AdS_7/CFT_6 anomaly match as a continuum limit

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Gauge theories in $d > 4$: non-renormalizable and strongly coupled in the UV.

For 6d $\mathcal{N} = (1, 0)$ gauge theories, a mixture of string theory and field theory arguments suggest that they can be UV-completed by interacting SCFT's.

The SCFT's are isolated. The gauge theories arise on the moduli space.

The case for a CFT is not very strong:

- there are no scales (but tensionless string excitations)
- the gauge theory can be made anomaly-free.

Today I will focus on a simple but rich class of 6d $\mathcal{N} = (1, 0)$ SCFT's which have a peculiar large N limit and admit AdS_7 dual descriptions.

I will provide a strong test of these dualities by matching the a conformal anomalies computed on the field theory and gravity side in the large N limit.

Plan

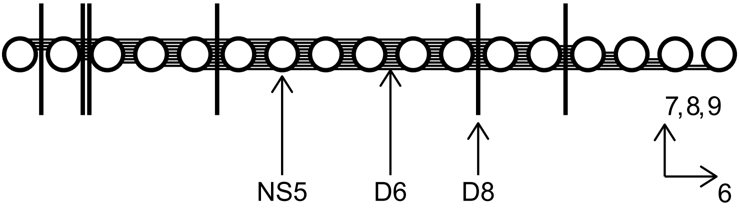
1. The brane construction
2. The field theory on the tensor branch
3. The gravity duals
4. The conformal anomaly a
5. Outlook

The brane construction

The brane construction

	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D6	–	–	–	–	–	–	–			
D8	–	–	–	–	–	–		–	–	–

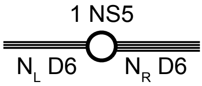
$SO(1,5) \times SO(3)_R$ bosonic symmetry, 8 supercharges: 6d $\mathcal{N} = (1,0)$.



Conservation laws

- **RR D6-brane charge** (in massive IIA) [Hanany, Zaffaroni 97]

$$S_{IIA}^{massive} \supset -\frac{F_0}{2\pi}\int B \wedge F_8 \ , \qquad \frac{F_0}{2\pi} \equiv n_0 \in \mathbb{Z}$$

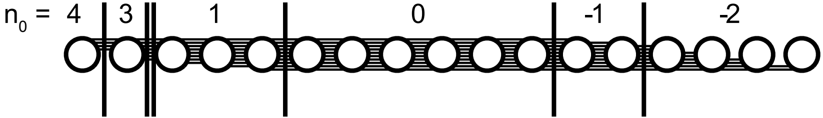


$$N_R - N_L = n_0$$

Bianchi id.:

$$dF_2 = d{*}F_8 = \left(N_L\theta(-x^6) + N_R\theta(x^6)\right)\delta^{(789)} - n_0 H$$

$$0 = d^2F_2 = (N_R - N_L)\delta^{(6789)} - n_0\delta^{(6789)}$$

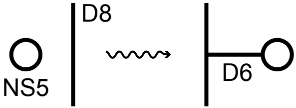


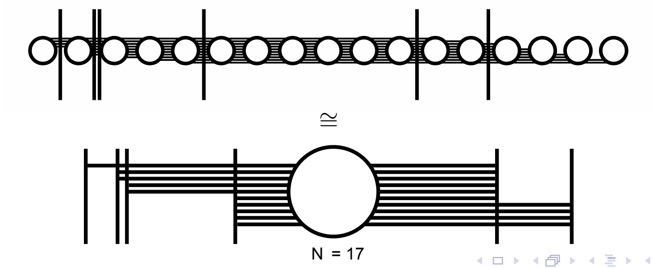
Conservation laws

- **‘Total linking number’ on D8** [Hanany, Witten 96]

$$L_{\mathrm{D8}} = \int_{\mathbb{R}^3_{\mathrm{D8}}} d(B + F^{\mathrm{D8}}_{U(1)}) = \frac{1}{2}(\mathrm{NS}_R - \mathrm{NS}_L) + (\mathrm{D6}^{end}_L - \mathrm{D6}^{end}_R)$$

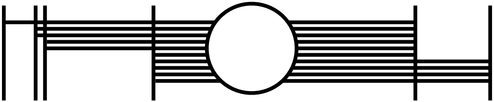
$\Rightarrow$ D6-brane creation effect:





Data of the configuration

[Gaiotto, Tomasiello 14]

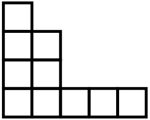


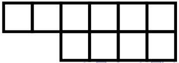
N coincident NS5-branes intersecting k D6-branes ending on D8-branes ($\rho_{L/R}$) $\longleftrightarrow$ 6d $\mathcal{N} = (1,0)$ SCFT $\mathcal{T}^N_{\rho_L,\rho_R}$

$\rho_{L/R}$: partitions of k encoding how the k D6 end on D8's on the left/right.

