Higgs Pair Production:
Choosing Benchmarks with Cluster Analysis

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Overview

Goal:
categorisation of the model parameter space into regions that share the same kinematical features in the context of Higgs pairs production.

Summary:

1) Higgs Effective Lagrangian
    - Parametrisation
    - Bounds on couplings

2) Cross section fit

3) **Clustering Technique**
    - Setup
    - The algorithm
    - Results

Higgs Effective Lagrangian

New physics residing at a scale $\Lambda > M_{EW}$ can be described by dim > 4 operators

$$\mathcal{L} = \mathcal{L}_{SM}^{D\leq 4} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i^{D=6}$$

Assuming L&B conservation and CP a relevant effective Lagrangian to gg → hh production is

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_H}{2\Lambda^2}(\partial^\mu |H|^2)^2 - \frac{c_6}{\Lambda^2}\lambda |H|^6$$ Pure Higgs

$$-\left(\frac{c_t}{\Lambda^2} y_t |H|^2 \bar{Q}_L H^c t_R + \frac{c_b}{\Lambda^2} y_b |H|^2 \bar{Q}_L H b_R + \frac{c_\tau}{\Lambda^2} y_\tau |H|^2 \bar{L}_L H \tau_R + h.c.\right)$$ Yukawa

$$+ \frac{\alpha_s c_g}{4\pi\Lambda^2}|H|^2 G^a_{\mu\nu}G^{\mu\nu}_a + \frac{\alpha' c_\gamma}{4\pi\Lambda^2}|H|^2 B_{\mu\nu}B^{\mu\nu}$$ Gluons, neutral vector

See e.g.: Elias-Miro, Espinosa, Masso, Pomarol, 1308.1879; Pomarol, Riva, 1308.2803; Corbett, Eboli, Gonzalez-Fraile, Gonzalez-Garcia 1207.1344, 1211.4580, 1304.1151; Falkowski, Riva, Urbano, 1303.1812; Contino, Ghezzi, Grojean, Muhlleitner, Spira, 1303.3876; Dumont, Fichet, von Gersdorff 1304.3369; Trott 1409.7605; Falkowski, Riva, 1411.0669; Corbett, Eboli, Goncalves, Gonzalez-Fraile, Plehn, Rauch 1505.05516, Goertz 1406.0102; HXSWG;

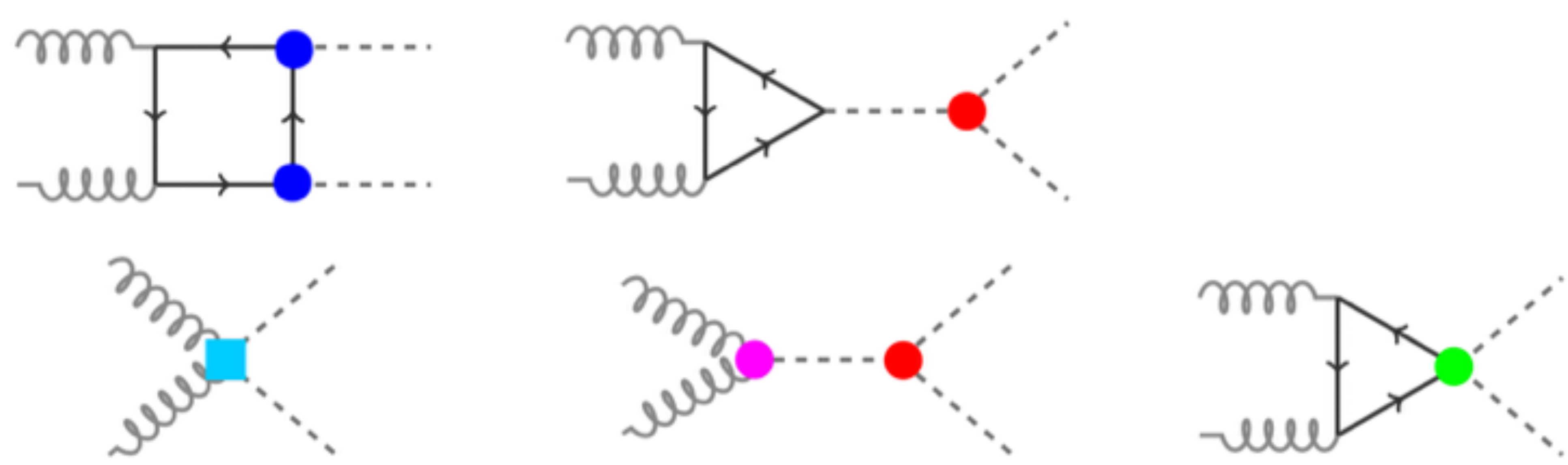
Higgs Effective Lagrangian

Higgs effective Lagrangian after EWSB, neglecting couplings with light fermions $H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \kappa_\lambda \lambda_{SM} v\, h^3 - \frac{m_t}{v}(v + \kappa_t\, h + \frac{c_2}{v}\, h\, h)\,(\bar{t}_L t_R + h.c.)$$
$$+ \frac{1}{4}\frac{\alpha_s}{3\pi v}(c_g\, h - \frac{c_{2g}}{2v}\, h\, h)\, G^{\mu\nu} G_{\mu\nu}$$

where in EFT linear realisation $c_{2g} = -c_g$. Constraint relaxed.

ggF Higgs pairs production diagrams



Bounds on couplings

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \kappa_\lambda \lambda_{SM} v h^3 - \frac{m_t}{v}(v + \kappa_t h + \frac{c_2}{v} h h)(\bar{t_L} t_R + h.c.) + \frac{1}{4}\frac{\alpha_s}{3\pi v}(c_g h - \frac{c_{2g}}{2v} h h) G^{\mu\nu} G_{\mu\nu}$$

κ_λ	anomalous trilinear	$\lvert\kappa_\lambda\rvert \sim 15$ (κ_λ only variation)(1)
κ_t	anomalous top Yukawa	$\kappa_t \in [0.5, 2.5]$(2)
c_2	tthh interaction	$\lvert c_2\rvert < 5$ if $\kappa_\lambda = 1$ and $\kappa_t \in [0.5, 2.5]$(1)
c_g	h-gluon contact int.	$c_g \sim O(1)$(3)
c_{2g}	hh-gg contact int.	$c_{2g} \sim O(1)$

5D parameter space

(1) ATLAS hh → γγbb, hh → bbbb 8 TeV analysis 1406.5053, 1506.00285

(2) from single h Run 1 study 1306.6464

(3) 1304.3369, 1308.1879, 1308.2803, 1405.0181, 1505.05516

Cross section fit

Parametrisation with 15 A_i coefficients

$$\frac{\sigma_{hh}}{\sigma_{hh}^{SM}} = A_1\,\kappa_t^4 + A_2\,c_2^2 + (A_3\,\kappa_t^2 + A_4\,c_g^2)\,\kappa_\lambda^2 + A_5\,c_{2g}^2 + (A_6\,c_2 + A_7\,\kappa_t\kappa_\lambda)\kappa_t^2 + (A_8\,\kappa_t\kappa_\lambda + A_9\,c_g\kappa_\lambda)c_2 + A_{10}\,c_2c_{2g} + (A_{11}\,c_g\kappa_\lambda + A_{12}\,c_{2g})\,\kappa_t^2 + (A_{13}\,\kappa_\lambda c_g + A_{14}\,c_{2g})\,\kappa_t\kappa_\lambda + A_{15}\,c_gc_{2g}\kappa_\lambda$$

Assuming

$$(M_iM_j^\dagger + h.c.)^{\text{higher order}} = k_{ij}\,(M_iM_j^\dagger + h.c.)^{(LO)} = k_{SM}\,(M_iM_j^\dagger + h.c.)^{(LO)}$$

on the amplitudes it was possible to calculate the cross section with MG5_aMC@NLO implementation of gg→hh (LO) in different point of the parameter space and then perform a fit on the A_is.

