrections to the vacuum solution, such as warping effects.
  [Grimm, Pugh, Weissenbacher `14] ; [Martucci `14]
- Our correction persists on singular CY_4! Are there more?
- Study the 4D scalar potential.