Matching NLO QCD with parton shower in Monte Carlo scheme - the KrkNLO method

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in collaboration with:
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based on:
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Outline

- Motivation/notation.
- KrkNLO approach to NLO+PS matching
- Results, comparison to:
  - fixed order NLO
  - other NLO matched calculations (MCatNLO and POWHEG)
  - fixed order NNLO
- Final remarks and outlook

What do general-purpose Monte Carlo generators do?

- An "event" is a list of particles (pions, protons, ...) with their momenta.
- The MCs generate events.
- The probability to generate an event is proportional to the (approximate!) cross section for such an event.
- Calculate Everything $\sim$ solve QCD $\rightarrow$ requires compromise!
- Improve lowest-order perturbation theory, by including the "most significant" corrections $\rightarrow$ complete events (can evaluate any observable you want)

The Workhorses: What are the Differences?

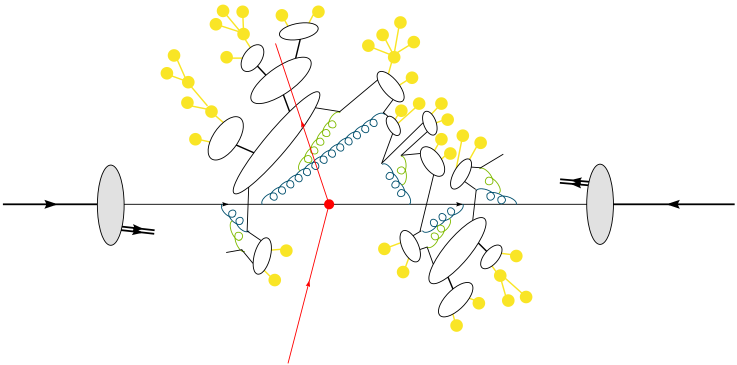
All offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA: Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.

HERWIG: Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering parton shower. Cluster model.

SHERPA: Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW.

Basics of Monte Carlo Generators



taken from Stefan Gieseke©

The general approach is the same in different programs but the models and approximations used are different.
In this talk I will focus on NLO matrix element + Parton Shower matching.

Fixed order calculations in QCD

General structure of NLO cross sections:

$$d\sigma = \Big[B + V(\alpha_s) + C(\alpha_s)\Big] d\phi_B + R(\alpha_s)\, d\phi_B d\phi_1$$

- B, R, V - Born, real and virtual part
- C - collinear subtraction counterterm (for initial state radiation case)

Each part: V, C and $\int R d\phi_1$ is separately divergent (soft and collinear). Divergences cancel in the sum.

Calculation possible e.g. by means of subtraction procedure

$$\begin{aligned} d\sigma &= \Big[B + V(\alpha_s) + \int_1 A(\alpha_s) d\phi_1 + C(\alpha_s)\Big] d\phi_B + \\ &\quad \int_1 \Big[R(\alpha_s) - A(\alpha_s)\Big]\, d\phi_1 d\phi_B\,, \end{aligned}$$

where $A \simeq R$, such that it reproduces collinear and soft singularities.

- Good for inclusive observables or distributions at high-p_T.

Parton shower

In the collinear region, fixed order calculation becomes unreliable because each α_s^n is accompanied by a large, logarithmic coefficient, $\ln^n$, and

$$(\alpha_s \ln)^n \sim 1 \text{ for all } n\,.$$

These terms must be summed to all orders and this is what the Parton Shower (PS) is aiming at. In the collinear limit

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} P(z) dz\,.$$

This can be iterated and used to resum all leading log contributions. In particular, non-emission probability (Sudakov form factor) is given by

$$\mathit{\Delta}(q_1, q_2) = \exp\left[- \int_{q_1}^{q_2} \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} \int_{z_0}^{1} P(z)\, dz \right].$$

In Monte Carlo event generators, the scale of i^{th} emission, q_i, is found by solving

$$\mathit{\Delta}(q_{i-1}, q_i) = R_i\,,$$

where $R_i \in [0, 1]$ is a random number and q_{i-1} is a scale of previous emission.

Parton shower and NLO

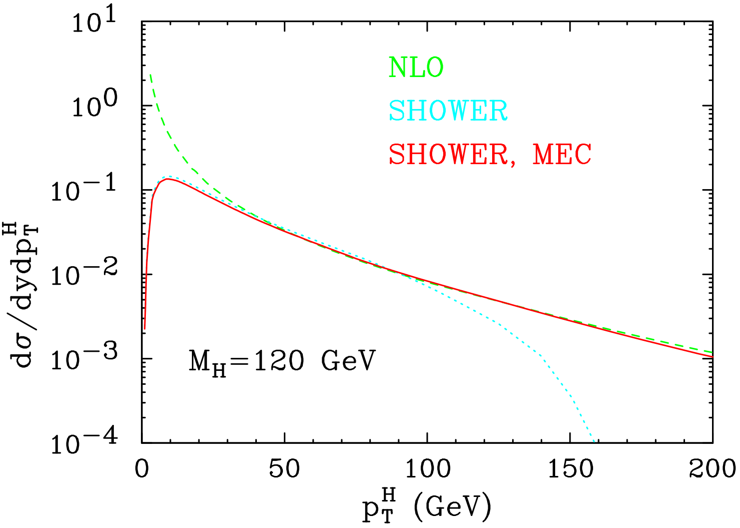


Figure from P. Nason and B. Webber [arxiv:1202.1251]

Motivation

I will talk about **a method for NLO+PS matching applied to Drell-Yan process.**

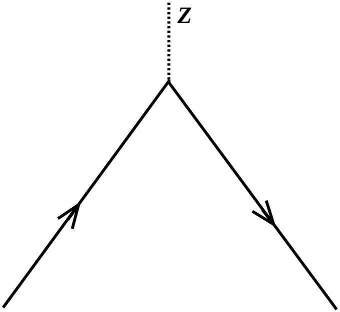
Key ingredients:

- new factorization scheme leading to new MC PDFs
- NLO correction applied to PS via reweighting of MC events

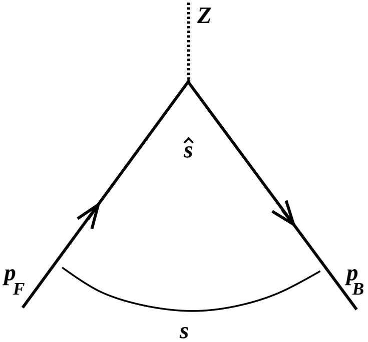
There are two well established methods MC@NLO and POWHEG...