PARAMETER SPACE SCAN GRID:

k_λ = 0, ± 1, ± 2.4, ± 3.5, ± 5, ± 10, ± 15
k_t = [0.5, 2.5] in steps of 0.25
c_2 = [-3.0, 3.0] in steps of 0.5
c_g, c_{2g} = [-1.0, 1.0] in steps of 0.2
+ plane with c2 = 0.5, k_t = k_λ = 1
+ 3D grid with c_g = c_{2g} = 0

=

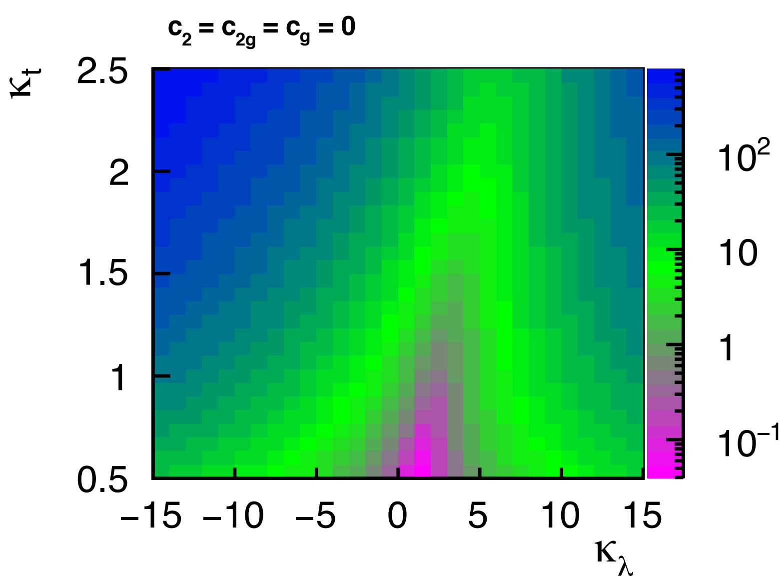
1507 LHE samples
13TeV, 20k hh events
the same samples used for the clustering technique

[updated, details in backup]

Motivation

Looking forward data analysis…

Different values of the couplings yield a **cross section** up to > 100 times the SM but also **change drastically the kinematics** requiring a custom analysis for each set of values.



σ/σ_{SM} on $\kappa_t \times \kappa_\lambda$ plane

Arbitrary number of parameter space points to be probed.

⇩

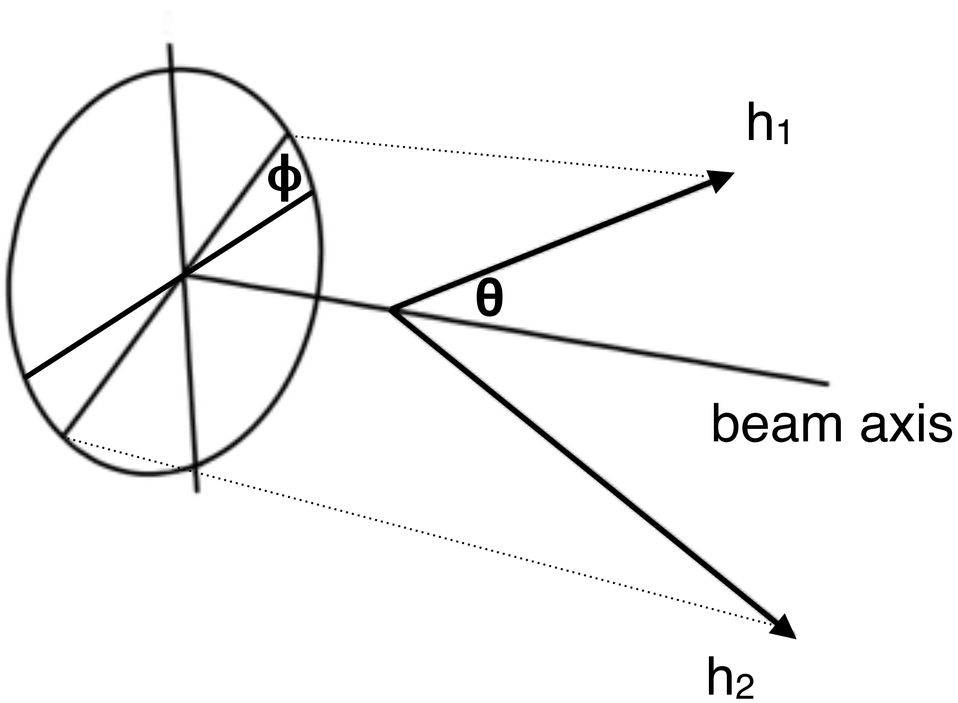
Clustering technique

⇩

A handful of points representing all the possible kinematic scenarios.

Clustering Setup

Variables choice



The bosons are back-to-back in ϕ (no ISR), so, disregarding of the particular azimuthal angle, we just need 3 variables to describe the system

p_T , $p_{z,1}$, $p_{z,2}$

Boost along the z from parton distribution functions we do not want to account for. So we study the process in the centre of mass frame with just two variables

$\mathbf{m_{hh}}$, **cosθ***

Parameter space point → Monte Carlo sample → 2D shape

Binning: sufficiently populated 50 (m_{hh}) x 5 (|cosθ*|) bins.

m_{hh} [0, 1500 GeV] 30 GeV wide-bin
|cosθ*| [0,1] 0.2 wide-bin.

Likelihood ratio test statistic [updated]

Several possible choices to test **samples similarity**: Kolmorov-Smirnov, Anderson-Darling, Zach-Aslan… Final choice: likelihood ratio based on Poisson counts.

Steps to build our likelihood ratio:

1. <u>If the two samples under test share the same parent distribution</u> the probability to observe $n_{1,i}$ and $n_{2,i}$ in the i-th bin is given by

$$Pois(n_{i,1}|\hat{\mu}_i) \times Pois(n_{i,2}|\hat{\mu}_i), \quad \hat{\mu}_i = (n_{i,1} + n_{i,2})/2$$

however there is an *ancillary* statistic

$$Pois(n_{i,1}) \times Pois(n_{i,2}) = Pois(n_{i,1} + n_{i,2}) \times Binomial(n_{i,1}/(n_{i,1} + n_{i,2}))$$

only the binomial term contains useful information

$$Binomial(n_{i,1}/(n_{i,1} + n_{i,1})) = \frac{(n_{i,1} + n_{i,2})!}{n_{i,1}!n_{i,1}!} \left(\frac{1}{2}\right)^{n_{i,1}} \left(\frac{1}{2}\right)^{n_{i,2}}$$

Let’s call L the likelihood built from this pdf.