Nahm pole bc $X^i \sim \frac{\rho(\sigma^i/2)}{x^6 - x^6_{D8}}$ for the $U(k)$ adjoint scalars $X^{1,2,3}$ on the k D6.

[Gaiotto, Witten 08]



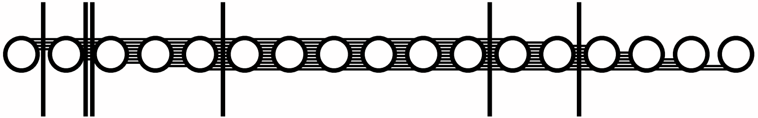


The field theory on the tensor branch

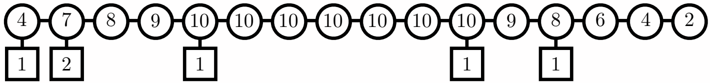
Linear quiver gauge theory on the tensor branch

[Brunner, Karch 97; Hanany, Zaffaroni 97]

Separate the NS5-branes in the 6 direction, have D8 with no D6 attached:



- r_i D6-branes in i^{th} interval: $U(r_i)$ vector multiplet
- f_i D8-branes in i^{th} interval: f_i flavors of fundamental hypermultiplets
- i^{th} NS5-brane: $U(r_{i-1}) \times U(r_i)$ bifundamental hyper
  + (1,0) tensor + (1,0) linear multiplet



NS5-brane worldvolume fields

(1,0) tensor + (1,0) linear associated to NS5-brane are dynamical in 6d.

(1,0) tensor

- Real scalar $\Phi = x^6_{NS5}/(g_s l_s^3)$
- 2-form B^- with ASD field strength
- Symplectic Majorana-Weyl spinor (SMW$^-$)

Dynamical gauge coupling $\frac{1}{g_i^2} = \Phi_{i+1} - \Phi_i$

- The quiver description holds at generic points on the tensor branch ($\Phi_i \neq \Phi_{i+1}$ for all i).
- The CFT sits at the origin of the tensor branch ($\Phi_i = \Phi_{i+1}$ for all i): strong coupling, tensionless strings.

NS5-brane worldvolume fields

(1,0) linear

- $SU(2)_R$ triplet $W^{1,2,3} = x^{7,8,9}_{NS5}/l_s^3$ + singlet $C =$ "x^{10}_{NS5}"$/l_s$ (periodic)
- SMW$^-$ spinor

Dynamical FI parameter $\vec{\xi}_i = \vec{W}_{i+1} - \vec{W}_i$

Dynamical "$\theta_{U(1)}$ angle" $\theta_i = C_{i+1} - C_i$

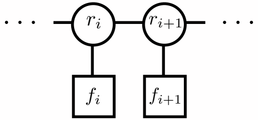
- θ_i are Stückelberg fields that give mass to anomalous gauge $U(1)$'s.
- Linear $L_{i+1} - L_i$ pairs with vector $\mathrm{Tr}(V_i)$ to form long massive vector.

$\hookrightarrow$ $SU(r_i)$ gauge groups at low energies.

Also discard decoupled (1,0) tensor + linear for center of mass of NS5's.

From the quiver to the partitions

Gauge anomaly cancellation for $SU(N_c)$ with N_f fundamentals: $N_f = 2N_c$.
(D6-charge conservation in the brane construction)



$$f_i = -r_{i-1} + 2r_i - r_{i+1} = -(\partial\partial^* r)_i \ ,$$
$$(\partial r)_i \equiv r_{i+1} - r_i$$
$$(\partial^* r)_i \equiv r_i - r_{i-1} \ .$$

$f_i \geq 0 \Longrightarrow r_i$ is a concave function.

