CAN A CHROMOSPHERE BE FORMED IN BLACK HOLE DECAYS?

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BLACK HOLES IN 4D SPACE-TIME

MacGibbon Carr & Page PRD 78, 064043 (2008)

Page Carr & MacGibbon PRD 78, 064044 (2008)

MOTIVATION

Heckler Model

A.F.Heckler PRD 55, 480 (1997); A.F.Heckler PRL 78, 3430 (1997)

- **QED/QCD bremsstrahlung and pair-production interactions between Hawking-radiated particles form photosphere/chromosphere**

Other 4D Photosphere/Chromosphere Models

- **Belyanin et al**
- **Bugaev et al**
- **D. Cline and Hong**
- **Kapusta and Daghigh**

BLACK HOLE THERMODYNAMICS

HAWKING TEMPERATURE:

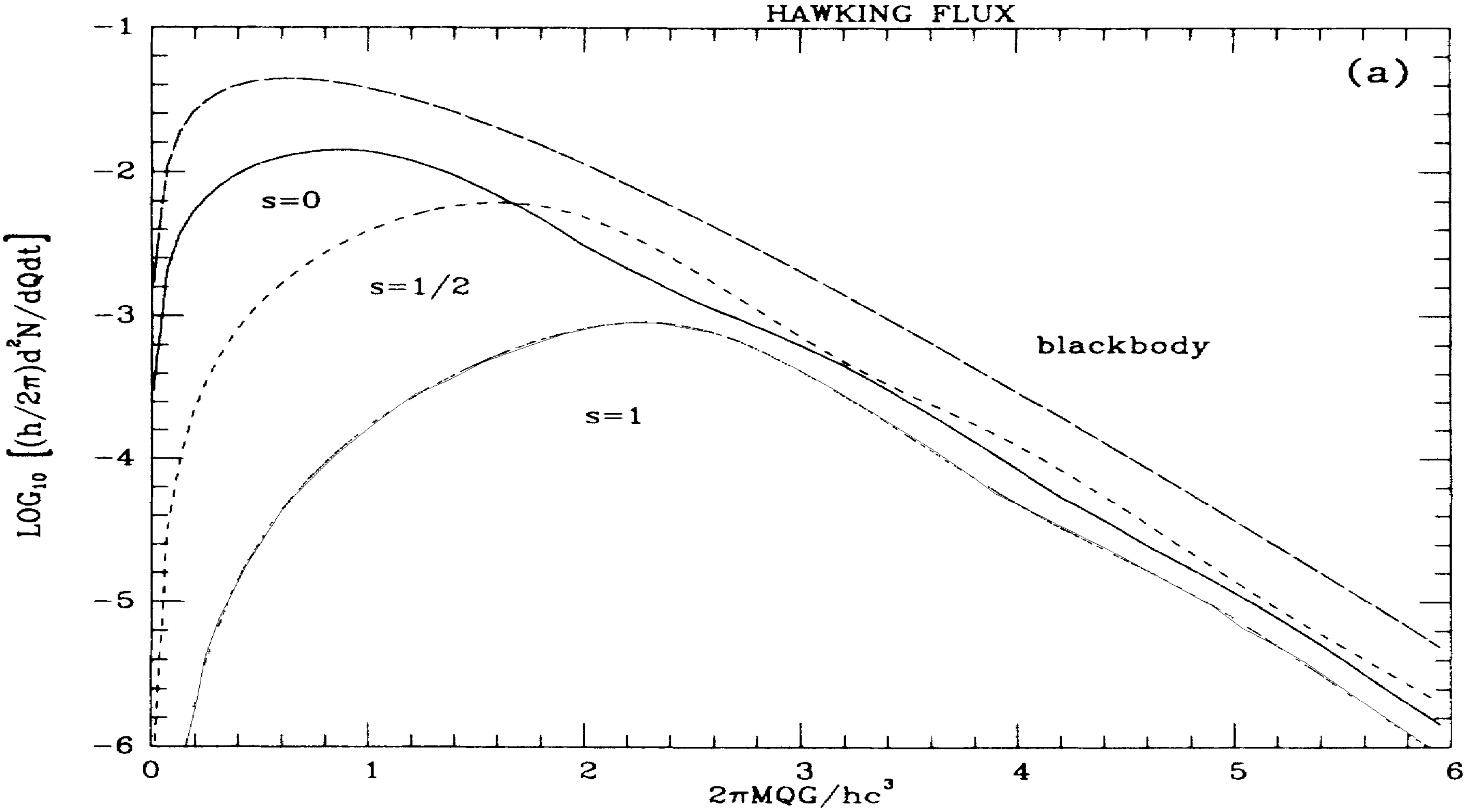
$$kT_{BH} = \frac{\hbar c^3}{8\pi G M_{BH}} = 1.06 \left(\frac{M_{BH}}{10^{13}\, g} \right) \text{GeV}$$

Solar Mass BH $T_{BH} \sim 10^{-7}\, K$ $M_{BH} \sim 10^{25}\, g$ $T_{BH} \sim 3\, K$ **CMB**

HAWKING RADIATION FLUX:

$$\frac{d^2 N_S}{dt\; dE} = \sum_{n,l} \frac{\Gamma_{snl}}{2\pi\hbar} \left[\exp\left[\frac{E - n\hbar\Omega - e\Phi}{\hbar\kappa / 2\pi c} \right] - (-1)^{2s} \right]^{-1}$$

4D HAWKING RADIATION



Sources: Page, Elster, Simkins

STANDARD PICTURE
(MacGibbon-Webber)