Likelihood ratio test statistic

2. If the <u>two samples are equal</u> (*saturated hypothesis*) the pdf is just

$$Binomial(n_{i,1} = n_{i,2} = \hat{\mu}_i) = \frac{(2\hat{\mu}_i)!}{(\hat{\mu}_i!)^2} \left(\frac{1}{2}\right)^{2\hat{\mu}_i}$$

Let's call L_S the likelihood associated to this pdf.

3. The (log-) likelihood ratio is defined as

$$TS = 2\ log\left(\frac{L}{L_S}\right) = 2\sum_{i=1}^{N_{bins}} log(n_{i,1}!) + log(n_{i,2}!) - 2log\left(\frac{n_{i,1}+n_{i,2}}{2}!\right)$$

thanks to Wilks theorem TS is χ^2 distributed and can be used <u>directly as an ordering parameter.</u>

Clustering algorithm

$TS_{ij} > TS_{kl}$ → i and j are more similar to each other then kl

Steps:

1) Identify each sample as one element cluster

2) define cluster-to-cluster similarity as $TS^{min} = min(TS_{ij})$ where *i* runs on first cluster elements and *j* on the second one

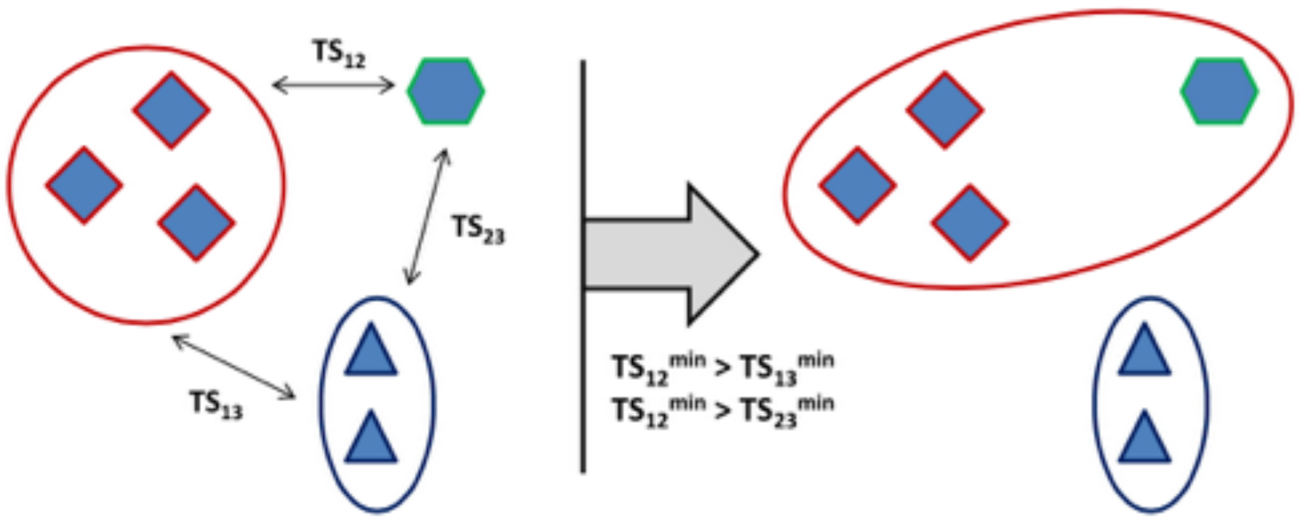
3) merge the pair of clusters with highest TS^{min}

4) repeat until the desired number of clusters $\mathbf{N_{clus}}$ is reached

5) identify the benchmark *k* of each cluster as the one with the highest

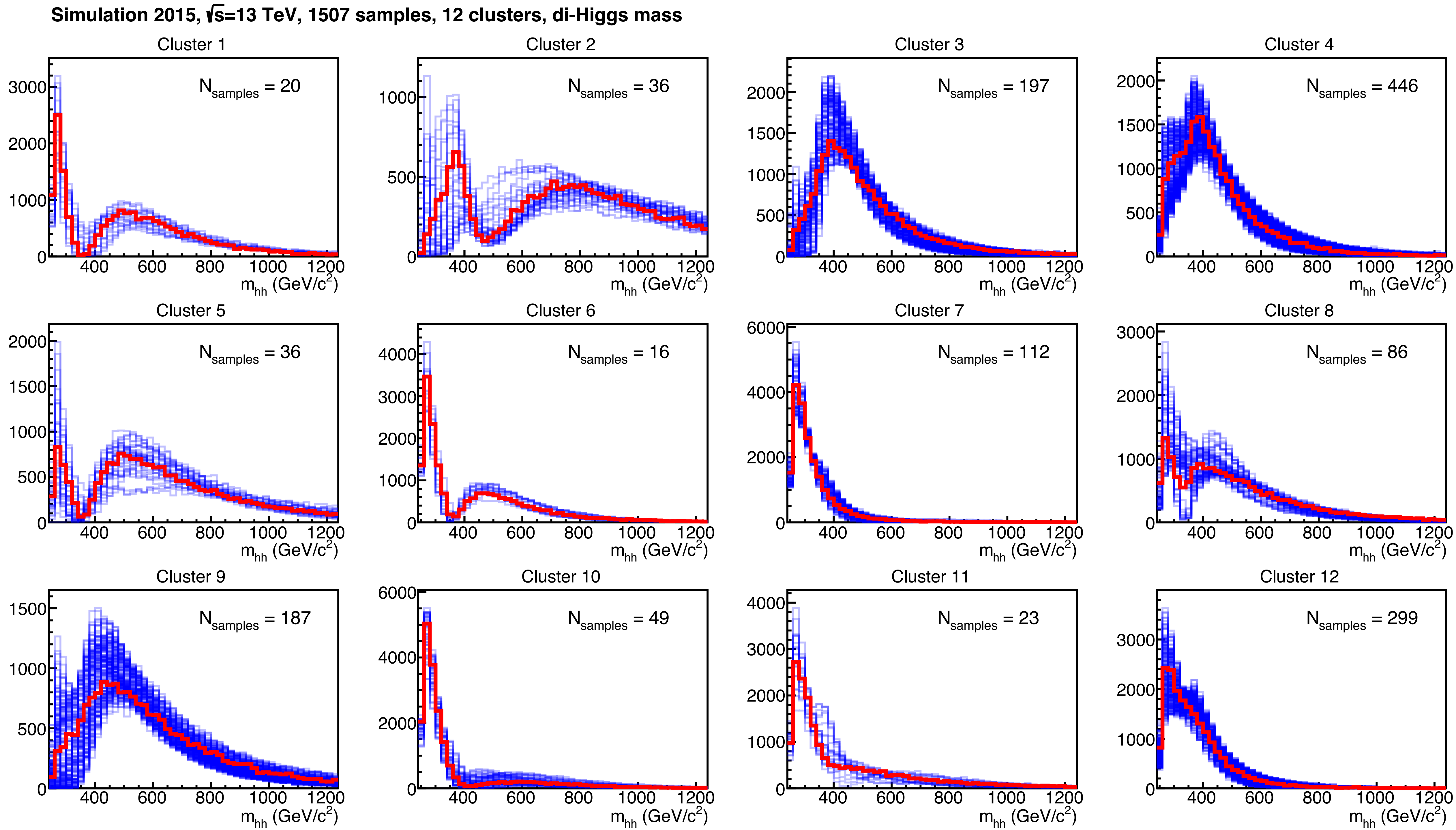
$$TS_k^{min} = \min_i(TS_{ki})$$

where *i* runs on the cluster elements.