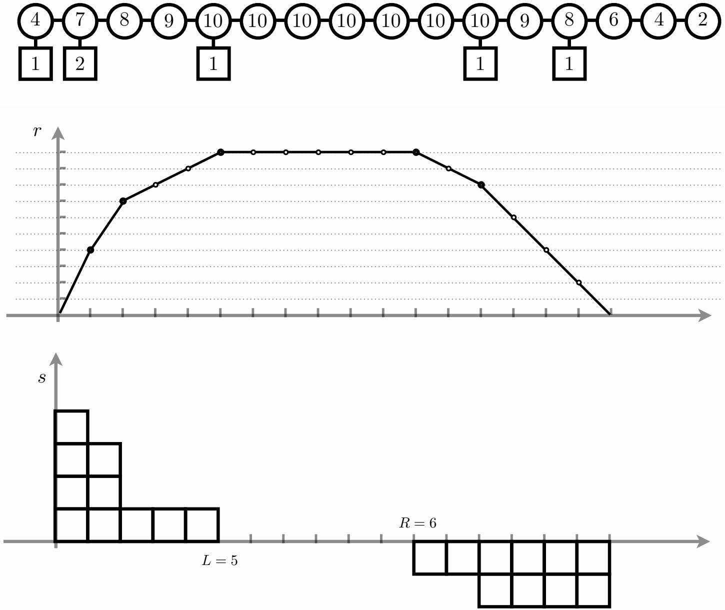
Let $r_0 = r_N = 0$ and introduce the

Slopes $\qquad s_i = r_i - r_{i-1} = (\partial^* r)_i$

so that $\qquad f_i = s_i - s_{i+1} = -(\partial s)_i$.

Let us plot these data...

From the quiver to the partitions



The gravity duals

The gravity duals

[Apruzzi, Fazzi, Rosa, Tomasiello 13; Apruzzi, Fazzi, Passias, Rota, Tomasiello 15]

Warped $AdS_7 \times M_3$ solutions of (massive) IIA supergravity. $M_3 \approx S^3$.

$$ds^2_{10} = e^{2A}\left(ds^2_{\mathrm{AdS}_7} - \frac{1}{16}\frac{\beta' dy^2}{\beta y} + \frac{\beta/4}{4\beta - y\beta'} ds^2_{S^2}\right) , \qquad e^{2A} \equiv \frac{4}{9}\sqrt{-\frac{\beta'}{y}}$$

$$e^{\phi} = \frac{(-\beta'/y)^{5/4}}{12\sqrt{4\beta - y\beta'}}$$

$$F_2 = y\frac{\sqrt{\beta}}{\beta'}\left(4 - \frac{F_0}{18y}\frac{(\beta')^2}{4\beta - y\beta'}\right)\mathrm{vol}_{S^2}$$

$$H = -9\left(-\frac{y}{\beta'}\right)^{1/4}\left(1 + \frac{F_0}{108y}\frac{(\beta')^2}{4\beta - y\beta'}\right)\mathrm{vol}_{M_3} \ .$$

The solution is locally determined in terms of F_0 and the function $\beta(y)$, s.t.

$$q(y) \equiv -4y\frac{\sqrt{\beta}}{\beta'} \qquad \text{obeys} \qquad \partial_y(q^2) = \frac{2}{9}F_0 \ .$$

Massless solutions

$$q(y) \equiv -4y\frac{\sqrt{\beta}}{\beta'} \qquad \text{obeys} \qquad \partial_y(q^2) = \frac{2}{9}F_0 = 0\ .$$

$F_0 = 0$

$$\sqrt{\beta} = \frac{2}{k}(R_0^2 - y^2)$$

- KK reduction of $AdS_7 \times S^4/\mathbb{Z}_k$
- k D6-branes at north pole $y = -R_0$, k $\overline{\text{D6}}$-branes at south pole $y = R_0$.

Massive solutions

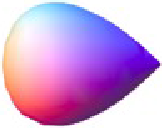
$$q(y) \equiv -4y\frac{\sqrt{\beta}}{\beta'} \qquad \text{obeys} \qquad \partial_y(q^2) = \frac{2}{9}F_0 \neq 0 \,.$$

$F_0 \neq 0$

$$\sqrt{\beta} = -2\int \frac{y dy}{\sqrt{\frac{2}{9}F_0(y-y_0)}} = \sqrt{\frac{8(y-y_0)}{F_0}}(-2y_0 - y) + \sqrt{\beta_0} \,.$$

Set $\beta_0 = 0$ and let $F_0 > 0$, $y_0 < 0$:

- North pole $y = y_0$ is a regular point.
- South pole $y = -2y_0$ is singular: stack of $\overline{\text{D6}}$-branes.