BH should directly Hawking evaporate those particles which appear non-composite compared to wavelength of the radiated energy (or equivalently BH size) at given T_{BH}

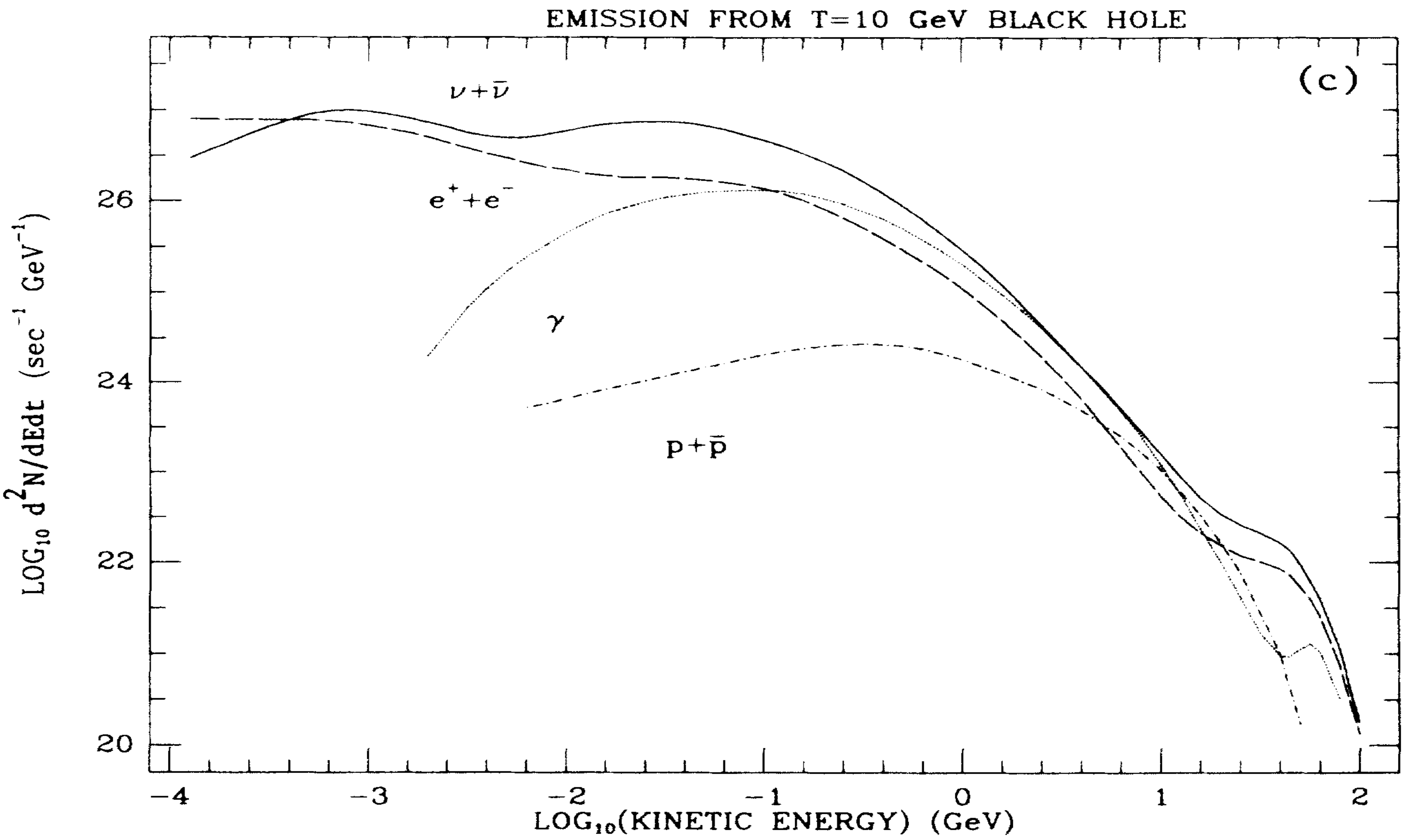
As T_{BH} increases:

BH directly emits photons + gravitons → + neutrinoes → + electrons → + muons → + pions

Once $T_{BH} >> \Lambda_{QCD}$:

BH directly emits quarks and gluons (not direct pions) which shower and hadronize into astrophysically stable γ, ν, p, pbar, e^-, e^+

4D HAWKING RADIATION



Source: MacGibbon and Webber (1990)

TOTAL BLACK HOLE EMISSION

MASS LOSS RATE: $\frac{dM_{BH}}{dt} \approx -5 \,\mathrm{x}\, 10^{25} \left(M_{BH} / \mathrm{g}\right)^{-2} f\left(M_{BH}\right) \,\mathrm{g\,s}^{-1}$

BLACK HOLE LIFETIME: $\tau_{evap} = 6.24 \mathrm{x} 10^{-27} M_i^3 f\left(M_i\right)^{-1} \,\mathrm{s}$

Mass of PBH whose lifetime equals age of Universe
(MacGibbon, Carr & Page 2008):

$$M_* \approx 5.00(\pm 0.04) \times 10^{14} \,\mathrm{gm}$$

HECKLER MODEL

Number Density at radius *r* from BH of e^- directly Hawking radiated by BH

$$n_0(r) \approx \frac{10^{-4}}{M_{BH} r^2} \quad \text{where } \hbar = k = G = c = 1$$

Two-body QED bremsstrahlung cross-section in BH center-of-mass frame

$$\sigma_{brem} \approx \frac{8\alpha^3}{m_e^2} \ln \frac{2E}{m_e} \qquad \text{for } e + e \rightarrow e + e + \gamma$$