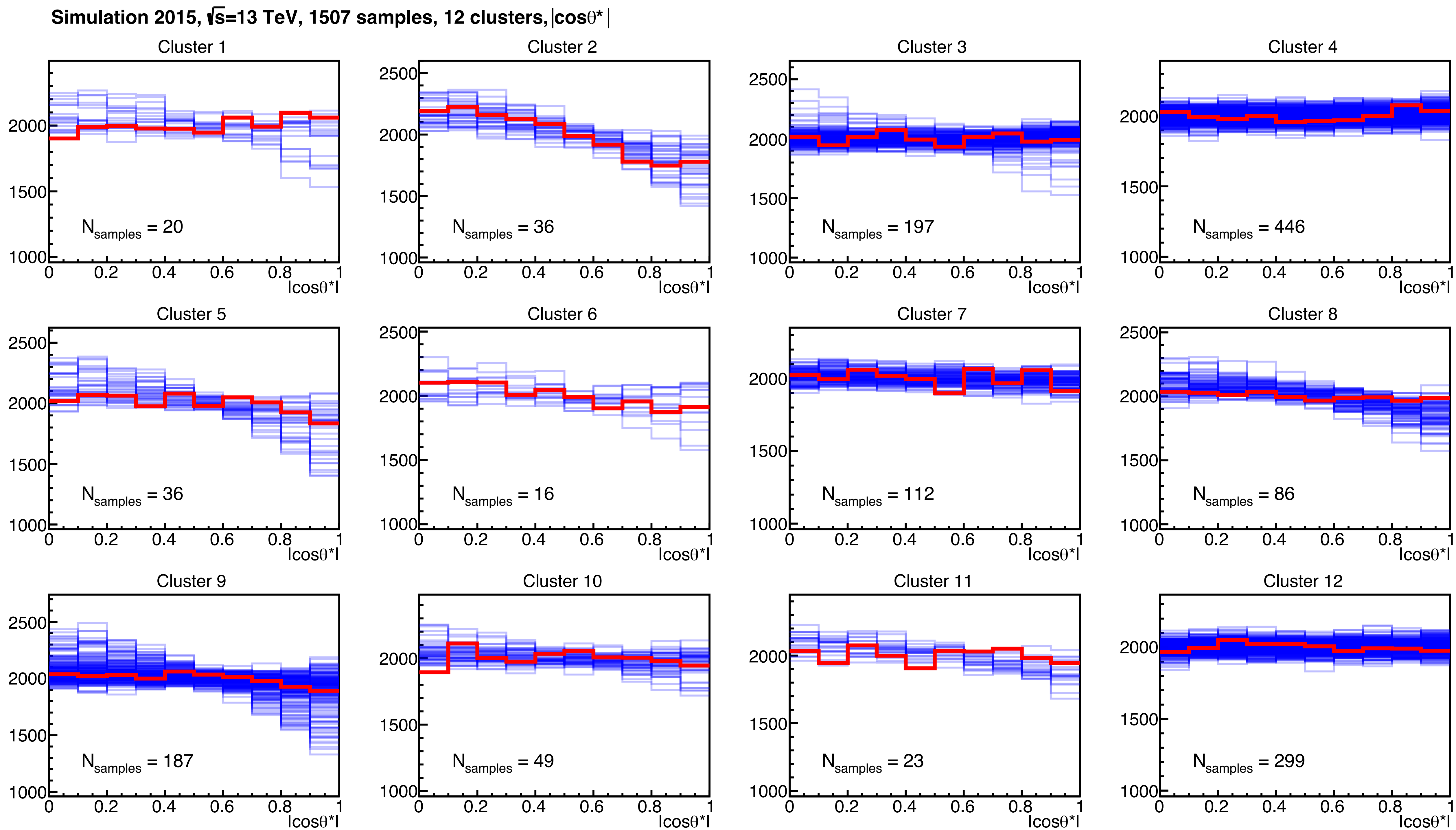
<u>$\mathbf{N_{clus}}$ is the only free parameter, fixed a posteriori.</u>