Solutions with D8-branes: *crescent rolls*

Glue local solutions

$$\sqrt{\beta} = \tfrac{2}{k}(R_0^2 - y^2) \qquad (F_0 = 0)$$

$$\sqrt{\beta} = \sqrt{\tfrac{8(y-y_0)}{F_0}}(-2y_0 - y) + \sqrt{\beta_0} \qquad (F_0 \neq 0)$$

by continuity at the locations of D8-branes

$$q|_{\rm D8} = \frac{1}{2}(-n_2 + \mu n_0) \ ,$$

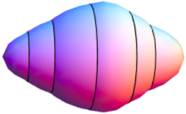
where

$$n_0 \equiv 2\pi F_0 \ , \qquad n_2 = \frac{1}{2\pi}\int_{S^2}(F_2 - BF_0) \ , \qquad \mu = \frac{1}{2\pi}\int_{S^2} F^{\rm D8}_{U(1)} \ .$$

D8 Page charge

(−)D6 Page charge

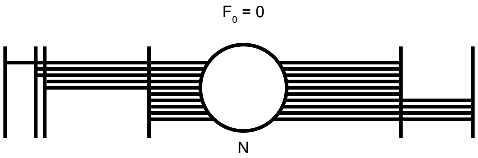
wv magnetic charge (D6-charge of D8)



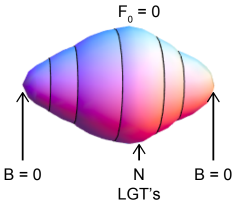
$AdS_7 \times M_3$ backgrounds from near-horizon of NS5–D6 brane intersection:

NS5, D6 on finite intervals $\rightsquigarrow$ fluxes

D8 (, D6 on half-lines) $\rightsquigarrow$ sources



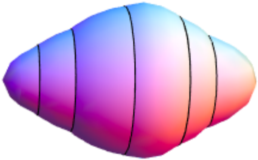
$\longleftrightarrow$



Net number of D6 attached to a D8 $\longleftrightarrow$ D6-charge of a D8

The gauge/gravity correspondence

[Gaiotto, Tomasiello 14]

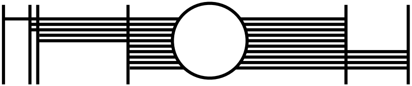


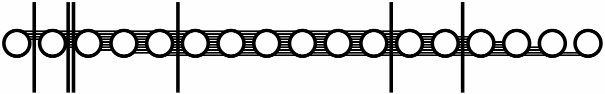
$AdS_7 \times M_3$ solution

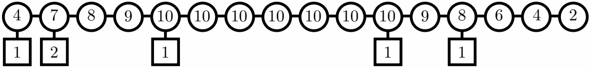
$$N = \frac{1}{4\pi^2}\int_{M_3} H$$

f_i D8's of D6-charge $\mu = \begin{cases} i & \text{(N)} \\ i - N & \text{(S)} \end{cases}$

D6-charge k in massless region







Linear quiver

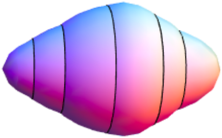
$$G = \times_{i=1}^{N-1} SU(r_i)$$

$$F = \times_{i=1}^{N-1} U(f_i)$$

$$k = \max(r_i)$$

The general solution

No D6 sources: replaced by D8 sources of D6-charge 1 on *small* S^2.



- (D8,D6) Page charges $(n_{0,i}, -n_{2,i})$ between $(i-1)^{th}$ and i^{th} stack of D8:

$$n_{0,i} = s_i = \sum_{j=i}^{L} f_j \ , \qquad -n_{2,i} = \sum_{j=1}^{i-1} j f_j \qquad (N)$$

$$n_{0,i} = s_i = -\sum_{j=N-i}^{R} f_{N-j} \ , \qquad -n_{2,i} = -\sum_{j=1}^{N-i-1} j f_{N-j} \qquad (S)$$

[Notation: i^{th} stack of D8's $:=$ stack of f_i D8's of D6-charge i (or $i-N$).]

The general solution

- $q|_{\text{D8}} = \frac{1}{2}(-n_2 + \mu n_0)$ of i^{th} stack of D8:

$$q_i = \frac{1}{2} r_i$$

Note: $2q|_{\text{D8}}$ is a D6-charge which is *quantized* and *gauge invariant*.