Plasma mass correction

$$m'_e = \sqrt{m_e^2 + m_{pm}^2} \quad \text{where} \quad m_{pm}^2 \approx \frac{4\pi\alpha n(r)}{E_{av}}$$

Total Number of Scatterings

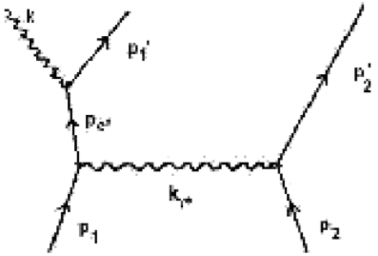
$$N(R) = \int_{r_{\min} = r_{BH}}^{r_{\max} = R} \frac{dr}{\lambda(r)} \quad \text{where } \lambda(r) = \left(n(r)\sigma_{brem} v_{rel}\right)^{-1} \text{ and } n(r) = \left(\frac{3}{2}\right)^{N(r)} n_0(r)$$

→QED Photosphere above T_{BH} ~ 45 GeV. Similarly

QCD $\sigma_{brem} \approx \frac{8\alpha_s^3}{m_q^2} \ln \frac{2E}{m_q}$ **→ Chromosphere above $T_{BH} \sim \Lambda_{QCD}$**

IS THE HECKLER MODEL CORRECT?

QED 3-vertex Bremsstrahlung



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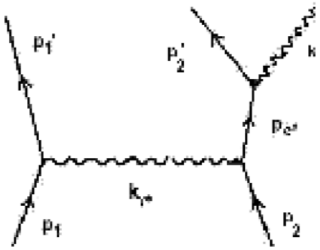

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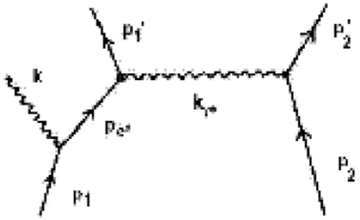
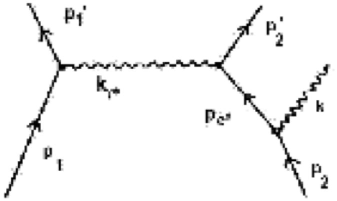


(c)



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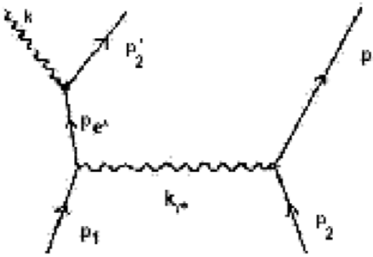

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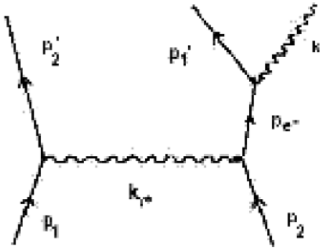

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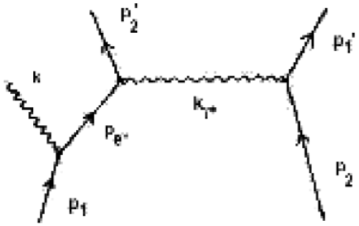

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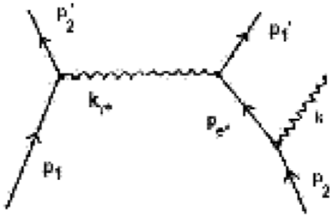


(h)

IS THE HECKLER MODEL CORRECT?

✓ Two-body bremsstrahlung cross-section

$$\sigma_{brem} = \frac{1}{E}\int_0^E \omega \frac{d\sigma_{brem}}{d\omega}\, d\omega \approx \frac{8\alpha^3}{m_e{}^2}\ln\frac{2E}{m_e}$$

Average momentum exchanged is $\sim m_e$ in center-of-momentum (CM) frame → particles must be within $\sim 1/m_e$ of each other to interact

Average angle between final on-shell electron and outgoing photon in CM frame is $\varphi_{av} \sim m_e / 2E$

Average energy of final on-shell electron and outgoing photon in CM frame is $E_e \sim \omega \sim E/2$

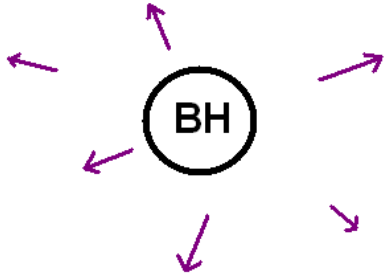
IS THE HECKLER MODEL CORRECT?

✓ Heckler assumes, even after photosphere / chromosphere develops, that most particles are moving radially out from BH

For random walk, particle emitted by T_{BH} ~ **1-10 GeV** BH would have to undergo $N \approx \left(\frac{\theta}{\varphi_{av}}\right)^{2} \approx 10^{7}$ scatterings to deviate $\theta \approx \mathrm{O}(0.1-1)$ from the radial

IS THE HECKLER MODEL CORRECT?