- **Why would you like another method of NLO+PS matching?**
  - The method is extremely simple.
  - No negative weight events.
  - In angular ordered PS - no need for a truncated shower.
  - Simple at NLO $\Rightarrow$ you may hope that pushing the method to NNLO+NLO PS should be possible.

Drell-Yan process

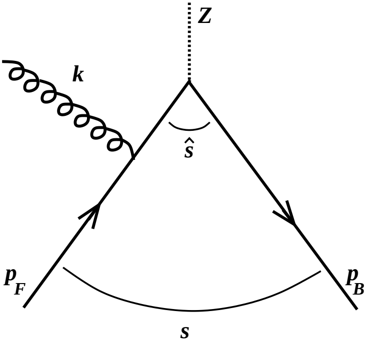


Drell-Yan process



$$s = (p_F + p_B)^2$$

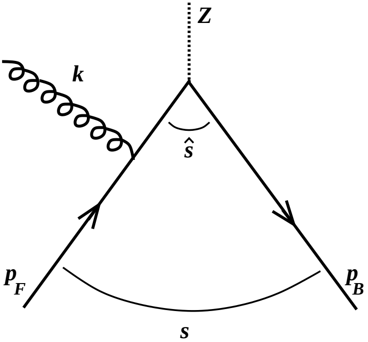
Drell-Yan process



$$s = (p_F + p_B)^2$$

$$z = \frac{\hat{s}}{s}$$

Drell-Yan process



$$s = (p_F + p_B)^2$$

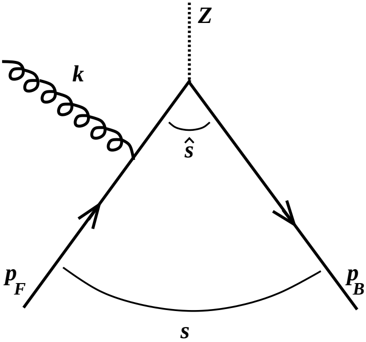
$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}$$

Drell-Yan process



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$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

Basic idea of the MC scheme

DY cross section at NLO in collinear $\overline{\text{MS}}$ factorization for the $q\bar{q}$ channel:

$$\sigma^1_{\text{DY}} - \sigma^B_{\text{DY}} = \sigma^B_{\text{DY}} \, D_1^{\overline{\text{MS}}}(x_1, \mu^2) \otimes \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2, \mu^2)\,,$$

where

$$C_q^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right) \right].$$

All solutions for NLO + PS matching which use $\overline{\text{MS}}$ PDFs, need to implement terms of the type $4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

The implementation is not easy since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.
The idea behind the MC scheme is to absorb those terms to PDF.

The KrkNLO method

The KrkNLO method

Two essential parts

1. **Change the factorization scheme from $\overline{\text{MS}}$ to MC**
   - produce new MC PDFs
   - differences at LO
   - universality: recovering $\overline{\text{MS}}$ NLO result

2. **Reweight parton shower**
   - correct hardest emission by 'real' weight
   - upgrade the cross section/distributions to NLO by multiplicative, constant 'soft+virtual' weight

KRK method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

1. Take a parton shower that covers the (α, β) phase space completely (no gaps, no overlaps) and produces emissions according to approx. real matrix element K.
2. Upgrade the real emissions to exact ME R by reweighting the PS events by $W_R = R/K$.
3. We define the coefficion function $C_2^R(z) = \int(R - K)$. To avoid unphysical artifacts of $\overline{\mathrm{MS}}$.
4. Transform PDF for $\overline{\mathrm{MS}}$ scheme to this new physical MC factorization scheme.
5. As a result the virtual+soft correction, Δ_{S+V}, is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

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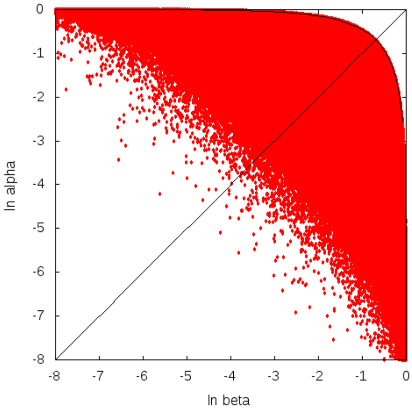
This has been shown to reproduce exactly the NLO result of fixed order collinear factorization, for the case of simplistic PS by means of analytical integration.
[S. Jadach at al. Phys.Rev. D87]

Could we implement the method in a popular, general purpose MC?

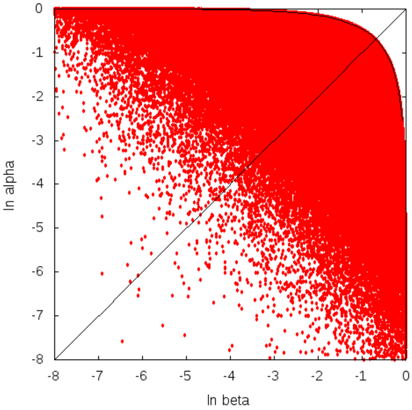
1. Take a PS that covers the (α, β) phase space

Herwig 7 (Dipole Shower)



The evolution variable:
$q^2 = k_T^2 = \alpha\,\beta\,s.$

Sherpa 2.0.0



The evolution variable:
$q^2 = (\alpha + \beta)\,\beta\,s.$

1. Take a PS that covers the (α, β) phase space

$\hookrightarrow$ We used Sherpa 2.0.0 implementation of the Catani-Seymour (CS) dipole shower.