Using $\partial_y(q^2) = \frac{2}{9} F_0$ and $(F_0)_i = \frac{n_{0,i}}{2\pi} = \frac{s_i}{2\pi}$, we obtain

$$q^2(y) = \frac{1}{9\pi} s_{i+1}(y - y_i) + \frac{1}{4} r_i^2 \,, \qquad y_i \leq y \leq y_{i+1}$$

The relative y-positions of the D8-brane stacks are

$$\Delta y_{i+1} \equiv y_{i+1} - y_i = \frac{9}{4}\pi(r_{i+1} + r_i) \qquad \text{(massive)}$$

$$y_{\text{R}} - y_{\text{L}} = \frac{9}{4} k\pi(N - L - R) \qquad \text{(massless)}$$

The integration constants are similarly determined in terms of quiver data.

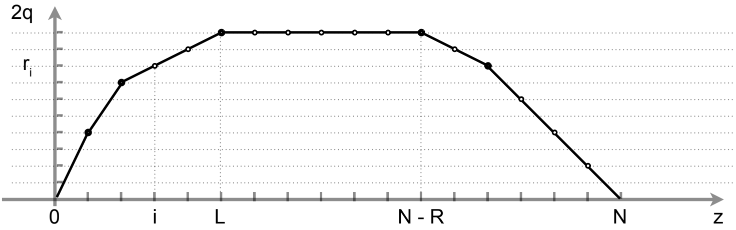
The z coordinate

We noticed that the function $2q$ interpolates the numbers of colors r_i.

Introduce a new coordinate z s.t. $2q(z)$ is piecewise linear with slopes $s_i = n_{0,i}$:

$$\begin{cases} 2dq = n_0 dz \\ 2q dq = \frac{n_0}{9\pi} dy \end{cases} \qquad \Rightarrow \qquad dz = \frac{1}{9\pi q} dy \; .$$

Then by construction the i^{th} D8-brane stack is at $z = i$ and the plot of $2q(z)$ is



The z coordinate

$$q = \frac{1}{9\pi}\partial_z y \,, \qquad\qquad y = -\frac{1}{18\pi}\partial_z\sqrt{\beta} \,,$$

so

$$\begin{aligned} 2q(z) &= r_i + s_{i+1}(z-i) \\ \frac{2}{9\pi}(y-y_i) &= r_i(z-i) + \frac{1}{2}s_{i+1}(z-i)^2 \\ -\frac{1}{(9\pi)^2}\left(\sqrt{\beta}-\sqrt{\beta_i}\right) &= \frac{2}{9\pi}y_i(z-i) + \frac{1}{2}r_i(z-i)^2 + \frac{1}{6}s_{i+1}(z-i)^3 \,. \end{aligned} \qquad z\in[i,i+1]$$

The $AdS_7 \times M_3$ dual is determined by the piecewise linear function $2q(z)$ of $z \in [0,N]$ interpolating the ranks r_i of the linear quiver on the tensor branch.

$$ds^2 = \pi\sqrt{2}\left(8\sqrt{-\frac{\alpha}{\ddot\alpha}}ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\ddot\alpha}{\alpha}}dz^2 + \frac{\alpha^{3/2}(-\ddot\alpha)^{1/2}}{\sqrt{2\alpha\ddot\alpha-\dot\alpha^2}}ds^2_{S^2}\right) \,, \qquad \alpha \equiv \sqrt{\beta}$$

$\sqrt{\beta}(z) \propto 2^{nd}$ primitive of $2q(z)$ that vanishes at $z = 0, N$.

The holographic limit

The supergravity solutions come from near horizon of N 5-branes.

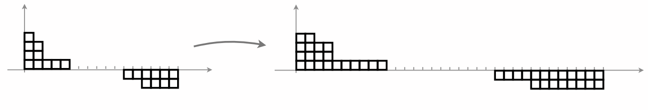
Take a large N limit to ensure small curvature. $(e^{2A} \sim N\ , \quad e^{\phi} \sim \frac{\sqrt{N}}{k})$

Holographic limit

$$N, L, R \to \infty \quad \text{with} \quad \frac{L}{N}, \frac{R}{N} \quad \text{constant.}$$

This can be achieved *e.g.* by a rescaling that keeps the number of D8 fixed:

$$e^{2A} \mapsto ne^{2A}\ , \qquad e^{2\phi} \mapsto \frac{1}{n}e^{2\phi} \qquad \Leftrightarrow \qquad N \mapsto nN\ , \qquad \mu_i \mapsto n\mu_i$$



but one can also take a large number of D8-branes and grow the Young diagrams both horizontally and vertically towards a limit shape.