✓ BH is center-of-momentum frame for most pairs of emitted charged particles

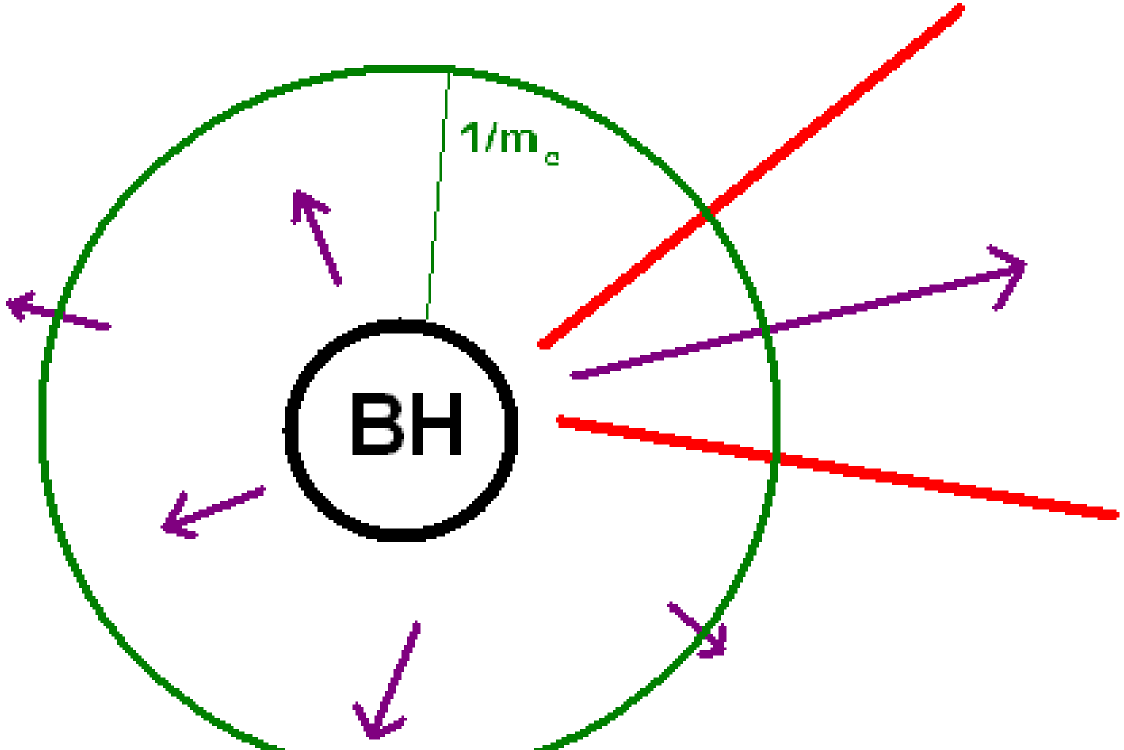


BUT two particles moving in similar direction will not interact near BH (because their center-of-momentum frame is highly Lorentz-boosted) →

✗ ‘Exclusion cone’ around emitted particle → once particle is a distance d from BH the transverse distance to nearest particle for interaction is $x_T \sim d$

→ particles must be within $\sim 1/m_e$ of BH to interact

IS THE HECKLER MODEL CORRECT?



IS THE HECKLER MODEL CORRECT?

✗ For radial emission $\lambda(r)=\left(n(r)\sigma_{brem}v_{rel}\right)^{-1}$ is not correct

→ must be replaced by radial description

✗ particles are Hawking emitted near BH so particles do not travelling in from ***minus*** ∞, past the BH and each other, then out to ***plus*** ∞

BUT bremsstrahlung cross-section assumes interacting particles travel in from ***minus*** ∞

→ interaction cross-section is decreased

IS THE HECKLER MODEL CORRECT?

✗ **Causality Constraint**

Two particles must be in casual contact to interact BUT negligible fraction of Hawking emitted particles are in causal contact with each other

Time between subsequent Hawking emissions is

$$\Delta t_e \sim 200 / E_{peak}$$

For causal contact within $\sim 1 / m_e$ of BH require

$$\Delta t_e < \Delta t_c \sim 1 / \gamma \, m_e \text{ where } \gamma \sim E_{peak} / m_e$$

→ $\Delta t_c \ll \Delta t_e$ for almost all emitted particles

IS THE HECKLER MODEL CORRECT?

✕ **Scale for Completion of Interaction**

Heckler assumed distance required for formation of final on-shell electron and outgoing photon is $d_{form} \sim 1/m_e$ in CM frame

BUT average angle between final on-shell electron and photon is $\varphi_{av} \sim m_e / 2E$

so $d_{form} \sim E/m_e^2$ in CM frame

→Electron must travel $d_{form} \sim E/m_e^2$ before it can undergo next on-shell interaction

→ Any multiple interactions of electron within $\sim 1/m_e$ of BH are off-shell interactions and so strongly suppressed by LPM effect

IS THE HECKLER MODEL CORRECT?

- The Heckler QED photosphere model does not work for 4D BHs because it neglects the requirement that the emitted particles must be in causal contact to interact and neglects LPM effects in any (very rare) multiple scatterings

QCD CHROMOSPHERE?

- ✗ when $T_{BH} >> \Lambda_{QCD}$ the causality constraint ($\Delta t_e \sim 20 / E_{peak}$) and LPM suppression in any (rare) multiple scatterings also prevent QCD chromosphere formation for 4D BHs

BUT could a QCD chromosphere form when $T_{BH} \sim \Lambda_{QCD}$?

QCD CHROMOSPHERE WHEN $T_{BH} \sim \Lambda_{QCD}$?

- Hawking emission damped (lower flux and greater Δt between emissions) near rest mass threshold (eg Λ_{QCD}) + low multiplicity per jet near Λ_{QCD}

- Δt between consecutive Hawking emissions increases around Λ_{QCD} → causality constraint is stronger

QCD CHROMOSPHERE WHEN $T_{BH} \sim \Lambda_{QCD}$?