* Clusters m_{hh} distributions

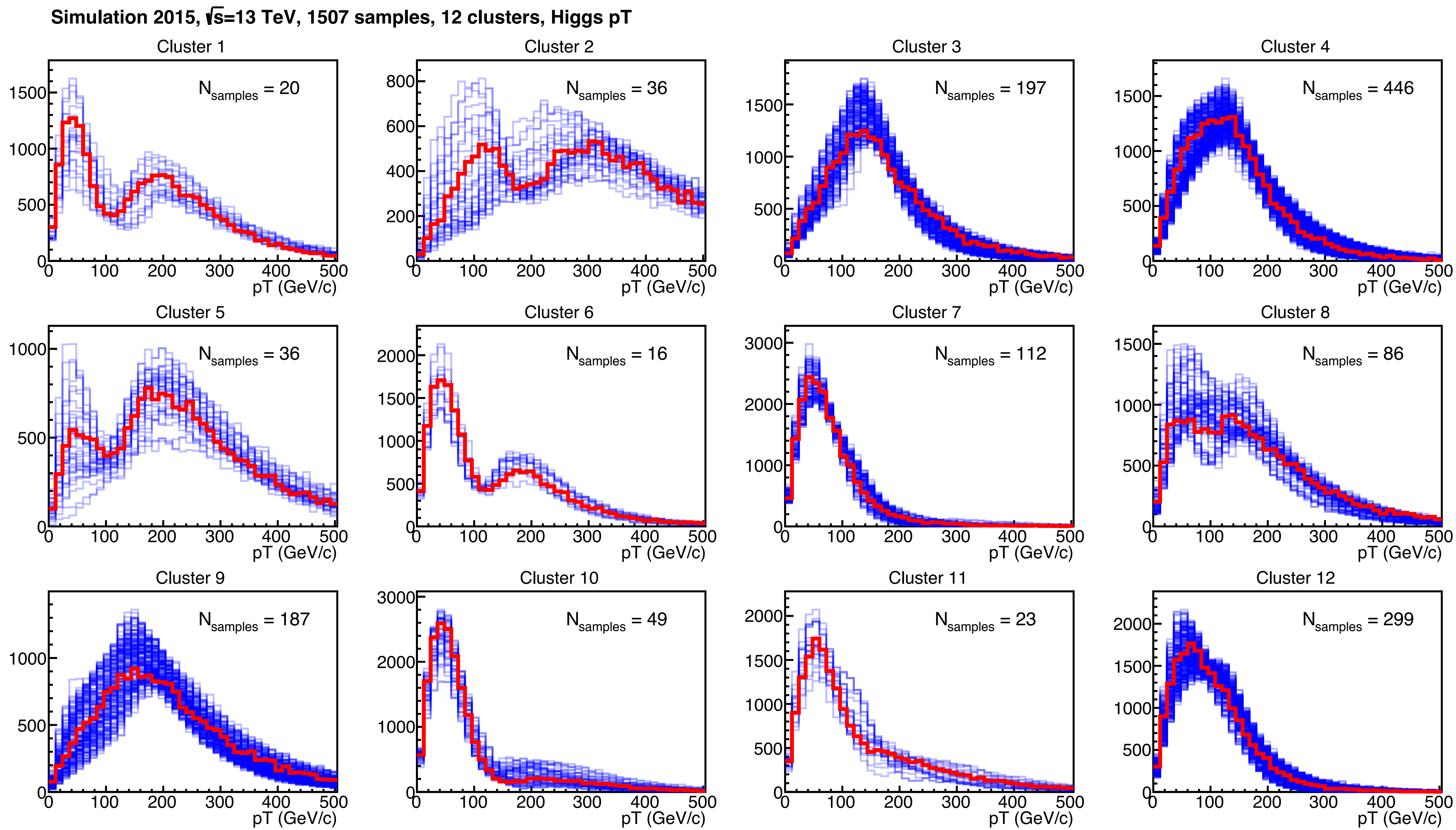


* suggested for Yellow report

* Clusters |cosθ*| distributions



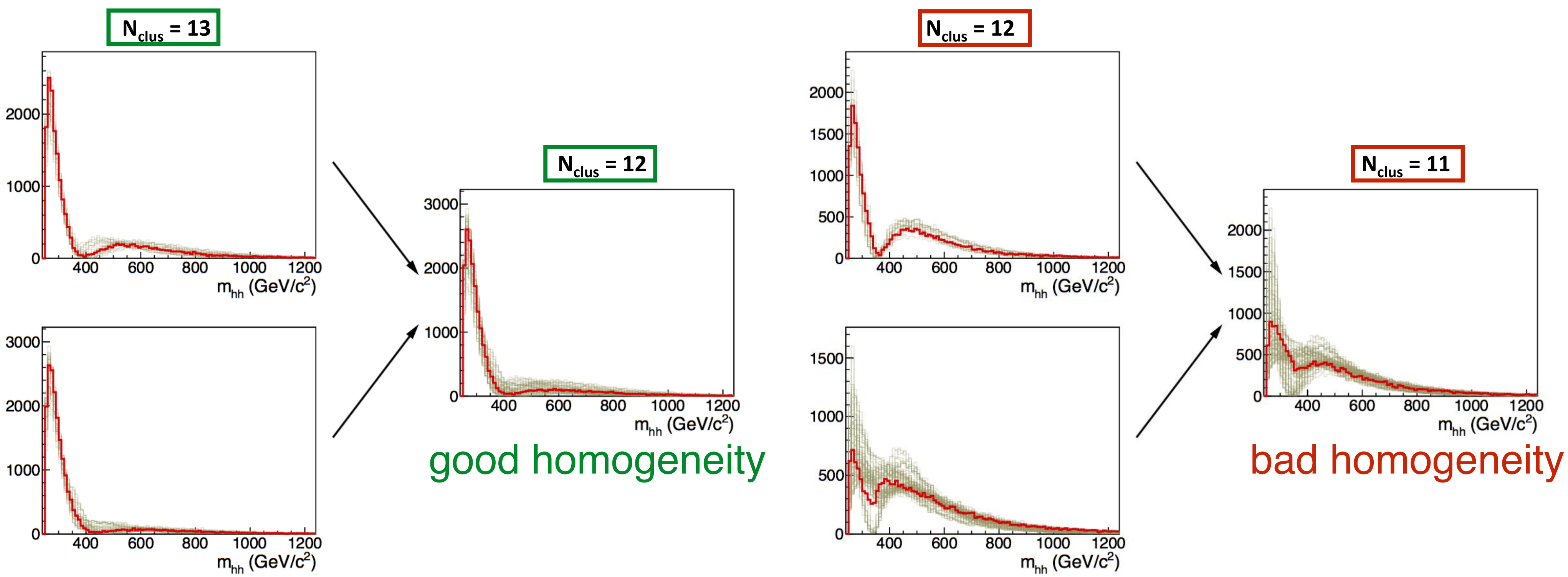
Clusters Higgs p_T distribution



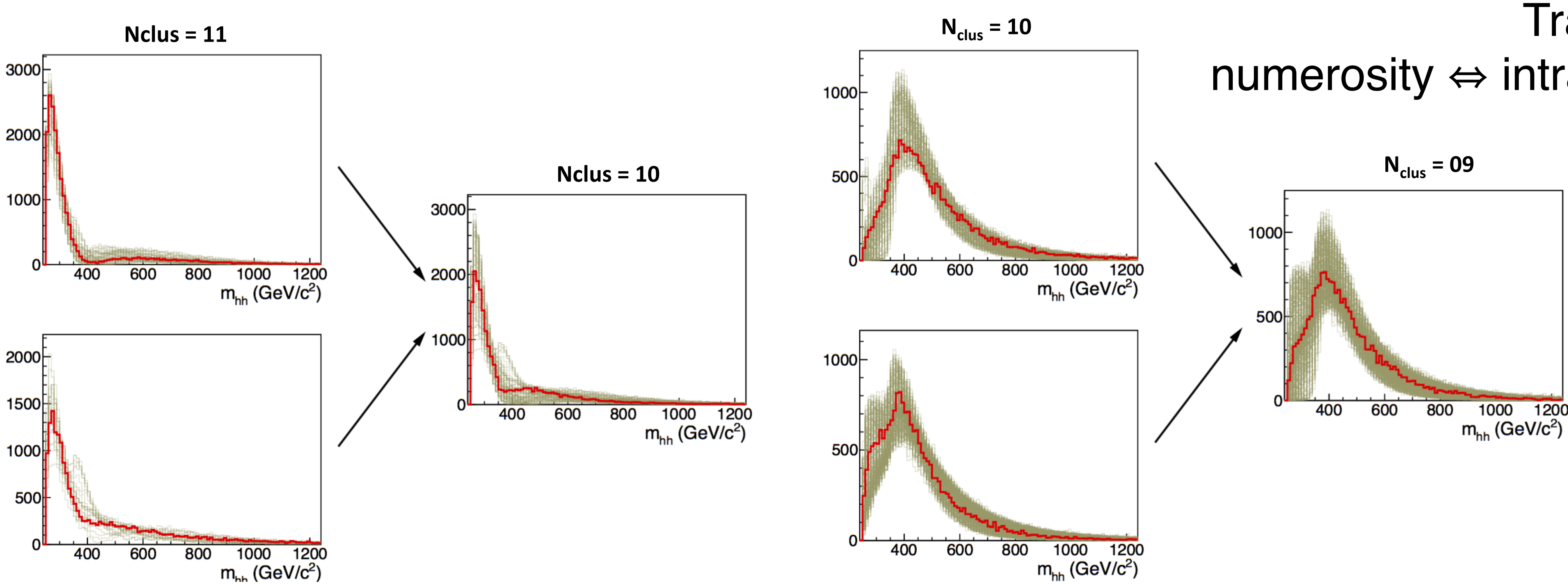
N_{clus} = 12 choice

Looking for $N_{clus} \sim O(10)$…

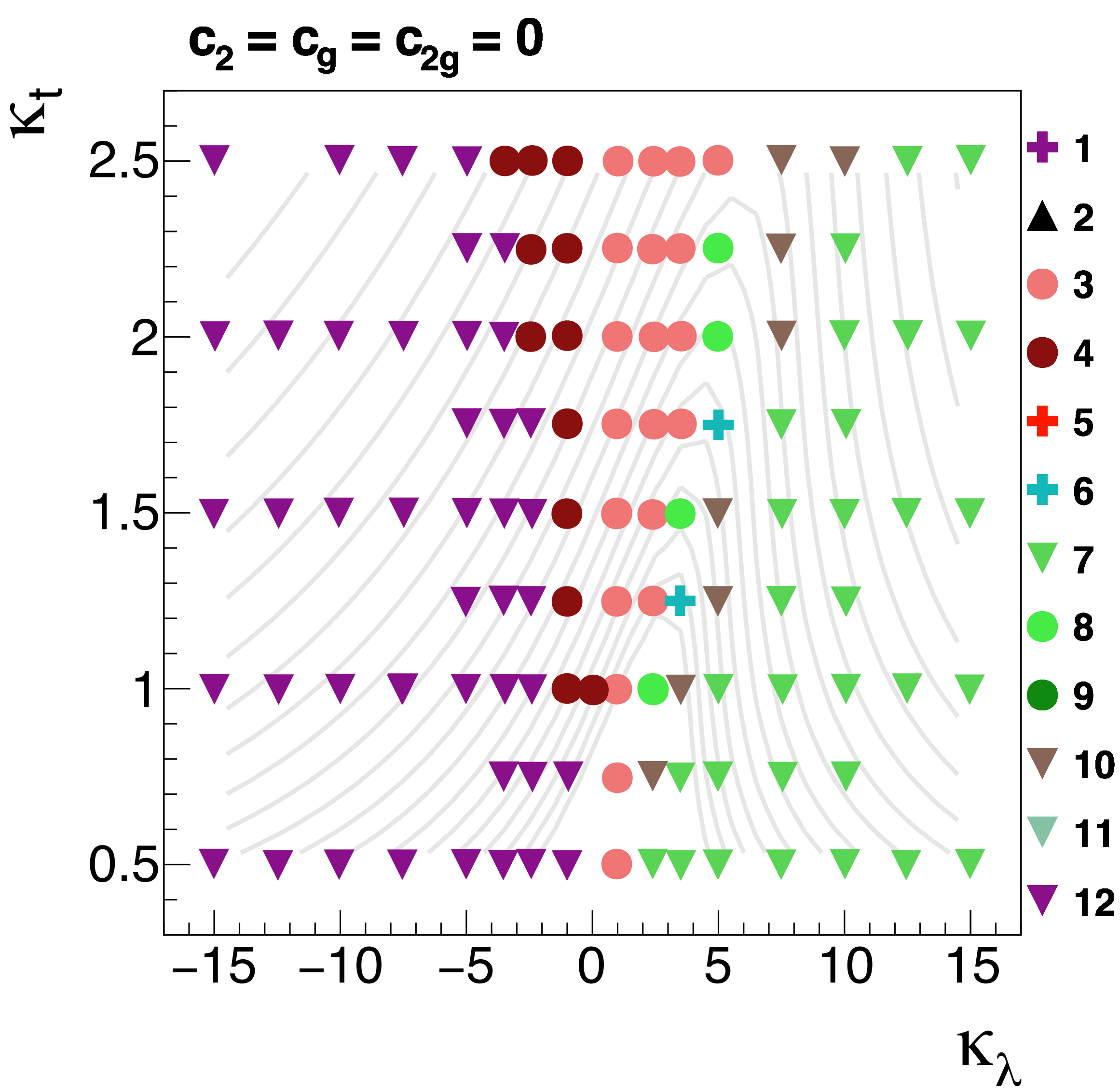
Clusters pair merged stepping from final cluster number N_{clus} to N_{clus} -1



Tradeoff:
numerosity ⇔ intra-cluster homogeneity



Clusters map in $\kappa_t \times \kappa_\lambda$ plane



samples in κ_t and κ_λ plane, one color per cluster.

▼ p_T peak < 50 GeV

● p_T peak ~ 100 GeV

▲ p_T peak > 150 GeV

✚ double peak in m_{hh}

great variability around SM point which is a cross section minimum

Conclusions

- An effective parametrisation of the gg→hh process leads to a five dimensional parameter space
- The parameter space is wide and only a limited number of analyses can be performed
- **A clustering technique has been developed and leads to a subdivision of the parameter space into 12 regions**