The conformal anomaly a

Weyl and 't Hooft anomalies in 6d $\mathcal{N} = (1,0)$ SCFTs

- **Weyl anomaly** $\langle T^\mu_\mu \rangle \sim aE + \sum_{i=1}^{3} c_i I_i + \nabla_\mu (\dots)^\mu$ [Deser, Schwimmer 93]

  a decreases in RG flows in 2d and 4d; in 6d partial results.

- The superconformal algebra relates the conformal anomalies a and c_i to 't Hooft **SU(2)$_R$ and gravitational anomalies**:

  [Cordova, Dumitrescu, Intriligator 15; Beccaria, Tseytlin '15]

$$a = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta \, ,$$

where

$$I_8 = \frac{1}{24}\left(\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)\right)$$

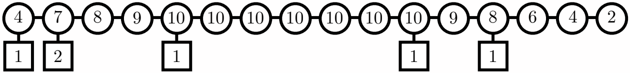
is the anomaly polynomial related to the anomaly I_6^1 by the descent eqn's

$$I_8 = dI_7 \, , \qquad \delta I_7 = dI_6^1 \, . \qquad \text{(locally)}$$

Anomaly polynomial of $\mathcal{T}^N_{\rho_L,\rho_R}$ from the linear quiver

Compute the anomaly polynomial of the SCFT by going to the tensor branch.

[Intriligator 14; Ohmori, Shimizu, Tachikawa, Yonekura 14]



The IR linear quiver gauge theory has $\sim N$ fields.
How can it match the expected $a \sim N^3$ anomaly of the SCFT?

The Green-Schwarz(-West-Sagnotti) mechanism that cancels gauge anomalies also gives a large contribution $\sim N^3$ to the $SU(2)_R$ anomaly α.

$$\alpha \sim N^3 \,, \quad \beta, \gamma, \delta \sim N \quad \Longrightarrow \quad c_1 \approx -\frac{7}{12} a \,, \quad c_2 \approx \frac{1}{4} c_1 \,, \quad c_3 \approx -\frac{1}{12} c_1$$

Naive anomaly polynomial

Chiral fields: SMW$^+$ in Vector, W$^-$ in Hyper, SMW$^-$ $+ B^-$ in Tensor.

$$
\begin{aligned}
I_8^{\text{naive}} = \frac{1}{24}\Bigg(&\sum_{i=1}^{N-1} \Big[\overbrace{(-2r_i + r_{i-1} + r_{i+1} + f_i)}^{=0} \left(\text{tr}F_i^4 + \frac{1}{2} p_1 \text{tr}F_i^2 \right) - 12 r_i c_2 \text{tr}F_i^2 \Big] \\
&- 3 \sum_{i,j} C_{ij} \text{tr}F_i^2 \text{tr}F_j^2 + \left(2(N-1) - \sum_i r_i^2 \right) \left(c_2^2 + \frac{1}{2} c_2 p_1 \right) + \frac{N-1}{240}(23p_1^2 - 116 p_2) \\
&+ \frac{7p_1^2 - 4p_2}{240} \Bigg(N - 1 + \frac{1}{2} \sum_i r_i \underbrace{(-2r_i + r_{i-1} + r_{i+1} + 2f_i)}_{=f_i} \Bigg) \Bigg) .
\end{aligned}
$$

with $C_{ij} = 2\delta_{ij} - \delta_{i,j-1} - \delta_{i,j+1}$ the Cartan matrix of A_{N-1}.

The terms $C_{ij}\text{tr}F_i^2\text{tr}F_j^2$ and $r_i c_2 \text{tr}F_i^2$ can be rewritten as

$$
-\frac{1}{8} C_{ij} I_{4,i} I_{4,j} + \frac{1}{2} C_{ij}^{-1} r_i r_j c_2^2 , \qquad I_{4,i} \equiv \text{tr}F_i^2 + 2c_2 C_{ij}^{-1} r_j .
$$