- ✗ e^+e^- accelerator collisions – smooth transition around Λ_{QCD} from direct π regime to quark/gluon mediated regime – sets in when π relativistic i.e. sets in when constituent quarks relativistic

→when BH goes from directly emitting π to directly emitting quarks and gluons and initial quarks and gluons are relativistic

QCD CHROMOSPHERE WHEN $T_{BH} \sim \Lambda_{QCD}$?

- ✗ number of final states from hadronization is limited by available energy ($E \sim \Lambda_{QCD}$ per Hawking emitted particle) + conservation laws → decays produce mainly π (and only a couple of π) around Λ_{QCD} and soft gluon bremsstrahlung is insignificant (because lowest colourless state from g is π)

QCD CHROMOSPHERE WHEN $T_{BH} \sim \Lambda_{QCD}$?

✕ $T_{BH} \sim \Lambda_{QCD}$ BH no simultaneous production of ultrahigh density QCD particles

→ $T_{BH} \sim \Lambda_{QCD}$ 4D BH can **NOT** form quark-gluon plasma

No analogy to RHIC quark-gluon plasma (RHIC ~ 200 GeV per nucleon, gluon-saturated, high baryon/antibaryon asymmetry)

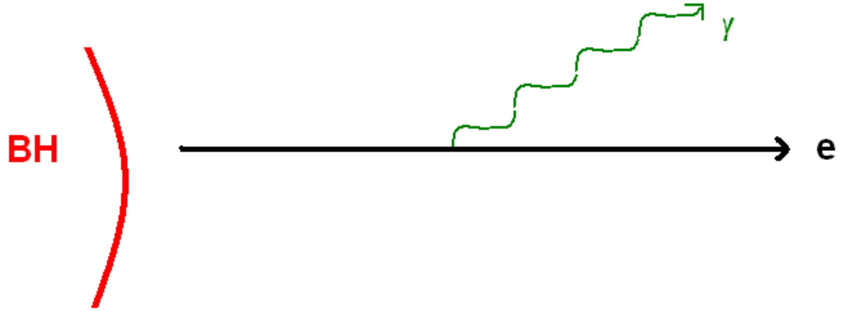
OTHER PHOTOSPHERE/ CHROMOSPHERE MODELS

- **Kapusta and Daghigh** – assumes plasma thermalized by QED and QCD bremsstrahlung and pair-production of Heckler model
- **Belyanin et al** – ‘collisionless’ QED plasma – omits Lorentz factors → no self-induced MHD photosphere but strong ambient magnetic field may induce (weak) photosphere
- **Bugaev et al** – ‘Stretched Horizon’ T_{pl} region just outside horizon → neglects LPM suppression (and thermalization scales)
- **D. Cline and Hong** – Hagedorn-type emission of remaining BH mass into exponentially growing number of states at $\boldsymbol{T_{BH} \sim \Lambda_{QCD}}$ → state occupancy should be determined by available energy $\boldsymbol{E \sim \Lambda_{QCD}}$ → model would require direct coupling of BH mass to Hagedorn states (but $\boldsymbol{T_{BH}}$ increases as $1/M_{BH}^{\ 2}$)

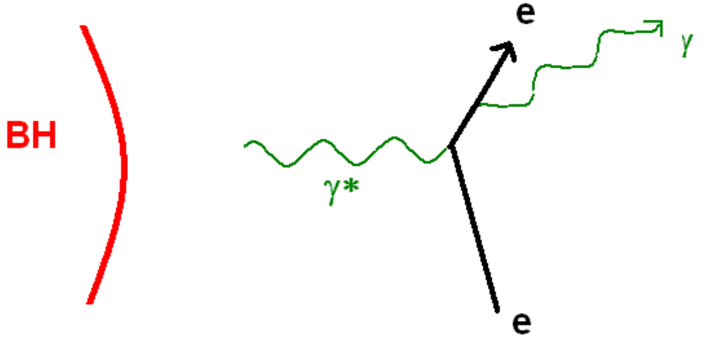
BREMSSTRAHLUNG EFFECTS

(Page, Carr and MacGibbon 2008)

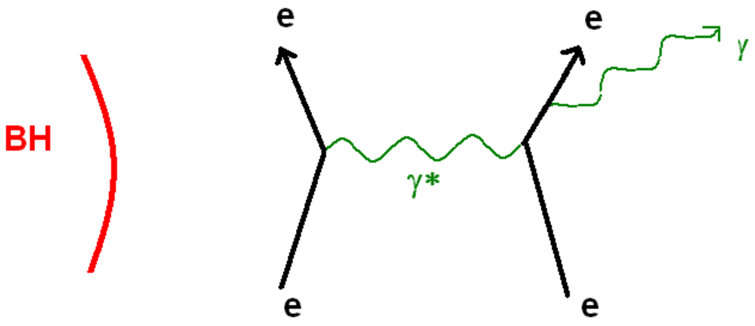
Inner Bremsstrahlung



2-vertex Bremsstrahlung



3-vertex Bremsstrahlung



INNER BREMSSTRAHLUNG

Number flux of inner bremsstrahlung photons radiated by charged particles of mass m and $\gamma_{av} \sim 4.20 T_{BH} / m$ emitted by BH with spectrum dN/dt:

$$\frac{d^2 N_{b\gamma}}{dt\,d\omega} \approx \frac{2\alpha}{\pi\omega}\left[\ln\left(2\gamma_{av}\right)-1\right]\frac{dN}{dt}$$