- Good uniformity of kinematical distributions in each cluster validates the method
- In each homogeneous region, a benchmark point is identified as the most similar to all the other samples in that cluster. Searches targeting those benchmarks are guaranteed to be sensitive to a large area of the parameter space.

Conclusions

The benchmark parameter points we propose are:

* Benchmark	κ_λ	κ_t	c_2	c_g	c_{2g}
1	7.5	1.0	-1.0	0.0	0.0
2	1.0	1.0	0.5	-0.8	0.6
3	1.0	1.0	-1.5	0.0	-0.8
4	-3.5	1.55	-3.0	0.0	0.0
5	1.0	1.0	0.0	0.8	-1
6	2.4	1.0	0.0	0.2	-0.2
7	5.0	1.0	0.0	0.2	-0.2
8	15.0	1.0	0.0	-1	1
9	1.0	1.0	1.0	-0.6	0.6
10	10.0	1.5	-1.0	0.0	0.0
11	2.4	1.0	0.0	1	-1
12	15.0	1.0	1.0	0.0	0.0
SM	1.0	1.0	0.0	0.0	0.0

Work documented in arXiv:1507.02245
Submitted to JHEP

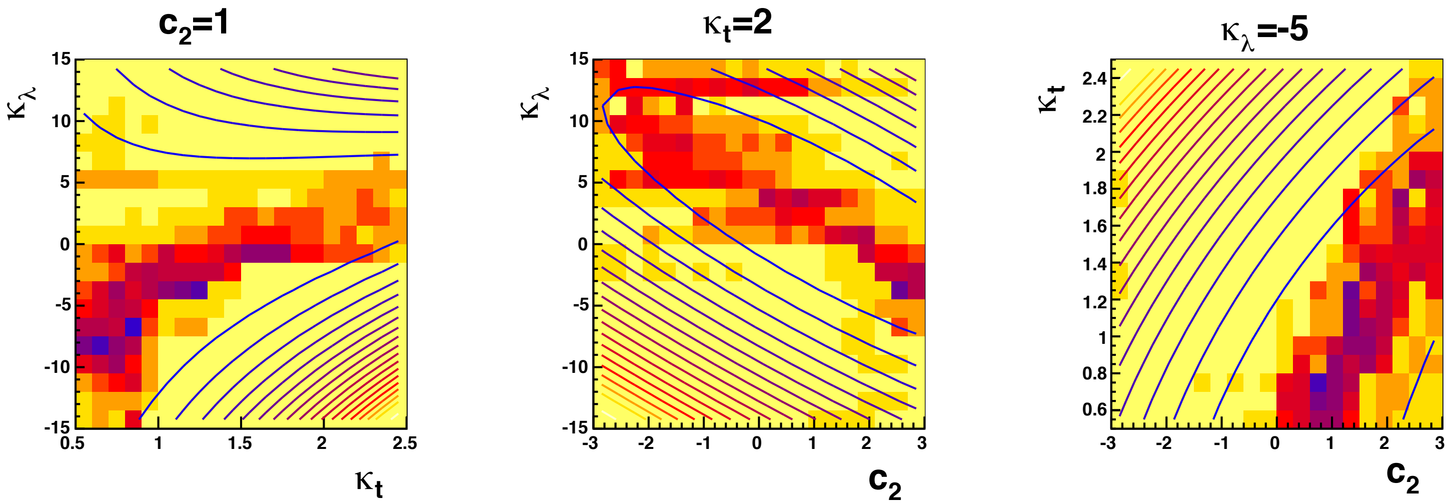
https://twiki.cern.ch/twiki/bin/view/Sandbox/NonResonantHHAtLHC

Backup

Parameter space sampling

Regular grid? → too many samples to be generated for an accurate scan!