Green-Schwarz anomaly cancellation

The factorized gauge anomaly polynomial

$$I_8^{\text{factor}} = -\frac{1}{8} C_{ij} I_{4,i} I_{4,j} \ , \qquad I_{4,i} \equiv \mathrm{tr} F_i^2 + 2 c_2 C_{ij}^{-1} r_j$$

encodes by descent the anomalous variation of the effective action

$$\delta\Gamma = \int I_6^{1,\text{factor}} = -\frac{1}{8} C_{ij} \int I_{2,i}^1 I_{4,j} \ .$$

$$I_{2,i}^1 = \mathrm{tr}(\lambda_i dA_i) + \mathrm{tr}(\lambda^{(\mathrm{R})} dA^{(\mathrm{R})}) C_{ij}^{-1} r_j$$

appears in the descent of $I_{4,i}$

$$I_{4,i} = dI_{3,i} \ , \qquad \delta I_{3,i} = dI_{2,i}^1 \ .$$

The anomaly is canceled by adding to the action the counterterm

$$S_{\mathrm{GS}} = \frac{1}{8} C_{ij} \int b_i I_{4,j} \ , \qquad (B_j - B_{j+1} = C_{ij} b_i)$$

with the gauge transformation $\delta b_i = I_{2,i}^1$.

Anomaly coefficients of $\mathcal{T}^N_{\rho_L,\rho_R}$

Adding up the terms of the anomaly polynomial that survive GS cancellation, we find the $SU(2)_R$ and gravitational anomaly coefficients

$$\alpha = 12 \sum_{i,j} C^{-1}_{ij} r_i r_j + 2(N-1) - \sum_i r_i^2 \,, \qquad \beta = N - 1 - \frac{1}{2} \sum_i r_i^2 \,,$$

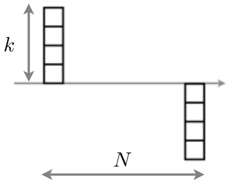
$$\gamma = \frac{1}{240} \left(\frac{7}{2} \sum_i r_i f_i + 30(N-1) \right) \,, \qquad \delta = -\frac{1}{120} \left(\sum_i r_i f_i + 60(N-1) \right)$$

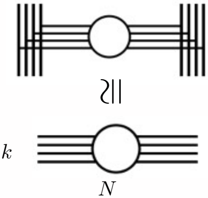
and the a Weyl anomaly

$$a = \frac{16}{7} \left(12 \sum_{i,j} C^{-1}_{ij} r_i r_j - \frac{1}{2} \sum_i r_i^2 + \frac{11}{960} \sum_i r_i f_i + \frac{15}{16}(N-1) \right) \,.$$

Leading N^3 dependence comes from $a \approx \frac{192}{7} \sum_{i,j} C^{-1}_{ij} r_i r_j$.

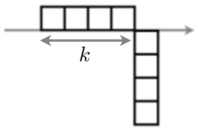
Examples

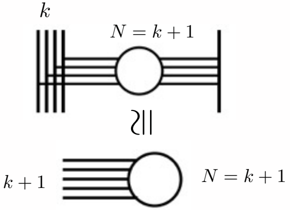


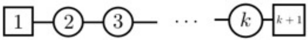


$$a \approx \frac{16}{7} N^3 k^2$$

Examples

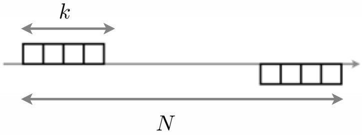


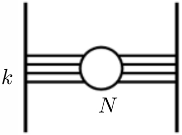


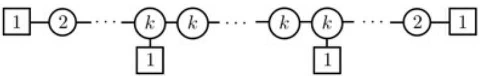


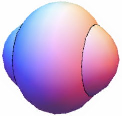
$$a \approx \frac{16}{7} \cdot \frac{4}{15} k^5$$

Examples









$$a \approx \frac{16}{7} k^2 \left(N^3 - 4Nk^2 + \frac{16}{5} k^3 \right)$$

Holographic a

- **Holographic Weyl anomaly** [Henningson, Skenderis 98]

$$S^E_{7d} = \frac{1}{16\pi G_{\mathrm{N},7}} \int d^7x \sqrt{g_7}(R_7 + \Lambda) + \text{boundary terms}$$

$$ds^2 = \frac{l^2}{r^2}(dr^2 + r^2 g^{(6)}_{ij} dx^i dx^j + \dots)$$

$$\langle T^\mu_\mu \rangle = \frac{l^5}{G_{\mathrm{N},7}} \times (\text{polynomial in Riemann of } g^{(6)} \text{ and derivatives})$$

$$\Longrightarrow \quad a_{\mathrm{hol}} \propto \frac{l^5}{G_{\mathrm{N},7}} \propto l^5 \mathrm{Vol}(M_3)$$