→ **Nearly flat power spectrum up to $\omega \sim E - m$ cut-off**

Total power in inner bremsstrahlung photons radiated by charged particles emitted by BH with power dE/dt:

$$\frac{dE_{b\gamma}}{dt} \approx \frac{2\alpha}{\pi\omega}\left[\ln\left(2\gamma_{av}\right)-1\right]\frac{dE}{dt}$$

INNER BREMSSTRAHLUNG

Total power in inner bremsstrahlung photons radiated by charged particle emitted by BH with power *dE/dt*:

$$\frac{dE_{b\gamma}}{dt} \approx \frac{2\alpha}{\pi\omega}\left[\ln\left(2\gamma_{av}\right)-1\right]\frac{dE}{dt}$$

Compare with power in direct photons:

$$\frac{dE_{d\gamma}}{dt} \approx 0.3364 \text{ x } 10^{-4} M_{BH}^{\ -2} \quad \text{At low } \omega \to 0, \quad \frac{d^2 E_{d\gamma}}{dt\,d\omega} = \frac{8}{3\pi^2} M^3 \omega^4$$

For $M_{BH} = 5\text{x}10^{14}\ g$ BH, $\frac{d^2 E_{b\gamma}}{dt\,d\omega} \approx 1.73 \text{ x } 10^{-19}\ \text{s}^{-1}$

→ inner bremsstrahlung photons dominate the directly Hawking emitted photons below 57 MeV

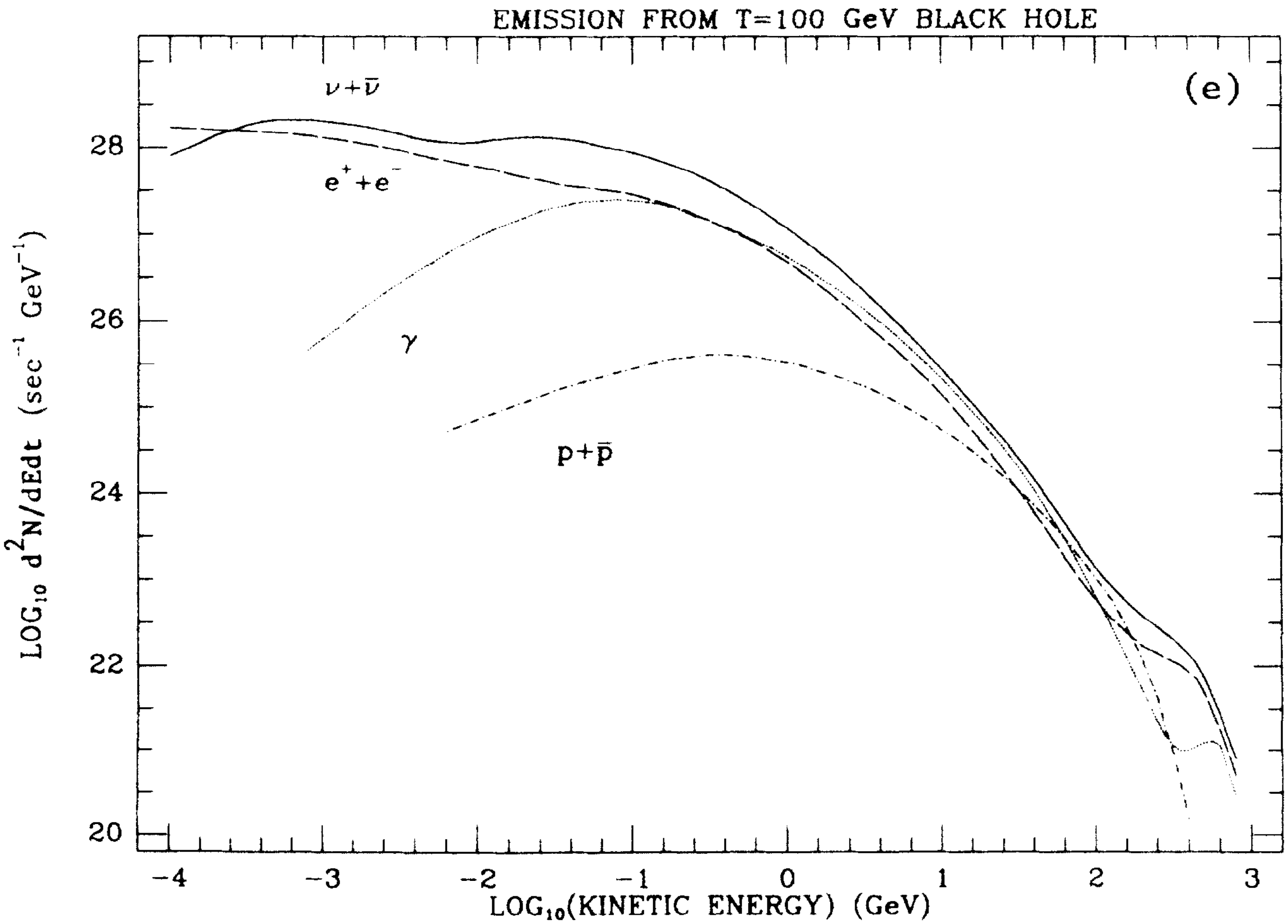
SUMMARY FOR 4D BLACK HOLES

MacGibbon, Carr and Page 2008:

- ✗ None of the photosphere/chromosphere models work because they neglect the requirement that the emitted particles must be in causal contact to interact and/or neglect LPM effects in any multiple scatterings and/or energy constraints