Smart grid:
look for greatest observables variability with parameter variation and increase sampling there.



TS variation rate higher near σ minima

- First grid:

$k_\lambda = 0, \pm 1, \pm 2.4, \pm 3.5, \pm 5, \pm 10, \pm 15$

k_t = [0.5, 2.5] in steps of 0.25 for $|k_\lambda| < 5$, 0.5 step elsewhere

c_2 = [-3.0, 3.0] in steps of 0.5

c_g, c_{2g} = [-1.0, 1.0] in steps of 0.2

- extra plane: $c_2 = 0.5$, $k_t = k_\lambda = 1$

In this region there is a strong cancellation between operators that leads to high m_{hh} tails*. Points added both to describe the topology and to probe the cancellation.

*Thanks to Andreas Papaefstathiou for checking the same behaviour in Herwig MC implementation

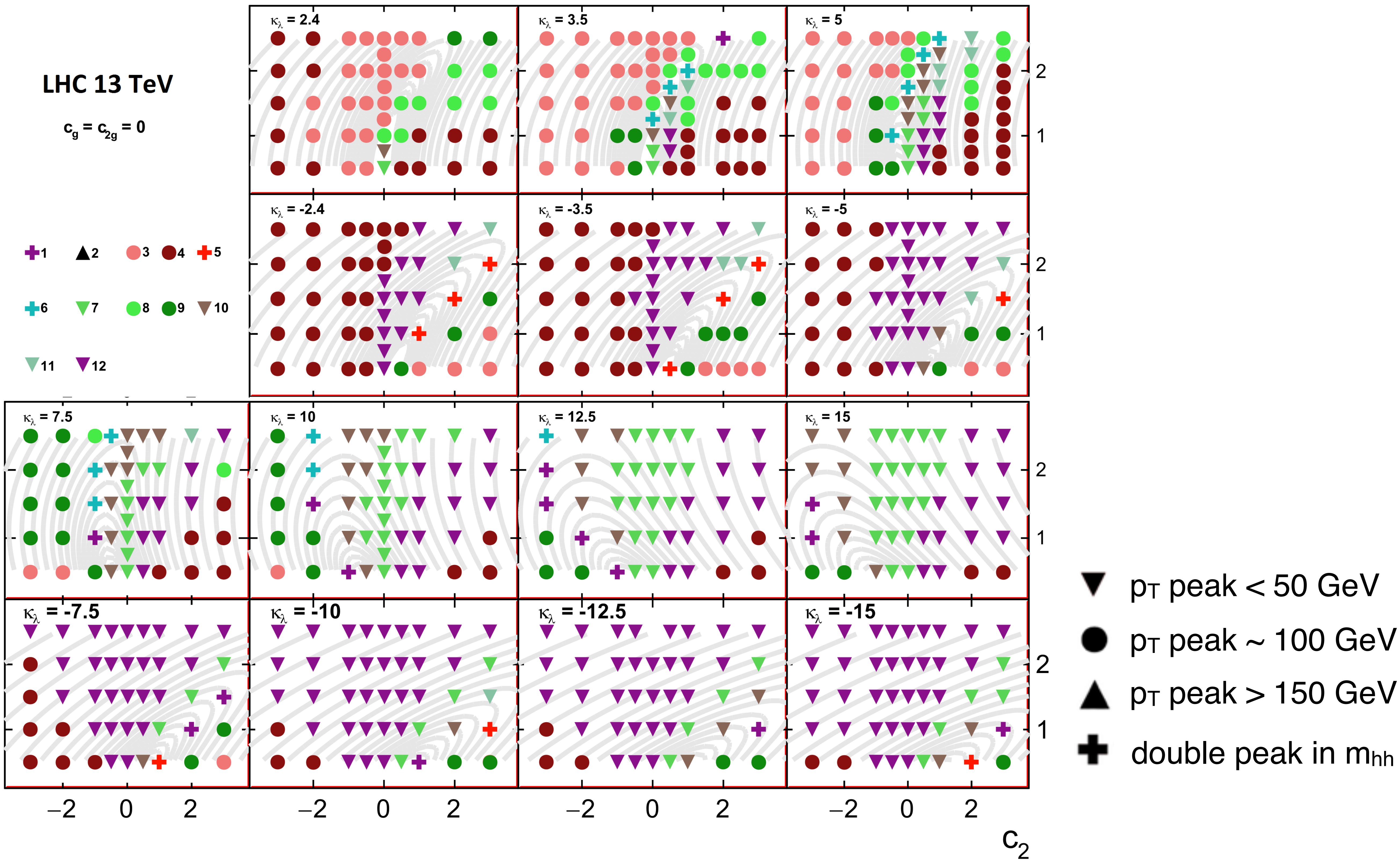
- 3D grid: $c_g = c_{2g} = 0$

$k_\lambda = 0, \pm 1, \pm 2.4, \pm 3.5, \pm 5, \pm 10, \pm 12.5, \pm 15$

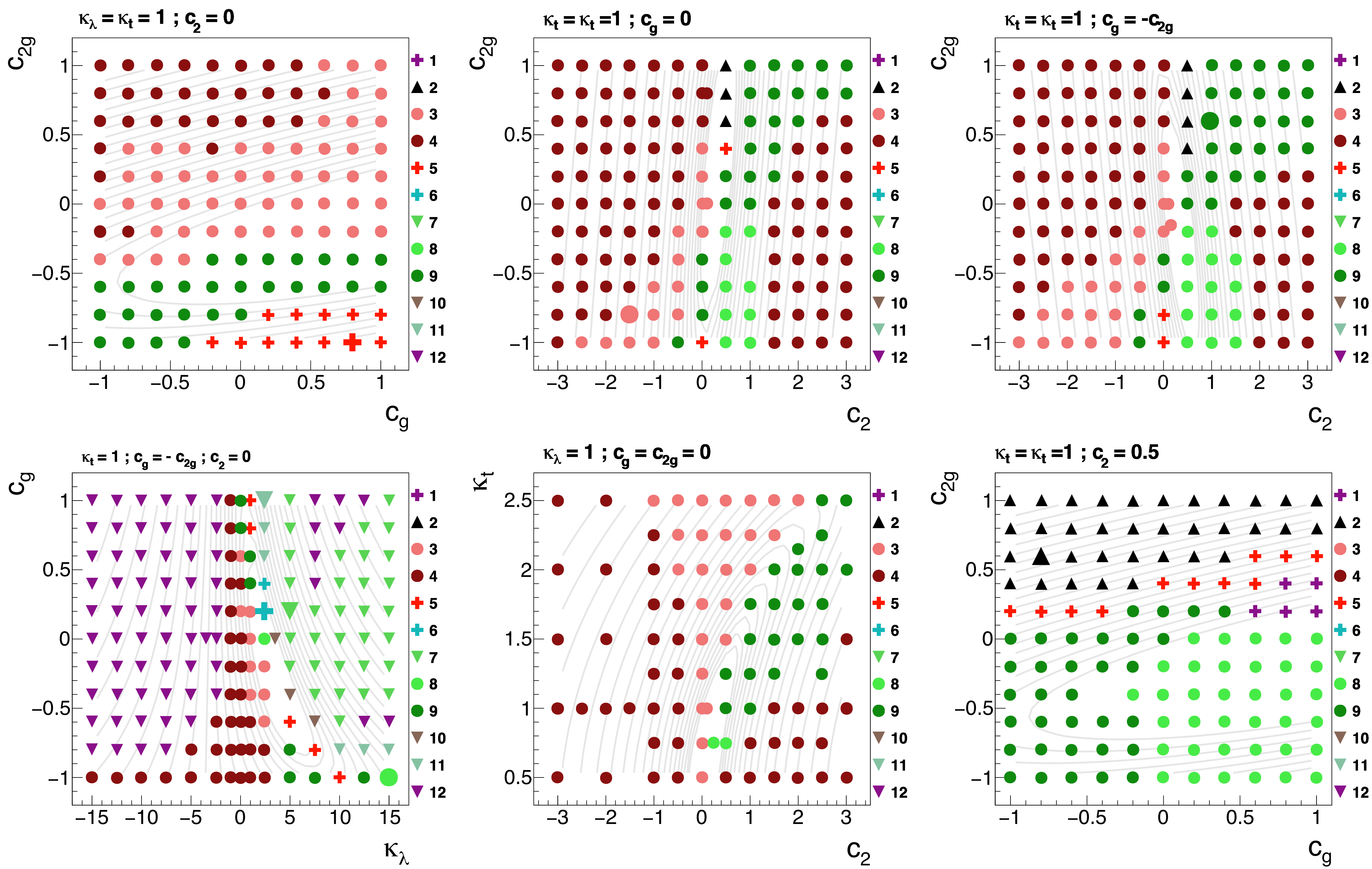
k_t = [0.5, 2.5] in steps of 0.25

c_2 = [-3.0, 3.0] in steps of 0.5

Clusters map in $c_2 \times \kappa_t$ planes



Clusters map



▼ p_T peak < 50 GeV

▲ p_T peak > 150 GeV

● p_T peak ~ 100 GeV

✚ double peak in m_{hh}

More on XS fit

Cross section coefficients calculated for 8, 13 and 14 TeV.