Phase space measure of emitted gluon

$$\frac{d\alpha}{\alpha}\frac{d\beta}{\beta} = \frac{d\alpha d\beta}{\beta(\alpha+\beta)} + \frac{d\alpha d\beta}{\alpha(\alpha+\beta)}$$

- The evolution variable:

$$q_F^2 = s(\alpha+\beta)\beta, \qquad q_B^2 = s(\alpha+\beta)\alpha\,,$$

hence

$$\frac{d\alpha d\beta}{\alpha\beta} = \frac{dq_F^2}{q_F^2}\frac{dz}{1-z} + \frac{dq_B^2}{q_B^2}\frac{dz}{1-z}.$$

- The CS shower covers all space of (α, β).

$$\alpha+\beta \leq 1 \quad \Rightarrow \quad z \geq 0 \quad \text{and} \quad q_{F,B}^2 \leq s$$

$$\alpha, \beta > 0 \quad \Rightarrow \quad (1-z)^2 > q_F^2/s \quad \text{or} \quad (1-z)^2 > q_B^2/s$$

2. Upgrade the real emissions to exact ME by reweighting.

The hardest real emission is upgraded to ME by reweighting:

$$W_R = R/K$$

Where the kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution. The "Sudakov" form factor for he CS shower

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

Real part:

$$\begin{aligned} W_R^{q\bar{q}}(\alpha, \beta) &= 1 - \frac{2\alpha\beta}{1 + (1-\alpha-\beta)^2} \\ W_R^{qg}(\alpha, \beta) &= 1 + \frac{\alpha(2-\alpha-2\beta)}{1 + 2\,(1-\alpha-\beta)(\alpha+\beta)} \end{aligned}$$

Note:

Very simple weight dependent only on the kinematics α, β. One can compute it on the fly, inside an MC, or outside, using information from event record.

3. The coefficient function $C_2(z)$

$\hookrightarrow$ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, CS $\equiv$ MC.

The $C_2(z)$ function:

$$C_2^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \int (R - K)$$

- For the $q\bar{q}$ channel:

$$C_{2q}^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right]$$

- For the qg channel:

$$C_{2g}^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi}\, T_R\, \frac{1}{2}(1-z)(1+3z)$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $C_{2q}^{\mathrm{MC}}(z)$ and $C_{2g}^{\mathrm{MC}}(z)$ from $\overline{\mathrm{MS}}$ PDFs
  - absorbing all z-dependent terms from $\overline{\mathrm{MS}}$ coefficient functions

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Simple form of the coefficient functions with no singular terms!

- Quark and anti-quark PDFs are redefined by:
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4. Redefine PDFs: MC PDF

Recipe: Make convolution of the LO PDF in $\overline{\mathrm{MS}}$ (q and $\bar{q}$) with the difference of collinear counterterms in $\overline{\mathrm{MS}}$ and MC schemes:

$$f^{\mathrm{MC}}_{q(\bar{q})}(x, Q^2) = f^{\overline{\mathrm{MS}}}_{q(\bar{q})}(x, Q^2) + \int_x^1 \frac{dz}{z}\, f^{\overline{\mathrm{MS}}}_{q(\bar{q})}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z) + \int_x^1 \frac{dz}{z}\, f^{\overline{\mathrm{MS}}}_{g}\left(\frac{x}{z}, Q^2\right) \Delta C_{2g}(z)$$

where

$$\Delta C_{2q}(z) = C^{\overline{\mathrm{MS}}}_{2q}(z) - C^{\mathrm{MC}}_{2q}(z) = \frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln\frac{(1-z)^2}{z} + 1 - z\right]_+$$

$$\Delta C_{2g}(z) = C^{\overline{\mathrm{MS}}}_{2g}(z) - C^{\mathrm{MC}}_{2g}(z) = \frac{\alpha_s}{2\pi}\, T_R \left\{\left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z)\right\}$$

The formula is valid for any process up to $\mathcal{O}(\alpha_s^2)$.

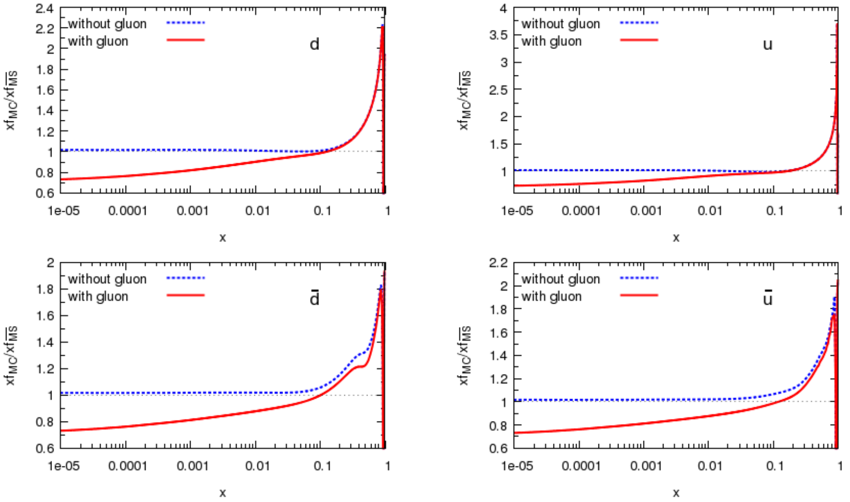
The gluon PDF for DY: $f^{\mathrm{MC}}_g(x, Q^2) = f^{\overline{\mathrm{MS}}}_g(x, Q^2)$

Notes:

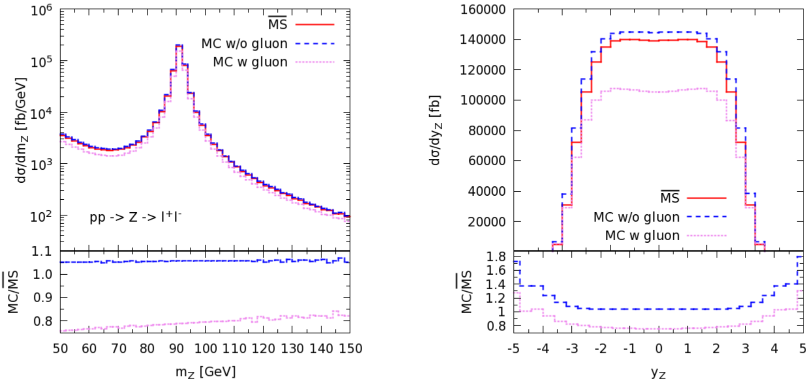
- The MC scheme has been validated by reproducing the scheme-independent relations between DY and DIS. [S. Jadach at al. Phys.Rev. D87]
- LHAPDF grid (easy to use by all PS MC) for the MC PDF. (As a source we used MSTW2008lo, other $\overline{\mathrm{MS}}$ PDF possible).