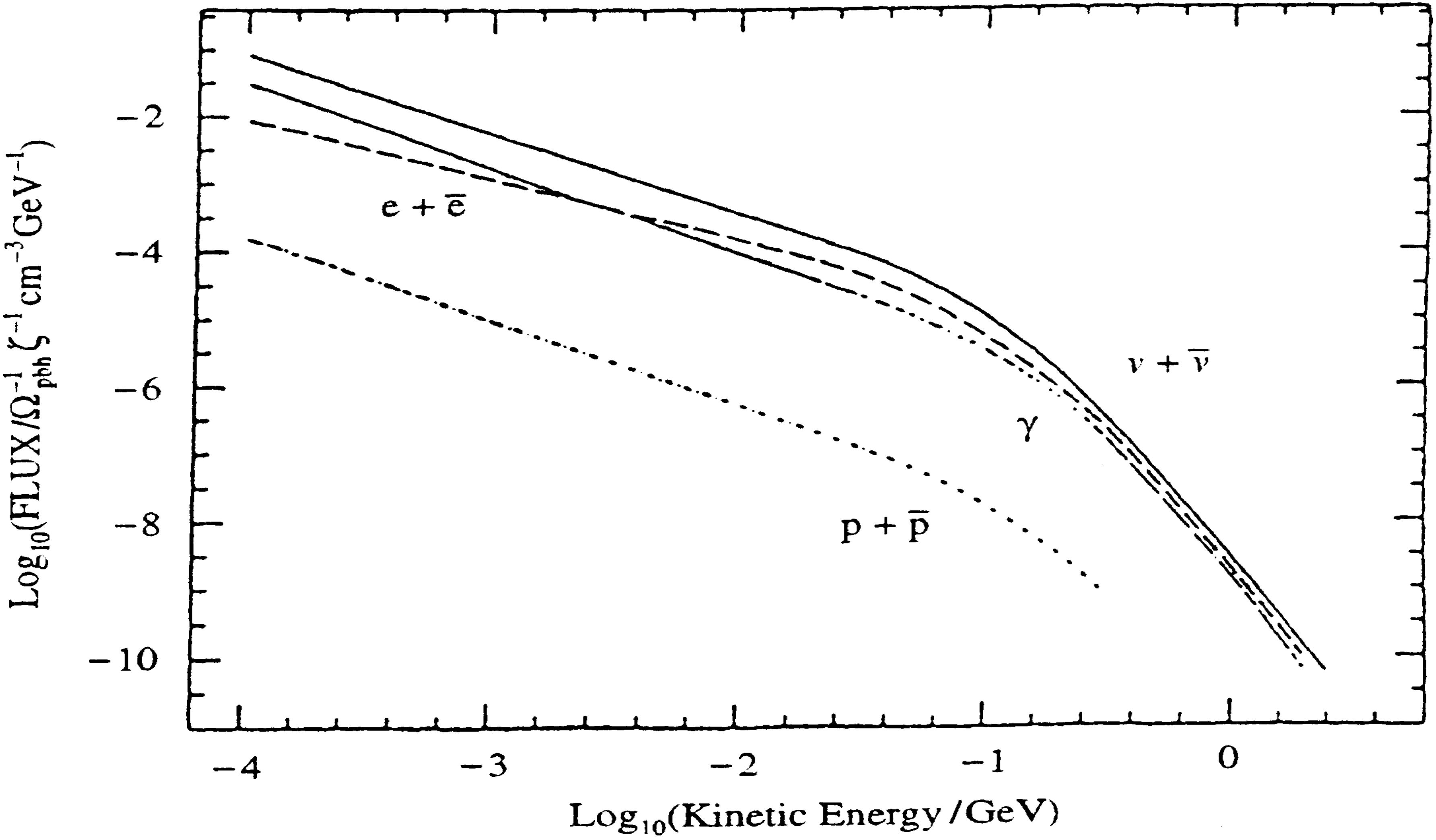
- ✗ Energy and quantum conservation laws prevent significant increase in particle states near $T_{BH} \sim \Lambda_{QCD}$ → no quark-gluon plasma near $T_{BH} \sim \Lambda_{QCD}$

Predicted 4D Black Hole Spectra



Source: MacGibbon and Webber (1990)

Astrophysical Spectra from Uniformly Distributed PBHs with $dn/dM_i \alpha M_i^{-2.5}$



Source: MacGibbon and Carr (1991)

HIGHER-DIMENSIONAL BLACK HOLES

Even one interaction (ie N ~ 1) could modify expected signal compared with experimental precision

Higher-D BHs Simulations:

Draggiotis, Masip and Mastromatteo arXiv:0805.1344v3;

Mastromatteo, Draggiotis and Masip arXiv:0901.0325v2

Alig, Drees and Oda JHEP 0612, 049 (2006)

4+n-DIMENSIONAL BLACK HOLES

$$kT_{BH} = \frac{1+n}{4\pi r_{BH}} \qquad \text{where} \quad r_{BH} = \frac{1}{M_D}\left[\frac{M_{BH}}{M_D}\,\frac{2^n \pi^{(n-3)/2}\,\Gamma\!\left(\frac{3+n}{2}\right)}{n+2}\right]^{1/(1+n)}$$

$$\text{lifetime} \qquad \tau_{BH} \sim \frac{0.2}{M_D}\left(\frac{M_{BH}}{M_D}\right)^{(n+3)/(n+1)} \qquad \text{4D } n = 0$$

Higher Planck scale M_D → higher T_{BH} so fewer, more energetic primary particles emitted over short BH lifetime → expect fewer interactions

HIGHER-DIMENSIONAL BLACK HOLES

Draggiotis, Masip and Mastromatteo

Discussion and simulation of $T_{BH} \sim$ 10 - 100 GeV $M_D \sim$ 1 TeV cosmogenic Higher-D BHs using MCP