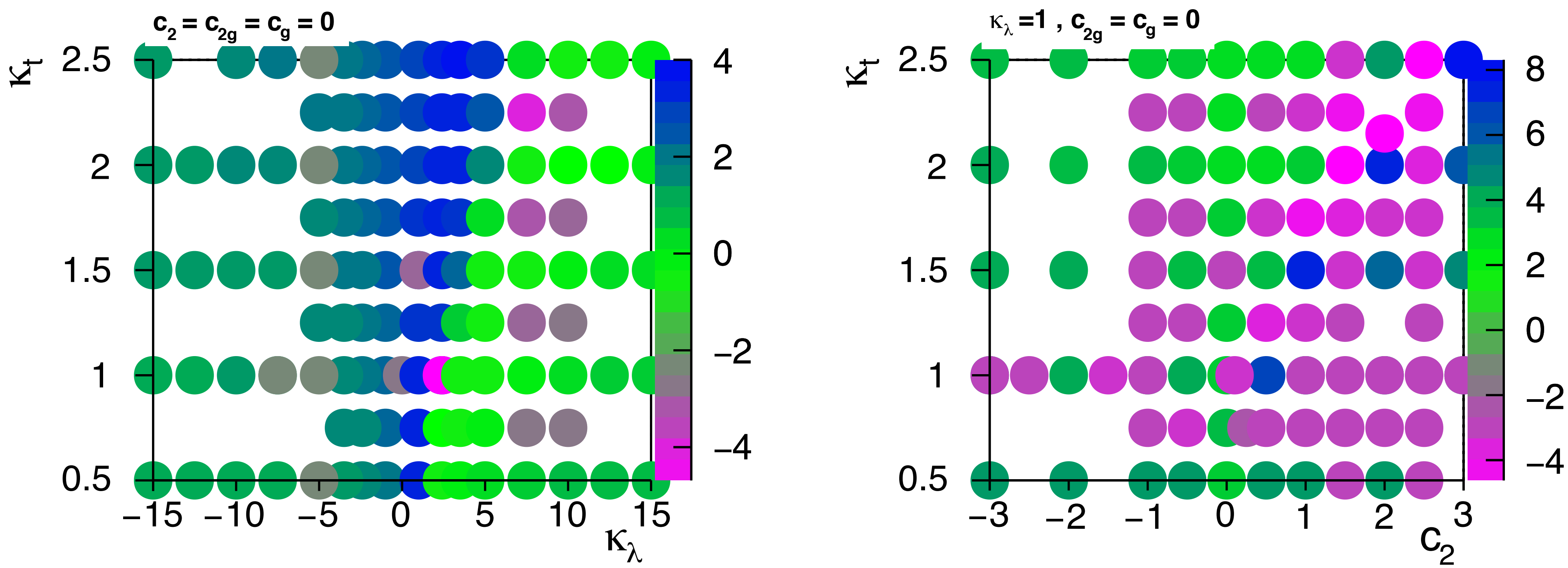
$\sqrt{s}$	8 TeV	13 TeV	14 TeV
A_1	2.188 ± 0.029	2.210 ± 0.016	2.010 ± 0.027
A_2	9.914 ± 0.090	10.598 ± 0.060	10.187 ± 0.094
A_3	0.3242 ± 0.0062	0.3046 ± 0.0022	0.2872 ± 0.0052
A_4	0.118 ± 0.014	0.109 ± 0.015	0.103 ± 0.015
A_5	1.179 ± 0.041	1.467 ± 0.044	1.360 ± 0.046
A_6	-8.73 ± 0.13	-8.968 ± 0.054	-8.54 ± 0.15
A_7	-1.512 ± 0.027	-1.471 ± 0.010	-1.388 ± 0.024
A_8	3.04 ± 0.15	3.017 ± 0.020	2.85 ± 0.16
A_9	1.61 ± 0.11	1.56 ± 0.13	1.48 ± 0.15
A_{10}	-5.11 ± 0.13	-5.23 ± 0.12	-4.93 ± 0.15
A_{11}	-0.770 ± 0.029	-0.707 ± 0.029	-0.685 ± 0.029
A_{12}	2.074 ± 0.063	1.989 ± 0.066	1.876 ± 0.064
A_{13}	0.374 ± 0.016	0.342 ± 0.015	0.328 ± 0.015
A_{14}	-0.932 ± 0.056	-0.887 ± 0.057	-0.847 ± 0.056
A_{15}	-0.611 ± 0.038	-0.595 ± 0.041	-0.576 ± 0.041

- 10 % error assigned to each point cross section
- max likelihood fit
- performed with both MINUIT and Mathematica

Points probed
13 TeV:
same employed for the clustering.

8, 14 TeV:
points from 6 orthogonal plans, 5k events samples

$|\sigma_{FIT} - \sigma_{simulation}|$



Discrepancies between the fit and the actual results of the simulation in percentage.

σ/σ_{SM}

σ/σ_{SM} on slices of the parameter space