MC PDFs

- Ratios with respect to standard $\overline{\text{MS}}$ PDFs for light quarks ($Q^2 = 100$ GeV).



$\overline{\text{MS}}$ vs MC at LO



- $+5\%$ effect at central rapidities in $q\bar{q}$ and -20% for both channels
- pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

MCFM $\overline{\text{MS}}$ vs MCFM modified MC scheme at NLO

Fixed order cross-check
(using modified MCFM: using MC PDF and MC C_2)

$$
\begin{aligned}
\sigma_{\rm tot}^{\overline{\rm MS}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\rm MS}}) \otimes f_{\bar q} \\
\sigma_{\rm tot}^{\rm MC} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\rm MC}) \otimes (f_{\bar q} + \alpha_s \Delta f_{\bar q}) \\
&= f_q \otimes f_{\bar q} + \alpha_s \left(\Delta f_q \otimes f_{\bar q} + \Delta f_{\bar q} \otimes f_q + C_q^{\rm MC} \otimes f_q \otimes f_{\bar q} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

At $\mathcal{O}(\alpha_s)$:

$$
C_q^{\overline{\rm MS}} \otimes f_q \otimes f_{\bar q} = \Delta f_q \otimes f_{\bar q} + \Delta f_{\bar q} \otimes f_q + C_q^{\rm MC} \otimes f_q \otimes f_{\bar q}
$$

Drell-Yan, $q\bar q$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$
(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}
$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

$\hookrightarrow$ Identical validation performed with both $q\bar q$ and qg channels.

5. Virtual+soft correction, Δ_{S+V}

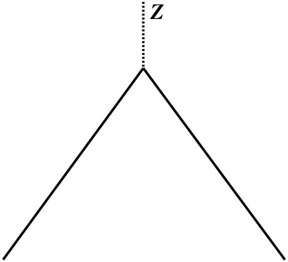
Virtual + soft:

$$
\begin{aligned}
W_{V+S}^{q\bar{q}} &= \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2}\right] \\
W_{V+S}^{qg} &= 0
\end{aligned}
$$

Notes:

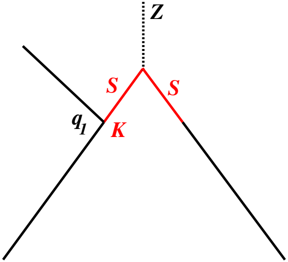
- Simple, kinematics independent!
- No need to generate strictly collinear contributions (like $d\Sigma^{c\pm}$ terms in MC@NLO).

Upgrading to NLO: the hardest emission



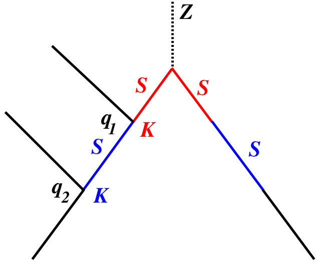
$$\sigma^{\text{LO}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



$$\sigma_{1+}^{\mathrm{PS}} = \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2) + S_\ominus(Q^2, q_1^2) K_\ominus(q_1^2, z_1) S_\oplus(Q^2, q_1^2) \Big\}$$

Upgrading to NLO: the hardest emission



$$\sigma_{2+}^{PS} = \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$

$$\otimes\Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2)$$

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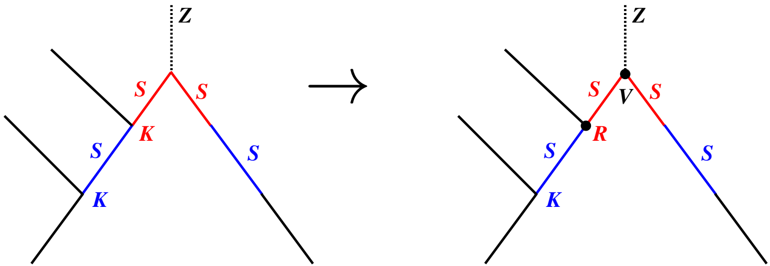
$$+ S_\ominus(Q^2, q_1^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(Q^2, q_1^2)$$

$$\otimes\Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\}\Big\}$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{NLO+PS}} \quad = \quad & \sigma_B\,(1+V)\otimes D_\oplus(Q^2,x_\oplus)\otimes D_\ominus(Q^2,x_\ominus) \\
& \otimes\Big\{ S_\oplus(Q^2,q_1^2)K_\oplus(q_1^2,z_1)S_\ominus(Q^2,q_1^2)\,R_\oplus(q_1^2,z_1)/K_\oplus(q_1^2,z_1) \\
& \quad \otimes\Big\{S_\oplus(q_2^2,q_1^2)K_\oplus(q_2^2,z_2)S_\ominus(q_2^2,q_1^2)+S_\oplus(q_2^2,q_1^2)K_\ominus(q_2^2,z_2)S_\ominus(q_2^2,q_1^2)\Big\} \\
& \quad + S_\ominus(Q^2,q_1^2)\otimes K_\ominus(q_1^2,z_1)\otimes S_\oplus(Q^2,q_1^2)\,R_\ominus(q_1^2,z_1)/K_\ominus(q_1^2,z_1) \\
& \quad \otimes\Big\{S_\oplus(q_2^2,q_1^2)K_\oplus(q_2^2,z_2)S_\ominus(q_2^2,q_1^2)+S_\oplus(q_2^2,q_1^2)K_\ominus(q_2^2,z_2)S_\ominus(q_2^2,q_1^2)\Big\}\Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission

Steps:

1. Run LO PS[1] (Herwig/Sherpa) using MC PDF (via LHAPDF interface)
2. Get and an event record (for example in the HepMC format).

```
GenEvent: #8 ID=0 SignalProcessGenVertex Barcode: 0
 Momenutm units:     GEV     Position units:      MM
 Cross Section: 697.653 +/- 206.627
 Entries this event: 1 vertices, 5 particles.
 Beam Particles are not defined.
 RndmState(0)=
 Wgts(9)=(0,3023.17) (1,0.17886) (2,3023.17) (3,9) (4,0) (5,1.14371) (6,0) (7,1) (8,1)
 EventScale -1 [energy]          alphaQCD=0.139387       alphaQED=-1
                                    GenParticle Legend
        Barcode   PDG ID      ( Px,       Py,       Pz,     E ) Stat  DecayVtx
________________________________________________________________________________
GenVertex:       -1 ID:    0 (X,cT):0
 I: 2     10001        1 +0.00e+00,+0.00e+00,+6.26e+02,+6.26e+02   2        -1
          10002       21 +0.00e+00,+0.00e+00,-1.84e+01,+1.84e+01   2        -1
 O: 3     10003        1 -1.82e+00,+5.68e-01,-1.50e+01,+1.51e+01   1
          10004       11 +2.58e+01,+9.16e+00,+5.71e+02,+5.71e+02   1
          10005      -11 -2.40e+01,-9.73e+00,+5.17e+01,+5.78e+01   1
```

3. Book histograms (for example using Rivet) with MC weight calculated from the event record (and information on α_s).

It is almost as fast as LO+PS calculation!

[1] Cover full Phase Space.

Results

NLO+PS results

KrkNLO

- Virtual: $\mu^2 = \mu_F^2 = \mu_R^2 = m_Z^2$
- Real: two choices
  - $\mu^2 = m_Z^2$
  - $\mu^2 = q^2$

  $\hookrightarrow$ differences formally beyond NLO, indicative of missing higher orders

Compared to:

- **MCFM**: pure NLO, $\mu^2 = m_Z^2$
- **MC@NLO**: from Sherpa, with the evolution variable q^2
- **POWHEG**: from Herwig 7, with the evolution variable k_T^2

Matched results: total cross section

$q\bar{q}$ channel

	$\sigma_{\text{tot}}^{q\bar{q}}$ [pb]
MCFM	1273.4 ± 0.1
MC@NLO	1273.4 ± 0.1
POWHEG	1272.1 ± 0.7
KrkNLO $\alpha_s(q^2)$	1282.6 ± 0.2
KrkNLO $\alpha_s(M_Z^2)$	1285.3 ± 0.2

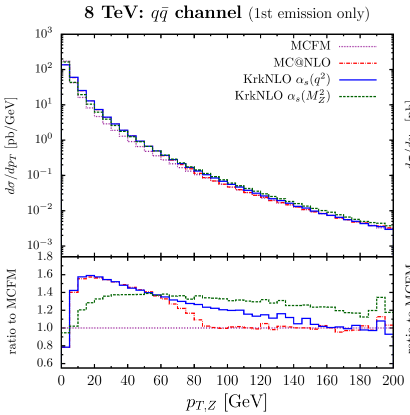
- sub-percent differences from beyond-NLO terms in the KrkNLO result (MC PDFs, mixed real-virtual)
- negligible difference between fixed and running coupling

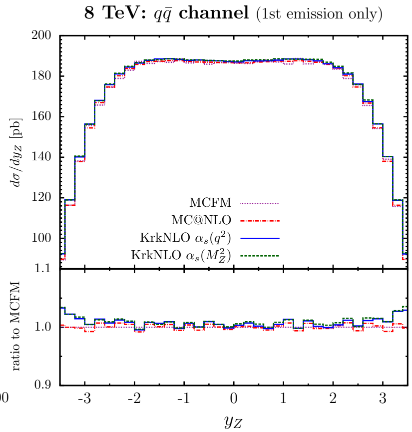
$q\bar{q} + qg$ channels

	$\sigma_{\text{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

- beyond-NLO terms reach up to 4% in the KrkNLO result
  $\hookrightarrow$ resulting from large gluon luminosity leading to $f^{\text{MC}}/f^{\overline{\text{MS}}} < 1$
- small differences between fixed and running coupling choices

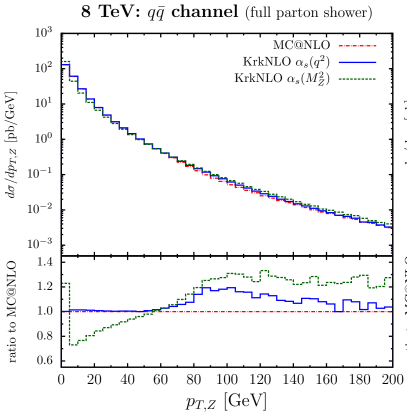
Matched results: $q\bar{q}$, 1st emission

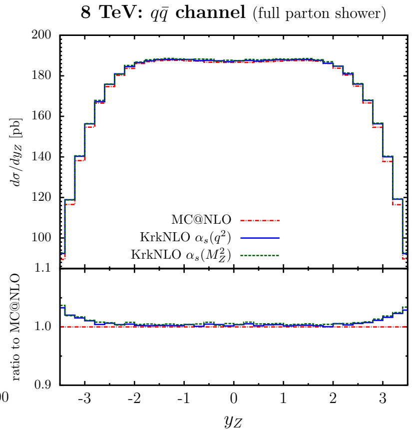




- Reproduction of y_Z distribution at NLO.
- Agreement of KrkNLO $\alpha_s(q^2)$ with MC@NLO at low $p_{T,Z}$: PS domination
- KrkNLO results above MC@NLO and MCFM at higher $p_{T,Z}$: $\mathcal{O}(\alpha_s^2)$ terms