[Gubser 98]

- For our warped product $(ds^2)^{\mathrm{str}}_{10} = e^{2A} ds^2_{AdS_7} + ds^2_{M_3}$,

$$a_{\mathrm{hol}} = \frac{3}{56\pi^4} \int_{M_3} e^{5A-2\phi} \mathrm{vol}_{M_3} = \frac{128}{189\pi^2} \int_0^N dz\, q\sqrt{\beta}$$

Field theory a in the holographic limit

$$a = \frac{16}{7}\left(12\sum_{i,j} C^{-1}_{ij} r_i r_j - \frac{1}{2}\sum_i r_i^2 + \frac{11}{960}\sum_i r_i f_i + \frac{15}{16}(N-1)\right) ,$$

$$\sum_{i,j} C^{-1}_{ij} r_i r_j = \frac{1}{N}\left(\sum_i i(N-i) r_i^2 + 2\sum_{i<j} i(N-j) r_i r_j\right) .$$

In the holographic limit $N \to \infty$ and the quiver becomes very long.

The leading order in N can be obtained from a continuum limit

$$i/N \rightsquigarrow x \in [0,1] , \qquad r_i \rightsquigarrow r(x) , \qquad \sum_i \rightsquigarrow N \int dx .$$

$$a = \frac{16}{7}\left(12\sum_{i,j} C^{-1}_{ij} r_i r_j - \frac{1}{2}\sum_i r_i^2 + \frac{11}{960}\sum_i r_i f_i + \frac{15}{16}(N-1)\right) ,$$

$\sim N^3$ $\qquad \sim N$ $\qquad \sim N$ $\qquad \sim N$

Field theory a in the holographic limit

The leading N^3 term in the holographic limit is

$$a \approx \frac{384}{7} N^3 \int_0^1 dy \int_0^y dx\, x(1-y) r(x) r(y)$$
$$= \frac{192}{7} N^3 \left[-\int_0^1 dx\, r(x) r^{(-2)}(x) + r^{(-1)}(x) r^{(-2)}(x) \Big|_0^1 - \left(r^{(-2)}(x) \Big|_0^1 \right)^2 \right]$$

where $r^{(-k)}(x)$ is a k^{th} primitive of $r(x)$. Setting $r^{(-2)}(0) = r^{(-2)}(1) = 0$,

$$a \approx -\frac{192}{7} N^3 \int_0^1 dx\, r(x) r^{(-2)}(x) \ .$$

This reproduces the holographic result

$$a_{\text{hol}} = \frac{128}{189\pi^2} \int_0^N dz\, q(z) \sqrt{\beta}(z)$$

using $z = Nx$, $2q(z) = r(x)$ and $q = -\frac{1}{2(9\pi)^2} \partial_z^2 \sqrt{\beta}$, with $\sqrt{\beta}|_{z=0,N} = 0$.

Outlook

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- The AdS_7 duals of 6d $\mathcal{N}=(1,0)$ SCFT's $\mathcal{T}^N_{\rho_L,\rho_R}$ are determined by the piecewise linear function $2q(z)$ interpolating the ranks r_i of the quiver.

- The holographic limit $N\to\infty$ is a continuum limit: $r_i \to r(x) = 2q(Nx)$.

  Discrete field theory data $\to$ Continuous functions in the geometry

- We provided a detailed test of this class of dualities by showing that

$$a \approx \frac{192}{7}\sum_{i,j} r_i C^{-1}_{ij} r_j \xrightarrow{\text{hol. limit}} a_{\text{hol}} = \frac{192}{7}\int 2q(z)\frac{1}{\partial_z^2}2q(z)dz\ .$$

Outlook

- The same pwl functions appear in AdS_5/CFT_4 duals [Gaiotto, Maldacena 09] as linear charge densities determining axisym sol'ns of $SU(\infty)$ Toda eq.

  $\rightarrow$ Does the $SU(\infty)$ Toda eq. arise from massive IIA EoM?

  $\rightarrow$ Holographic match of a_{4d} anomaly from continuum limit?

- It would be interesting to characterize the general sol'n with orientifolds:

  $\rightarrow$ Do $\beta, \gamma \sim N^3$?

  $\rightarrow$ O8 planes and enhanced global symmetries?

- We are now (more) confident of these AdS_7/CFT_6 dualities. What can we learn about the CFT_6's from the AdS_7 duals?