For n = 2 - 6

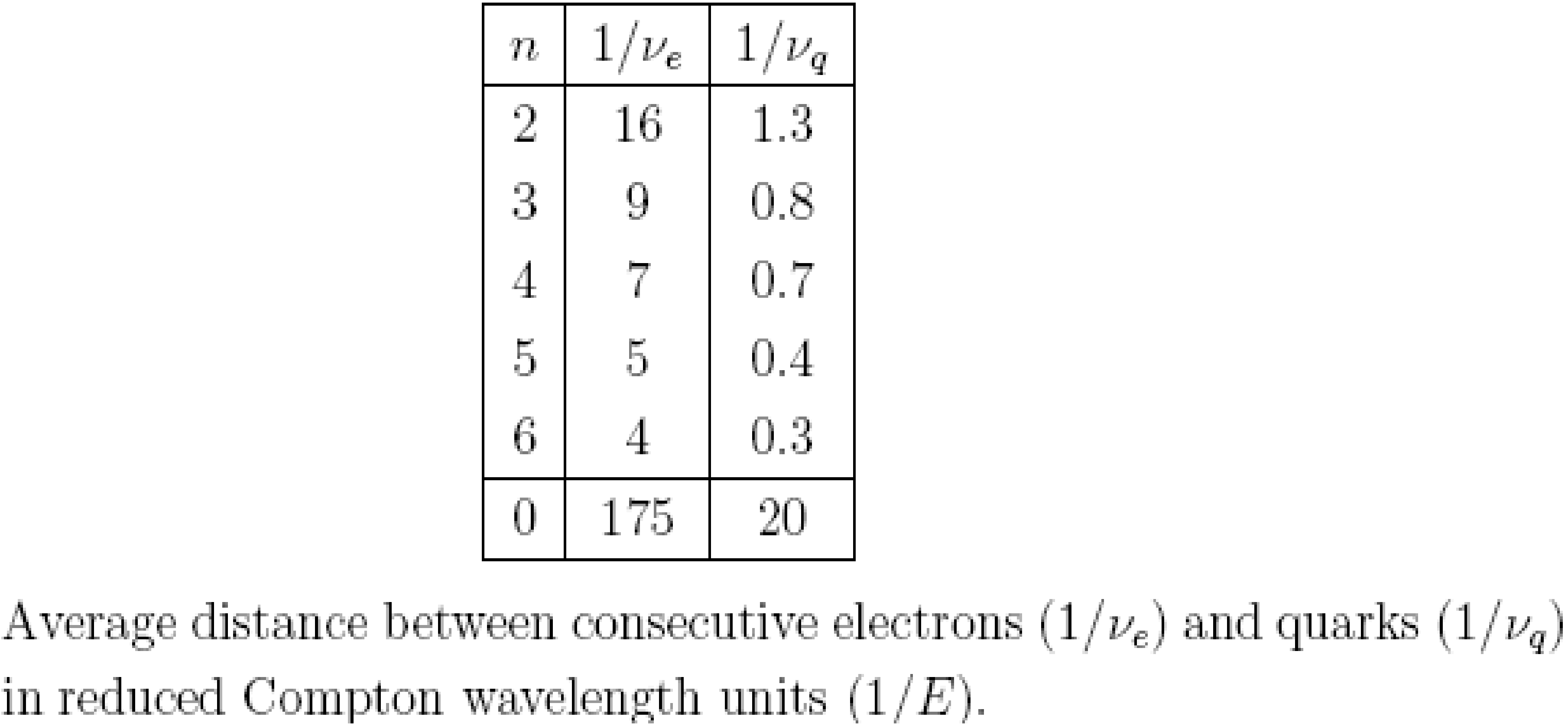
→ $\Delta t_c << \Delta t_e$ for QED so no QED photosphere

→ but $\Delta t_c \sim \Delta t_e$ for QCD. So formation distance argument used to justify no QCD chromosphere

BUT Draggiotis et al MC simulations **assume** no photosphere/chromosphere interactions (don't prove it)

HIGHER-DIMENSIONAL BLACK HOLES

Draggiotis, Masip and Mastromatteo



n	$1/\nu_e$	$1/\nu_q$
2	16	1.3
3	9	0.8
4	7	0.7
5	5	0.4
6	4	0.3
0	175	20

Average distance between consecutive electrons $(1/\nu_e)$ and quarks $(1/\nu_q)$ in reduced Compton wavelength units $(1/E)$.

HIGHER-DIMENSIONAL BLACK HOLES

Alig, Drees and Oda

Discussion and simulation of $n = 6$ $T_{BH} \sim$ 100 GeV $M_D \sim$ 0.65 TeV accelerator Higher-D BHs

includes formation distance constraint and truncation of particle histories

MC simulation of space-time evolution of particle decays (not virtuality in parton showers)

→ jet structure maintained with some soft scattering, no dense chromosphere (doesn’t use Heckler definition of ‘radial’ chromosphere)

BUT cuts off p_T below 1 GeV and only considers pairs of particles which approach each other

HIGHER-DIMENSIONAL BLACK HOLES

QCD PHOTOSPHERE FOR HIGHER-D BHs?

NEED

QCD simulation which includes p_T ~ m$_\pi$ - 1 GeV

and considers all 4π directions for particles

radiated by BH

Even one interaction (ie N ~ 1) per QCD particle could modify expected signal compared with accelerator detector precision. $<E>_{2Q\ \mathrm{jet}} \neq 2<E>_{Q\ \mathrm{jet}}$