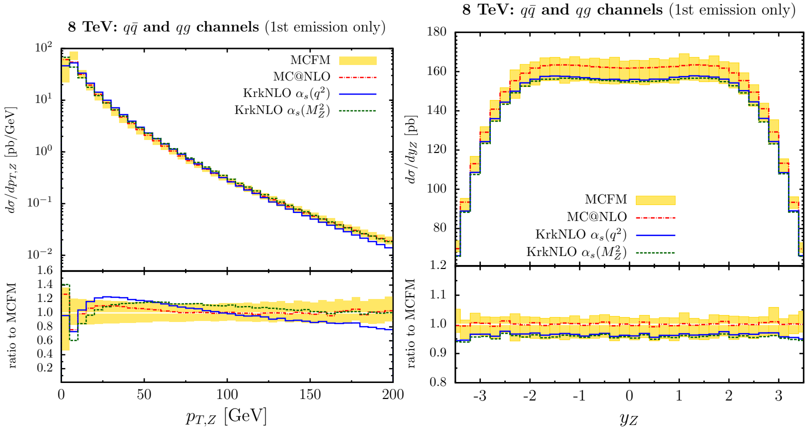
Matched results: $q\bar{q}$, full PS





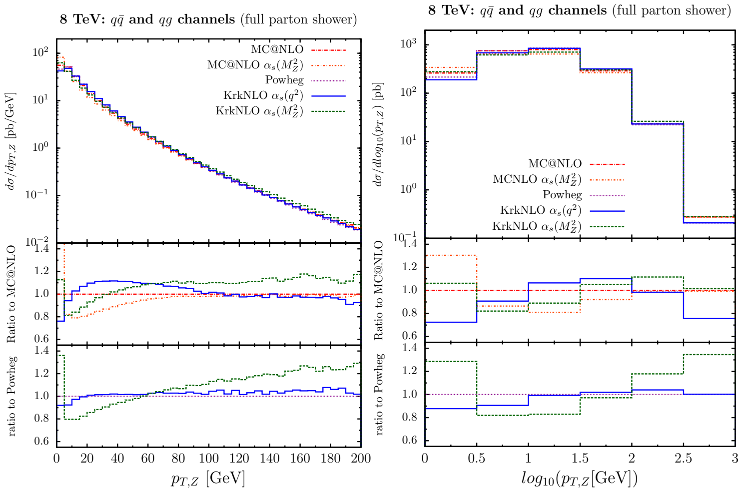
- Low $p_{T,Z}$ part of the spectrum changes but KrkNLO $\alpha_s(q^2)$ with MC@NLO agree there because of shower domination
- KrkNLO results above pure NLO at high $p_{T,Z}$: admixture of NNLO terms
- Diffs between two KrkNLO result at high $p_{T,Z}$: running coupling effects

Matched results: botch channels, 1st emission



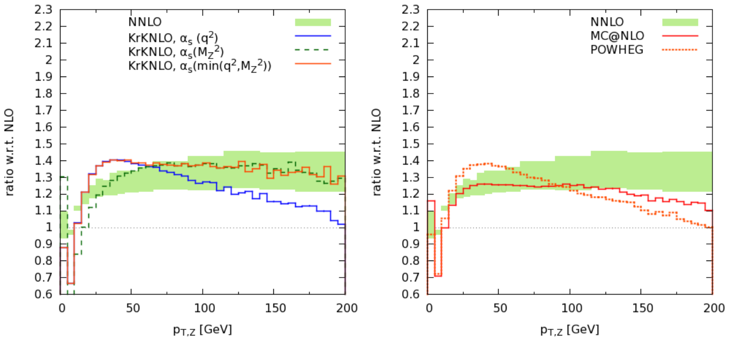
- MCFM band is an uncertainty estimate obtained by independent variation of μ_F and μ_R by a factor 1/2 and 2
- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

Matched results: both channels, full PS



- KrkNLO $\alpha_s(q^2)$ stays overall very close to MC@NLO
- KrkNLO $\alpha_s(q^2)$ almost coincides with POWHEG $p_{T,Z}$ distributions

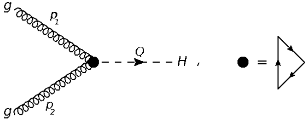
Comparison with fixed order NNLO results (DYNNLO)



- DYNNLO green band is an uncertainty estimate obtained by independent variation of μ_F and μ_R by a factor 1/2 and 2
- KrkNLO $\alpha_s(min(q^2, M_Z))$ and NNLO results show the same trends (left).
- Similar comparisons for POWHEG and MCatNLO are also shown (right).

Near Future

KrkNLO for Higgs-boson production in gluon–gluon fusion



As expected we get simple weights:

1. $g + g \longrightarrow H + g$:

$$W_R^{gg}(\alpha, \beta) = \frac{1 + z^4 + \alpha^4 + \beta^4}{1 + z^4 + (1-z)^4} \quad (2)$$

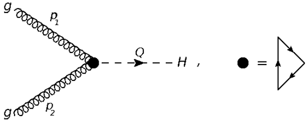
2. $g + q \longrightarrow H + q$:

$$W_R^{gq}(\alpha, \beta) = \frac{1 + \beta^2}{1 + (1-z)^2} \quad (3)$$

and for the process with exchanged initial-state partons we have: $W_R^{qg}(\alpha, \beta) = W_R^{gq}(\beta, \alpha)$.

Near Future

KrkNLO for Higgs-boson production in gluon–gluon fusion



Full definition (including gluon) PDFs in the MC and the $\overline{\mathrm{MS}}$ factorisation schemes:

$$g_{\mathrm{MC}}(x, Q^2) = g_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, g_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2gg}(z) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2gq}(z),$$

A similar relation for the quark (antiquark) PDFs reads

$$q_{\mathrm{MC}}(x, Q^2) = q_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q\bar{q}}(z) + \int_x^1 \frac{dz}{z}\, g_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2qg}(z),$$

- In the format of LHAPDF6 obtained from different MS bar PDF sets.

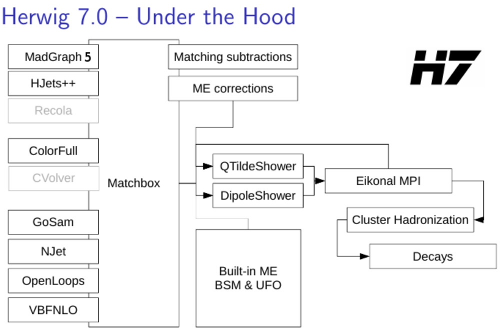
Near Future

Implementation of KrkNLO in Herwig 7

"Herwig 7.0 / Herwig++ 3.0 Release Note", arXiv:1512.01178

J. Bellm, S. Gieseke, D. Grellscheid, S Platzer, M. Rauch, Ch. Reuschle, P. Richardson, P. Schichtel, M. H. Seymour, AS, A Wilcock, N. Fischer, M. A. Harrendorf, G. Nail, A. Papaefstathiou, D. Rauch

- NLO matched to parton showers as default for the hard process.
- Two showers: Angular-ordered and dipole shower.
- Two matching algorithms: Subtractive (MC@NLO-type) and multiplicative (Powheg-type) matching.
- Vastly improved documentation, usage and installation + new tunes.
- and much more ...



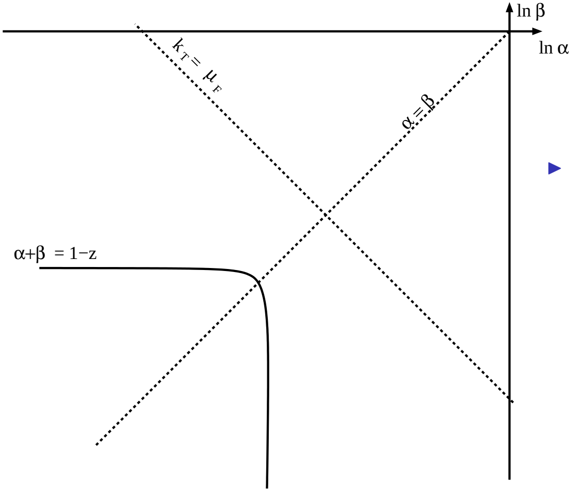
Conclusions

- I have discussed a method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented on top of Catani-Seymour shower.
- It has been validated against fixed order NLO for Drell-Yan process.
- First comparisons to MC@NLO and POWHEG.
- KrkNLO $\alpha_s(min(q^2, M_Z))$ and NNLO results show the same trends.

Near future: Higgs production, full definition of the MC pdf in LHAPDF6 format, public version implemented in Herwig 7, diboson production, correction of n emissions.
Next: work on extension of the method to NNLO+NLO PS.

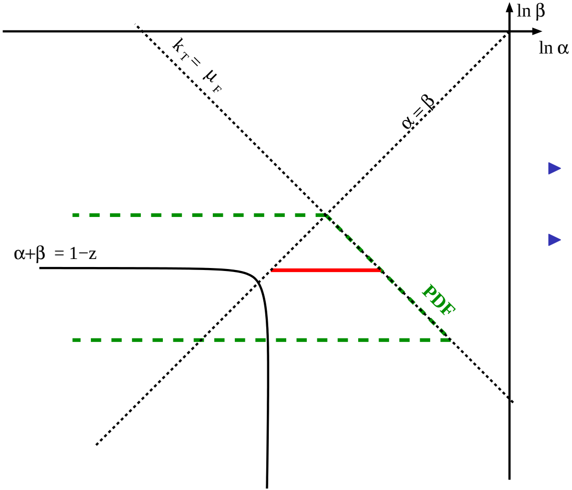
Thank you for the attention!

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\mathrm{MS}}$



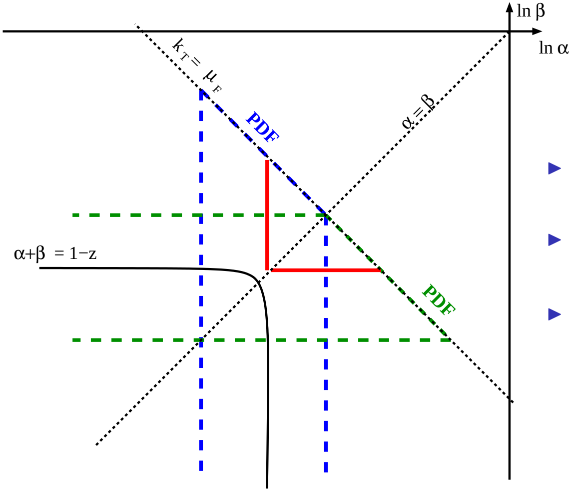
- Integration extends up to a fixed $k_T = \mu_F$.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
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- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

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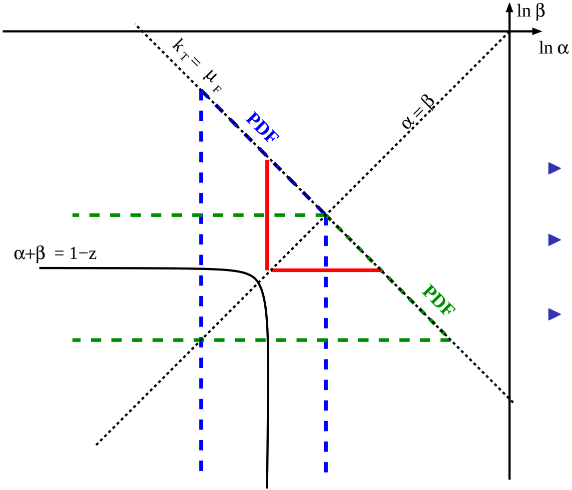

- Integration extends up to a fixed $k_T = \mu_F$.
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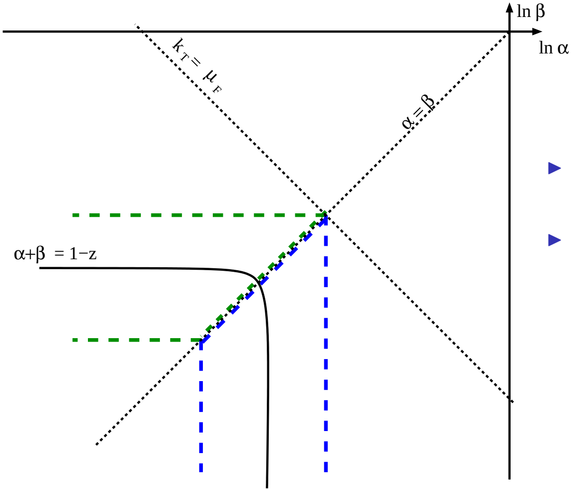
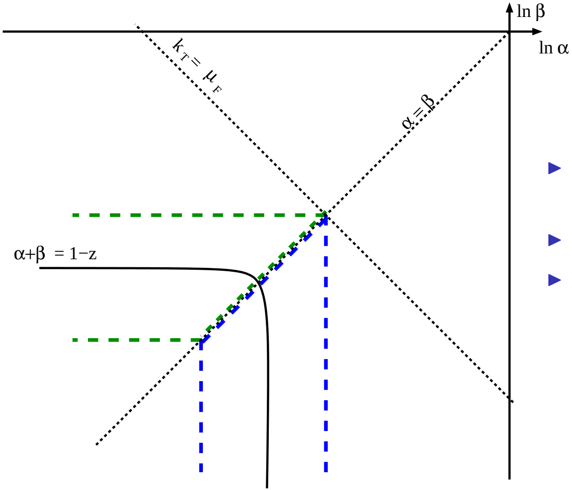
Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



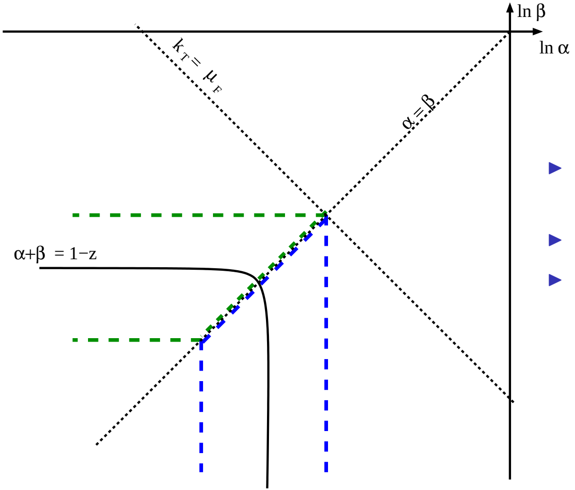
- Integration in angle rather than k_T.
- No overcounting.

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Could the change of factorization scheme help us to simplify NLO+PS matching?

More on Δ_{V+S} virtual+soft correction

$$\Delta_{V+S} = D_{DY}^{\overline{MS}}(z) - 2C_{ct}^{psMC}(z)$$

where we use $\overline{MS}$ results, eq. (89) in Altarelli+Ellis+Martinelli (1979):

$$D_{DY}^{\overline{MS}}(z), = \delta(1-z) + \delta(1-z)\frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) +$$
$$+ 2\frac{C_F\alpha_s}{\pi}\left(\frac{\hat{s}}{\mu^2}\right)^{\varepsilon}\left(\frac{\bar{P}(z)}{1-z}\right)_+\left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + [2\ln(1-z) - \ln z]\right)$$

and collinear counterterm of psMC (one gluon in psMC in $d = 4 + 2\varepsilon$):

$$C_{ct}^{psMC}(z) = \frac{C_F\alpha_s}{\pi}\int_{\beta<\alpha}\frac{d\alpha d\beta}{\alpha\beta}\int d\Omega_{1+2\varepsilon}\left(\frac{s\alpha\beta}{\mu_F^2}\right)^{\varepsilon}\bar{P}(1-\alpha,\varepsilon)\delta_{1-z=\alpha} =$$
$$= \frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}'(z,\varepsilon)}{1-z}\right)_+\left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + \ln\frac{s}{\mu_F^2}\right),$$
$$\bar{P}'(z,\varepsilon) = \bar{P}(z) + \frac{1}{2}\varepsilon(1-z)^2 + \varepsilon\ln(1-z).$$

NLO Monte Carlo weight
This is Yennie-Frautschi-Suura (YFS) style!

Once LO MC is re-designed, introduction of the complete NLO to hard process part is done with help of simple positive MC weight:

$$W^{NLO}_{MC} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

where the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F,\hat{p}_B;q_1,q_2,k) = \left[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{B2})\right]$$
$$-\,\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\,\frac{1}{2}$$

Next slide more on Δ_{V+S}.

Notation: CS parton shower

The "Sudakov" form factor

$$S(Q^2, \Lambda^2, x) = \int\limits_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int\limits_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \;\frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Convolution:

$$(f \otimes g)(x) \equiv \int_0^1 dx_1 \int_0^1 dx_2\ \delta(x - x_1 x_2)\ f(x_1) f(x_2). \tag{4}$$

Eliminating x_2 and delta function we obtain[2]

$$(f \otimes g)(x) \equiv \int_x^1 \frac{dx_1}{x_1}\ f(x_1) f(x/x_1). \tag{5}$$

$$C(z) = \tilde{C}(z) + \{\Delta C(z)\}_+. \tag{6}$$

$$\begin{aligned}[C \otimes D_1 \otimes D_2](x) &= [\tilde{C} \otimes D_1 \otimes D_2](x) \\ &+ \frac{C_F \alpha_s}{\pi} \Big[\Big(\Big\{\frac{1}{2}\Delta C(z)\Big\}_+ \otimes D_1) \otimes D_2\Big](x) + \frac{C_F \alpha_s}{\pi} \Big[D_1 \otimes \Big(\Big\{\frac{1}{2}\Delta C(z)\Big\}_+ \otimes D_2\Big)\Big](x)\end{aligned} \tag{7}$$

Denoting

$$\begin{aligned}\Delta D(x) &= \frac{C_F \alpha_s}{\pi} \Big[\Big\{\frac{1}{2}\Delta C(z)\Big\}_+ \otimes D\Big](x), \\ \tilde{D}(x) &= D(x) + \Delta D(x),\end{aligned} \tag{8}$$

the above formula can be expressed at the NLO precision level (i.e. dropping NNLO terms) as follows:

$$\begin{aligned}[C \otimes D_1 \otimes D_2](x) &= [\tilde{C} \otimes D_1 \otimes D_2](x) + [\Delta D_1 \otimes D_2](x) + [D_1 \otimes \Delta D_2](x) \\ &= [\tilde{C} \otimes \tilde{D}_1 \otimes \tilde{D}_2](x) + \mathcal{O}(\alpha_s^2).\end{aligned} \tag{9}$$

[2]Note the importance of $x/x_1 < 1$ condition when eliminating delta.